Faddeev Equations and Coulomb Effects in ^{3}He

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(Received 2 October 1967; revised manuscript received 12 February 1968)

When two particles interact via a potential V which is the sum of a separable potential V_S and a Coulomb potential V_C, the two-body T matrix can be obtained exactly and can be split in the usual way into a pure Coulomb T matrix and a "nuclear" T matrix which also contains Coulomb effects. Using the fact that the "nuclear" T matrix is still separable in the off-shell variables, the formalism of Alt, Grassberger, and Sandhas is applied to the calculation of Coulomb effects in the three-nucleon system, with the contribution of the nonseparable Coulomb T matrix fully taken into account. The Coulomb energy Δ_C of ^{3}He and the probability $P_{3/2}$ of finding the ^{3}He in an $I=\frac{3}{2}$ state are calculated, using s-wave spin-dependent potentials of the Yamaguchi type to describe the two-nucleon interaction. This model yields values for Δ_C which are in reasonable agreement with the binding-energy difference of ^{3}H and ^{3}He, but predicts a negligible admixture of the $I=\frac{3}{2}$ state in the ^{3}He wave function. The effect of the nonseparable part of the p-p T matrix on the binding energy and wave function of ^{3}He is discussed. Finally, the possible relevance of hard-core effects is pointed out.

I. INTRODUCTION

IT is a well-known fact that a nonlocal separable potential yields a two-body T matrix which is also separable in the off-shell variables.[1] In the last few years, these potentials have been used extensively in three-body calculations; the reason being that the Faddeev equations[2] with separable two-body T-matrices become Fredholm integral equations in only one variable after angular momentum decomposition.

The application of the separable approximation to the three-nucleon system has had a reasonable success.[3–19] The two-nucleon system contains, at low energies, the deuteron bound state with quantum numbers $J=1$, $I=0$, and a singlet virtual state (denoted by s) with quantum numbers $J=0$, $I=1$. The simplest assumption, as discussed by Lovelace,[1] is that the two-body T matrix is dominated by central forces in these two states. Tensor forces have also been taken into account—either phenomenologically,[7,13] or in an explicit fashion[12,17,19]—in calculations of the ^{3}H bound state and n-d scattering lengths. Perhaps the most complete calculation has been carried out by Schrenk and Mitra.[19] These authors include tensor forces in the triplet state and hard-core effects in the singlet state using a variety of separable potentials that fit the two-body data, and conclude that the separable approximation provides a reasonable description of bound-state and scattering properties of the three-nucleon system at low energies.

We are concerned in this paper with a nonperturbative calculation of Coulomb effects in the three-nucleon system. The problem of two particles interacting via a potential V which is the sum of a separable potential V_S and a Coulomb potential V_C has been discussed by Harrington.[20] This author showed that the problem can be solved exactly and that the resulting T matrix can be split in the usual way in a pure Coulomb and a "nuclear" T matrix.[21] Needless to say, the "nuclear" T matrix also contains Coulomb effects. However, the crucial point is that it is still separable in the off-shell

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variables. In Sec. II we briefly review the results by Harrington which are relevant for our purposes and discuss the two-body interactions used as an input in our model.

In writing the equations for p-d scattering, one has to face the problem of the nonseparable Coulomb part of the p-p T matrix. This is accomplished in our calculation by using an elegant formalism devised to take into account nonseparable contributions to the Lovelace equations, due to Alt, Grassberger, and Sandhas.[22] In Sec. III we briefly review their formalism and derive the Lovelace equations for p-d scattering.

Since the Coulomb potential violates isospin symmetry in a very definite way, we discuss to some extent the $SU(2)$ symmetry breaking in the three-nucleon system and write the equations in a basis in Hilbert space in which the breaking is more transparent. The bound-state scattering amplitudes are no longer diagonal in I, the total isospin, and for a given value of I_Z we obtain a set of coupled equations in I. When $I_Z=-\frac{1}{2}$, they reduce to the well-known equations for n-d scattering,[1] but for $I_Z=+\frac{1}{2}$ they show explicitly the Coulomb effects.

In Sec. IV, we discuss the bound-state properties of the three-nucleon system and use a procedure suggested by Amado[23] to obtain the ^{3}He wave function from the Lovelace equations for the p-d scattering amplitude. Section V contains the numerical results. The binding energy difference of ^{3}H and ^{3}He is calculated by computing the shift of the pole of the p-d scattering amplitude with respect to the pole of the n-d scattering amplitude. It should be emphasized that, although our calculation of the ^{3}He Coulomb energy is based on the exact solution of a model, we are not saying that perturbation theory calculations of such a quantity are unreliable.[24] Our interest in this model has been raised because it allows an exact computation of both ^{3}He binding energy and its wave function. The isospin symmetry breaking due to the Coulomb interaction produces not only the splitting of isospin multiplets but also configuration mixing, and therefore the ^{3}He wave function acquires a small $I=\frac{3}{2}$ component. We also discuss the calculation of the probabilities of finding the ^{3}He in the $I=\frac{3}{2}$ state, and in the S' state of mixed spatial symmetry with $I=\frac{1}{2}$.

Finally, Sec. V also contains a discussion of our results.

It should be emphasized that, although our calculations are restricted to bound-state properties of the three-nucleon system, our equations can be immediately used as they stand to perform calculations of p-d scattering with the Coulomb corrections fully taken into account.

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[23] R. D. Amado, Phys. Rev. **141**, 902 (1966).
[24] In these calculations Δ_C is obtained by computing the expectation value of the Coulomb potential, using ^{3}H wave functions. See, for example, Ref. 36.

II. TWO-BODY INTERACTIONS

The simplest model used previously to describe the two-nucleon interaction assumes a spin-dependent separable potential of the Yamaguchi[25,26] type, defined by

$$V(\mathbf{p}',\mathbf{p})=\lambda_s g_s(\mathbf{p}')g_s(\mathbf{p})P_s+\lambda_d g_d(\mathbf{p}')g_d(\mathbf{p})P_d, \quad (2.1)$$

where

$$g_i(\mathbf{p})=1/(\mathbf{p}^2+\beta_i^2), \quad i=s,d \quad (2.2)$$

and P_s, P_d are the appropriate spin-isospin projection operators. The two-nucleon T matrix is given in this case by

$$T(\mathbf{p}',\mathbf{p};E)=g_s(\mathbf{p}')t_s(E)g_s(\mathbf{p})P_s + g_d(\mathbf{p}')t_d(E)g_d(\mathbf{p})P_d, \quad (2.3)$$

where

$$t_i^{-1}(E)=\frac{1}{\lambda_i}+4\pi\int_0^\infty dp\frac{p^2g_i^2(p)}{p^2-E} =\frac{1}{\lambda_i}+\frac{\pi^2}{\beta_i(\beta_i-i\sqrt{E})}. \quad (2.4)$$

The singlet and triplet parameters in the potentials are determined by fitting the two-body data in a way described in Sec. V. In order to take into account Coulomb effects, one has to add to the potentials given by Eq. (2.1) a term of the form $V_C P_{1,1}$, where V_C is the Fourier transform of the Coulomb potential and $P_{1,1}$ is the projection operator

$$P_{1,1}=|I=1;\ I_Z=+1\rangle\langle I=1,\ I_Z=+1|. \quad (2.5)$$

Since the Coulomb potential acts only on $I=1$ states, it modifies only the singlet part of the two-nucleon T matrix. It is a well-known fact that if isospin is an exact symmetry, the T matrix is not only diagonal in I, I_Z but also independent of I_Z. The Coulomb potential breaks the symmetry in such a way that the singlet T matrix is still diagonal, but acquires an I_Z dependence. Consequently, the two-nucleon T matrix is given by[20]

$$T(\mathbf{p}',\mathbf{p};E)=g_d(\mathbf{p}')t_d(E)g_d(\mathbf{p})P_d +g_s(\mathbf{p}')t_s(E)g_s(\mathbf{p})[P_{1,0}+P_{1,-1}] +[T_C(\mathbf{p}',\mathbf{p})+g_C(\mathbf{p}')t_C(E)g_C(\mathbf{p})]P_{1,1}. \quad (2.6)$$

