Radiative Neutron-Deuteron Capture and the Bound and Scattering States of the Three-Nucleon System

A. C. PHILLIPS*
Physics Department, Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 7 February 1968)

The capture of thermal neutrons by deuterons is found to be sensitive to three-body scattering effects in the initial neutron-deuteron state. The interaction effect is also important. But theoretical errors associated with this effect prevent any detailed conclusions with regard to the implications of the capture reaction for the 3-nucleon bound-state wave function. However, a simple phenomenological representation of the interaction effect, which is consistent with the 3-nucleon exchange magnetic moments and neutron-proton capture, is sufficient to reconcile the results of neutron-deuteron capture with present knowledge of the 3-nucleon bound state.

I. INTRODUCTION

THE elastic scattering of el ectrons on H^3 and He has furnished useful information on the charge and magnetic properties of the 3-nucleon bound states. The most striking result is that the rms radii and form factors for the H^3 and He^3 charge distributions are substantially different, the radii being $R(H^3)=1.70\pm0.05$ F and $R(He^3)=1.87\pm0.05$ F.[1] Provided the neutron charge form factor is small, this difference indicates that there is a significant lack of spatial symmetry in the H^3 and He^3 states. One possibility is the existence of an appreciable S' state. This state has mixed spatial symmetry and is generated by the spin dependence of the 2-nucleon interaction. However, a complete analysis of the form-factor data is also sensitive to the probability densities of the isobaric $T=\frac{3}{2}$ and D-wave admixtures of H^3 and He^3.[2] This situation has led to a renewal of interest in the radiative capture of thermal neutrons by deuterons which, in principle, provides independent information on the properties of the triton S' state.[3,4]

The capture of thermal neutrons by deuterons is predominately a magnetic dipole transition. The amplitude for the transitions due to the nuclear spin magnetic moments is independent of the dominant spatially symmetric S state of the triton, and the most important terms correspond to capture from the doublet and quartet neutron-deuteron states to the S' triton state. In addition to this dependence on the S' state, the capture rate of thermal neutrons by deuterons is sensitive to the interaction effect,[4,5] i.e., the distortion of the electromagnetic properties of the free nucleons due to the nuclear interaction. It is necessary to invoke the existence of this effect in order to account for the experimental magnetic moments of the H^3 and He^3 nuclei.[6] Furthermore, the interaction effect is probably responsible for the 10% discrepancy between the theoretical and experimental cross sections for neutron-proton capture.[7] Here it should be noted that, whereas the neutron-deuteron amplitude due to the nuclear spins is limited by the S' probability, the corresponding amplitude for neutron-proton capture is sensitive to the $^1S_0$ np wave function in the region of the singlet antibound state and hence is large. Accordingly, the interaction effect is expected to have a much more important role in neutron-deuteron capture than in neutron-proton capture.

An assessment of the importance in neutron-deuteron capture of the interaction effect and of the structure of the triton wave function requires an accurate representation of the low-energy 3-nucleon scattering state. In particular, it is important to treat the three-particle aspects of the problem exactly; that is to include the basic analytic structure implied by the existence of the 3-nucleon bound state and the three-particle unitarity relations. A practical theory which is capable of accommodating this requirement is the separable interaction approximation of the Faddeev equations.[8] A complete investigation of the regions of validity of the various separable approximations must await the solution of the Faddeev equations for realistic interactions. However, central forces of separable type, fitted to the low-energy 2-nucleon data, have been found to have some physical justification.[8] In addition, this model gives a good representation of the quartet scattering length and the elastic and inelastic neutron-deuteron cross sections up to 14 MeV and fair results for the triton binding energy and doublet scattering

* Work supported in part by the U. S. Atomic Energy Commission under Contract No. At(30-1) 2098.

[1] H. Collard, R. Hofstadter, E. B. Hughes, H. Johanson, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. **138**, B59 (1965).

[2] L. I. Schiff, Phys. Rev. **133**, B802 (1964); B. F. Gibson and L. I. Schiff, *ibid.* **138**, B26 (1965); B. F. Gibson, *ibid.* **139**, B1153 (1965). For a discussion of the sensitivity of the 3-nucleon form factors to the neutron-charge form factor, see J. S. Levinger and B. K. Srivastava, *ibid.* **137**, B426 (1965).

[3] N. T. Meister, T. K. Radha, and L. I. Schiff, Phys. Rev. Letters **12**, 509 (1964).

