Analysis of the ΛΛ Hypernucleus $_{\Lambda\Lambda}C^{14}$

S. Ali,* L. P. Kok,† and M. E. Grypeos‡
International Atomic Energy Agency, International Centre for Theoretical Physics, Trieste, Italy
(Received 23 January 1968)

An analysis has been made of the yet unobserved $_{\Lambda\Lambda}C^{14}$ system in a Λ-Λ-C^{12} model, using Λ-Λ interactions quite reliably determined from the analysis of $_{\Lambda\Lambda}He^6$. Allowance has been made for distortion of the core by the two Λ particles. The sensitivity of the results for $\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}-2B_\Lambda$ to the nuclear compressibility coefficient is explored and the role of the ΛΛ hypernucleus as a possible probe into this coefficient is discussed. Finally, the importance of an accurate experimental determination of $B_{\Lambda\Lambda}(_{\Lambda\Lambda}C^{14})$ has been stressed.

1. INTRODUCTION

In a recent paper[1] by the present authors, hereafter referred to as I, a preliminary theoretical investigation was made of the yet unobserved ΛΛ hypernucleus $_{\Lambda\Lambda}C^{14}$, treating the latter as a system consisting of two Λ particles and a C^{12} core. Although such a treatment may not be very unrealistic in view of the fair rigidity of the core, the calculations of Herndon and Tang[2] show that the C^{12} core may be rigid but not completely so. From their analysis of $_\Lambda C^{13}$ on an α-α-α-Λ model, they find that the Λ causes a decrease of about 8% in the rms value of the α-α separation distance in the free C^{12} core. Thus in $_{\Lambda\Lambda}C^{14}$, one may expect some contribution to the additional binding energy $\Delta B_{\Lambda\Lambda}$ $(=B_{\Lambda\Lambda}-2B_\Lambda)$ due to core distortion by the two Λ particles, although this contribution is likely to be considerably less than in $_{\Lambda\Lambda}Be^{10}$ and other possible ΛΛ hypernuclei, excepting $_{\Lambda\Lambda}He^6$, whose core has rather exceptional rigidity. Since, after $_{\Lambda\Lambda}He^6$, the next rather rigid core ΛΛ hypernucleus is $_{\Lambda\Lambda}C^{14}$, it seems worthwhile to make a detailed investigation of the latter with the inclusion of the above-mentioned distortion effect. It is hoped that of all ΛΛ hypernuclei, a combined analysis of $_{\Lambda\Lambda}He^6$ and $_{\Lambda\Lambda}C^{14}$ is likely to give comparatively more significant information about Λ-Λ, Λ-N interactions, besides throwing light on the structure of the core nuclei.[3–5]

In the present investigation, we shall analyze $_{\Lambda\Lambda}C^{14}$ by the same C^{12}-Λ-Λ model as used in I but shall, however, include the core-distortion effects. Furthermore, the present analysis will differ from I in that besides using completely attractive Λ-Λ potentials, we have also used here the meson theoretical hard-core Λ-Λ potentials. The use of the latter is important, especially if one wants to obtain information about the Σ-Λ-π coupling constant $f_{\Sigma\Lambda}$. This coupling constant seems to be extremely important in determining the strength of the ΛΣ interaction.[6,7] For both above-mentioned types of potentials we have built up information on $_{\Lambda\Lambda}C^{14}$, basing our results on those quite reliably determined from $_{\Lambda\Lambda}He^6$. Finally, we have also considered the role of ΛΛ *hypernuclei* as a possible probe into the nuclear cores.

2. CALCULATIONAL PROCEDURES

We consider the following two Λ-Λ potentials.

(a) Completely attractive Yukawa potential:

$$V_{\Lambda\Lambda}(r)=-U_{\Lambda\Lambda}(\mu^3/4\pi)(e^{-\mu r}/\mu r),$$

where $U_{\Lambda\Lambda}$ is the volume integral of the Λ-Λ potential and μ the inverse range.

(b) Meson theoretical hard-core potential:

$$V_{\Lambda\Lambda}(r)=\begin{cases}\infty\,, & r<r_c\\ =3f_{\Sigma\Lambda}{}^4W(r)\,, & r>r_c\end{cases}$$

where r_c is the hard-core radius and $W(r)$ the shape function.

The two Λ's in the ΛΛ hypernuclei are in the singlet configuration $^1S_0$ (Pauli principle) for which the coupling with the ΣΣ channel is weak and we have therefore used only the lowest fourth-order potential, the second-order potentials being zero because of zero isospin of the Λ hyperon. Our potential (b) (for even ΣΛ parity and $f_{\Sigma\Sigma}=0$) thus corresponds to the static limit of the graphs in Fig. 1. The shape function $W(r)$ is given by

$$W(r)=XV_1{}^{(4)}-3XV_\sigma{}^{(4)}+IIV_1{}^{(4)}-3IIV_\sigma{}^{(4)}\,,\quad(1)$$

* On deputation from Atomic Energy Centre, Dacca, East Pakistan.

