Λ-Σ Conversion and the Hypertriton*

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The binding energy of $_\Lambda H^3$ is calculated using a Faddeev-type multiple-scattering formalism. The effects of virtual Λ-Σ conversion via $\Lambda N \leftrightarrow \Sigma N$ are included in a full two-channel representation of the YN interaction. Nonlocal separable, S-wave, spin-dependent potentials are used for each two-body interaction. Calculations are performed for two sets of low-energy ΛN scattering parameters and two different symmetry models for the YN potential. The introduction of Λ-Σ conversion in the YN spin-triplet channel increases the binding energy. The binding is decreased by the use of Λ-Σ conversion in the YN spin-singlet channel. When incorporated into both YN spin channels, the effect of Λ-Σ conversion is to reduce the $_\Lambda H^3$ binding energy.

I. INTRODUCTION

THIS paper is one of a series[1–3] in which the Λ-nucleon (ΛN) interaction is investigated by means of the Faddeev equations[4] applied to the hypernucleus $_\Lambda H^3$. The purpose of these investigations is to find an appropriate set of nonlocal separable (NLS) potentials which fits the low-energy ΛN scattering data and the binding energy of $_\Lambda H^3$. In this paper the effect of virtual Λ-Σ conversion on the binding energy of the hypertriton is examined.

Recent studies of the ΛN interaction in hypernuclei with $A>3$ have been complicated by the existence of charge symmetry breaking (CSB) forces,[5] possible three-body ΛNN forces,[6,7] and in certain cases suppression of Λ-Σ conversion due to isospin conservation.[8] Because the third component of the $_\Lambda H^3$ isopin is zero, CSB effects cancel out. In addition, the calculations of Gal,[6] and of Bhaduri, Loiseau, and Nogami[7] have indicated that the three-body ΛNN forces play only a minor role in the loosely bound $_\Lambda H^3$ system, so that a phenomenological treatment of them is not unwarranted. With the inclusion of the Σ channel in the problem, isospin must and can be explicitly taken into account.

For free ΛN scattering[9] and for the ΛN interaction in light hypernuclei[10] a single channel (the Λ channel), energy-independent, ΛN potential is usually used. One effect of taking $\Lambda\Sigma$ conversion *explicitly* into account is to make the ΛN potential energy-dependent. A second effect is to change the single-channel ΛN potential into a 2×2 $YN(Y=\Lambda,\Sigma)$ potential matrix whose diagonal (off-diagonal) elements represent the potential for $YN \leftrightarrow Y'N$ with $Y=Y'(Y\neq Y')$.[11] This is known as the two-channel model. A third effect is to introduce a three-body ΛNN force via $\Lambda N \to \Sigma N$, followed by $\Sigma N' \to \Lambda N'$, where N and N' are different nucleons. The ΛNN potential may also be handled in a one- or a two-channel model.

The effects of Λ-Σ conversion on the binding energy of the hypertriton have been previously considered by Rajasekaran and Biswas,[12] Vashakidze and Chilashvili,[13] and BLN.[14]

Rajasekaran and Biswas used a two-channel ΛN model with NLS potentials and global symmetry to determine the $\Lambda N \to \Lambda N$ scattering lengths and effective ranges. In their variation calculations of hypernuclear binding energies, however, they merely used single-channel, energy-independent local ΛN potentials adjusted to yield these same low-energy scattering parameters.

Vashakidze and Chilashvili also used NLS potentials and global symmetry but with a full two-channel formalism throughout. Within their model they obtained an exact set of coupled integral equations for the hypertriton, but in solving this set of equations the angular dependence of part of the kernel was neglected. According to their calculations, the net effect of the Σ channel was a considerable reduction (from 4.6 to 2.9 MeV) in the binding energy of $_\Lambda H^3$. As global symmetry yields low-energy $\Lambda N \to \Lambda N$ scattering parameters in disagreement with recent Λp scattering data and light

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[5] R. C. Herndon and Y. C. Tang, Phys. Rev. **153**, 1091 (1967); **159**, 853 (1967).
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[7] R. K. Bhaduri, B. A. Loiseau, and Y. Nogami, Ann. Phys. (N. Y.) **44**, 57 (1967). Hereafter referred to as BLN.
[8] A. R. Bodmer, Phys. Rev. **141**, 1387 (1966).
[9] S. Ali, M. E. Grypeos, and L. P. Kok, Phys. Letters **24B**, 543 (1967).
[10] See, for example, R. C. Herndon, Y. C. Tang, and E. W. Schmid, Phys. Rev. **137**, B924 (1965) and references cited therein.