[4] T. K. Radha and N. T. Meister, Phys. Rev. **136**, B388 (1964); **138**, AB7(E) (1965).

[5] N. Austern, Phys. Rev. **83**, 672 (1951); **85**, 147 (1951).

[6] See, e.g., R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1953).

[7] H. Pierre Noyes, Nucl. Phys. **74**, 508 (1965); **A95**, 705(E) (1967).

[8] For a discussion of the Faddeev equations and a very complete list of references on separable interactions see C. Lovelace, Phys. Rev. **135**, B1225 (1964).

length.[9,10] The latter results are improved by the introduction of tensor forces[11–13] or by the introduction of a fictitious three-body force as a phenomenological representation of tensor and odd-parity forces, short-range repulsion, and genuine three-body forces.[10] Another aspect is the phase-shift-analysis prediction for the doublet neutron-deuteron effective-range expansion.[14] This expansion shows that $k \cot\delta$ has a pole at small negative k^2 and both the existence and approximate position of this pole are predicted well by the separable model.[15] Thus the low-energy separable interaction model is in general agreement with the basic features of the low-energy 3-nucleon system.

The use of the separable interaction model leads to a considerable improvement over previous neutron-deuteron capture calculations. First, the initial state is consistent with the neutron-deuteron unitarity relation. Barucchi, Bosco, and Nata[16] used a two-body model to investigate the unitarity corrections. Their results are confirmed in this paper and effectively render meaningless calculations which represent the neutron-deuteron wave function by a plane wave. Second, the distortion of the deuteron in the presence of a third nucleon is automatically included by the exact treatment of the 3-body aspects of the neutron-deuteron system. We find that the distortion of the initial neutron-deuteron state also plays a very significant role, and hence the model of Barucchi *et al.*[16] is not sufficiently accurate to yield quantitative results.

Section II describes the three-particle equations that are the basis of this investigation. The separable interaction model is briefly discussed in Sec. III. Section IV contains a description of the relevant properties of the 3-nucleon wave functions. The interaction effect is discussed in Sec. V. The final equations and results are given in Sec. VI.

II. BASIC EQUATIONS

Consider a transition from the asymptotic state $|\Phi_{\alpha n}\rangle$ describing the free motion of a particle α and a bound state of particles β and γ (denoted by n) to a three-particle bound state $|\Phi_T\rangle$. To first order in the electromagnetic Hamiltonian H_ω the amplitude is

$$\langle\Phi_T|U\alpha(E+i\epsilon)|\Phi_{\alpha n}\rangle\equiv\langle\Phi_T|H_\omega|\Psi_{\alpha n}\rangle_+. \qquad (2.1)$$

[9] R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. **140**, B1291 (1965).
[10] A. C. Phillips, Phys. Rev. **142**, 984 (1966).
[11] A. G. Sitenko and V. F. Karchenko, Yadern. Fiz. **1**, 944 (1965) [English transl.: Soviet J. Nucl. Phys. **1**, 708 (1965)]; A. G. Sitenko, V. K. Karchenko, and N. M. Petrov, Phys. Letters **21**, 54 (1966).
[12] B. S. Bhakar and A. N. Mitra, Phys. Rev. Letters **14**, 143 (1965); A. N. Mitra, G. L. Schrenk, and V. S. Bhasin, Ann. Phys. (N.Y.) **40**, 357 (1966).
[13] A. C. Phillips, Nucl. Phys. **A107**, 209 (1968).
[14] W. T. H. van Oers and J. D. Seagrave, Phys. Letters, **24B**, 562 (1967).
[15] A. C. Phillips (unpublished).
[16] G. Baracchi, B. Bosco, and P. Nata, Phys. Letters **15**, 252 (1965). See also F. Eradas, C. Milani, A. Pompei, and S. Seatzu, Nuovo Cimento **45**, B72 (1966).