† On leave of absence from the State University, Groningen, The Netherlands.

‡ Address during the academic year 1967/68: Department of Physics, University of Surrey, Battersea Park Rd., London SW11, England.

[1] S. Ali, L. P. Kok, and M. E. Grypeos, Nuovo Cimento **50B**, 373 (1967).

[2] R. C. Herndon and Y. C. Tang, Phys. Rev. **149**, 735 (1966).

[3] As far as conclusions from the analysis of $_{\Lambda\Lambda}Be^{10}$ (or $_{\Lambda\Lambda}Be^{11}$) about the range of the Λ-N interaction are concerned, it has been recently shown in Ref. 4 that these are dependent on the interpretation of the event of Danysz *et al.* (see Ref. 5). It was found that if this event is really $_{\Lambda\Lambda}Be^{10}$, then one has support for a short range of the order of μ_K^{-1} (the range corresponding to the K-meson exchange mechanism of the Λ-N interaction) or even less, whereas if the event is interpreted as $_{\Lambda\Lambda}Be^{11}$, then both μ_K^{-1} as well as $\mu_{2\pi}^{-1}$ (two-pion exchange) seem almost equally likely. There was thus emphasis on the unambiguous identification of some other ΛΛ hypernucleus besides $_{\Lambda\Lambda}He^6$.

[4] S. Ali and L. P. Kok, Nucl. Phys. **B3**, 543 (1967).

[5] M. Danysz *et al.*, Nucl. Phys. **49**, 121 (1963).

[6] Very recently Pappademos (see Ref. 7) has studied the low-energy ΞN and ΛΣ interactions in connection with a search for dibaryon bound states and resonances in these systems. He comes to the conclusion that, for the most likely values of the coupling parameters and the core radii, no bound states in these systems are indicated.

[7] J. Pappademos, Phys. Rev. **163**, 1788 (1967).

TABLE I. Results for $_{\Lambda\Lambda}C^{14}$ as a function of the volume integral $U_{\Lambda\Lambda}$ of the Yukawa $\Lambda\Lambda$ potential (a) and the nuclear compressibility coefficient K. (All energies are in MeV, lengths in F; Δa is defined as $a_{\Lambda\Lambda}-a_0$). Figures in parentheses indicate rigid-core results.[a]

K	Range	$U_{\Lambda\Lambda}$ (MeV F³)	-300	0	300	500
100	$\mu_{2\pi}$	$B_{\Lambda\Lambda}$	19.51 (18.79)	22.01 (20.91)	26.82 (25.01)	33.28
		$\Delta B_{\Lambda\Lambda}$	-1.36 (-2.05)	1.14 (0.07)	5.95 (4.17)	12.40
		Δa	-0.092	-0.105	-0.127	-0.148
	μ_K	$B_{\Lambda\Lambda}$	19.66 (18.64)	22.50 (20.90)	27.93 (25.20)	35.04
		$\Delta B_{\Lambda\Lambda}$	-1.17 (-2.20)	1.67 (0.06)	7.10 (4.36)	14.21
		Δa	-0.115	-0.133	-0.164	-0.193
150	$\mu_{2\pi}$	$B_{\Lambda\Lambda}$	19.22 (18.79)	21.57 (20.91)	26.08 (25.01)	32.19
		$\Delta B_{\Lambda\Lambda}$	-1.65 (-2.05)	0.70 (0.07)	5.22 (4.17)	11.33
		Δa	-0.058	-0.066	-0.078	-0.090
	μ_K	$B_{\Lambda\Lambda}$	19.25 (18.64)	21.85 (20.90)	26.79 (25.20)	33.32
		$\Delta B_{\Lambda\Lambda}$	-1.59 (-2.20)	1.01 (0.06)	5.95 (4.36)	12.48
		Δa	-0.071	-0.085	-0.096	-0.112

[a] In both Tables I and II, in the calculations of $\Delta B_{\Lambda\Lambda}$, we used the variational results for B_Λ rather than the experimental value of B_Λ.

where X refers to the crossed graphs and II to uncrossed ones and σ refers to the spin-dependent contribution. The explicit expressions for the component parts involving Bessel functions of order zero and one have been taken from Ref. 8.

