# Two-Neutron-Hole Configurations in $Pb^{206}$ from Isobaric Analog Measurements*

PATRICK RICHARD
*University of Washington, Seattle, Washington 98105*

AND

NELSON STEIN†
*University of Washington, Seattle, Washington 98105*
*and*
*Yale University, New Haven, Connecticut 06520*

AND

C. D. KAVALOSKI‡ AND J. S. LILLEY§
*University of Washington, Seattle, Washington 98105*
(Received 5 January 1968)

Two-neutron-hole configurations involving $p_{1/2}$, $f_{5/2}$, and $p_{3/2}$ neutrons are obtained for low-lying states of $Pb^{206}$ from proton elastic and inelastic scattering from the isobaric analog states (IAS) of $Pb^{207}$. Excitation functions for inelastic scattering to four of the final states have very little off-resonance cross section and resonate at only one or two of the $Bi^{207}$ analog resonances, which is characteristic of the simple two-neutron-hole structure of the states. Experimental widths for decay from the IAS of the $Pb^{208}$ ground state to the first three hole states of $Pb^{207}$ are used to obtain the model widths needed to analyze the data. The two-neutron-hole configurations obtained for the $Pb^{206}$ wave functions compare very well with the recent shell-model calculations of True.

IN applying the shell model to nuclear structure, $Pb^{206}$ is of considerable interest for studying the behavior of two interacting particles moving in a nuclear potential. Starting with the well-established single-hole structure[1] of $Pb^{207}$, the low-lying levels in $Pb^{206}$ constitute the first step toward the more complex structure that exists away from closed shells. Calculations based on two interacting neutron holes have been performed for $Pb^{206}$ and energies[2–5] and wave functions[2,3] predicted. One purpose of this paper is to present quantitative experimental information about two-hole configurations in $Pb^{206}$. These results were obtained from excitation functions and angular distributions for the $Pb^{206}(p,p')$ reaction at the isobaric analog states (IAS) of $Pb^{207}$. A further aim here is to describe the simple method by which these data were analyzed to yield spectroscopic information.

Previous spectroscopic information about $Pb^{206}$ is based on stripping and pickup experiments[6] for which only a rough comparison with theory was possible, and from which only the ground-state wave function was obtained.[6,7] More recently, two-neutron transfer reactions leading to $Pb^{206}$ were studied,[8] but cross-section predictions for these reactions depend upon the wave functions in a complicated fashion, making it difficult to deduce them from the measurements. The present results are of particular interest since they provide information, independent of nuclear-transfer-reaction theories, which can be compared directly with shell-model calculations.

Proton scattering from $Pb^{206}$ at the position of IAS of $Pb^{207}$ has been reported by several groups.[9–13] In the present work, we have studied in much greater detail than previously the analog states corresponding to single-hole states of $Pb^{207}$ and their decay to two-hole states in $Pb^{206}$. In the formation of the IAS of a neutron-hole state $J^{-1}$ of $Pb^{207}$ as a resonance in the $Bi^{207}$ compound system, we are able to study final states in $Pb^{206}$ which have two-neutron-hole configurations $(J^{-1},j^{-1})_I$,