Here $|\Psi_{\alpha n}\rangle_+$ is the outgoing scattering state in the αn channel and is given by the Faddeev equations

$$|\Psi_{\alpha n}\rangle_+=\sum_{\beta=1}^{3}|\Psi_{\alpha n}{}^{(\beta)}\rangle_+,$$
$$|\Psi_{\alpha n}{}^{(\beta)}\rangle_+=|\Phi_{\alpha n}{}^{(\beta)}\rangle-G_0(E+i\epsilon)T_\beta(E+i\epsilon)\times\sum_{\gamma\neq\beta}|\Psi_{\alpha n}{}^{(\gamma)}\rangle_+,$$
$$|\Phi_{\alpha n}{}^{(\beta)}\rangle=|\Phi_{\alpha n}\rangle\delta_{\alpha\beta}, \qquad (2.2)$$

where $G_0(E)$ is the resolvent operator for three free particles and $T_\beta(E)$ is the transition operator in the three-particle Hilbert space of the γ-α subsystem. Straightforward manipulations then give the transition operator

$$U_\alpha(E)=H_\omega-\sum_{\beta\neq\alpha}U_\beta(E)G_0(E)T_\beta(E). \qquad (2.3)$$

If $\alpha=0$, then (2.3) represents the transition operator from the asymptotic state of three free particles to the bound state.

III. LOW-ENERGY SEPARABLE APPROXIMATION

Our basic assumption, following Ref. 8, is that the kernel of the three-particle Faddeev equation is mainly sensitive to the structure of the two-particle amplitudes in the vicinity of the singularities associated with the two-particle bound states and resonances. Defining $\mathbf{p}_\alpha$ as the relative momentum of the β-γ subsystem and $\mathbf{q}_\alpha$ as the momentum of particle α with respect to this subsystem, then the two-particle amplitude is

$$\langle\mathbf{p}_\alpha\mathbf{q}_\alpha|T_\alpha(E)|\mathbf{p}_\alpha'\mathbf{q}_\alpha'\rangle=\sum_n\langle\mathbf{p}_\alpha|\alpha n\rangle t_{\alpha n}(E-q_\alpha^2/2m_\alpha)\times\langle\alpha n|\mathbf{p}_\alpha'\rangle\delta(\mathbf{q}_\alpha-\mathbf{q}_\alpha'), \qquad (3.1)$$

where $t_{\alpha n}(E-q_\alpha^2/2m_\alpha)$ represents the propagation of a two-particle bound state or resonance with energy $E-q_\alpha^2/2m_\alpha$ (m_α is the appropriate reduced mass) and $\langle\mathbf{p}_\alpha|\alpha n\rangle$ is closely related to the wave function of the bound state or resonance.

For identical particles the amplitude for the transition from the asymptotic state of a particle and two-particle bound state (n) to a three-particle bound state may be defined as

$$\langle\Phi_T|X_n(E)|\mathbf{q}\rangle=\frac{-1}{\sqrt{3}}\sum_{\alpha=1}^{3}\langle\Phi_T|U_\alpha(E)G_0(E)|\alpha n,\mathbf{q}\rangle. \qquad (3.2)$$

Using (2.3), (3.1), and (3.2) we obtain the integral equation

$$\langle\Phi_T|X_n(E)|\mathbf{q}\rangle=\langle\Phi_T|B_n(E)|\mathbf{q}\rangle+\sum_m\int d^3\mathbf{q}'(\langle\Phi_T|X_m(E)|\mathbf{q}'\rangle\times t_m(E-\mathbf{q}'^2/2m)\langle\mathbf{q}'|Z_{mn}(E)|\mathbf{q}\rangle), \qquad (3.3)$$

where

$$\langle\Phi_T|B_n(E)|\mathbf{q}\rangle=\frac{-1}{\sqrt{3}}\sum_{\alpha=1}^{3}\langle\Phi_T|H_\omega G_0(E)|\alpha n,\mathbf{q}\rangle, \tag{3.4}$$

$$Z_{mn}(E)=-\tfrac{1}{3}\sum_{\alpha,\beta}(1-\delta_{\alpha\beta})\langle\alpha m|G_0(E)|\beta n\rangle. \tag{3.5}$$

For the 3-nucleon system the dominance of the deuteron and singlet antibound state (i.e., $n=d$ or s) is assumed. We take

$$\langle\mathbf{p}|\alpha n\rangle=\frac{n_n}{p^2+\mu_n{}^2}|I_{\beta\gamma}=I_n,I_\alpha;Ii\rangle \times|S_{\beta\gamma}=S_n,S_\alpha;Ss\rangle, \tag{3.6}$$

$$t_n(E)=\left[\frac{1}{\lambda_n}+\int\frac{d^3p n_n{}^2}{(p^2+\mu_n{}^2)^2(p^2-E)}\right]^{-1}. \tag{3.7}$$

The parameters μ_s, n_s λ_s, μ_d, n_d, and λ_d are adjusted to give a good representation of the low-energy ${}^1S_0$ and ${}^3S_1$ systems.