[11] The connection between these two effects is given in the Appendix. The energy dependence is usually ignored because the energy range of interest lies well below the ΣN threshold.
[12] G. Rajasekaran and S. N. Biswas, Phys. Rev. **122**, 712 (1961).
[13] I. Sh. Vashakidze and G. A. Chilashvili, Dokl. Akad. Nauk SSSR **157**, 557 (1964) [English transl.: Soviet Phys.—Doklady **9**, 576 (1965)].
[14] For other authors who have attacked this problem see BLN. The results of these authors are contained in or are special cases of the BLN work.

hypernuclear-binding-energy analyses, the validity of the results of this work and that described directly above is suspect.

BLN, using dispersion theoretic techniques, calculated the two-pion exchange contribution to the ΛNN potential (i.e., Λ-Σ conversion via one-pion exchange repeated on two different nucleons). They used a single-channel, energy-independent ΛNN model in their calculations of the effect of the ΛNN forces on the binding energy of ${}_\Lambda H^3$ and ${}_\Lambda He^5$. The effect on the hypertriton was calculated by the averaging of their three-body potential over the ${}_\Lambda H^3$ wave function of Downs, Smith and Truong.[15] This wave function is one obtained on the basis of a single-channel, energy-independent ΛN potential. Their results indicate a repulsive effect of about the same size (~0.15 MeV) as B_Λ, the binding energy of the Λ in ${}_\Lambda H^3$.

The present calculation makes use of the Faddeev multiparticle scattering formalism along with NLS S-wave potentials to obtain a set of coupled, one-dimensional, linear integral equations for the Λ-deuteron spin-$\frac{1}{2}$, S-wave, elastic-scattering amplitude. Nonrelativistic kinematics were used in all calculations. Within these limitations, an *exact* calculation of B_Λ was made by finding the pole in the Λ-d scattering amplitude. A full two-channel formalism for the interaction of the hyperons with the nucleons was used throughout. No attempt was made to separate the contributions to B_Λ of the different effects of Λ-Σ conversion described above.

Unlike the previous calculations described above, the YN potential parameters used were not completely determined by appeal to more fundamental considerations. In each case considered in this work there was one parameter left undetermined which was used to vary the coupling between the YN Λ and Σ channels. For the range of couplings for which calculations were performed, the change in B_Λ never exceeded 65% of B_Λ.

Section II describes the two-body calculations. The hyperon-nucleon (YN) potentials were written as 2×2 matrices in each spin state. The ΛN potentials were fit to two sets of shape-independent parameters, one determined from the low-energy Λ-p scattering data[16] and the other from the binding energies[5] of three- and four-body hypernuclei. In addition, two symmetry models were used to relate the ΣN potential parameter to the ΛN interactions. One assumed equal strength in the YN interactions and the other was based upon restricted symmetry.

In Sec. III, the effects of Λ-Σ conversion in each spin state of the YN interaction on the hypertriton binding energy are given. It was found that the inclusion of Λ-Σ conversion in the spin-zero $(S=0)YN$ state alone gives a decrease in the ${}_\Lambda H^3$ binding energy, the inclusion of Λ-Σ conversion in the spin-one $(S=1)YN$ state alone gives a very slight increase in binding energy, and the inclusion of hyperon conversion in both spin states results in an over-all decrease in binding energy which is larger than that calculated for the spin-zero case alone.

The mathematical details of the two-body calculations are covered in the Appendix.

II. TWO-BODY INTERACTIONS

All particles were taken to have spin $\frac{1}{2}$. The n and p were assumed to be members of an isospin doublet; the Λ, an isospin singlet; and the Σ, an isospin triplet. The two-body interactions were assumed to be charge symmetric. Coulomb forces were neglected throughout the work. The values used for the particle masses were (in MeV) $M_N=938.9$, $M_\Lambda=1115.4$, and $M_\Sigma=1193.0$.

The two-body interactions were taken to be NLS S-wave potentials of the Yamaguchi type.[17] In the single-channel Λ-deuteron scattering problem,[1] the NN S-wave interactions were restricted to $S=1$ by isospin conservation. With the inclusion of the Σ channel, both spin-0 and -1 NN interactions must be considered. The NN $S=1$ parameters were fit to the binding energy of the deuteron, $B_D=2.225$ MeV, and a scattering length of 5.39 F. The NN $S=0$ parameters were fit to a scattering length of -23.678 F and an effective range of 2.51 F.[18] Details of these calculations may be found in Refs. 1 and 17.