In this equation $T_C(\mathbf{p}',\mathbf{p})$ represents the pure Coulomb T matrix, and the remaining coefficient of $P_{1,1}$ represents the "nuclear" T matrix for p-p scattering. The modified form factors $g_C(\mathbf{p})$ have been calculated by Harrington,[20] with the following result:

$$g_C(\mathbf{p})=g_s(\mathbf{p})C_0(\eta)\exp[2\eta\tan^{-1}(p\beta_s^{-1})], \quad (2.7)$$

where $\eta=me^2/p$, m is the reduced mass of the two-

[25] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).
[26] Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. **95**, 1635 (1954).

particle system, and

$$C_0(\eta)=\{2\pi\eta[\exp(2\pi\eta)-1]^{-1}\}^{1/2}. \quad (2.8)$$

The function $t_C(E)$ is given by an expression analogous to Eq. (2.4), except that it is the modified form factor which appears in the integrand. Therefore, one is faced with the problem of evaluating the integral

$$I(E)=4\pi\int_0^\infty dp\frac{p^2C_0^2(\eta)\exp[4\eta\tan^{-1}(p\beta_s^{-1})]}{(p^2+\beta_s^2)^2(p^2-E)}. \quad (2.9)$$

The value of β_s necessary to fit the two-body data is such that $(me^2/\beta_s)\simeq 0.06$. Consequently, this quantity can be used as an expansion parameter in the calculation of $I(E)$, and terms of order $(me^2/\beta_s)^2$ can safely be neglected. We refer to Harrington's paper for details.[20] As a result of the calculation one can get relations between p-p and n-n low-energy scattering parameters which are indeed very reasonable. If the parameters β_s and λ_s are fixed so as to yield $a_{pp}=-7.81$ F and $r_{pp}=2.80$ F, one gets for the corresponding parameters with the Coulomb effects removed the values $a_{nn}=-18.0$ F and $r_{nn}=2.93$ F. Although these values for a_{nn} and r_{nn} are a little large,[27] this result indicates that the Coulomb interaction is rather well described by this formalism at low energies. Anyway, this point is not of extreme importance to us since we are not taking the attitude of introducing as an input for the ^{3}He calculation the p-n scattering data with Coulomb corrections. Our inputs are the "nuclear" two-body T matrix plus the pure Coulomb T matrix. The parameters will be determined as indicated in Sec. V by fitting the singlet effective range and scattering length. Without Coulomb corrections, this gives a reasonable model for the ^{3}H. Our attitude is to test the stability of the three-body model against electromagnetic corrections to the strong interactions.

The calculations we carry out in this paper are restricted to the bound-state region of the three-nucleon system. Since the two-particle propagators $t_i(E)$ appear in the Lovelace equations in the form $t_i(E-q^2)$, E being the total energy, it is clear that we need the function $t_C(E)$ below its cut on the positive real axis. For $E<0$, the integral (2.9) can be evaluated,[20] and we obtain, setting $k=i\sqrt{-E}$,

$$t_C^{-1}(k)=\frac{1}{\lambda_s}+\frac{\pi^2}{\beta_s(\beta_s+k)^2}+\frac{4\pi^2me^2}{(\beta_s^2-k^2)^2}\times\left\{1-\frac{k}{me^2+k}+\ln\left[\frac{4k^3}{\beta_s(\beta_s+k)^2}\right]+\frac{\beta_s^2-k^2}{\beta_s^2}\right\}. \quad (2.10)$$

Going back to the modified form factor $g_C(\mathbf{p})$, it follows from Eqs. (2.7) and (2.8) that it has an essential singularity at $p=0$ owing to the behavior of $C_0(\eta)$. This essential singularity merely reflects the long-range nature of the Coulomb potential.[28] In order to avoid this difficulty, the Coulomb potential is cut off at a distance R much greater than the range β_s^{-1} of the separable potential. In this condition, Eq. (2.7) is not valid all the way down to $p=0$, and ceases to be valid for values of p such that

$$pR\simeq m^2e^4/p^2. \quad (2.11)$$

In our calculations we have kept R as a free parameter and have used a form factor given by Eq. (2.7) for $p>p_0$, but defined for $p\leqslant p_0$ by

$$g_C(\mathbf{p})=g_s(\mathbf{p})C_0(\eta_0)\exp[2\eta_0\tan^{-1}(p_0\beta_s^{-1})], \quad (2.12)$$

where $p_0^3=m^2e^4R^{-1}$, $\eta_0=\eta(p_0)$.

All the results we quote in Sec. V correspond to a value of $R=20$ F. As a matter of fact, we have checked that they are quite insensitive to variations of this parameter up to 50 F. The range of the singlet potential that we use is of the order of 1 F.

III. THREE-BODY EQUATIONS

The problem of modifying the Lovelace equations in the case in which the two-body T matrix contains nonseparable terms such as, for example, the Coulomb T matrix given by Eq. (2.6), has been discussed by Alt, Grassberger, and Sandhas (AGS).[22] These authors introduce transition operators for bound-state elastic rearrangement scattering, $\mathcal{U}_{\beta\alpha}(E)$, which are slightly different from those used by Lovelace.[1] However, they yield the same scattering amplitude when their matrix elements are put on the energy shell. In terms of the Lovelace transition operators $\mathcal{U}_{\beta\alpha}^{(\pm)}(E)$, the $\mathcal{U}_{\beta\alpha}(E)$ are defined by

$$\begin{aligned}\mathcal{U}_{\beta\alpha}&=-(1-\delta_{\beta\alpha})(H_\alpha-E)+\mathcal{U}_{\beta\alpha}^{(+)}\\&=-(1-\delta_{\beta\alpha})(H_\beta-E)+\mathcal{U}_{\beta\alpha}^{(-)},\end{aligned} \quad (3.1)$$

where $H_\alpha=H_0+V_\alpha$; $\alpha,\beta=0,1,2,3$; and $V_0\equiv 0$. These operators satisfy Faddeev-like equations,

$$\mathcal{U}_{\beta\alpha}(E)=-(1-\delta_{\beta\alpha})G_0^{-1}(E)-\sum_{\gamma\neq\beta}T_\gamma(E)G_0(E)U_{\gamma\alpha}(E), \quad (3.2)$$

or

$$\mathcal{U}_{\beta\alpha}(E)=-(1-\delta_{\beta\alpha})G_0^{-1}(E)-\sum_{\gamma\neq\alpha}\mathcal{U}_{\beta\gamma}(E)G_0(E)T_\gamma(E), \quad (3.3)$$

where $G_0(E)=(H_0-E)^{-1}$ and T_γ is the two-body T matrix for the pair γ in the three-body Hilbert space. The bound-state elastic or rearrangement scattering operators are defined, as usual, by[29]

$$X_{\alpha n,\beta m}(E)=\langle\alpha n|G_0(E)\mathcal{U}_{\alpha\beta}(E)G_0(E)|\beta m\rangle, \quad (3.4)$$

[27] See Ref. 37 for a discussion of the experimental situation with respect to nucleon-nucleon scattering lengths.

[28] J. D. Jackson and J. M. Blatt, Rev. Mod. Phys. 22, 77 (1950).

[29] We are using here the same notation as in Ref. 1.

where $\langle \mathbf{p}_\alpha | \alpha n \rangle = g_{\alpha n}(\mathbf{p}_\alpha)$, and the corresponding scattering amplitudes are calculated by taking matrix elements of $X_{\alpha n,\beta m}(E)$ between plane waves representing the relative motion of the particle α (or β) and the nth (or mth) bound state of the pair $(\beta\gamma)$ [or $(\alpha\gamma)$].