The binding energies of 1 and 2 Λ particles to the core (exclusive of core energy) were expanded about the free harmonic-oscillator size parameter a_0 characterizing the density distribution of the free core as follows:

$$b_\Lambda(a)=b_\Lambda(a_0)-b_1\frac{(a-a_0)}{a_0}+\tfrac{1}{2}b_2\frac{(a-a_0)^2}{a_0^2}, \qquad (2)$$

$$b_{\Lambda\Lambda}(a)=b_{\Lambda\Lambda}(a_0)-c_1\frac{(a-a_0)}{a_0}+\tfrac{1}{2}c_2\frac{(a-a_0)^2}{a_0^2}. \qquad (3)$$

The total binding energies of 1 and 2 Λ particles to the core are then given by

$$B_\Lambda(a)=b_\Lambda(a)-[E_c(a)-E_c(a_0)] \qquad (4)$$

and

$$B_{\Lambda\Lambda}(a)=b_{\Lambda\Lambda}(a)-[E_c(a)-E_c(a_0)], \qquad (5)$$

where the core energy E_c is represented by the quadratic approximation

$$E_c(a)=E_c(a_0)+\tfrac{1}{2}\epsilon_2[(a-a_0)^2/a_0^2], \qquad (6)$$

ϵ_2 being related to the compressibility (stiffness) coefficient K by

$$\epsilon_2=AK=a_0^2(d^2E_c/da^2), \quad a=a_0. \qquad (7)$$

A is the mass number. $B_{\Lambda\Lambda}$ is thus characterized by the compressibility coefficient through expressions (3), (5), and (7).[9] Maximization of B_Λ and $B_{\Lambda\Lambda}$ with respect to a yield the equilibrium sizes a_Λ and $a_{\Lambda\Lambda}$ in the ordinary hypernucleus and the double hypernucleus configuration, respectively. One readily obtains the convenient expression

$$\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}(a_{\Lambda\Lambda})-2B_\Lambda(a_\Lambda)$$
$$=\Delta b_{\Lambda\Lambda}(a_0)+b_1^2/(b_2-\epsilon_2)-c_1^2/2(c_2-\epsilon_2), \qquad (8)$$

where $\Delta b_{\Lambda\Lambda}(a_0)=b_{\Lambda\Lambda}(a_0)-2b_\Lambda(a_0)$ would have been the total additional binding energy if there was no distortion, while the remainder in (8) gives the contribution to $\Delta B_{\Lambda\Lambda}$ of core distortion which is, as expected, K-dependent. For a given value of K, $\Delta B_{\Lambda\Lambda}$ is thus readily obtained as a function of the volume integral $U_{\Lambda\Lambda}$ of potential (a) or of the coupling constant $f_{\Sigma\Lambda}$ in potential (b), the coefficients b_1, b_2, c_1, c_2, however, varying in each case.

The calculation of $b_{\Lambda\Lambda}$ was made with the equivalent two-body method of Ref. 10. Since this method has been applied before in a number of other problems, we shall not go into details of the method but shall outline its application in the present problem very briefly. According to this method appropriate to the best S-state variational wave function of the product form $g_1(r_1)g_2(r_2)g_3(r_3)$, where the r's are interparticle separations, one obtains the following two-body equation for the radial Schrödinger function $f_{\Lambda\Lambda}(r)$ describing Λ-Λ

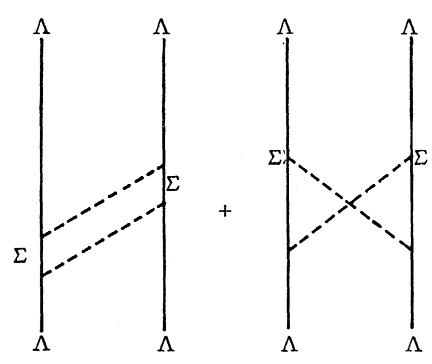

FIG. 1. Graphs showing the two-pion exchange processes contributing to the $\Lambda\Lambda$ potential in the lowest order.

[8] J. J. De Swart and C. Iddings, Phys. Rev. 128, 2810 (1962); R. H. Dalitz, Phys. Letters 5, 53 (1963).

[9] These expansions were also used in the Λ-Λ core model studies (see Ref. 4) of $_{\Lambda\Lambda}Be^{11}$. In the present case, these are even more suitable in view of the better rigidity of the C^{12} core.

[10] A. R. Bodmer and S. Ali, Nucl. Phys. 56, 657 (1964).

TABLE II. Results for $_{\Lambda\Lambda}C^{14}$ for the meson theoretical hard-core ΛΛ potential (b) (with $r_c=0.3\mu_\pi^{-1}$) as a function of the coupling constant $f_{\Sigma\Lambda}$ and the nuclear compressibility coefficient K. Figures in parentheses indicate rigid-core results. Again $\Delta a=a_{\Lambda\Lambda}-a_0$.