IV. THREE-NUCLEON WAVE FUNCTIONS

The first problem is the adjustment of the neutron-deuteron scattering parameters. Here the chief difficulty is that while the separable interaction model gives a good representation of the quartet scattering state, the absence of tensor and short-range forces in this model results in too much attraction in the doublet scattering state. In order to bring the theoretical doublet scattering length into agreement with experiment, the phenomenological device of an effective three-body force is used.[10] This approach has the merit of simplicity and the important advantage that the three-particle unitarity relations are obeyed. Two weak repulsive forces of this kind are considered such that the theoretical doublet scattering lengths take on the values ${}^2a=0.1$ and ${}^2a=0.7$ F. The former agrees with the experimental result of van Oers and Seagrave,[14] and the latter with Hurst and Alcock.[17]

Corresponding to these two situations, bound-state wave functions are constructed so as to be orthogonal to the zero energy doublet scattering state. The separable model yields an exact numerical bound-state wave function. However, for convenience simple analytic expressions are considered. The spacially symmetric state is represented by the function $u(p,q)$, and the S' state by v_1 and v_2 which may be constructed in the usual way from the function $g(p_\alpha,q_\alpha)$. The two wave functions with which we work are the Irving wave function

$$u(p,q)=[1+(8/3\alpha^2)(p^2+\tfrac{3}{4}q^2)]^{-7/2},$$
$$g(p,q)=\left[1+\frac{8}{3\alpha'^2}\left(\frac{3\alpha'^2}{\alpha'^2+2\beta'^2}p^2+\tfrac{3}{4}q^2\right)\right]^{-7/2}, \tag{4.1}$$

[17] D. G. Hurst and J. Alcock, Can. J. Phys. 29, 26 (1951).

TABLE I. The 3-nucleon bound-state wave functions. In calculating the radii, proton and neutron mean-square charge radii of 0.64 and −0.12 F² have been taken.

Wave function	α (F^{-1})	α' (F^{-1})	β' (F^{-1})	Radii for $P_{S'}=1.4\%$ R(He³)	R(H³)	Orthogonal to nd state with doublet scattering length (F)
Irving:						
A	1.295	1.150	1.950	1.86	1.69	0.1
B	1.271	0.910	1.710	1.89	1.72	0.7
Modified Irving:						
C	1.470	0.882	3.760	1.81	1.63	0.1
D	1.380	0.828	3.530	1.87	1.69	0.7

and a modification of the Irving wave function which has the correct asymptotic behavior for the case when all three particles are far apart:

$$u(p,q)=[E_T+p^2+\tfrac{3}{4}q^2]^{-1}[1+(8/3\alpha^2)(p^2+\tfrac{3}{4}q^2)]^{-2},$$
$$g(p,q)=[E_T+p^2+\tfrac{3}{4}q^2]^{-1}\times\left[1+\frac{8}{3\alpha'^2}\left(\frac{3\alpha'^2}{\alpha'^2+2\beta'^2}p^2+\tfrac{3}{4}q^2\right)\right]^{-2}, \tag{4.2}$$

where E_T is the 3-nucleon binding energy.

For small S' probability densities, the orthogonality conditions are not particularly sensitive to the size parameters (α' and β') of the S' state. We have determined α' and β' by making a rough fit to the numerical 3-nucleon wave function of the separable interaction model. The parameters of the bound-state wave functions are given in Table I. Note that, contrary to what is often assumed, β' is much greater than α'. This result agrees with the most recent variational calculations.[18]

The charge radii of the He³ and H³ nuclei are given in column 5 of Table I. Proton and neutron mean-square charge radii of 0.64 and −0.12 F², respectively, have been assumed. These results illustrate a defect of the separable interaction model. The charge radii of the wave functions which are orthogonal to the neutron-deuteron state with the older and less reliable doublet scattering length of 0.7 F are closest to the experimental radii. A related problem is discussed in Ref. 13.