In the case in which the two-body T matrices are separable, Lovelace has shown[1] that Eqs. (3.2) or (3.3) can be recast in the form of a set of coupled Lippmann-Schwinger equations for the operators $X_{\alpha n,\beta m}(E)$, with energy-dependent potentials which become complex above the three-particle threshold. Let us now assume that T_α can be written in the form

$$T_\alpha(E) = T_\alpha{}^S(E) + T_\alpha{}'(E), \tag{3.5}$$

where $T_\alpha{}'(E)$ represents the nonseparable contribution. In order to handle this piece of the two-body T matrix, AGS define a new set of operators $\mathfrak{U}_{\alpha\beta}{}'(E)$, as the solution of Eqs. (3.2) and (3.3) with only the nonseparable part of $T_\alpha(E)$:

$$\mathfrak{U}_{\alpha\beta}{}'(E) = -(1-\delta_{\alpha\beta})G_0{}^{-1}(E) - \sum_{\gamma\neq\beta} T_\gamma{}'(E)G_0(E)\mathfrak{U}_{\gamma\beta}{}'(E) \tag{3.6}$$

and

$$\mathfrak{U}_{\alpha\beta}{}'(E) = -(1-\delta_{\alpha\beta})G_0{}^{-1}(E) - \sum_{\gamma\neq\alpha} \mathfrak{U}_{\alpha\gamma}{}'(E)G_0(E)T_\gamma{}'(E). \tag{3.7}$$

After doing this, AGS proceed to show that Eqs. (2.2) and (2.3) can again be reduced to a set of multichannel Lippmann-Schwinger equations, namely,

$$X_{\alpha n,\beta m}(E) = Z_{\alpha n,\beta m}(E) - \sum_{r,\gamma\neq\alpha} Z_{\alpha n,\gamma r}(E)t_{\gamma r}(E)X_{\gamma r,\beta m}(E), \tag{3.8}$$

where $t_{\gamma n}(E)$ are the Lovelace propagators, but the potentials are different and defined by

$$Z_{\alpha n,\beta m}(E) = \langle \alpha n | G_0(E)\mathfrak{U}_{\alpha\beta}{}'(E)G_0(E) | \beta m \rangle. \tag{3.9}$$

It is clear from Eqs. (3.6) and (3.7) that if $T_\gamma{}' \equiv 0$, these potentials reduce to those defined by Lovelace. AGS suggest the use of perturbation theory to compute them. This means that one can solve Eqs. (3.6) or (3.7) by iteration and replace the result in Eq. (3.9) to obtain, to first order,

$$Z_{\alpha n,\beta m}(E) = -(1-\delta_{\alpha\beta})\langle \alpha n | G_0(E) | \beta m \rangle + \sum_{\gamma\neq\alpha,\beta} \langle \alpha n | G_0(E)T_\gamma{}'(E)G_0(E) | \beta m \rangle - \cdots. \tag{3.10}$$

However, AGS also point out that in the case in which there is a nonseparable interaction only between one pair of particles, Eqs. (3.6) and (3.7) can be trivially solved and the operators $\mathfrak{U}_{\alpha\beta}{}'(E)$ can be coumpted exactly. Let us assume, for the sake of definiteness, that $T_1{}' \neq 0$ and $T_2{}' \equiv T_3{}' \equiv 0$. In this case, Eqs. (3.6) and (3.7) imply that

$$\mathfrak{U}_{\alpha 1}{}'(E) = -(1-\delta_{\alpha 1})G_0{}^{-1}(E) \text{ for any } \alpha, \tag{3.11}$$

$$\mathfrak{U}_{1\beta}{}'(E) = -(1-\delta_{1\beta})G_0{}^{-1}(E) \text{ for any } \beta, \tag{3.12}$$

respectively. Therefore, we can summarize this result by stating that

$$\mathfrak{U}_{\alpha\beta}{}'(E) = -(1-\delta_{\beta\alpha})G_0{}^{-1}(E) \quad \text{if } \alpha=1 \text{ or } \beta=1. \tag{3.13}$$

Using again this information in Eqs. (3.6) and (3.7) we find

$$\mathfrak{U}_{\alpha\beta}{}'(E) = -(1-\delta_{\alpha\beta})G_0{}^{-1}(E) - T_1{}'(E) \quad \text{if } \alpha,\beta\neq 1. \tag{3.14}$$

Therefore, only the potentials $Z_{\alpha n,\beta m}(E)$ such that α, $\beta \neq 1$ are modified and the final *exact* result is

$$Z_{\alpha n,\beta m}(E) = -(1-\delta_{\alpha\beta})\langle \alpha n | G_0(E) | \beta m \rangle + (1-\delta_{\alpha 1}) \times (1-\delta_{\beta 1})\langle \alpha n | G_0(E)T_1{}'(E)G_0(E) | \beta m \rangle. \tag{3.15}$$

It is obvious that these results can be immediately applied to our problem, $T_1{}'(E)$ being the pure Coulomb T matrix. When we treat the three particles as identical by introducing the isospin formalism, the fact that the series solution of AGS reduces to just the first and second term is guaranteed by the isospin transformation properties of the Coulomb T matrix as given, for example, by Eq. (2.6).

In what follows we shall keep in the potentials only terms of order e^2, and therefore we replace $T_1{}'$ by $V_1{}^C$, the Coulomb potential. This is consistent with our previous approximation for the Coulomb-corrected form factor $g_C(\mathbf{p})$, where we have also retained the terms of order e^2. Therefore, all the Coulomb corrections to the potentials $Z_{\alpha n,\beta m}(E)$ will be of first order in e^2, because it can be checked immediately that the condition α, $\beta \neq 1$ in Eq. (3.15) implies that the matrix element of $G_0V_1{}^CG_0$ never has to be calculated between states that contain Coulomb-corrected form factors. Moreover, this matrix element can be computed analytically if the initial and final states contain form factors of the Yamaguchi type[25] discussed in Sec. II. Such a calculation is described in Appendix A.

Having established where we stand, we proceed with our calculation along the usual patterns. In the first place, the identity of the particles can be used to reduce the number of equations. As a result, they no longer depend on the index α that labels pairs of particles. Since we are including neither tensor nor L-S forces, the orbital and spin angular momentum decouple and we can discuss the equations in the L-S representation. The ^{3}H and ^{3}He are, in this model, bound states in the $L=0$, $S=\frac{1}{2}$ partial wave. The spin dependence of the two-body forces that we use can be taken into account in the three-body equations by means of recoupling coefficients.[1]

As far as the treatment of isospin is concerned, instead of working in a basis in which the individual isospins of the bound state and the remaining particle are diagonal, we choose to work in a basis in which the total isospin I is diagonal. Since the Coulomb interaction violates I but not I_z, the bound-state scattering operators will still be diagonal in I_z, but will have nonvanishing matrix elements between states of different I. Therefore, for a given eigenvalue of I_z, let us say M, we must expect to obtain a set of coupled equations in I. Setting $M=\frac{1}{2}$ or $(-\frac{1}{2})$, we shall obtain the equations for p-d or n-d scattering, respectively.

The eigenstates of I available for the three-nucleon system are $I=\frac{1}{2}$ and $I=\frac{3}{2}$. If a pair of nucleons are in a d state, the total isospin can only be $I=\frac{1}{2}$. However, if the pair of nucleons are in an s state, the total isospin can take both values $\frac{1}{2}$ or $\frac{3}{2}$. Therefore, it is clear that we are dealing here with a three-channel system.

Our formulation of the problem, as well as the kinematics we use, is the same as in Ref. 1. We shall label $\mathbf{q}_\alpha$ the relative momentum of the pair $(\beta\gamma)$ and the particle α in the total center-of-mass system, and $\mathbf{p}_\alpha$ the relative momentum of the particles β and γ in their own center-of-mass system. There are factors absorbed in the definition of $\mathbf{q}_\alpha$ and $\mathbf{p}_\alpha$ in such a way that the kinetic energy of three free particles is $E=\mathbf{p}_\alpha^2+\mathbf{q}_\alpha^2$. The equations for our problem are of the form

$$X_{n'I',nI}{}^M(\mathbf{q}',\mathbf{q};E)=-2Z_{n'I',nI}{}^M(\mathbf{q}',\mathbf{q};E) -2\sum_{n''I''}\int d\mathbf{q}''K_{n'I',n''I''}{}^M(\mathbf{q}',\mathbf{q}'';E) \times X_{n''I'',nI}{}^M(\mathbf{q}'',\mathbf{q};E), \quad (3.16)$$

where the channel indices (n,I) can take the values $(d,\frac{1}{2})$, $(s,\frac{1}{2})$, and $(s,\frac{3}{2})$. It only remains to specify the potentials and the kernel of Eq. (3.16). Let us define, for the sake of simplicity, the functions

$$f_{ij}(\mathbf{q}',\mathbf{q};E)=(\tfrac{4}{3})^{3/2}\frac{g_i(\mathbf{p}')g_j(\mathbf{p})}{p^2+q^2-E} \quad (3.17)$$