K	Range	$f_{\Sigma\Lambda}$	0.1	0.2	0.25	0.275
100	$\mu_{2\pi}$	$B_{\Lambda\Lambda}$	18.99 (18.34)	19.93 (19.15)	22.63 (21.46)	27.96 (26.04)
		$\Delta B_{\Lambda\Lambda}$	−1.89 (−2.50)	−0.95 (−1.69)	1.76 (0.62)	7.08 (5.20)
		Δa	−0.086	−0.091	−0.106	−0.128
	μ_K	$B_{\Lambda\Lambda}$	19.00 (18.15)	20.07 (19.02)	23.15 (21.48)	29.16 (26.28)
		$\Delta B_{\Lambda\Lambda}$	−1.83 (−2.69)	−0.76 (−1.82)	2.32 (0.62)	8.33 (5.44)
		Δa	−0.107	−0.114	−0.134	−0.166
150	$\mu_{2\pi}$	$B_{\Lambda\Lambda}$	18.76 (18.34)	19.65 (19.15)	22.19 (21.46)	27.21 (26.04)
		$\Delta B_{\Lambda\Lambda}$	−2.11 (−2.50)	−1.22 (−1.69)	1.33 (0.62)	6.35 (5.20)
		Δa	−0.055	−0.058	−0.066	−0.079
	μ_K	$B_{\Lambda\Lambda}$	18.66 (18.15)	19.65 (19.02)	22.47 (21.48)	27.92 (26.28)
		$\Delta B_{\Lambda\Lambda}$	−2.18 (−2.69)	−1.19 (−1.82)	1.62 (0.62)	7.08 (5.44)
		Δa	−0.066	−0.070	−0.081	−0.097

relative motion:

$$(d^2f_{\Lambda\Lambda}/dr^2)-(2\mu_{\Lambda-\Lambda}/\hbar^2)\times\{b_{\Lambda\Lambda}+[V_{\Lambda\Lambda}(r)+W_{\Lambda\Lambda}^{(3)}(r)]\}f_{\Lambda\Lambda}=0. \quad (9)$$

The third particle (i.e., the core in the present case) appears through $W_{\Lambda\Lambda}^{(3)}$ which is solely due to its presence. $W_{\Lambda\Lambda}^{(3)}$ (for details of the definition and computation of $W_{\Lambda\Lambda}^{(3)}$, see Ref. 10) is a functional[10] of the relative Λ-C^{12} function $g_{\Lambda\text{-}C^{12}}$ for which we choose a three-parameter trial function $g(r)=e^{-\alpha r}+se^{-\beta r}$ and of the effective Λ-C^{12} potential $V_{\Lambda\text{-}C^{12}}$ which was generated by folding a Yukawa Λ-N interaction into the normalized spherical density distribution of the core $\rho_c(r)$ represented by

$$\rho_c(r)=(3\pi^{3/2}a^3)^{-1}[1+(4r^2/3a^2)]e^{-r^2/a^2}. \quad (10)$$

The electron-scattering data for C^{12} are well fitted[11] by expression (10) with $a=a_0=1.64\pm0.05$ F. For the Λ-N interaction we have considered two ranges $\mu_{2\pi}^{-1}=0.7$ F ($b_{\Lambda N}=1.5$ F) and $\mu_K^{-1}=0.4$ F ($b_{\Lambda N}=0.85$ F) appropriate to two-pion and K-meson exchange (see Refs. 12–19), respectively. Variational calculations with the above Λ-core function and the generated Λ-core potentials have been made. These calculations yield binding energies which agree within 1% of the values obtained by numerical solution of the two-body Λ-core Schrödinger equation with the same potential.

The numerical solution of the Schrödinger eigenvalue problem (9) with the effective potential $V_{\Lambda\Lambda}+W_{\Lambda\Lambda}^{(3)}$ gives $b_{\Lambda\Lambda}$ as a function of the parameters α,s,β of the Λ-core function. For a given strength of the Λ-Λ potential, the maximum of this function gives the required $b_{\Lambda\Lambda}$ for this strength.

For the estimation of distortion effects we started, for a given K, with a value of U_4 (four times the spin-averaged volume integral of the Λ-N interaction) occurring through the volume integral of the Λ-core potential U_{12} $(=3U_4)$ which gives $b_\Lambda(a_0)=10.51$ MeV, the recent experimental value[20] of $B_\Lambda({}_\Lambda C^{13})$. Maximization of B_Λ in Eq. (4) with respect to a then gave a value of $B_\Lambda(a_\Lambda)$ which was obviously higher than 10.51 MeV. However, to achieve stabilization of the Λ-core system at the experimental value of $B_\Lambda(a_\Lambda)$, the value of U_4 was lessened somewhat. The procedure was repeated till $B_\Lambda(a_\Lambda)=10.51$ MeV was obtained. The final value of U_4 thus fixed, which corresponded to a weakening of the Λ-N interaction due to core distortion by one Λ particle, was kept the same in all subsequent calculations of $b_{\Lambda\Lambda}$ for various values of a.