V. INTERACTION EFFECT

The electromagnetic interaction, with a photon moving in direction **k** and with polarization vector **u**, is taken to be

$$H_\omega=\mathbf{u}\wedge\mathbf{k}\cdot\mathbf{D}+\mathbf{u}\wedge\mathbf{k}\cdot\mathbf{I}.$$

Here **D** is the usual direct magnetic-moment operator due to the nucleon spins. The vector **I** is the interaction effect or exchange operator. Because present theoretical techniques do not allow reliable calculations of the

[18] L. M. Delves and J. M. Blatt, Nucl. Phys. A98, 503 (1967).

interaction effect in terms of meson currents, we adopt a phenomenological representation which is consistent with the magnetic moments of H^3 and He^3 and the neutron-proton capture reaction. The magnitudes of the isovector and isoscalar exchange magnetic moments of the He^3 and H^3 doublet depend on the percentages of the D, P, S', and $T=\frac{3}{2}$ admixtures in the bound states.[6] Keeping only the D-state contribution the isovector exchange moments are 0.27, 0.32, and 0.37 μ_N for D-state probabilities of 4, 7, and 10%, respectively. The corresponding isoscalar moments are 0.0, 0.15, and 0.025 μ_N. Thus the neglect of the isoscalar part of the operator $\mathbf{I}$ seems reasonable. For the isovector part we take

$$\langle \mathbf{pq}|\mathbf{I}|\mathbf{p}'\mathbf{q}'\rangle=\sum \lambda_I(\tau_{\alpha z}-\tau_{\beta z})(\boldsymbol{\sigma}_\alpha-\boldsymbol{\sigma}_\beta)\times w_\gamma(\mathbf{p}_\gamma,\mathbf{p}_\gamma')\delta_3(\mathbf{q}_\gamma-\mathbf{q}_\gamma'). \quad (5.1)$$

A Yukawa form for the potential function is assumed;

$$w_\gamma(\mathbf{p}_\gamma,\mathbf{p}_\gamma')=\frac{1}{(\mathbf{p}_\gamma-\mathbf{p}_\gamma')^2+\mu_I^2}, \quad (5.2)$$

where μ_I^{-1} is taken to be 1.18 F.

It should be emphasized that (5.1) and (5.2) for $\mathbf{I}$ comprise only one of several possibilities for the isovector exchange operator.[6] For a given fit to the magnetic moments of the 3-nucleon bound states the contributions to neutron-deuteron capture of several of the permitted types of expressions are identical.[4] However, the possibility that (5.1) and (5.2) constitute a poor representation of the interaction effect cannot be ruled out. For example, the existence of a three-body interaction effect is possible.[19] Nevertheless, the approximate nature of the experimental evidence for the interaction effect does not warrant the use of a complicated representation.

VI. FINAL EQUATIONS, RESULTS, AND DISCUSSION

The neutron-deuteron capture cross section is

$$\sigma=\tfrac{1}{3}(2\pi)^3\alpha(\hbar/Mc)^4(E_\gamma^3/q_L)\times(|\langle\Phi_T|X_d^{1/2}|\mathbf{q}\rangle|^2+|\langle\Phi_T|X_d^{3/2}|\mathbf{q}\rangle|^2),$$

where E_γ is the photon energy (F^{-2}) and q_L the laboratory momentum of the neutron (F^{-1}).

The two physical transition amplitudes are the quartet $\langle\Phi_T|X_d^{3/2}|\mathbf{q}\rangle$ and the doublet $\langle\Phi_T|X_d^{1/2}|\mathbf{q}\rangle$. The former satisfies a single-channel integral equation and the latter, together with an unphysical amplitude $\langle\Phi_T|X_s^{1/2}|\mathbf{q}\rangle$, satisfies a coupled integral equation [see (3.3)–(3.5)]. The amplitude $\langle\Phi_T|X_S^{1/2}|\mathbf{q}\rangle$ corresponds to capture from a configuration of a nucleon and a singlet antibound state. Using (3.4) the corresponding inhomogeneous or plane-wave-approximation terms are

$$\langle\Phi_T|B_d^{3/2}(E)|\mathbf{q}\rangle=-\tfrac{2}{3}\sqrt{3}(\mu_p-\mu_n)\langle v_1f_{1d}\rangle+(\tfrac{2}{3}\sqrt{3})\lambda_I\langle u|2w_1-w_2-w_3|f_{1d}\rangle, \quad (6.1)$$

$$\langle\Phi_T|B_d^{1/2}(E)|\mathbf{q}\rangle=-(\tfrac{1}{3}\sqrt{6})(\mu_p-\mu_n)\langle v_1f_{1d}\rangle+(\tfrac{2}{3}\sqrt{6})\lambda_I\langle u|w_1+w_2+w_3|f_{1d}\rangle, \quad (6.2)$$