The YN potential was written as a 2×2 matrix in each spin state. For a given spin, the matrix element of the YN potential taken between two plane-wave states of momenta $\mathbf{k}_A$ in hyperon channel A and $\mathbf{k}_B$ in channel B is

$$\langle \mathbf{k}_A | V | \mathbf{k}_B \rangle = \lambda_{AB} v_A(k_A) v_B(k_B) \quad (A, B = \Lambda, \Sigma), \quad (1)$$

where $v_A = 1/(k_A{}^2 + \beta_A{}^2)$. The notation $\lambda\Lambda \equiv \lambda_{\Lambda\Lambda}$, $\lambda_\Sigma \equiv \lambda_{\Sigma\Sigma}$, $\lambda_X \equiv \lambda_{\Lambda\Sigma} = \lambda_{\Sigma\Lambda}$ is introduced for convenience. Each element of the potential matrix involves a range parameter β_A and a strength parameter λ_A. The entire potential matrix is block diagonal in the spin variable. The YN isospin coordinate is ignored in the following discussion because the problem at hand restricts the total isospin of the hyperon and nucleon to the $I=\frac{1}{2}$ channel. Details of the derivation of the YN t-matrix from the Lippmann-Schwinger equation are given in the Appendix.

The dynamical input parameters to the two-body YN calculations were the $S=0$ and 1, ΛN scattering lengths and effective ranges. Two sets of data were used and are given in Table I. The result of Alexander *et al.*[16] (referred to in Tables I–IV as AEA) was obtained as a

[15] B. W. Downs, D. R. Smith, and T. N. Truong, Phys. Rev. **129**, 2730 (1963).

[16] G. Alexander, O. Benary, U. Karshon, A. Shapiro, G. Yekutieli, R. Englemann, H. Filthuth, A. Fridman, and B. Schilby, Phys. Letters **19**, 715 (1966).

[17] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).

[18] H. P. Noyes, Phys. Rev. **130**, 2025 (1963); Phys. Rev. Letters **12**, 171 (1964). To enchance the effect of the NN $S=0$ interaction on B_Λ, the most attractive interaction (i.e., the one with most negative scattering length and smallest effective range) of the S-wave singlet np and nn interactions was used.

TABLE I. Two-body ΛN low-energy parameters and corresponding single-channel range parameter.

Input	Spin	a_Λ (F)	r_Λ (F)	β_0^{-1} (F)
AEA	0	-2.46	3.87	0.8750
	1	-2.07	4.50	0.9359
HTCS	0	-2.75	2.98	0.7329
	1	-1.95	3.42	0.7527

best fit to the low-energy Λ-p cross sections in the effective range approximation. The HTCS values are those corresponding to the *charge-symmetric part* of the Λ-N potential "H" obtained by Herndon and Tang,[5,19] who used an exponential well outside a hard core to fit the binding energies of the S-shell hypernuclei and the Λ-p scattering data. [*Note added in proof.* Our calculations for the HTCS scattering parameters were not correct. The values for r_Λ are too small by 2.5%. The use of the correct values would not change our conclusions.] As was shown in Ref. 3 (for no explicit Σ channel), the AEA parameters used with a single attractive NLS potential for each ΛN interaction give B_Λ in agreement with the experimental value of 0.20 ± 0.12 MeV.[20] Because the HTCS parameters were determined from a local potential that included a hard core, when a purely attractive NLS potential (again with no explicit Σ channel) is matched to these parameters, a too large value of B_Λ results. Thus, by the use of both these sets of input parameters, any dependence of the three-body results on the specific value of B_Λ may be seen.

In each spin state there were five parameters to be fixed, the strength parameters λ_Λ, λ_Σ, λ_X, and the ranges β_Λ and β_Σ. To determine these parameters, the perfect $SU3$ symmetry model of Deloff and Wyld[21] was used. Here, however, the potentials used were purely attractive and the ΛN effective ranges as well as the scattering lengths were taken as known. With this model $\beta_\Lambda=\beta_\Sigma=\beta_{NN}$, where β_{NN} is the range parameter of the nucleon-nucleon potential whose value in each spin state was determined by the methods described above. From Ref. 22 and Eq. (1) the YN strength parameters in this model are related to λ_{NN}, the nucleon-nucleon strength parameter, by