3. RESULTS AND DISCUSSION

For the Λ-core distortion, we obtained volume integrals U_4 of magnitude 948.4 and 955.7 MeV F^3 for $\mu_{\Lambda N}=\mu_{2\pi}$ for $K=100$ and $K=150$ MeV, respectively, the original rigid-core value of U_4 being $U_4=970.0$

[11] M. F. Ehrenberg *et al.*, Phys. Rev. **113**, 666 (1959); U. Meyer-Berkhout, K. W. Ford, and A. E. S. Green, Ann. Phys. (N. Y.) **8**, 119 (1959).

[12] Although somewhat larger intrinsic ranges than these have been found to be more suitable for describing the Λ-p scattering data (see Refs. 13–15) we restrict ourselves to these, the reason being mainly that we are then able to compare the present results with the existing ones (obtained with the same method) for other ΛΛ hypernuclei (see Refs. 16–18) (especially $_{\Lambda\Lambda}He^6$) for which no calculations have been made with $b_{\Lambda N}>1.5$ F. However, the appropriateness of a Λ-N potential with $b_{\Lambda N}>1.5$ F is more for the scattering data than for hypernuclear analyses which in fact often tend to favor a shorter range (see Refs. 19 and 2). Recently Herndon and Tang (see Ref. 14) have proposed a hard-core Λ-N potential (with $b_{\Lambda N}=2.1$ F) which gives agreement with the scattering data as well as with the binding energies of S-shell hypernuclei. The essential feature which this potential has is that the attractive part of the potential when centered at the origin has an intrinsic range of 1.5 F or less. As mentioned by Herndon and Tang, the longest range for the attractive part consistent with charge symmetry corresponds to the range $\mu_{2\pi}^{-1}$ for a Yukawa potential without a hard core ($b_{\Lambda N}=1.5$ F).

[13] S. Ali, M. E. Grypeos, and L. P. Kok, Phys. Letters **24B**, 543 (1967).

[14] R. C. Herndon and Y. C. Tang, Phys. Rev. **159**, 853 (1967); **153**, 1091 (1967).

[15] G. Alexander and U. Karshan, invited talk on low-energy hyperon interaction in *Proceedings of the Second International Conference on High-Energy Physics and Nuclear Structure, Rehovoth, Israel* (North-Holland Publishing Co., Amsterdam, 1967).

[16] A. R. Bodmer and S. Ali, Phys. Rev. **138**, B644 (1965).

[17] S. Ali and A. R. Bodmer, Phys. Letters **24B**, 343 (1967).

[18] S. Ali and A. R. Bodmer, Nuovo Cimento **50A**, 511 (1967).

[19] S. Ali, J. W. Murphy, and A. R. Bodmer, Phys. Rev. Letters **15**, 534 (1965).

[20] W. Gajewski *et al.*, Nucl. Phys. **B1**, 105 (1967).

TABLE III. Results for $_{\Lambda\Lambda}He^6$ for the meson theoretical hard-core Λ-Λ potentials for $r_c = 0.3\mu_\pi^{-1}$ as a function of the coupling constant $f_{\Sigma\Lambda}$. α_m, β_m, s_m are optimum parameters of the Λ-core function.

$f_{\Sigma\Lambda}$	$B_{\Lambda\Lambda}$(MeV)	α_m (F^{-1})	β_m (F^{-1})	s_m
0.1	3.59	0.62	2.93	−0.365
0.2	4.33	0.64	2.88	−0.367
0.25	6.74	0.70	2.84	−0.371
0.275	12.31	0.82	2.24	−0.490

MeV F^3. Thus the values of U_4 were lessened by about 2.23% and 1.48%, respectively. For μ_K, the corresponding values were 3.00% and 1.95% for $K=100$ and 150 MeV, respectively. These values are consistent with the estimates of Bodmer and Murphy,[21] who also studied $_\Lambda C^{13}$ with a two-body model. The values of the equilibrium sizes were found to be $a_\Lambda = 1.593$ and 1.610 F for $\mu_{2\pi}$ and $a_\Lambda = 1.584$ and 1.604 F for μ_K, the values in both cases corresponding to $K=100$ and $K=150$, respectively.