$$\langle\Phi_T|B_s^{1/2}(E)|\mathbf{q}\rangle=-(\tfrac{1}{3}\sqrt{6})(\mu_p-\mu_n)\langle v_1f_{1s}\rangle-(\tfrac{2}{3}\sqrt{6})\lambda_I\langle u|w_1+w_2+w_3|f_{1s}\rangle, \quad (6.3)$$

where f_{1n} is the continuation off the energy shell of the asymptotic n-d or n-s wave functions

$$\langle p_1q_1|f_{1n}\rangle=\frac{-1}{p_1^2+\frac{3}{4}q_1^2-E}\,\frac{n_n}{p_1^2+\mu_n^2}\delta_3(\mathbf{q}_1-\mathbf{q}).$$

The functions u, v_1, and v_2 represent the triton wave function. In the evaluation of the matrix element of the interaction effect operator (5.1), only the fully symmetric S state for the bound state has been retained.

The expressions (6.1)–(6.3) do not include the contributions due to the deuteron and triton D states. Austern[5] found that the electric quadrapole transition from the continuum S state to the triton $^4D_{1/2}$ state and the transition from the D state of the deuteron to the triton S state are very small. However, there has been no estimate of the magnetic dipole transition amplitude from the deuteron D state to the $^4D_{1/2}$ state of the triton. We estimate this amplitude in the plane-wave approximation. The only component of the $^4D_{1/2}$ triton state that contributes corresponds to a configuration of two nucleons in a $^3D_1$ state and a third nucleon in a relative S wave. It is a simple matter to extract this component from the numerical bound-state wave function that is given by the separable interaction model. It is found that, in the plane-wave approximation, the introduction of the D to D-state transitions gives rise to a quartet and doublet amplitude that interfere destructively with the corresponding S' amplitudes for capture. The modifications are usefully expressed as effective S' states. The 4, 5.5, and 7% deuteron D states correspond approximately to at most 0.16, 0.29, and 0.50% S' states, respectively, for both the doublet and quartet inhomogeneous terms (6.1) and (6.2). However, it will be shown that scattering and interaction effects lead to a result in which the term (6.3), which is independent of the D to D-state transitions, gives rise to the dominant contribution to the capture cross section.

We now turn to the role played in neutron-deuteron capture by the scattering in the initial state. For the quartet contribution (for various $P_{S'}$, and with and without the inclusion of interaction effect) the complete amplitude $X_d^{3/2}$ is about three times smaller than the corresponding plane-wave term $B_d^{3/2}$. That is, the quartet contribution to the cross section is decreased by a factor of $\sim$10. Similar reductions have been obtained before using a two-body model for the neutron-

[19] D. W. Padgett, W. M. Frank, and J. G. Brennan, Nucl. Phys. 73, 424 (1965).

TABLE II. Three-body scattering effects in neutron-deuteron capture for modified Irving wave function D of Table I and an S' probability of 1.4%.

	Plane-wave approximation			Complete solution		
Exchange moment of H^3-He^3 (μ_N)	0.0	0.27	0.37	0.0	0.27	0.37
Enhancement of n-p capture (%)	0.0	5.5	7.5	0.0	5.5	7.5
Quartet amplitude	0.786	0.685	0.648	0.262	0.243	0.235
Doublet amplitude	0.556	0.245	0.133	0.311	0.652	0.778
Capture cross section (mb)	1.078	0.6127	0.5143	0.1920	0.5622	0.7683
Ratio of quartet to doublet cross sections	2.00	7.77	19.01	0.714	0.138	0.091

TABLE III. The 3-nucleon S' probability corresponding to the experimental capture cross section as a function of the interaction effect and wave function.

Wave function (Table I)	Interaction effect: Exchange moment of He^3, H^3 (μ_N)	Interaction effect: Enhancement of np capture (%)	S' probability corresponding to experimental cross section (%)
A	0.0	0.0	7.8 ±0.40
	0.27	6.4	2.5 ±0.40
	0.37	8.8	1.4 ±0.30
B	0.0	0.0	3.0 ±0.25
	0.27	6.7	1.1 ±0.15
	0.37	9.2	0.44±0.16
C	0.0	0.0	5.0 ±0.60
	0.27	5.0	1.9 ±0.25
	0.37	6.9	1.0 ±0.12
D	0.0	0.0	4.4 ±0.40
	0.27	5.5	1.6 ±0.20
	0.37	7.5	0.75±0.20

deuteron scattering system.[16] One would expect that such a model would be reasonable for the quartet state since the Pauli principle keeps the nucleons far apart in this state and prevents excessive distortion of the deuteron. However, this is not the case for the doublet state. Furthermore, because of the huge reduction in the quartet capture amplitude, it is the doublet state that provides the dominant contribution to the total capture cross section.