* Research supported in part by the U. S. Atomic Energy Commission.
† Present address: Physics Department, Yale University, New Haven, Conn.
‡ Present address: Physics Department, Lowell Institute of Technology, Lowell, Mass.
§ Present address: Physics Department, University of Minnesota, Minneapolis, Minn.
[1] G. Muehllehner, A. S. Poltorak, W. C. Parkinson, and R. M. Bassel, Phys. Rev. **159**, 1039 (1967).
[2] W. W. True and K. W. Ford, Phys. Rev. **109**, 1675 (1958).
[3] W. W. True, Phys. Rev. **168**, 1388 (1968).
[4] D. M. Clement and E. V. Baranger, Nucl. Phys. **89**, 145 (1966).
[5] A. Plastino, R. Arview, and S. A. Moskowski, Phys. Rev. **145**, 838 (1966).
[6] P. Mukherjee and B. L. Cohen, Phys. Rev. **127**, 1284 (1962).
[7] R. Tickle and J. Bardwick (to be published) have recently restudied the $Pb^{207}(d,t)$ reaction, from which they derive wave functions which are in fair agreement with most of those obtained from the present experiment.
[8] E. R. Flynn, G. J. Igo, R. Woods, P. D. Barnes, and N. K. Glendenning, Phys. Rev. Letters **19**, 798 (1967); G. E. Holland, N. Stein, C. A. Whitten, and D. A. Bromley, Bull. Am. Phys. Soc. **12**, 537 (1967).
[9] P. Richard, C. D. Kavaloski, J. S. Lilley, and N. Stein, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 307.
[10] A. Adam, G. Bassani, N. Cindor, C. Levi, M. Mermaz, and L. Papineau, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 366.
[11] G. H. Lenz and G. M. Temmer, Phys. Letters **24B**, 368 (1967); Nucl. Phys. (to be published).
[12] E. C. Booth and B. S. Madsen (private communication).
[13] N. Stein, C. A. Whitten, Jr., and D. A. Bromley, Phys. Rev. Letters **20**, 113 (1968); and (to be published).

where $j^{-1}$ is the second neutron hole and $I$ is the spin of the final state. Neglecting nonresonant contributions to the scattering, the total cross section $\sigma_{pp'}$ for a final excited state at a particular resonance peak is assumed to be of the Breit-Wigner form[14,15]

$$\sigma_{pp'}=(2\pi/k^2)[(2J+1)/\Gamma^2]\Gamma_{p_0}\Gamma_{p'}. \quad (1)$$

$\Gamma_{p_0}$ and $\Gamma_{p'}$ are the widths for decay to the ground state (elastic width) and final excited state (inelastic width), respectively. The total width $\Gamma$ is obtained directly from the width of the inelastic scattering resonances. If a $Pb^{206}$ wave function is expressed as a sum over two-hole configurations as

$$\Psi^I=\sum C_{Jj}{}^I(J^{-1},j^{-1})_I, \quad (2)$$

then for the decay of a hole state $(J^{-1})$ with outgoing particle $j$, the inelastic width may be written as

$$\Gamma_{p'}=\sum_j \Gamma_{IjJ}=\frac{2I+1}{2J+1}\sum_j \Gamma_{\mathrm{mod}}{}^j(C_{Jj}{}^I)^2, \quad (3)$$

where $j\neq J$. For $j=J$ the numerator is multiplied by 2. The quantities $\Gamma_{\mathrm{mod}}{}^j$ are single-particle decay widths from the $j$ shell-model orbital, and our technique for analysis rests on the choice for their values. At this stage of the development of the theory for inelastic scattering via analog resonances, we regard the *experimental* single-particle widths obtained from the decay of the IAS of the $Pb^{208}$ ground state to the hole states in $Pb^{207}$, which have been measured in the $Pb^{207}(p,p')$ reaction,[15,16] as the best values to use for $\Gamma_{\mathrm{mod}}$. This procedure avoids the uncertainties inherent in theoretically calculated single-particle widths. It rests, rather, on two assumptions for which there is ample experimental evidence[1,6]: (1) a good closed shell for $Pb^{208}$ and (2) pure one-hole configurations in $Pb^{207}$.

Five $(p,p')$ excitation functions leading to excited states in $Pb^{206}$ are shown in Fig. 1. The resonances of interest for the present discussion correspond to IAS of the first three states of $Pb^{207}$: $3p_{1/2}{}^{-1}$ ground state, $2f_{5/2}{}^{-1}$ state at 0.57 MeV, and $3p_{3/2}{}^{-1}$ state at 0.89 MeV. The higher-energy resonances in the 0.803-MeV $2^+$ excitation function are discussed elsewhere.[9,11,17] The state at 1.68 MeV with spin $4^+$ was not resolvable from the 1.71-MeV $1^+$ state, but as described below, the resonance behavior is undoubtedly due to the $1^+$ state, while the off-resonance behavior, especially above 14 MeV, is probably due to the $4^+$ state. With the exception of the first $2^+$ state, the excitation functions are simple and allow one to infer qualitatively which configurations