and

$$F_{ij}(\mathbf{q}',\mathbf{q};E)=\frac{8}{3\sqrt{3}}\frac{e^2}{\pi^2}\int d\mathbf{p}\frac{1}{[\mathbf{p}-\mathbf{k}_2]^2+\beta_i^2}\frac{1}{[\mathbf{p}-\mathbf{k}_2]^2+q'^2-E} \times\frac{1}{p^2}\frac{1}{[\mathbf{p}-\mathbf{k}_1]^2+q^2-E}\frac{1}{[\mathbf{p}-\mathbf{k}_1]^2+\beta_i^2}+\frac{4}{3\sqrt{3}}\frac{e^2}{\pi^2}\frac{1}{\tau^2}\int d\mathbf{p} \times\frac{1}{(\mathbf{p}-\frac{1}{4}\tau)^2+\beta_i^2}\frac{1}{(\mathbf{p}-\frac{1}{4}\tau)^2+-q'^2E}\frac{1}{(\mathbf{p}+\frac{1}{4}\tau)^2+\beta_j^2} \times\frac{1}{(\mathbf{p}+\frac{1}{4}\tau)^2+q^2-E}, \quad (3.18)$$

where

$$\mathbf{k}_1=(1/\sqrt{3})\mathbf{q}+(2/\sqrt{3})\mathbf{q}', \quad \mathbf{k}_2=(2/\sqrt{3})\mathbf{q}+(1/\sqrt{3})\mathbf{q}'$$

and

$$\tau=(2/\sqrt{3})(\mathbf{q}'-\mathbf{q}).$$

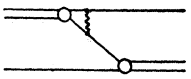

FIG. 1. Graphical representation of particle-bound-state potentials that contain a Coulomb-corrected form factor.

In Eq. (3.17) $g_i(\mathbf{p})$ are the form factors defined in Sec. II, and the indices i, j can take the values d, s, or C. In Eq. (3.18) the indices i, j can only take the values d or s. The method of computing the integrals is discussed in Appendix A, where a more explicit formula for $F_{ij}(\mathbf{q}',\mathbf{q};E)$ is given. With these definitions, it is shown in Appendix B that the potentials are given by

$$Z_{d\frac{1}{2},d\frac{1}{2}}(\mathbf{q}',\mathbf{q};E)=-\tfrac{1}{2}[f_{da}-F_{dd}], \quad (3.19)$$

$$Z_{d\frac{1}{2},s\frac{1}{2}}(\mathbf{q}',\mathbf{q};E)=-(1/\sqrt{3})f_{dC} -(1/2\sqrt{3})[f_{ds}-F_{ds}], \quad (3.20)$$

$$Z_{d\frac{1}{2},s\frac{3}{2}}(\mathbf{q}',\mathbf{q};E)=-(1/\sqrt{6})f_{dC} +(1/\sqrt{6})[f_{ds}-F_{ds}]. \quad (3.21)$$

$$Z_{s\frac{1}{2},s\frac{1}{2}}(\mathbf{q}',\mathbf{q};E)=\tfrac{1}{6}[f_{ss}-F_{ss}]-\tfrac{1}{3}[f_{sC}+f_{Cs}], \quad (3.22)$$

$$Z_{s\frac{1}{2},s\frac{3}{2}}(\mathbf{q}',\mathbf{q};E)=-\tfrac{1}{6}\sqrt{2}[f_{ss}-F_{ss}] -\tfrac{1}{6}\sqrt{2}f_{sC}+\tfrac{1}{3}\sqrt{2}f_{Cs}, \quad (3.23)$$

$$Z_{s\frac{3}{2},s\frac{3}{2}}(\mathbf{q}',\mathbf{q};E)=\tfrac{1}{3}[(f_{ss}-F_{ss})+f_{Cs}+f_{sC}]. \quad (3.24)$$

These potentials must still be multiplied by the spin recoupling coefficients. Those not listed here can be obtained without difficulty by recalling that they are symmetric under exchange of all the variables (including channel indices) in the initial and final states.

We can use Feynman-like diagrams to represent these particle-bound-state potentials. The function $f_{c,i}$ can be represented by the diagram of Fig. 1, and the function $(f_{ij}-F_{ij})$ $(i,\ j\neq c)$ by that of Fig. 2. These diagrams will be helpful when we compare our method with perturbation theory.

The calculation of the propagators is also discussed in Appendix B. Since two nucleons in an s state and a third one can be in two total isospin states, the propagator $t_s(E)$ now becomes a matrix in total isospin space:

$$t_{sI,sI'}{}^{1/2}(E) =\begin{pmatrix}\frac{2}{3}t_C(E)+\frac{1}{3}t_s(E) & \frac{1}{3}\sqrt{2}[t_C(E)-t_s(E)]\\ \frac{1}{3}\sqrt{2}[t_C(E)-t_s(E)] & \frac{1}{3}t_C(E)+\frac{2}{3}t_s(E)\end{pmatrix}. \quad (3.25)$$

The first and second row, or column, correspond to $I=\frac{1}{2}$ or $\frac{3}{2}$, respectively. The deuteron propagator remains unchanged, and is given by

$$t_{dI,dI'}{}^{1/2}(E)=\delta_{II'}\delta_{I\frac{1}{2}}t_d(E). \quad (3.26)$$

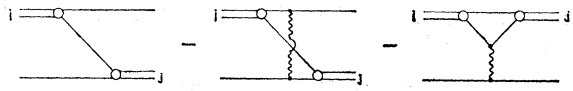

FIG. 2. Graphical representation of the nonseparable corrections to the potentials that do not contain a Coulomb-corrected form factor.

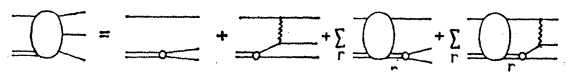

FIG. 3. Graphical representation of the form of the p-d breakup amplitude.

Finally, the kernel of Eq. (3.16) is

$$K_{n'I',nI}{}^{M}(\mathbf{q}',\mathbf{q},E) = \sum_{I''} Z_{n'I',n,I''}{}^{M}(\mathbf{q}',\mathbf{q};E)t_{nI'',nI}{}^{M}(E-q^2). \quad (3.27)$$

We have written our equations for the case $M=\frac{1}{2}$; that is to say, they are valid to discuss p-d scattering or the ^{3}He bound state. It is not necessary to write explicitly the equations for $M=-\frac{1}{2}$ because they can be obtained from the previous ones by switching off the Coulomb interaction. If this is the case, the modified form factors $g_C(\mathbf{p})$ coincide with the singlet form factors $g_s(\mathbf{p})$, and the same thing happens with the propagators $t_C(E)$ and $t_s(E)$. Therefore, the indices C and s in all our previous equations became indistinguishable, and it is evident that:

(i) Both the potentials and propagators are now diagonal in I. As a matter of fact, $t_{s,I;s,I'}{}^{1/2}(E)$ reduces to the singlet propagator times the unit matrix. Consequently, the two-channel equations for $I=\frac{1}{2}$ are decoupled from the one-channel equation for $I=\frac{3}{2}$.

(ii) The numerical coefficients that multiply the functions f_{dd}, f_{ds}, and f_{ss} are precisely the isospin recoupling coefficients quoted by Lovelace.[1]

This is a consistency check on our results.

IV. ^{3}He WAVE FUNCTION

We are interested in calculating not only the Coulomb energy of ^{3}He, but also its wave function. There are several ways of doing this, and we choose to follow a procedure suggested by Amado[23] to extract the wave function from the Lippmann-Schwinger equations for the T matrix. This method was used by Amado to calculate the ^{3}H wave function.[23]

It is a well-known fact that the off-shell matrix elements have a pole in the parameter E at the binding energy E_B of the two-particle system, and that the residues factorize in the off-shell variables,[1] namely,

$$\langle \mathbf{p}'|T(E)|\mathbf{p}\rangle \simeq \frac{g(\mathbf{p}')g(\mathbf{p})}{E+E_B} + \text{terms regular at } E=E_B. \quad (4.1)$$

The bound-state wave function is given by the product of the bound-state form factor and the free Green's function, that is,

$$\Psi(\mathbf{p}) = (p^2+E_B)^{-1}g(\mathbf{p}). \quad (4.2)$$

The same is true for the three-body problem; the bound-state scattering amplitudes $\langle \mathbf{q}'|X_{\alpha n,\beta m}(E)|\mathbf{q}\rangle$ have a pole at the binding energy of the three-body bound state, and their residues factorize in the off-shell variables.[30] However, if we calculate the residue of these amplitudes and generalize the preceding argument, we do not obtain the full three-body wave function but rather the vertices for finding the bound state as a system composed of a nucleon and a d or s state.[23] It is rather easy to convince oneself that, in order to obtain the three-body wave function, one has to calculate the residue at the pole in a breakup amplitude, namely, an amplitude in which there are three free particles in the final state.