The doublet capture amplitude is obtained by solving a coupled integral equation. That is, capture can be conceived of as occurring from the S-wave configuration of a nucleon and a 2-nucleon subsystem in a $^3S_1$ state and the configuration of a nucleon and a 2-nucleon subsystem in a $^1S_0$ state. The final result depends on the relative magnitudes of $B_d^{1/2}$ and $B_s^{1/2}$. With zero-interaction effect the magnitude of $X_d^{1/2}$ is slightly less than $B_d^{1/2}$. Inclusion of the interaction effect results in a drastic reduction in the value for $B_d^{1/2}$ but an increase for $B_s^{1/2}$. In this situation the scattering corrections manifest themselves by giving a large value for $X_d^{1/2}$ even though $B_d^{1/2}$ is fairly small. The mechanism for doublet capture corresponds to a severe distortion of the initial state by the nuclear strong interaction and subsequent capture from a configuration corresponding to nucleon scattering off a 2-nucleon subsystem in a $^1S_0$ state. Thus, even though the interaction effect interferes destructively with the spin magnetic moment in the plane wave amplitude $B_d^{1/2}$, the inclusion of three-body aspects of the neutron-deuteron wave function results in an over-all constructive interference. We also note that the dominant contribution, corresponding to capture from a nucleon and singlet 2-nucleon configuration, is not sensitive to the triton and deuteron D states and hence the neglect of these states is expected to be a good approximation.

Typical three-particle scattering effects are summarized in Table II. These results correspond to the modified Irving wave function, and Table I, which is orthogonal to the doublet state with scattering length 0.7 F. The plane-wave approximation and the complete solution for several values of the interaction effect are examined. It is clear from Table II that radiative neutron-deuteron capture, because of its dependence on nuclear scattering effects and the interaction effect, is an exceedingly complex problem. This is illustrated particularly in the prediction for the ratio of the quartet to doublet contributions to the cross section. For the neutron-deuteron system this ratio is only accessible experimentally if polarized deuterons and neutrons are used. However, proton-deuteron fusion in the muonic molecule $p\mu d$ allows an estimate of the corresponding ratio for the radiative capture of protons by deuterons.[20] This ratio is <0.25 and provides at least qualitative support for our predictions for the case that includes both scattering and interaction effects.

The theoretical complexity of the neutron-deuteron capture reaction prevents any detailed conclusions with regard to the structure of the 3-nucleon bound state. The chief source of uncertainty is in the treatment of the interaction effect. But in the absence of a reliable theory of strong interactions nothing more than a phenomenological approach with a minimum of unknown parameters is feasible. In this spirit we adjust the simple representation of the interaction effect by the isovector exchange operator, (5.1) and (5.2), to account for the 3-nucleon magnetic moments and neutron-proton capture and investigate whether the results of neutron-deuteron capture are consistent with our knowledge of the 3-nucleon system. The relevant properties of the 3-nucleon system are the probability of the S' state and the difference in the He^3 and H^3 charge radii.

Variational calculations using a Hamada-Johnson

[20] B. P. Carter, Phys. Rev. **141**, 863 (1966); **153**, 1358(E) (1967).

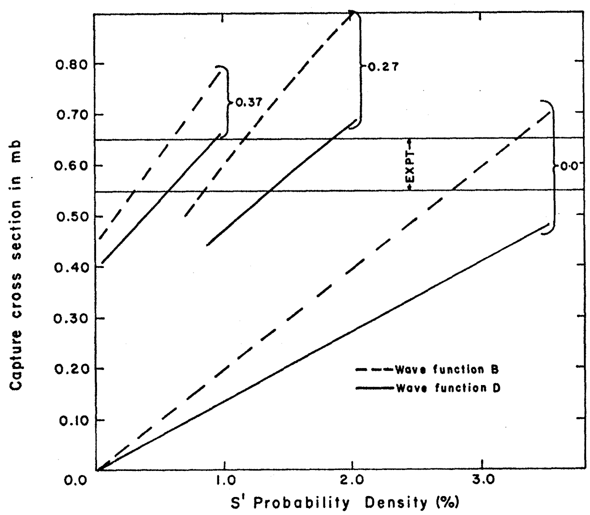

FIG. 1. The capture cross section as a function of the S' probability for wave functions B and D of Table I. Three values of the interaction effect, corresponding to 3-nucleon exchange moments of 0.0, 0.27, and $0.37\mu_N$ are considered.