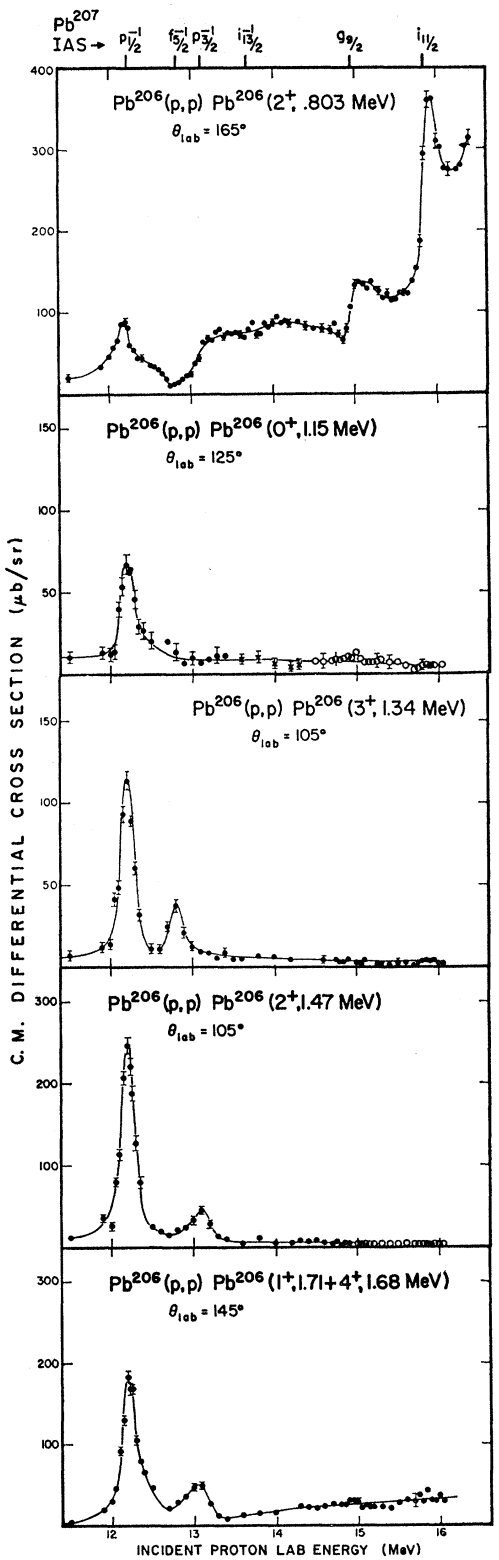


FIG. 1. Proton inelastic scattering differential cross section to the lowest six levels in $Pb^{206}$. The positions of the analog states of $Pb^{207}$ are indicated at the top of the figure.

[14] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
[15] B. L. Anderson, J. P. Bondorf, and B. S. Madsen, Phys. Letters **22**, 651 (1966).
[16] G. M. Temmer, G. H. Lenz, and G. T. Garvey, in *Proceedings of the International Conference on Nuclear Physics, Gatlinburg, Tennessee, 1966* (Academic Press Inc., New York, 1967).
[17] Naftali Auerbach and Nelson Stein (to be published).

TABLE I. Values of $(C_{Jj})^2$ in the $Pb^{206}$ wave functions.