The breakup amplitude for the process $1+(2,3)\to 1+2+3$ can be obtained in the AGS formalism[22] in terms of the bound-state scattering amplitudes given by the integral equation (3.8). Such a breakup amplitude is the matrix element $\langle \mathbf{q}_1|X_{1n,p_1'}|\mathbf{q}_1'\rangle$ of an operator defined by

$$X_{1n,\mathbf{p}_1'}(E) = -Z_{1n,\mathbf{p}_1'}(E) - \sum_{\alpha,m} X_{1n,\alpha m}(E)t_{\gamma m}(E)Z_{\gamma m,\mathbf{p}_1'}(E). \quad (4.3)$$

Notice that this is not an integral equation. In the case in which there are nonseparable interactions only between one pair of particles, let us say γ, the operators $Z_{1n,\mathbf{p}_1'}(E)$ are given *exactly* by

$$Z_{1n,\mathbf{p}_1'}(E) = -\langle \alpha n|\mathbf{p}_1'\rangle + (1-\delta_{\alpha\gamma})\langle \alpha n|G_0(E)T_\gamma'|\mathbf{p}_1'\rangle. \quad (4.4)$$

The meaning of Eq. (4.3) is exhibited in terms of Feynman-type diagrams in Fig. 3. Since we are interested in obtaining the residue of the matrix element of $X_{1n,\mathbf{p}_1'}(E)$ at $E=E_B$ (binding energy of the three-body bound state), we can drop the first term of the right-hand side of Eq. (4.3) because only the bound-state scattering amplitudes are singular at $E=E_B$.

The effect of the nonseparable interactions on the residue of the breakup amplitude is taken into account by: (a) the bound-state scattering amplitudes, because they are calculated with potentials that include the nonseparable contributions, and (b) the second term of the right-hand side of Eq. (4.4), in which the nonseparable terms produce a "final-state interaction," as is obvious from Fig. 3. This term complicates the analysis of the three-body wave function. We have carried out the analysis as far as possible by replacing T_γ' by the Coulomb potential in the final-state interaction term, but in spite of the fact that some of the integrals involved can be computed analytically, we are left with quintuple integrals to be computed numerically in the final formulas for the probabilities we want to calculate. Therefore, we have neglected the contribution of such "final-state interaction" terms, since we cannot estimate them in a reliable way. They are of the

[30] C. Lovelace, in *Strong Interactions and High-Energy Physics*, edited by R. C. Moorhouse (Oliver and Boyd, London, 1964).

same order as other contributions we have taken into account, and therefore our final numerical results should be considered only as estimates of the order of magnitude of the different probabilities we calculate.

Taking into account the identity of the particles, neglecting the "final-state interaction" terms, and introducing spin and isospin, we compute the right-hand residue of $\langle \mathbf{q}_1 | X_{1n,p_1'} | \mathbf{q}_1' \rangle$, which we call $\Gamma(\mathbf{p}_1,\mathbf{q}_1)$. The wave function is obtained by multiplying the residue by the Green's function $(p_1^2+q_1^2-E_B)^{-1}$. However, before giving our results it is convenient to recall some properties of the spin and isospin wave functions of three nucleons. We shall use here the same notation as Schiff[31] and write the doublet spin states of three nucleons in the form

$$\chi_1=\frac{1}{\sqrt{6}}[\alpha_1\alpha_2\beta_3+\alpha_1\beta_2\alpha_3-2\beta_1\alpha_2\alpha_3], \tag{4.5}$$

$$\chi_2=\frac{1}{\sqrt{2}}[\alpha_1\alpha_2\beta_3-\alpha_1\beta_2\alpha_3]. \tag{4.6}$$

These spin functions correspond to $S=\frac{1}{2}$, $S_Z=\frac{1}{2}$. Since spin is conserved in our problem, the results are independent of S_Z. The spin functions χ_1 and χ_2 correspond to coupling first the pair (2,3) in a state with $S_{23}=1$ or $S_{23}=0$, respectively. These two functions are the basis of the two-dimensional representation of the permutation group of three objects and transform under permutations into linear combinations of themselves, as discussed in Ref. 31.

We also need the isospin wave function. There are two functions corresponding to $I=\frac{1}{2}$, $I_Z=\frac{1}{2}$, which we call η_1 and η_2. They have the same structure and transformation properties under permutations as χ_1, χ_2. There is also the isospin wave function corresponding to $I=\frac{3}{2}$, $I_Z=\frac{1}{2}$, which we call $\eta_{3/2}{}^{1/2}$. Its explicit form is not needed; we must only remember that it is a totally symmetry function.

Going back to our problem, it is clear that when we solve the homogeneous integral equations with the same kernel as Eq. (3.16) at the bound-state energy $E=E_B$, we obtain the vertices for finding the ^{3}He as a d-N bound state with $I=\frac{1}{2}$ or as an S-N bound state with $I=\frac{1}{2}$ or $\frac{3}{2}$. We call this function $\Gamma_d(\frac{1}{2},q)$, $\Gamma_s(\frac{1}{2},q)$, and $\Gamma_s(\frac{3}{2},q)$, respectively. Of course, it is only the overall normalization of these three functions which is arbitrary, since they satisfy coupled integral equations.

A straightforward calculation gives the wave function as

$$\psi(\mathbf{p}_1,\mathbf{q}_1)=\tfrac{1}{3}(P_{12}+P_{23}+P_{31})[F_1(q_1,p_1)\chi_1\eta_2 +F_2(q_1,p_1)\chi_2\eta_1+G(q_1,p_1)\chi_2\eta_{3/2}{}^{1/2}], \tag{4.7}$$

where P_{ij} are permutation operators that interchange the coordinates (including spin and isospin) of particles i and j, and the functions F_1, F_2, and G are given by

$$(p^2+q^2-E_B)F_1(q,p)=\Gamma_d(\tfrac{1}{2},q)t_d(E_B-q^2)g_d(p), \tag{4.8}$$

$$3(p^2+q^2-E_B)F_2(q,p) =\Gamma_s(\tfrac{1}{2},q)[2t_C(E_B-q^2)g_C(p)+t_s(E_B-q^2)g_s(p)] +\sqrt{2}\Gamma_s(\tfrac{3}{2},q)[t_C(E_B-q^2)g_C(p)-t_s(E_B-q^2)g_s(p)], \tag{4.9}$$

$$3(p^2+q^2-E_B)G(q,p) =\sqrt{2}\Gamma_s(\tfrac{1}{2},q)[t_C(E_B-q^2)g_C(p)-t_s(E_B-q^2)g_s(p)] +\Gamma_s(\tfrac{3}{2},q)[t_C(E_B-q^2)g_C(p)+2t_s(E_B-q^2)g_s(p)]. \tag{4.10}$$

The complete antisymmetry of the wave function given by Eq. (4.7) is guaranteed by the properties of the spin and isospin wave functions and the fact that the functions F_1, F_2, G depend only on the modulus of $\mathbf{p}$ and $\mathbf{q}$. Here again we can check very easily that, if the Coulomb interaction is switched off, these results coincide with the expression written by Amado for the ^{3}H wave function.[23] Carrying out the symmetrization required by Eq. (4.13) we obtain

$$\psi(\mathbf{p}_i,\mathbf{q}_i)=\frac{1}{\sqrt{2}}u\phi_0+(\sqrt{\tfrac{3}{8}})(w_1\phi_2-w_2\phi_1) +\tfrac{1}{2}\sqrt{3}(v_1\chi_2-v_2\chi_1)\eta_{3/2}{}^{1/2}, \tag{4.11}$$

where the spin-isospin wave function ϕ_0, ϕ_1, and ϕ_2 are defined in Ref. 31 as

$$\phi_0=\frac{1}{\sqrt{2}}(\chi_2\eta_1-\chi_1\eta_2), \tag{4.12}$$

$$\phi_1=\frac{1}{\sqrt{2}}(\chi_2\eta_2-\chi_1\eta_1), \tag{4.13}$$

$$\phi_2=\frac{1}{\sqrt{2}}(\chi_2\eta_1+\chi_1\eta_2) \tag{4.14}$$

and the functions u, w_1, w_2, v_1, and v_2 can be calculated in terms of F_1, F_2, and G. Let us define

$$f_\pm(q,p)=F_1(q,p)\pm F_2(q,p). \tag{4.15}$$

Then those functions are given by the following formulas:

$$u=f_-(q_1,p_1)+f_-(q_2,p_2)+f_-(q_3,p_3), \tag{4.16}$$

$$w_1=\frac{1}{\sqrt{3}}[f_+(q_2,p_2)+f_+(q_3,p_3)-2f_+(q_1,p_1)], \tag{4.17}$$

$$w_2=f_+(q_3,p_3)-f_+(q_2,p_2). \tag{4.18}$$

The functions v_1 and v_2 depend on $G(q,p)$ in the same way as w_1, w_2 depend on $f_+(q,p)$.