$$\lambda_\Lambda=\lambda_\Sigma=\tfrac{1}{2}(1+x)\lambda_{NN},\quad \lambda_X=\tfrac{1}{2}(-1+x)\lambda_{NN} \tag{2}$$

for the spin-one channel and

$$\lambda_\Lambda=\tfrac{1}{10}(9+y)\lambda_{NN},\quad \lambda_\Sigma=\tfrac{1}{10}(1+9y)\lambda_{NN},\quad \lambda_X=\tfrac{1}{10}(3-3y)\lambda_{NN} \tag{3}$$

for the spin-zero channel. The parameters x and y were fixed by fitting the appropriate ΛN scattering length. The singlet and triplet effective ranges were then calculated and found to be significantly smaller than they should have been. For example, with $S=1$ and $a_\Lambda=-2.07$ F, it was found that $r_\Lambda=2.50$ F rather than 4.50 F as listed in Table I. Because of the assumption of *perfect SU*3 symmetry and the use of potentials without repulsive cores, no great significance is attached to this failure to fit both the ΛN scattering lengths and effective ranges.

A "patched up" version of the above $SU3$ model (e.g., one with the range parameter variable) that allowed the desired agreement to be obtained was not used. Rather, this model was discarded in favor of two other, more blatantly phenomenological models. In both of these, as a means of reducing the number of undetermined parameters, the ranges in a given spin state were taken to be equal and the resulting β was left as a free parameter. The expression (A6)[22] for the $\Lambda N\rightarrow\Lambda N$ t-matrix together with Eq. (A13) and the standard effective range expansion gave two relations connecting the three λ's. The models used to obtain a third relation were an "equal-strength" model and a model based upon restricted symmetry in the hyperon-nucleon coupling constants.

The equal-strength model assumed that $\lambda_\Lambda=\lambda_\Sigma\equiv\lambda$. With this assumption λ_X could be varied by allowing β to vary. As β approached its maximum value β_0 (cf. Table I and the Appendix), λ_X would approach zero. Hence, the degree of coupling between the hyperon channels could be varied by changing β. The parameter

$$\eta\equiv(\beta_0/\beta)-1 \tag{4}$$

given in the tables was used as a measure of this coupling. It was felt that this parameter rather than some more direct measure of the coupling (e.g., the ratio $\lambda_X/\lambda_\Lambda$) offered the best way to compare the results of the two models used. Table II lists the parameters corresponding to the equal-strength model which were used in the three-body calculation.

The restricted-symmetry model was based upon the field-theoretic calculation of ΛN and ΣN scattering carried out by de Swart and Iddings[23] to fourth order in proper pion-exchange diagrams. According to their calculations, if the Σ-Λ mass difference is neglected and if, in the usual notation, $f_{\Lambda\Sigma\pi}=f_{\Sigma\Sigma\pi}$ (but unlike global symmetry $f_{\Sigma\Sigma\pi}\neq f_{NN\pi}$), then

$$V_X=\tfrac{1}{2}\sqrt{3}(V_\Sigma-V_\Lambda). \tag{5}$$

Since it is assumed here that $\beta_\Sigma=\beta_\Lambda\equiv\beta$, Eq. (3) becomes

$$\lambda_X=\tfrac{1}{2}\sqrt{3}(\lambda_\Sigma-\lambda_\Lambda). \tag{6}$$

As the V_Σ and V_Λ used here did not include a repulsive

[19] The scattering lengths and effective ranges given by Herndon and Tang (Ref. 5) for their "H" potential are appropriate for the Λ-p interaction only. The values given in Table I were calculated from the CS potential parameters given for potential "H" in Eq. (16) of the second paper listed in Ref. 5.

[20] W. Gajewski, C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. Harmsen, R. Levi-Setti, M. Raymund, J. Zakrzewski, D. Stanley, D. H. Davis, E. R. Fletcher, J. E. Allen, V. A. Bull, A. P. Conway, and P. V. March, Nucl. Phys. B1, 105 (1967).

[21] A. Deloff and H. W. Wyld, Jr., Phys. Letters 12, 245 (1964).

[22] All equations whose number begins with A are given in the Appendix.

[23] J. J. de Swart and C. K. Iddings, Phys. Rev. 128, 2810 (1962).

TABLE II. YN potential parameters from the equal-strength model.