Tables I and II show the three-body results for potentials (a) and (b), respectively. We have, for ease of reference, also included in brackets the rigid-core results [obtained with $a_\Lambda = a_{\Lambda\Lambda} = a_0$: $B_\Lambda(a_0) = b_\Lambda(a_0)$, $B_{\Lambda\Lambda}(a_0) = b_{\Lambda\Lambda}(a_0)$]. Let us first discuss the results for potential (a) for which we have considered $\mu^{-1} = \mu_{2\pi}^{-1} = 0.7$ F, corresponding to the two-pion exchange mechanism of the Λ-Λ interaction. The general features of the three-body rigid-core results (insensitivity to the core size so long as the strength of the Λ-core potential is adjusted to reproduce B_Λ correctly, etc.) have already been discussed in I and hence we shall not discuss them further. By plotting the results (including core distortion) for $B_{\Lambda\Lambda}$ as a function of $U_{\Lambda\Lambda}$ for both $\mu_{2\pi}$ and μ_K, one notices that the behavior of $B_{\Lambda\Lambda}$ is as expected. For a given U, $B_{\Lambda\Lambda}$ increases with decreasing K, the rate of increase being larger for larger U. Although the rigid-core results for $\mu_{2\pi}$ and μ_K differ much less, reflecting the fact that the over-all differences between the Λ-C^{12} potentials for these two ranges are small, one has that for a given U (especially for a larger one) and a given K, the μ_K results for $B_{\Lambda\Lambda}$ are somewhat larger than the $\mu_{2\pi}$ ones. This may be understood in the following way: The presence of the two Λ particles causes a radial core compression. For the compressed core size which is smaller than the free core size, the density distribution is effectively pushed inward and has a shorter range [note from Eq. (10) that a decrease in a implies a faster falling off of the density]. Now since the Λ-N interaction for the K-meson range is also deep near the origin and shallow outside compared to that for the 2π range which is less deep near the origin but more extended outside, the Λ-core wave functions are now pulled in and the two Λ's are allowed to interact more effectively. Thus the K-meson potential contributes proportionally more to the binding energy $B_{\Lambda\Lambda}$ than the two-pion one. However, as the value of K is increased, corresponding to comparatively less distortion and hence a lesser decrease in the core size, the disparity between the $\mu_{2\pi}$ and the μ_K results decreases. All this is also reflected in $|\Delta a_{\Lambda\Lambda}|$ $(=|a_{\Lambda\Lambda}-a_\Lambda|)$, which assumes comparatively larger values in the case of μ_K. For the values of the volume integrals $U_{\Lambda\Lambda}$ of the Λ-Λ potential as are determined from the experimentally observed value of $B_{\Lambda\Lambda}(_{\Lambda\Lambda}He^6) = 10.8 \pm 0.6$ MeV,[22] namely for $U_{\Lambda\Lambda} = 310_{-28}^{+23}$ MeV F^3 corresponding to $\mu_{2\pi}$ and for $U_{\Lambda\Lambda} = 265 \pm 25$ MeV F^3 corresponding to μ_K, one obtains $\Delta B_{\Lambda\Lambda}(_{\Lambda\Lambda}C^{14}) = 6.18_{-0.62}^{+0.55}$ and $6.22_{-0.50}^{+0.65}$ MeV, respectively, for $K=100$ MeV. For $K=150$ MeV, the corresponding values are $5.43_{-0.6}^{+0.5}$ and 5.21 ± 0.51 MeV. Thus, although the volume integral of the Λ-Λ potential for the larger Λ-N range $\mu_{2\pi}^{-1}$ is larger than that for the shorter range μ_K^{-1}, the results for $\Delta B_{\Lambda\Lambda}$ are about the same for these two ranges because of the above-discussed situation arising due to the core distortion. If the results for the Λ-Λ interaction are taken to be reliably determined from $_{\Lambda\Lambda}He^6$ (and in fact this should be the case since distortion effects for ΛΛ hypernuclei are at their minimum[23] in $_{\Lambda\Lambda}He^6$) and if the experimental observation of $_{\Lambda\Lambda}C^{14}$ gives a value of $\Delta B_{\Lambda\Lambda}$, which is approximately equal to or a little less than $\Delta B_{\Lambda\Lambda}(_{\Lambda\Lambda}He^6) = 4.6 \pm 0.6$ MeV, then this could be taken to imply that for a given reasonable value of the compressibility coefficient, one would perhaps need a Λ-N interaction range which might be even shorter than μ_K^{-1} (note that for a larger K, $B_{\Lambda\Lambda}$ tends to decrease with $\mu_{\Lambda\text{-}N}^{-1}$). If, however, one assumes that the 2π range and the K range are about equally compatible (in fact, as we have discussed earlier, the analysis of $_{\Lambda\Lambda}Be^{11}$ did not seem to differentiate between these two ranges, for the $_{\Lambda\Lambda}Be^{11}$ interpretation of Danysz *et al.*'s event), then one would require rather high magnitudes of the compressibility coefficient—the one needed for $\mu_{2\pi}$ would be somewhat larger than μ_K. In the limiting case when K becomes infinite (rigid core) the $\Delta B_{\Lambda\Lambda}$ values for $\mu_{2\pi}$ and μ_K become equal to $4.36_{-0.50}^{+0.46}$ and $3.71_{-0.42}^{+0.45}$ MeV, respectively. Thus, depending on the experimental determination of $\Delta B_{\Lambda\Lambda}(_{\Lambda\Lambda}C^{14})$ one could associate varying amounts of compressibility with $\mu_{2\pi}$ and μ_K. The present analysis thus gives an indication that, given an accurate determination of $B_{\Lambda\Lambda}$, the ΛΛ hypernucleus can be employed profitably as a probe into the nuclear compressibility. In the present case, even if one expects that the value of $\Delta B_{\Lambda\Lambda}$ would be insensitive to the mass number in the known range of ΛΛ hypernuclei, i.e., $_{\Lambda\Lambda}He^6$ and $_{\Lambda\Lambda}Be^{10}$ (or $_{\Lambda\Lambda}Be^{11}$) for which the experimental $\Delta B_{\Lambda\Lambda}$ values are about the same, one would need to consider a "quasihard" C^{12} core. On the basis of the present results, one would expect a value of $K \gtrsim 150$ MeV which is in not too great disagreement with the observed values of K determined from isotope shift and also with other estimates.[24]