| | Experimental | | Theoretical[a] |
|---|---|---|---|
| $0^+$ g.s. | $0.65(\pm0.13)p_{1/2}^{-2}+0.25(\pm0.07)f_{5/2}^{-2}+0.20(\pm0.08)p_{3/2}^{-2}$ $0.54p_{1/2}^{-2}+0.20f_{5/2}^{-2}+0.12p_{3/2}^{-2}$[b] | $0^+$ g.s. | $0.68p_{1/2}^{-2}+0.16f_{5/2}^{-2}+0.13p_{3/2}^{-2}$ |
| $2^+$ 0.803 MeV[c] | ... | $2^+$ 0.64 MeV | $0.55p_{1/2}^{-1}f_{5/2}^{-1}+0.27p_{1/2}^{-1}p_{3/2}^{-1}+0.08f_{5/2}^{-2}$ |
| $0^+$ 1.15 MeV | $0.22(\pm0.05)p_{1/2}^{-2}+\leqslant0.78f_{5/2}^{-2}+\leqslant0.29p_{3/2}^{-2}$ | $0^+$ 1.31 MeV | $0.24p_{1/2}^{-2}+0.70f_{5/2}^{-2}+0.01p_{3/2}^{-2}$ |
| $3^+$ 1.34 MeV | $1.00p_{1/2}^{-1}f_{5/2}^{-1}$ | $3^+$ 1.43 MeV | $0.99p_{1/2}^{-1}f_{5/2}^{-1}$ |
| $2^+$ 1.47 MeV | $0.39(\pm0.10)p_{1/2}^{-1}f_{5/2}^{-1}+0.61(\pm0.18)p_{1/2}^{-1}p_{3/2}^{-1}$ | $2^+$ 1.44 MeV | $0.38p_{1/2}^{-1}f_{5/2}^{-1}+0.59p_{1/2}^{-1}p_{3/2}^{-1}$ |
| $4^+$ 1.68 MeV[d] | ... | $4^+$ 1.60 MeV | $0.32f_{5/2}^{-2}+0.53f_{5/2}^{-1}p_{3/2}^{-1}+0.086p_{1/2}^{-1}f_{7/2}^{-1}$ |
| $1^+$ 1.71 MeV | $1.00(\pm0.10)p_{1/2}^{-1}p_{3/2}^{-1}$ | $1^+$ 1.78 MeV | $0.99p_{1/2}^{-1}p_{3/2}^{-1}$ |

[a] True (Ref. 3). In some cases very small amplitudes have been omitted.
[b] Determined from $(d,p)$ and $(d,t)$ measurements by Mukherjee and Cohen (Ref. 6).
[c] Resonances were observed implying $p_{1/2}$, $f_{5/2}$, and $p_{3/2}$ contributions. These were not analyzed because of substantial interference and contributions from direct scattering (see text).
[d] Resonances for this state were not observed (see text).

are likely to be important in the final-state wave function. By noting the location of the $Pb^{207}$ IAS $J^{-1}$ states (at the top of Fig. 1), we see that the $3^+$ state is probably dominated by the configuration $(p_{1/2}^{-1},f_{5/2}^{-1})$ and the $1^+$ state by $(p_{1/2}^{-1},p_{3/2}^{-1})$. In addition, the angular distributions in Fig. 2 are consistent with this, since the isotropy at the $f_{5/2}^{-1}$ IAS for the $3^+$ distribution and at the $p_{3/2}^{-1}$ IAS for the $1^+$ distribution imply outgoing $p_{1/2}$ protons in each case. The four isotropic angular distributions at the g.s. $p_{1/2}^{-1}$ IAS verify that the resonance spin is $\frac{1}{2}$, while the forward peaking of the 0.803-MeV $2^+$ angular distribution indicates a direct $(p,p')$ component in the cross section. This direct component in the cross section, as well as the interference character of the resonances that is evident in Fig. 1, prevent the collective first $2^+$ state from being analyzed by the method outlined above. An appropriate analysis must take into account the interference between direct and compound scattering.

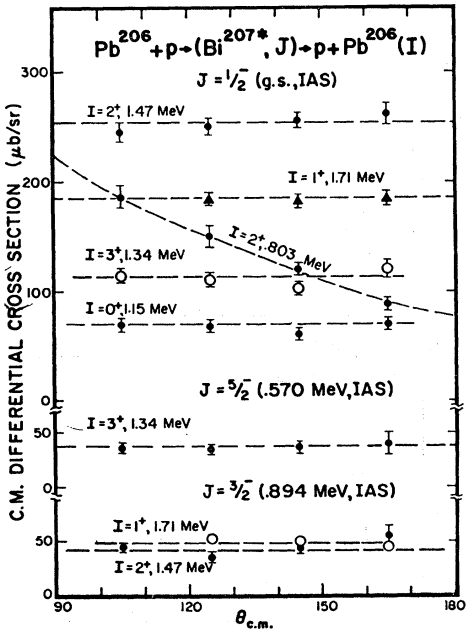


FIG. 2. Angular distributions of the inelastic scattering to states in $Pb^{206}$ at the incident proton energy for the $p_{1/2}^{-1}$, $f_{5/2}^{-1}$, and $p_{3/2}^{-1}$ analog states in $Bi^{207*}$.