Input	Case	λ_Λ (MeV)$^2/(20\pi)^3$	λ_X (MeV)$^2/(20\pi)^3$	β^{-1} (F)	$10^2\eta$	B_Λ (MeV)
AEA Spin 0	A	−0.6659	0.0000	0.8750	0.00	0.21
	B	−0.6615	0.1247	0.8755	0.06	0.21
	C	−0.6526	0.2178	0.8765	0.17	0.19
	D	−0.6215	0.3971	0.8800	0.57	0.17
	E	−0.5780	0.5583	0.8850	1.14	0.14
AEA Spin 1	A	−0.4892	0.0000	0.9359	0.00	0.21
	B	−0.4609	0.3064	0.9400	0.44	0.21
	C	−0.4274	0.4522	0.9450	0.97	0.22
	D	−0.2700	0.8456	0.9700	3.64	0.23
	E	0.0000	1.2450	1.0201	9.00	0.26
HTCS Spin 0	A	−1.2651	0.0000	0.7329	0.00	0.65
	B	−1.2315	0.3804	0.7350	0.29	0.62
	C	−1.1528	0.6944	0.7400	0.97	0.55
	D	−1.0760	0.8999	0.7450	1.65	0.49
HTCS Spin 1	A	−1.0105	0.0000	0.7527	0.00	0.65
	B	−0.8752	0.7612	0.7630	1.37	0.66
	C	−0.4464	1.5311	0.8000	6.28	0.68
	D	0.0000	2.0099	0.8478	12.64	0.71

core, no attempt was made to relate the parameters λ to the coupling constant f. Note that this model is quite different from the equal-strength model in that the coupling between the hyperon channels vanishes if $\lambda_\Lambda=\lambda_\Sigma$ is used in Eq. (6).

The resulting set of equations for λ_Λ, λ_Σ, and λ_X are given in matrix form in Eq. (A16). As shown in the Appendix, the restricted-symmetry model yields two sets of solutions for the strength parameters, denoted by RS(+) and RS(−). This ambiguity arises from the combination of Eq. (6), a linear relation that connects λ_X to the other two strength parameters, with the relations (that connect λ_X^2 to λ_Λ and λ_Σ) which follow from fitting the model to the low-energy ΛN scattering parameters. These same two solutions would be obtained if the phase convention used in Ref. 23 were changed so that the sign of V_X relative to that of $V_\Sigma - V_\Lambda$ in Eq. (5) were reversed. With the sign convention of Eq. (5), if the absolute sign of λ_X is chosen to be positive, the set of solutions RS(+) results. The choice $\lambda_X<0$ yields the set RS(−). Of course $|\lambda_X|$, λ_Σ, and λ_Λ do not have the same values for the RS(+) solutions as for the RS(−) solutions.[24] The RS(−) solutions were discarded because they yielded infinite solutions for certain values of β. As β is allowed to approach β_0, λ_X again becomes small and Eqs. (A16) decouple. Table III lists the parameters corresponding to RS(+) which were used in the three-body calculations.

Further details of Tables II and III are included in Sec. III.

III. THREE-BODY CALCULATIONS

The application of the Faddeev formalism to Λ-d scattering has been discussed previously.[1] Addition of the Σ channel to the problem results in a set of six coupled linear integral equations corresponding to the possible two-body ΛN, ΣN, and NN interactions in the singlet and triplet spin states. The binding energy of the hypertrition was computed[25] by finding the pole in the S-wave t-matrix element corresponding to Λ-d elastic scattering in the doublet state. The values obtained for B_Λ are estimated to be accurate only to within 0.01 MeV, owing to computational inaccuracies.

The shift in the binding energy of ${}_\Lambda H^3$ due to virtual Λ-Σ conversion in the YN $S=0$ channel alone was calculated for each set of singlet YN parameters listed in Tables II and III. In these calculations, Λ-Σ conversion was neglected in the $S=1$ channel, the triplet ΛN interactions being given by the potentials labelled A in the tables. In addition, the NN $S=0$ interaction was neglected, i.e., the nucleons were not allowed to scatter in the presence of the Σ hyperon. With these assumptions, the original set of six coupled equations was reduced to four. Analogous calculations showing the effects of Λ-Σ conversion in the YN $S=1$ channel alone were also made. The results are presented in Tables II and III under the heading B_Λ, the difference in the binding energies of ${}_\Lambda H^3$ and the deuteron.