[21] A. R. Bodmer and J. W. Murphy, Nucl. Phys. **64**, 593 (1965).

[22] D. J. Prowse, Phys. Rev. Letters **17**, 782 (1966).

[23] Y. C. Tang and R. C. Herndon, Phys. Rev. Letters **14**, 991 (1965).

[24] T. Kohmura, Progr. Theoret. Phys. (Kyoto) **34**, 956 (1965).

We now discuss the results obtained with the meson theoretical Λ-Λ potential for which we choose a hard-core radius of $r_c=0.3\mu_\pi^{-1}=0.42$ F.[25,26] For this potential, we had to perform calculations on $_{\Lambda\Lambda}He^6$ for the K range which were not performed in Ref. 12. The results are shown in Table III. The value of the coupling constant which is determined for μ_K from $B_{\Lambda\Lambda}(_{\Lambda\Lambda}He^6)=10.8\pm0.6$ MeV is found to be $f_{\Sigma\Lambda}=0.2711\pm0.002$, while the value obtained for $\mu_{2\pi}$ is $f_{\Sigma\Lambda}=0.2729\pm0.002$.[27] For these values of the coupling constants, one obtains $\Delta B_{\Lambda\Lambda}=6.23_{-0.6}^{+0.8}$ and $6.82_{-0.70}^{+0.75}$ MeV, respectively, for $K=100$ MeV; and $5.18_{-0.55}^{+0.75}$ and $5.26_{-0.65}^{+0.7}$ MeV, respectively, for $K=150$ MeV. One notices here in the predicted values of $\Delta B_{\Lambda\Lambda}$, for the values of K considered, a slightly different role of the two ranges $\mu_{2\pi}^{-1}$ and μ_K^{-1} as compared to the situation for the Yukawa Λ-Λ potential—one now has a little more binding with the K range rather than with the 2π range. This is presumably because the meson theoretical potential outside the hard core is extremely deep and rapidly varying, falling off to zero after about 2 F and thus the Λ-core wave function for the K range which, as discussed earlier, is expected to experience more attraction in the present problem than that for the 2π range, at short distances, feels even stronger attraction for the rapidly varying meson theoretical potential and hence makes $(\Delta B_{\Lambda\Lambda})_{\mu_K}$ greater than $(\Delta B_{\Lambda\Lambda})_{\mu_{2\pi}}$, especially for small K. However, again, if it turns out that the results for $\Delta B_{\Lambda\Lambda}$ are about the same for $_{\Lambda\Lambda}He^6$ and $_{\Lambda\Lambda}C^{14}$, then, for a given low K value, one would now favor $\mu_{2\pi}^{-1}$ rather than μ_K^{-1}. Thus the conclusions about the range of the Λ-N interaction are seen to depend somewhat on the type of the Λ-Λ potential. Nevertheless, if the Λ-N interaction range is fixed from some other considerations, e.g., from a thorough and combined analysis of *S and P shell hypernuclei*[28,29]; then, for this given range, the analysis of $_{\Lambda\Lambda}C^{14}$ would, besides selecting an adequate K value, also shed considerable light on the form and strength of the Λ-Λ potential. It is believed that the results of the present investigation will serve as a useful guide in understanding these points in greater detail when an accurate determination of $B_{\Lambda\Lambda}(_{\Lambda\Lambda}C^{14})$ has been made.