The squares of the amplitudes (defined in Eq. 2) in the $Pb^{206}$ wave functions that are derived from the experimental cross sections using Eqs. (1) and (3) are listed in Table I. Only two-hole configurations based on the $p_{1/2}$, $f_{5/2}$, and $p_{3/2}$ orbitals were considered in the analysis. From the single-hole-state energies in $Pb^{207}$ it is unlikely that other orbitals are important below 2-MeV excitation in $Pb^{206}$, and the detailed shell-model calculations support this.[2,3]

The values of $\Gamma_{\mathrm{mod}}{}^j$ used in Eq. (3) were obtained by averaging the experimental results for $Pb^{207}(p,p')$ at the $Pb^{208}$ g.s. analog.[15,16] They are: for $p_{1/2}$, 31 keV at 11.49 MeV; for $f_{5/2}$, 3 keV at 10.92 MeV; and for $p_{3/2}$, 12 keV at 10.60 MeV. Small corrections to these numbers were made for the slightly different energies in the present experiment by assuming that $\Gamma_{\mathrm{mod}}{}^j$ is proportional to the transmission coefficient $T_j$ and using the tabulated values of $T_j$ by Mani *et al.*[18]

The reason for the large difference between $\Gamma_{\mathrm{mod}}{}^{p1/2}$ and $\Gamma_{\mathrm{mod}}{}^{p3/2}$ obtained in both Refs. 15 and 16 is not presently understood. It could imply that there are difficulties in extracting the elastic resonance width $\Gamma_{\mathrm{mod}}{}^{p1/2}$ at the $Pb^{208}$ g. s. analog, which would then affect the values of $\Gamma_{\mathrm{mod}}{}^{p3/2}$ and $\Gamma_{\mathrm{mod}}{}^{f5/2}$ derived according to Eq. (1). It should be emphasized, however, that this problem would not affect the results in Table I, with the exception of the excited $0^+$ state. The coefficients for the other states all depend on products of the above $\Gamma_{\mathrm{mod}}{}^j$ which *are accurately* determined from the $Pb^{207}$ $(p,p')$ experiment.

The 1.34-MeV $3^+$ state was taken to have a pure $(p_{1/2}^{-1},f_{5/2}^{-1})$ configuration in order to obtain an overall normalization. The validity of this is suggested by the excitation function in Fig. 1, by the theoretical calculations,[2,3] and by the recent $(d,t)$ experiment.[7] Use of this normalization limits the uncertainties in the

[18] G. S. Mani, M. A. Melkanoff, and I. Iori, Commissariat a L'Énergie Atomique Report No. 2379, Saclay, France, 1963 (unpublished).

configurations of the other states to the values shown in Table I. An alternative approach, independent of any normalization, would have created greater uncertainties by its dependence on both the absolute values of $\Gamma_{mod}{}^{j}$ and the extraction of the elastic resonance widths (the latter being subject to the difficulty mentioned above).

Results from the most recent shell-model calculation by True[3] are compared in Table I with the experimentally determined wave functions. The theoretical column gives the energies and wave functions for the first seven predicted states in $Pb^{206}$, and the experimental column shows that at least five are in excellent agreement with the measurements. If the known $4^+$ state at 1.68 MeV is similar to the predicted $4^+$ state at 1.60 MeV, then resonances for this state would not have been observed. This is consistent with our assigning the resonances in the cross section at the bottom of Fig. 1 entirely to the $1^+$ state at 1.71 MeV which, as Table I shows, is then in accord with the predicted $1^+$ state.

The conclusions drawn from the results presented here may be summarized as follows:

(1) The low-energy structure of $Pb^{206}$ appears to be well described in terms of two-neutron holes moving in the shell-model potential of the $Pb^{208}$ core.