The first term in the right-hand side of Eq. (4.11) represents the dominant S state of the three-nucleon system, the function u being totally symmetric. The

[31] L. I. Schiff, Phys. Rev. 133, B802 (1964).

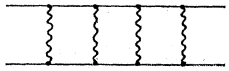

FIG. 4. Set of p-p perturbation diagrams that contribute to the pure Coulomb T matrix. Wavy crossed lines represent the Coulomb potential.

second term represents the S' state of mixed spatial symmetry, and the third term the $I=\frac{3}{2}$ component of the wave function which is present here due to the isospin symmetry breaking induced by the Coulomb interaction.

V. NUMERICAL RESULTS AND CONCLUSIONS

In order to calculate the Coulomb energy Δ_C of ^{3}He, we consider the Faddeev-Lovelace equations (3.16) and project the $l=0$ partial wave. The ^{3}H and ^{3}He binding energies are computed by searching for the zeros of the Fredholm determinants of the equations corresponding to $M=-\frac{1}{2}$ and $M=\frac{1}{2}$, respectively. Their difference yields the desired Coulomb energy Δ_C.

We have carried out two different calculations of Δ_C by choosing two different sets of parameters λ_s, β_s, λ_d, β_d to describe the two-body interactions. In the first case we have used the same parameters as Sitenko and Kharchenko[5] (SK) in their calculation of the triton binding energy. These parameters are obtained by fitting the deuteron binding energy and the scattering length in the $^3S_1$ state, and the scattering length and effective range in the $^1S_0$ state. This model overestimates the ^{3}H binding energy, which comes out to be -11.96 MeV. In the second case we have used the parameters corresponding to the Naqvi[32] potentials defined as (C_N+S_N) in Ref. 12. They are obtained by fitting the two-body data with a central plus a tensor separable interaction in the $^1S_0$ state. Only the central part of the separable potential is retained in the three-body calculation. The ^{3}H binding energy is underestimated by these potentials, which yield the value of -6.95 MeV for that quantity. In both cases the singlet scattering length takes the value $a_s=-23.7$ F.

We have used both sets of separable potentials with the same purpose in mind as Gupta and Mitra[33] in a recent perturbative calculation of Δ_C. These authors calculate Δ_C by computing the expectation value of the Coulomb potential, using exact ^{3}H wave functions given by this kind of model. As they point out, the difference of the results for the above two cases is expected to give an indication of the explicit contribution of the tensor forces to this calculation.

As far as the ^{3}He wave function is concerned, it is calculated by using the formulas developed in Sec. IV and by solving numerically the homogeneous equations corresponding to Eq. (3.16) to compute $\Gamma_d(\frac{1}{2},q)$, $\Gamma_s(\frac{1}{2},q)$, and $\Gamma_s(\frac{3}{2},q)$. This is equivalent to obtaining an eigenfunction of the kernel of Eq. (3.16) corresponding to an eigenvalue equal to 1. Most of the analysis carried out by Weinberg[34] concerning the properties of the eigenvalues and eigenvectors of the kernel of the Lippmann-Schwinger equations for real potentials can be extended to the case of complex potentials. We exploit the general properties of the eigenvalues discussed by Weinberg and solve numerically the homogeneous equations on a computer by a rapid convergent iterative method.[35]

The probabilities of the states S, S', and $I=\frac{3}{2}$, which we call P_S, $P_{S'}$, and $P_{3/2}$, respectively, can be calculated at once from Eq. (4.11) by recalling that the spin and isospin wave functions are defined in such a way that they are normalized to 1.[31] The integrals involved in the calculation of the probabilities can be reduced to a triple integral: one integral over angles and two integrals over the modulus of two independent momenta. Gaussian quadratures are used to evaluate them, with a mesh of seven points for the angular integral and 20 points for the momentum integrals. Because of our rather crude momentum mesh, the errors in the final results for the probabilities have been estimated in the order of 30%. These big errors, together with the fact that we have neglected what we called "final-state interaction" terms in the previous section, indicate that our results should be considered as estimates of the probabilities.

The results we obtained for Δ_C and $P_{3/2}$ are shown in Table I. In order to study the effect of the nonseparable T matrix on the binding energy and wave function of ^{3}He, we have recalculated Δ_C and $P_{3/2}$ by neglecting everywhere the contribution of the Coulomb T matrix to the Faddeev-Lovelace equations. It turns out that its contribution is quite small; it only accounts for a few percent of the predicted Δ_C for both sets of parameters. The estimates of $P_{3/2}$ also remain unchanged. Therefore, as far as Δ_C and $P_{3/2}$ are concerned, the main Coulomb effects are contained in the nuclear p-p T matrix.

TABLE I. Results of the calculation of the Coulomb energy Δ_C of ^{3}He and of the probability $P_{3/2}$ of the $I=\frac{3}{2}$ state. The notation for the potentials is explained in Sec. V.

Potential	Δ_C (MeV)	$P_{3/2}$(%)
SK	0.68±0.05	~0.01
C_N+S_N	0.84±0.05	~0.01

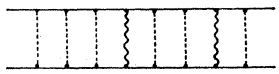

FIG. 5. Set of p-p perturbation diagrams that contribute to the "nuclear" T matrix. Dotted crossed lines represent the nuclear potential.

[32] J. H. Naqvi, Nucl. Phys. **58**, 289 (1964); A. N. Mitra and J. H. Naqvi, *ibid.* **25**, 307 (1961).
[33] V. Gupta and A. Mitra, Phys. Letters **24B**, 27 (1967).
[34] S. Weinberg, Phys. Rev. **131**, 440 (1963).
[35] H. Fanchiotti, thesis, University of La Plata, Argentina (unpublished).

These results seem to contradict one's intuition, because it can be immediately argued that, had we chosen to include the contribution of T_C in a perturbative way, we should probably have got an effect of the order of $\Delta E=(\psi_{^3\mathrm{He}},T_C\psi_{^3\mathrm{He}})$, where $\psi_{^3\mathrm{He}}$ is the wave function calculated by neglecting T_C. Then it is obvious that, since $\psi_{^3\mathrm{He}}\sim\psi_{^3\mathrm{H}}$ and, to first order in e^2, $T_C\simeq V_C$, the contribution expected from T_C is precisely the perturbation-theory result. Therefore, one is led to the conclusion that, as far as Δ_C is concerned, it should be accounted for mainly the by effects of the nonseparable Coulomb part of the p-p T matrix.

However, it is easy to convince oneself that in this case intuition fails and leads to double-counting. Let us first consider a perturbation expansion of the p-p transition matrix and make the rather trivial remark that perturbation diagrams containing only a certain number of Coulomb potentials (Fig. 4) contribute to T_C, but diagrams containing interference terms between the Coulomb and the nuclear potential (Fig. 5) contribute to the "nuclear" T matrix, that is to say, to the Coulomb-corrected form factor. If we consider now all the possible perturbation-theory diagrams for the three-nucleon system of first order in V_C, it is clear that the contribution of the class of diagrams shown in Fig. 6 is taken into account by T_C, but the contribution of the class of diagrams shown in Fig. 7 is included in the Coulomb-corrected form factors. This example shows clearly that the usual perturbation-theory contributions are shared in our approach by the Coulomb-corrected form factors and the corrections introduced by T_C, and that one cannot *a priori* decide which one, if any, will give the dominant contributions. This one is a model-dependent question; in our case it happens that the nuclear potentials are such that the corrections to the Lovelace potentials introduced by T_C are strongly depressed.