From Tables II and III it is easily seen how the effect of Λ-Σ conversion on B_Λ increases as the coupling between the Λ and Σ channels, as measured by η, increases. In Table II, the maximum values of η used for the $S=1$ state were set by going to the extreme case of $\lambda_\Lambda=0$. The maximum value of η in the $S=0$ state was arbitrarily chosen to give a change ΔB_Λ due to Λ-Σ conversion in this state that was large enough ($\Delta B_\Lambda/B_\Lambda \approx 25$–$33\%$) to make comparison of the various cases meaningful, yet small enough to assure $B_\Lambda>0$ when the $S=0$ and $S=1$ Λ-Σ conversion effects were combined as in Table IV. As is well known, B_Λ determines mainly the $S=0$ YN interaction. Thus, for a given absolute change in B_Λ, the $S=0$ value of η is much

[24] There are also two sets of solutions in the equal-strength model. With that model, however, the linear relation ($\lambda_\Lambda=\lambda_\Sigma$) does not contain λ_X. The two sets are identical except for the sign of λ_X. This sign was chosen to be positive.

[25] All numerical work was performed on the Honeywell H-800 computer at the University of Southern California Computer Science Laboratory.

TABLE III. YN potential parameters from RS(+) model.

Input	Case	λ_Λ (MeV)^2/(20π)^3	λ_Σ (MeV)^2/(20π)^3	λ_X (MeV)^2/(20π)^3	β^{-1} (F)	$10^2\eta$	B_Λ (MeV)
AEA Spin 0	B	−0.6568	−0.4431	0.1851	0.8760	0.11	0.21
	E	−0.5686	0.1734	0.6425	0.8850	1.14	0.14
AEA Spin 1	F	−0.4882	−0.4214	0.0579	0.9360	0.01	0.21
	C	−0.4213	0.1703	0.5124	0.9450	0.97	0.21
HTCS Spin 0	F	−1.2634	−1.2635	0.0843	0.7330	0.01	0.65
	D	−1.0530	0.1825	1.0699	0.7450	1.65	0.47
HTCS Spin 1	F	−1.0058	−0.8386	0.1448	0.7530	0.04	0.65
	B	−0.8611	0.1573	0.8820	0.7630	1.37	0.66

smaller than the corresponding $S=1$ value. The other values of η used in Table II were chosen to give representative results of how B_Λ varied with η. The values of η used in Table III were chosen to facilitate comparison between the two models used.

Table IV lists the results of the binding energy calculations where Λ-Σ conversion was considered in both YN spin channels. The YN potentials are identified in each case by a pair of Roman capitals referring to the potentials listed in Tables II and III under the appropriate model and input headings. In most of these calculations, the NN $S=0$ scattering was again neglected. An asterisk denotes calculations which did not make this approximation, with the singlet NN parameters determined as described in Sec. II.

A comparison of the results of Table IV with each other as well as with those of Tables II and III, yields the following information:

1. Independent of the model or the ΛN input parameters, the effect of Λ-Σ conversion is to reduce B_Λ. In fact, for the HTCS ΛN parameters, the reduction is sufficient to bring B_Λ into agreement with the experimental value without the use of repulsive core potentials.

2. Independent of model or ΛN input parameters, the effect of Λ-Σ conversion in the YN $S=1$ state is to increase B_Λ slightly—$\Delta B_\Lambda=0.06$ MeV at most. Yet, with Λ-Σ conversion in both YN spin states, the decrease in B_Λ is greater than it is when Λ-Σ conversion is present only in the YN $S=0$ state. This effect could be due to the presence of terms in the Fredholm determinant that lower B_Λ, which are absent unless Λ-Σ conversion is present in both YN spin channels. Such terms exist because the YN pairs in $_\Lambda H^3$ are not completely both in the singlet or both in the triplet spin state; i.e., in terms of a multiple-scattering expansion of the Faddeev equations, a YN spin singlet scattering may be followed by a YN spin triplet scattering and vice versa. That these terms do indeed contribute a repulsive effect is part of the subject of a further analysis of the hypertriton which will be published at a later date.

3. For a given set of ΛN input parameters and a given Λ-Σ coupling, the values obtained for B_Λ are model-independent; e.g., compare the E+C cases in Table IV.

4. The contribution of the NN $S=0$ interaction to B_Λ is attractive; i.e., B_Λ is increased. This contribution is, however, very small [e.g., compare cases D+E with (D+E)* in Table IV], being completely negligible at all except the largest values of η considered. This suggests that a large fraction of the change in B_Λ due to Λ-Σ conversion is due to the change in the two-body ΛN interaction rather than to the introduction of three-body ΛNN forces. That the NN $S=0$ potential may be dropped from consideration—thus reducing the number of coupled integral equations in the three-body problem—becomes significant when, in order to include the effects of repulsive cores, it becomes necessary to represent each basic two-body potential by a sum of NLS potentials.