(2) It is possible to extract quantitative spectroscopic information from inelastic scattering at isobaric analog resonances, in a relatively easy manner, at least in those regions of the Periodic Table where the simple shell model provides a good description.

PHYSICAL REVIEW VOLUME 171, NUMBER 4 20 JULY 1968

# Neutron Groups in the Spectrum of a $PuF_4$ Source*

RICHARD L. LEHMAN
*Department of Biophysics, Laboratory of Nuclear Medicine and Radiation Biology*
*University of California School of Medicine, Los Angeles, California 90024*

(Received 28 December 1967)

The neutron spectrum of a 1-kg $PuF_4$ laboratory neutron source was measured by use of nuclear track emulsion and a differentiation technique. The spectrum consists of a minor peak at 0.91 MeV and a major one at 1.55 MeV. The mean energy was found to be 1.35 MeV and the maximum energy 2.8 MeV. Special analysis of the track distribution revealed peaks at 0.91, 1.26, 1.44, 1.64, 1.8, and 2.1 MeV, and weaker ones at 0.5 and 2.5 MeV. The measured spectrum does not differ in principal features from an analytical spectrum obtained by a graphico-numerical study of published thin-target measurements. The analytical study predicts the effects on $F(\alpha,n)$-source spectra when $\alpha$ emitters of different energy from $Pu^{239}$ are employed, and permits an interpretation of some previously unexplained observations by Chadwick and Constable.

## 1. INTRODUCTION

FLUORINE is one of the few elements that produce fast neutrons in good yield when bombarded with polonium-210 $\alpha$ rays. Because of this, the $F(\alpha,n)$ reaction was the subject of many early studies. Ion- and cloud-chamber measurements in the 1930's established the existence of several groups of neutrons, in the energy range 0.4 to 2.5 MeV, that were associated with $\alpha$-particle resonances to levels in the resulting $Na^{22}$ nucleus.[1,2] More recent studies of the $F^{19}(\alpha,n)Na^{22}$ reaction, by use of $\alpha$-particle beams and $CaF_2$ targets, have provided detailed information on the neutron[3–6] and $\gamma$-ray[6–8] yields associated with resonances through the closely spaced 12.5- to 14.5-MeV levels in the compound $Na^{23}$ nucleus. There is current interest in this reaction because it is a means of studying excited states in $Na^{22}$ and $Na^{23}$, and because it is the basis for important laboratory neutron sources.[9–12]

Neutron spectral measurements have been reported for a small PoF source,[11] and one for the same 1-kg $PuF_4$ source used in this study.[12] The agreement between these spectra, however, was poor and the

* These studies were supported in part by Contract No. AT(04-1)GEN-12 and in part by Contract No. W-7405-eng 48 between the U. S. Atomic Energy Commission and the University of California.

[1] J. Chadwick and J. E. R. Constable, Proc. Roy. Soc. (London) **A135**, 48 (1932).
[2] T. W. Bonner and L. M. Mott-Smith, Phys. Rev. **46**, 258 (1934).
[3] W. T. Doyle and A. R. Quinton, Phys. Rev. **97**, 252 (1955); **101**, 669 (1956).
[4] R. M. Williamson, T. Katman, and B. S. Burton, Phys. Rev. **117**, 1325 (1960).
[5] R. Batchelor and J. H. Towle, Proc. Phys. Soc. (London) **73**, 307 (1959).
[6] R. M. Freeman and G. S. Mani, Nucl. Phys. **51**, 593 (1964).
[7] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **111**, 1303 (1958).
[8] T. A. Rabson, T. W. Bonner, R. Castillo-Bahena, M. V. Harlow, Jr., H. P. Haenni, and W. A. Ranken, Nucl. Phys. **19**, 314 (1960).
[9] E. Tochilin and R. V. Alves, Nucleonics **16**, 445 (1958).
[10] K. W. Geiger, Can. J. Phys. **37**, 550 (1959).
[11] A. Szilvasi, K. W. Geiger, and W. R. Dixon, J. Nucl. Energy **11**, 131 (1960).
[12] E. Tochilin, in *Neutron Dosimetry* (International Atomic Energy Agency, Vienna, 1963).