potential give an S' probability of 1.2%.[21] Separable interactions give a similar result.[12,22] The sensitivity of the separable interaction results to the 2-nucleon singlet effective range (r_s) and the deuteron D state (P_D) has been investiated.[15] As expected, the variation of $P_{S'}$ with r_s is fairly large, corresponding, when $P_D=0.0$, to 1.6% for $r_s=2.85$ F and 1.4% for $r_s=2.7$ F. With $r_s=2.7$ F and $P_D=7\%$, $P_{S'}$ is 1.27%. Thus a realistic S' probability density is in the range 0.8–1.6%.

Experimentally the difference in the charge radii is 0.17 ± 0.1 F.[1] For the Irving wave functions of Table I, $P_{S'}=1.2\%$ gives a difference of 0.12 F if the neutron charge form factor is taken to be zero. A neutron form factor of $-0.02q^2$ leads to a difference of 0.16 F. Of course the radii difference also depends on the particular analytic form of the S and S' wave functions[23] and the properties of the 3-nucleon D and $T=\frac{3}{2}$ states.[2] However, the contributions from these effects are not sufficiently important to lead to a rejection of the estimate of $P_{S'}$ between 0.8 and 1.6%.

The dependence of neutron-deuteron capture on the structure of the 3-nucleon bound state is now examined. The cross sections for the wave functions B and D of Table I as a function of $P_{S'}$ are shown in Fig. 1. The continuous curve corresponds to the modified Irving wave function (4.2) and the dotted curve to the Irving wave function (4.1). For each wave function, three values of the interaction effect are considered, corresponding to 0.0, 0.27, and 0.37 μ_N for the exchange magnetic moments of 3-nucleon bound state. Inspection of Fig. 1 shows that $0.8<P_{S'}<1.6\%$ and the experimental capture cross section of 0.60 ± 0.05 mb[24] are only compatible with the capture results which include the interaction effect. This is also the case for wave functions A and C.

The results of all four wave functions of Table I are summarized in Table III. The 3-nucleon exchange magnetic moments and the interaction effect enhancements for the neutron-proton capture cross section are given in columns 2 and 3. The interaction effect in neutron-proton capture amounts to 10%.[7] But it should be noted that the enhancement given by exchange operator, (5.1) and (5.2), is particularly sensitive to the range (μ_I^{-1}) of the potential function $w(\mathbf{p},\mathbf{p}')$. Accordingly, exchange moments of 0.27 and 0.37 μ_N are acceptable. Column 4 contains the 3-nucleon S' state which will give the experimental capture cross section. The results which include the interaction effect give $P_{S'}$ in range 0.4–2.5% and are consistent with our knowledge of the 3-nucleon bound-state wave function.

In conclusion we emphasize the following. Until the interaction effect is properly understood, radiative neutron-deuteron capture can yield no useful information on the structure of the 3-nucleon bound-state wave function. This reaction does, however, provide an interesting example of the importance of three-body scattering effects. These effects are markedly different for the doublet and quartet capture amplitudes and for the doublet case reverse the plane-wave-approximation result (and also the zero-distortion result) that the interaction effect and nucleon spin amplitudes interfere destructively.

Though scattering effects are not expected to be so large in the photodisintegration and the electrodisintegration of the 3-nucleon bound state, the results of this paper are sufficient to cast doubt on the reliability of the existing calculations for these reactions.

ACKNOWLEDGMENTS

I would like to thank Dr. I. M. Barbour, Professor R. J. Blin-Stoyle, and Dr. L. M. Delves of the University of Sussex for several useful discussions.

[21] B. Davies, University of New South Wales report (unpublished).

[22] J. Borysowicz and J. Dabrowski, Phys. Letters **24B**, 125 (1967).

[23] R. H. Dalitz and T. W. Thacker, Phys. Rev. Letters **15**, 204 (1965).

[24] This is the cross section for neutrons of velocity 22 km/sec; see F. T. Jurney and N. T. Motz (unpublished).