TABLE IV. Values of B_Λ for Λ-Σ conversion in both YN spin states. The potentials in the second column are given in Tables II and III.

Input/Model	YN potentials (spin 0 + spin 1)	B_Λ (MeV)
AEA	C+D	0.17
$\lambda_\Lambda=\lambda_\Sigma$	C+E	0.17
	(D+B)[a]	0.14
	D+B	0.14
	(D+E)[a]	0.11
	D+E	0.09
	E+C	0.08
HTCS	B+D	0.51
$\lambda_\Lambda=\lambda_\Sigma$	(C+D)[a]	0.36
	C+D	0.31
	D+B	0.36
AEA RS(+)	B+F	0.20
	E+C	0.08
HTCS RS(+)	F+F	0.65
	D+B	0.33

[a] Denotes calculations which included NN, $S=0$ interaction.

In sum, the exact three-body calculations performed here with phenomenological YN potentials strongly reinforce the result of previous approximate three-body calculations with more fundamental YN potentials that the explicit introduction of the Σ channel reduces B_Λ. This work furthermore shows the strong dependence of this reduction on the amount of coupling of the YN Λ and Σ channels, especially in the singlet spin state.

How much the change in B_Λ discussed here is modified when repulsive cores are included in the two-body potentials, what part of this change comes from modification of the ΛN potential due to Λ-Σ conversion rather than from the ΛNN potential generated by this conversion, and just what the size of this change is for more fundamental models of the YN potential, are all topics to be subjected to further investigation.

APPENDIX

The Lippmann-Schwinger equation for the YN t-matrix of the energy E in a given spin channel may be written

$$t_E = V + Vg(E)t_E\,, \tag{A1}$$

where

$$t_E = \begin{pmatrix} t_\Lambda & t_X \\ t_X & t_\Sigma \end{pmatrix}, \quad V = \begin{pmatrix} V_\Lambda & V_X \\ V_X & V_\Sigma \end{pmatrix}, \tag{A2}$$

and

$$g(E) = \begin{pmatrix} g_\Lambda(E_\Lambda) & 0 \\ 0 & g_\Sigma(E_\Sigma) \end{pmatrix}. \tag{A3}$$

Here $E_A = E - E_{TA}$, where E_{TA} is the threshold energy in channel A, $A = \Sigma$, Λ. The convention $E_{T\Lambda} = 0$ is used for convenience. It follows that $k_\Sigma^2 = R^2(k_\Lambda^2 - k_0^2)$, where $R^2 = \mu_\Sigma/\mu_\Lambda$ and $k_0^2 = 2\mu_\Lambda\Delta$, with μ_A being the reduced mass in channel A, Δ the Σ-Λ mass difference, and $k_A = |\mathbf{k}_A|$ the relative momentum in channel A. In momentum space, the kernel $g_A(E_A, q_A)$ of the Green's-function operator $g_A(E_A)$ is that appropriate to outgoing waves in hyperon channel A:

$$g_A(E_A, q_A) = [E_A - q_A^2/2\mu_A + i\epsilon]^{-1}, \quad \epsilon \to 0^+. \tag{A4}$$

With τ_{AB} defined by

$$\langle \mathbf{k}_A | t_E | \mathbf{k}_B \rangle = v_A(k_A)\tau_{AB}v_B(k_B)\,, \tag{A5}$$

Eq. (1) of Sec. II, and Eqs. (A1)-(A3) yield

$$\begin{aligned} \tau_{\Lambda\Lambda}(E) &= [\lambda_\Lambda(1-\lambda_\Sigma\bar{g}_\Sigma) + \lambda_X^2\bar{g}_\Sigma]/D(E)\,, \\ \tau_{\Sigma\Sigma}(E) &= [\lambda_\Sigma(1-\lambda_\Lambda\bar{g}_\Lambda) + \lambda_X^2\bar{g}_\Lambda]/D(E)\,, \\ \tau_{\Lambda\Sigma}(E) &= \lambda_X/D(E)\,, \end{aligned} \tag{A6}$$

where

$$\bar{g}_A = [1/(2\pi)^3]\int d\xi\, g_A(E,\xi)v_A^2(\xi) \tag{A7}$$

and

$$D(E) = (1-\lambda_\Lambda\bar{g}_\Lambda)(1-\lambda_\Sigma\bar{g}_\Sigma) - \lambda_X^2\bar{g}_\Lambda\bar{g}_\Sigma. \tag{A8}$$

For a Yamaguchi potential,

$$\bar{g}_A = (\mu_A/4\pi\beta_A)[(2\mu_A E_A)^{1/2} + i\beta_A]^{-2}. \tag{A9}$$

Below the channel-A threshold the positive imaginary root in Eq. (A9) is used which places $\bar{g}_A$ on the first, or physical, sheet of the E plane cut from E_A to ∞.