Our values for Δ_C are reasonably close to the experimental value of 0.764 MeV for the electromagnetic mass difference of ^{3}H and ^{3}He. The first and second sets of parameters slightly underestimate and overestimate such a mass, respectively. These results do not agree with the work of Gupta and Mitra,[33] in the sense that in their perturbative calculation, in addition to getting larger values for Δ_C, they found that the first set of parameters predicts a larger value than the second. We have no comments to make on this point because a close comparison of our results with perturbation theory is certainly not meaningful. We determine the potentials that appear in the Faddeev-Lovelace equations to first order in e^2, but afterwards we solve the equations numerically. Therefore, our results are *not* first-order results.

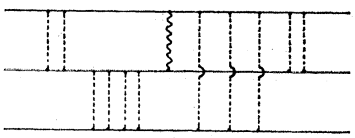

FIG. 6. Set of three-body perturbation diagrams, to first order in V_C, whose effect is taken into account by the nonseparable corrections to the bound-state potentials.

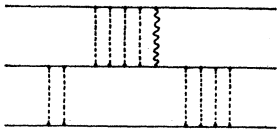

FIG. 7. Set of three-body perturbation diagrams, to first order in V_C, whose effect is taken into account by the Coulomb corrections to the two-body form factors.

It has been suggested recently that the Coulomb energy of ^{3}He may be considerably smaller than the binding energy difference of ^{3}H and ^{3}He.[36] A recent calculation that includes in the ^{3}H wave function the effect of a soft repulsive core of the nuclear forces[37] indicates that this discrepancy may be of the order of 18%. Needless to say, if this discrepancy is a real effect, it implies the existence of a charge asymmetry of the nuclear forces, in particular that the n-n interaction is stronger than the p-p interaction.[37] Moreover, it has been shown that in separable-potential models the inclusion of hard-core effects, which we have neglected, tends to reduce the binding energy of the three-nucleon system by approximately 10–15%.[19] It seems plausible to us that our results will tend to agree with those of Ref. 37 if hard-core effects in the nucleon-nucleon force are included.

We also find that for both sets of parameters $P_{S'}\sim 1\%$, in agreement with previous calculations,[11] and $P_{3/2}\sim 0.01\%$.[38] As far as $P_{3/2}$ is concerned, such a small value represents a negligible mixing of the $I=\frac{3}{2}$ state in the ^{3}He wave function. These results are not very surprising; we know that wave functions are much more sensitive to the details of the input forces than binding energies or scattering cross sections.[23] The separable potentials of the Yamaguchi type used in this model are purely attractive, and that is certainly not the case for the nucleon-nucleon system. The probability $P_{S'}$ has proved to be insensitive to the inclusion of tensor forces,[11] but the inclusion of hard-core effects can raise $P_{S'}$ up to a value of approximately 2%.[18] This is roughly the amount of S' state admixture compatible with inelastic electron scattering from ^{3}H and ^{3}He,[39] with the rate of slow neutron capture in deuterium,[40] and with the nuclear binding energy.[41] However, such an admixture of the S' state cannot account

[36] K. Okamoto, Phys. Letters **11**, 150 (1964); Progr. Theoret. Phys. (Kyoto) **34**, 326 (1965); J. N. Pappademos, Nucl. Phys. **56**, 351 (1964); D. A. Krueger and A. Goldberg, Phys. Rev. **135**, B934 (1964); L. M. Delves, *ibid.* **135**, B1316 (1964); Y. C. Tang and R. C. Herndon, Phys. Letters **18**, 42 (1965); B. K. Srivastava, Nucl. Phys. **67**, 236 (1965).

[37] K. Okamoto and M. Lucas, Nucl. Phys. **B2**, 347 (1967).

[38] See for example, T. Ohmura, Progr. Theoret. Phys. (Kyoto) **38**, 626 (1967).

[39] T. A. Griffy and R. J. Oakes, Phys. Rev. **135**, B1161 (1964).

[40] T. K. Radha and N. T. Meister, Phys. Rev. **136**, B388 (1964).

[41] J. M. Blatt and L. M. Delves, Phys. Rev. Letters **12**, 544 (1964).

for the striking difference between the charge form factors of both nuclei. Gibson and Schiff[42] consider that the most promising candidate to explain the remaining discrepancy between both the form factors is the $I=\frac{3}{2}$ state.

The fact that our model predicts such a small admixture of the $I=\frac{3}{2}$ state should not be taken as an argument against that possibility. It remains to be seen what happens to $P_{3/2}$ when more realistic two-body forces are considered. The next step is to include hard-core effects in the two-nucleon system. Work is in progress in this direction and we shall report on it in the near future. Finally, it must be emphasized that the methods we have discussed in this paper can be immediately applied to the study of p-d scattering, with the Coulomb corrections fully taken into account.

ACKNOWLEDGMENTS

We are grateful to the Laboratorio de Matematica Aplicada of the University of La Plata and to the Centro di Calcolo dell'Università di Trieste for allowing us extensive use of their computing facilities and to Dr. P. Grassberger and Dr. W. Sandhas for several valuable comments on this calculation. Two of us (V. A. A. and C. A. G.) are greatly indebted to Professor Abdus Salam and Professor P. Budini for the kind hospitality extended to us at the International Centre for Theoretical Physics, Trieste.

APPENDIX A

Here we discuss briefly the computation of one of the integrals involved in the definition (3.18); the other is much simpler and can be computed in a similar way. Consider

$$J_{ij}(\mathbf{q}',\mathbf{q};E)=\frac{8}{3\sqrt{3}}\frac{e^2}{\pi^2}\int d\mathbf{p}\frac{1}{[\mathbf{p}-\mathbf{k}_2]^2+\beta_1^2}\frac{1}{[\mathbf{p}-\mathbf{k}_2]^2+\mu_2^2}\times\frac{1}{p^2}\frac{1}{[\mathbf{p}-\mathbf{k}_1]^2+\mu_1^2}\frac{1}{[\mathbf{p}-\mathbf{k}_1]^2+\beta_i^2},\quad\text{(A1)}$$

where

$$\mathbf{k}_1=(1/\sqrt{3})\mathbf{q}+(2/\sqrt{3})\mathbf{q}',\quad \mathbf{k}_2=(2/\sqrt{3})\mathbf{q}+(1/\sqrt{3})\mathbf{q}',$$
$$\mu_1^2=q^2-s,\qquad \mu_2^2=q'^2-s.\quad\text{(A2)}$$

By making a decomposition in simple fractions, we can write (A1) in the form

$$J_{ij}(\mathbf{q}',\mathbf{q};E)=\frac{8}{3\sqrt{3}}\frac{e^2}{\pi^2}\frac{1}{(\mu_1^2-\beta_i^2)(\mu_2^2-\beta_j^2)}\times[I(\mathbf{k}_1,\beta_i;\mathbf{k}_2,\beta_j)-I(\mathbf{k}_1,\mu_1;\mathbf{k}_2,\beta_j)-I(\mathbf{k}_1,\beta_i;\mathbf{k}_2,\mu_2)+I(\mathbf{k}_1,\mu_1;\mathbf{k}_2,\mu_2)],\quad\text{(A3)}$$

where

$$I(\mathbf{k}_1,\eta_1;\mathbf{k}_2,\eta_2)=\int d\mathbf{p}\frac{1}{(\mathbf{p}-\mathbf{k}_1)^2+\eta_1^2}\frac{1}{p^2}\frac{1}{(\mathbf{p}-\mathbf{k}_2)^2+\eta_2^2}.\quad\text{(A4)}$$

An integral of this form with $1/p^2$ replaced by $1/(p^2-\lambda^2)$ was computed by Lewis.[43] We have checked that there are no difficulties in setting $\lambda^2=0$ in his result, which in our case reads

$$I(\mathbf{k}_1,\eta_1;\mathbf{k}_2,\eta_2)=\frac{\pi^2}{(\delta^2-\alpha\gamma)^{1/2}}\ln\left[\frac{\delta+(\delta^2-\alpha\gamma)^{1/2}}{\delta-(\delta^2-\alpha\gamma)^{1/2}}\right],\quad\text{(A5)}$$

where

$$\alpha\gamma=[(\mathbf{k}_1-\mathbf{k}_2)^2+(\eta_1+\eta_2)^2][k_1^2+\eta_1^2][k_2^2+\eta_2^2],$$
$$\delta=\eta_2(k_1^2+\eta_1^2)+\eta_1(k_2^2+\eta_2^2).\quad\text{(A6)}$$