A more dynamical approach than the one presented here would be to study the $_{\Lambda\Lambda}C^{14}$ system as being of α-α-α-Λ-Λ structure. In this case, it would be necessary to use suitable α-α potentials which give a fair representation of the ground state of the C^{12} system as a 3α system—one would probably have to allow for the existence of a possible D-wave α-α wave function component in the $J=0^+$ ground state of C^{12}. Thus one would need an angular momentum projection of the α-α potential onto the various partial waves.[30] As mentioned before, Herndon and Tang have used a 3α model of C^{12} in their α-α-α-Λ model studies of $_{\Lambda}C^{13}$. They however introduce, besides using a two-body α-α interaction, a completely attractive three-body potential which is parametrized. It is rather difficult to see the justification for introducing such an attractive three-body term and the significance of its parametrization.

After the work reported in this paper was completed, a report by Ananthanarayan[31] came to our attention in which the $_{\Lambda\Lambda}C^{14}$ system was studied using the different method of Dawson, Talmi, and Walecka. His results indicate a rather low value of $\Delta B_{\Lambda\Lambda}$(3.75 MeV) which was based on the Λ-Λ interaction deduced from $_{\Lambda\Lambda}He^6$ for which the method of Dawson *et al.* was not very appropriate but, nevertheless, the possible modification of the results due to uncertainties resulting from his analysis of $_{\Lambda\Lambda}He^6$ was also discussed. In any case, the results of Ananthanarayan correspond more to our rigid-core results, supporting a near rigid structure for C^{12}.

ACKNOWLEDGMENTS

We are very grateful to Professor Abdus Salam and Professor P. Budini and to the IAEA for their hospitality at the International Centre for Theoretical Physics, Trieste. We are also grateful to the staff of the Centro di Calcolo of the University of Trieste for providing machine time on their IBM 7040 computer. One of us (L.P.K.) has carried out this work as a staff member of the Foundation FOM (Fundamental Research on Matter) at Utrecht, The Netherlands.

---

[25] The behavior of $B_{\Lambda\Lambda}$ as a function of r_c was studied for $_{\Lambda\Lambda}He^6$, where it was found that, so long as the change in the hard-core radius is not too large, the results are not expected to depend dramatically on the hard-core radius (see Refs. 18 and 26). Incidentally, for $_{\Lambda\Lambda}He^6$, potential (a) with $\mu=\mu_{2\pi}$ and potential (b) with $r_c=0.3\mu_\pi^{-1}$ were found to be equivalent in the sense that the values of the binding energy $B_{\Lambda\Lambda}$, the scattering length $a_{\Lambda\Lambda}$, and the effective range $r_{0\Lambda\Lambda}$ were found to be about the same. This equivalence was attributed to the fact that potentials (a) and (b) for the above values of range and hard core have the same intrinsic range of ~1.5 F. Such an equivalence, although existing in the present problem for the range $\mu_{2\pi}^{-1}$ of the Λ-N interaction, does not seriously hold for μ_K^{-1}. Since the calculations of Ref. 18 were mostly performed for $\mu_{2\pi}$, we conclude from a comparison of the present investigation with that in Ref. 18 that the intrinsic range equivalence of the Λ-Λ potential in ΛΛ hypernuclei may not be absolute but may depend on the range of the Λ-N interaction.

[26] R. H. Dalitz and A. Rajasekaran, Nucl. Phys. **50**, 450 (1964).

[27] Note that these values are consistent with the observation of Pappademos that the strengths of the Λ-Σ interactions are not large enough to form any dibaryon bound states. The ΛΛ scattering length, the effective range, and the well-depth parameter for $f_{\Sigma\Lambda}=0.2711$ are -1.6 F, 2.55 F, and 0.815, respectively. The scattering parameters for the other ΛΛ potentials are given in Refs. 1 and 17.

[28] See Refs. 29 and 14 where possibilities of reconciliation of the scattering data with hypernuclear analyses have been discussed in some detail.

[29] R. H. Dalitz, invited paper presented at the *Topical Conference on The Use of Elementary Particles in Nuclear Structure Studies, Brussels, 1965* (Interuniversitaire des Sciences Nucléaires, Brussels, 1966); A. R. Bodmer, invited talk in *Proceedings of the Second International Conference on High-Energy Physics and Nuclear Structure, Rehovoth, Israel* (North-Holland Publishing Co., Amsterdam, 1967).

[30] One may note that the α-α potential can be regarded at low energies as local but *l-dependent*. See, e.g., S. Okai and S. C. Park, Phys. Rev. **145**, 787 (1966), and other references contained therein.

[31] K. Ananthanarayan, Stanford University Report No. ITP-277 (unpublished).