From Eqs. (A6) and (A8), it follows that $\tau_{\Lambda\Lambda}(E)$ may be expressed in the form it takes on when there is no Σ channel (i.e., $\lambda_X = 0$), but with an energy-dependent strength; namely

$$\tau_{\Lambda\Lambda}(E) = \gamma_{\Lambda\Lambda}[1 - \gamma_{\Lambda\Lambda}\bar{g}_\Lambda]^{-1}\,, \tag{A10}$$

where

$$\gamma_{\Lambda\Lambda} = \lambda_\Lambda + \lambda_X^2\bar{g}_\Sigma[1 - \lambda_\Sigma\bar{g}_\Sigma]^{-1} \tag{A11}$$

is energy-dependent through the dependence of $\bar{g}_\Sigma$ on E.

It is necessary for convergence of the Fredholm determinant in the three-body problem that the two-body YN t-matrices have no poles lying along the positive imaginary k_Λ axis. Such poles correspond to YN bound states. In order to check this, the zeros of $D(E)$ were calculated for each set of YN parameters used in the three-body calculations.

The S-wave phase shift for ΛN elastic scattering is related to the t-matrix through the equation

$$\langle \mathbf{k}_\Lambda' | t_\Lambda | \mathbf{k}_\Lambda \rangle = -(2\pi/\mu_\Lambda)(k_\Lambda \cot\delta_\Lambda - ik_\Lambda)^{-1}. \tag{A12}$$

It follows that

$$k_\Lambda \cot\delta_\Lambda = ik_\Lambda - 2\pi(k_\Lambda^2 + \beta_\Lambda^2)^2/(\mu_\Lambda\tau_{\Lambda\Lambda})\,, \tag{A13}$$

and the usual effective range expansion $k_\Lambda \cot\delta_\Lambda = -a_\Lambda^{-1} + \frac{1}{2}r_\Lambda k_\Lambda^2$ gives two equations relating the three λ's and the two β's to the ΛN shape-independent parameters.

With the assumption of equal strength in the hyperon-nucleon interactions, there results

$$\begin{aligned} \lambda_\Lambda = \lambda_\Sigma \equiv \lambda &= (pr - qs)/(s - p^2 r)\,, \\ \lambda_X &= \pm(r/s)^{1/2}(1 + \lambda p)\,, \end{aligned} \tag{A14}$$

where

$$\begin{aligned} p &= \mu_\Sigma(4\pi\beta)^{-1}(Rk_0 + \beta)^{-2}\,, \\ q &= 4\pi a_\Lambda\beta^4[\mu_\Lambda(a_\Lambda\beta - 2)]^{-1}\,, \\ r &= q^2(\beta_0 - \beta)[3\beta\beta_0 - (4/a_\Lambda)(\beta + \beta_0)]/(\beta\beta_0)^2\,, \\ s &= R^3\beta^2(k_0)^{-1}(Rk_0 + \beta)^{-3}. \end{aligned} \tag{A15}$$

The constant β_0 is the range parameter β that is obtained from the same a_Λ and r_Λ with $\lambda_X = 0$. The requirement that the λ's be real implies $r \geq 0$, so that with $a_\Lambda < 0$ there necessarily results $\beta \leq \beta_0$. If $\beta = \beta_0$, the equations in (A15) decouple.

Restricted symmetry leads to the equations

$$\begin{bmatrix} s & -p^2 r & 0 \\ 0 & -p^2 r & \pm p\sqrt{(rs)} \\ -\sqrt{3} & \sqrt{3} & -2 \end{bmatrix} \begin{bmatrix} \lambda_\Lambda \\ \lambda_\Sigma \\ \lambda_X \end{bmatrix} = \begin{bmatrix} pr - qs \\ pr \\ 0 \end{bmatrix}. \tag{A16}$$

There are two solutions to Eq. (A14) corresponding to the choice of sign of the radical. The appearance of r under the radical leads to the same requirement on β as in Eqs. (A15).