From these relations we find

$$\Delta^2=\delta^2-\alpha\gamma=-[\mathbf{k}_2(\eta_1^2+k_1^2)-\mathbf{k}_1(\eta_2^2+k_2^2)]^2<0.\quad\text{(A7)}$$

Therefore, using (A7), Eq. (A5) can be rewritten in the form

$$I(\mathbf{k}_1,\eta_1;\mathbf{k}_2,\eta_2)=\frac{\pi^2}{|\Delta|}\tan^{-1}\left(\frac{2\delta|\Delta|}{\delta^2-\Delta^2}\right).\quad\text{(A8)}$$

APPENDIX B

We want to discuss here the isospin dependence of the potentials and propagators that appear in Eqs. (3.16). We shall neglect spin indices because we are working with central forces and the spin dependence of the potentials can be taken into account in the way described in Ref. 1. From Eq. (3.15) we can see that the potentials are basically matrix elements of the Green's function $G_0(E)$. If there were no isospin in the problem, the potentials called $Z_{n'n}(\mathbf{q}',\mathbf{q};E)$ would be given by matrix elements of $G_0(E)$ between an initial and a final state in which there is a particle and a bound state called n (or n') with relative momentum $\mathbf{q}$ (or $\mathbf{q}'$) in the total center-of-mass system, respectively. For equal-mass particles, such a potential is given explicitly by[1]

$$Z_{n'n}(\mathbf{q}',\mathbf{q};E)=\langle\mathbf{q}',n'|G_0(E)|\mathbf{q},n\rangle=(\tfrac{4}{3})^{3/2}\frac{g_{n'}(\mathbf{p}')g_n(\mathbf{p})}{p^2+q^2-E},\quad\text{(B1)}$$

where $\mathbf{p}=-\frac{1}{2}\sqrt{3}\mathbf{q}-\frac{2}{3}\sqrt{3}\mathbf{q}'$; $\mathbf{p}'=\frac{2}{3}\sqrt{3}\mathbf{q}+\frac{1}{2}\sqrt{3}\mathbf{q}'$ and $g_n(\mathbf{p})$ are the form factors of the bound states. The same is true in the case in which, instead of bound states, we have virtual or resonant states of two particles.

When isospin is introduced and the second term in the right-hand side of Eq. (3.15) is taken into account (with $V_\gamma{}^C$, the Born approximation, instead of $T_\gamma{}'$), we

[42] B. F. Gibson and L. I. Schiff, Phys. Rev. 138, B26 (1965).

[43] R. R. Lewis, Jr., Phys. Rev. 102, 537 (1956).

use a basis in Hilbert space in which the three isospins of the individual particles, the isospin I_n of the bound state n, the total isospin I, and its z component I_Z are diagonal. The eigenvalues of I_Z are labeled M. Therefore, the potentials we need are given by

$$Z_{n'I',nI}{}^{M}(\mathbf{q}',\mathbf{q};E)=\langle(\tfrac{1}{2}\ \tfrac{1}{2})I_{n'},\tfrac{1}{2},I',M;\mathbf{q}',n'|G_0(E)\times|\tfrac{1}{2},(\tfrac{1}{2}\ \tfrac{1}{2})I_n,I,M,\mathbf{q},n\rangle-\langle(\tfrac{1}{2}\ \tfrac{1}{2})I_{n'},\tfrac{1}{2},I',M,\mathbf{q}',n'|\times G_0(E)V_\gamma{}^{C}G_0(E)|\tfrac{1}{2},(\tfrac{1}{2}\ \tfrac{1}{2})I_n,I,M,\mathbf{q},n\rangle. \quad \text{(B2)}$$

These matrix elements can be calculated by recalling that $G_0(E)$ is the identity in isospin space. Using Eq. (B1), everything reduces to Clebsch-Gordan algebra. However, one has to be careful because we have seen in Sec. II that in the case where there is isospin symmetry breaking, the two-body T matrix, and consequently the form factors $g_n(\mathbf{p})$, depend on the z component of the isospin I_n of the two-body subsystem. We label M_n the eigenvalues of I_{nz}, and denote the form factors by $g_{M_n}(\mathbf{p})$. For the singlet state $I_n=1$ and it is clear that

$$g_1(\mathbf{p})=g_C(\mathbf{p});\quad g_0(\mathbf{p})=g_{-1}(\mathbf{p})=g_s(\mathbf{p}). \quad \text{(B3)}$$

Therefore, the potentials are given by

$$Z_{n'I',nI}{}^{M}(\mathbf{q}',\mathbf{q};E)=\sum_{M=M_{n'}+m_2}\sum_{m_3}\sum_{M=M_n+m_1} f_{M_{n'}M_n}(\mathbf{q}',\mathbf{q};E)\langle I_{n'},\tfrac{1}{2},I',M|I_{n'},M_{n'},\tfrac{1}{2},m_2\rangle\langle I_{n'}M_{n'},\tfrac{1}{2},\tfrac{1}{2}|\tfrac{1}{2},m_1,\tfrac{1}{2},m_3\rangle$$
$$\times\langle\tfrac{1}{2},m_2,\tfrac{1}{2},m_3|\tfrac{1}{2},\tfrac{1}{2},I_n,M_n\rangle\langle I_n,M_n,\tfrac{1}{2},m_1|I_n,\tfrac{1}{2},I,M\rangle-\sum_{M=M_{n'}+m_2}\sum_{m_3}\sum_{M=M_n+m_1} F_{M_{n'},M_n}(\mathbf{q}',\mathbf{q};E)\langle I_{n'},\tfrac{1}{2},I',M|I_{n'}M_{n'},\tfrac{1}{2},m_2\rangle$$
$$\times\langle I_{n'},M_{n'},\tfrac{1}{2},\tfrac{1}{2}|\tfrac{1}{2},m_1,\tfrac{1}{2},m_3\rangle\langle\tfrac{1}{2},m_1,\tfrac{1}{2},m_2|1,+1\rangle\langle1,+1|\tfrac{1}{2},m_1,\tfrac{1}{2},m_2\rangle$$
$$\times\langle\tfrac{1}{2},m_3,\tfrac{1}{2},m_2|I_n,M_n,\tfrac{1}{2},\tfrac{1}{2}\rangle\langle I_n,M_n,\tfrac{1}{2},m_1|I_n,\tfrac{1}{2},I',M\rangle, \quad \text{(B4)}$$

where the functions $f_{M_{n'},M_n}(\mathbf{q}',\mathbf{q};E)$, and $F_{M_{n'},M_n}(\mathbf{q}'\mathbf{q},E)$ are given in Eqs. (3.17) and (3.18), respectively.

It is interesting to discuss in general what happens when the Coulomb interaction is switched off. In this case, obviously, the second term of the right-hand side of Eq. (B4) vanishes, but also in this case the form factors no longer depend upon M_n, and all the functions of $\mathbf{q}'$, $\mathbf{q}$ can be factored outside the sum over Clebsch-Gordan coefficients. What remains is by definition an isospin recoupling coefficient,[1] which of course vanishes unless $I=I'$. Therefore, Eq. (A4) shows explicitly how the isospin violation in the three-body system is induced by the corresponding violation in the two-body subsystems.

As far as the propagators $t_n(E-q^2)$ are concerned, they described the propagation of a particle and a bound state labeled n. However, it is clear from our discussion in Sec. II that in the case in which there are Coulomb interactions they also depend on M_n. We denote the propagators by $t_{M_n}(E-q^2)$. When $I_n=1$, it is clear that

$$t_1(E)=t_C(E);\quad t_0(E)=t_{-1}(E)=t_s(E). \quad \text{(B5)}$$

In the basis we are using, the propagators are

$$t_{nI,nI'}{}^{M}(E)=\sum_{M_n} t_{M_n}(E)\langle I_n,\tfrac{1}{2},I,M|I_n,M_n,\tfrac{1}{2},M-M_n\rangle\times\langle I_n,M_n,\tfrac{1}{2},M-M_n|I_n,\tfrac{1}{2},I',M\rangle. \quad \text{(B6)}$$

From this equation, one can easily obtain Eq. (3.25).