be an incoherent sum of the pickup and knockout contributions. The admixture would, of course, be determined by the character of the two states involved; consequently, under appropriate circumstances, the contribution of one mechanism or the other might be negligible for the production of different final states in the same nucleus. If one ignores the failure of the "single-particle" wave function to account for the large cross section associated with production of the second excited state of C^{11}, an interpretation of the results of these DWBA analyses from this point of view leads to the following conclusions concerning the states of C^{11}: (a) The first and third excited states have a strong α-cluster character; (b) the second excited state has a dominant "single-particle" character; (c) because of different over-all structure of the α_0 angular distribution, the ground state may be of "single-particle" character with a non-negligible α-cluster admixture. Of course, this interpretation presumes that interference effects are negligible, an assumption which may be valid, but for which there is no *a priori* justification. Within the context of the present study, these ideas are audacious speculations; however, they are intriguing, and an extended theoretical and experimental investigation would be of considerable interest.

ACKNOWLEDGMENTS

We wish to thank Dr. B. T. Lucas for helpful discussions during the analysis of the experimental data. We also wish to acknowledge the assistance rendered by Professor D. J. Tendam in connection with certain accelerator-related problems encountered during the experimental phases of this study. The operation of the cyclotron by K. R. Runck and F. G. Hobaugh was greatly appreciated. We would also like to express our gratitude to H. E. Larson for his able assistance in the fabrication of the multidetector counter assembly.

PHYSICAL REVIEW VOLUME 170, NUMBER 4 20 JUNE 1968

Theoretical Fission-Antineutrino Spectrum and Cross Section of the Reaction ^{3}He$(\bar{\nu}_e,e^+)^3$H[†]

F. T. Avignone, III, S. M. Blakenship, and C. W. Darden, III
Department of Physics, University of South Carolina, Columbia, South Carolina

(Received December 29, 1967)

The energy spectrum of antineutrinos emitted by binary fission fragments of uranium-235 in secular equilibrium was calculated. The primary fission yields used in this work were calculated, using the primary fission product charge distributions due to A. C. Wahl. The total number of antineutrinos per fission predicted with the calculated spectrum is 6.06. The antineutrino spectrum derived in this work was used to calculate the average cross section for the reaction $p(\bar{\nu}_e,e^+)n$ and ^{3}He$(\bar{\nu}_e,e^+)^3$H. The cross sections are 1.09×10^{-43} cm^2/(fission $\bar{\nu}$) and 2.46×10^{-43} cm^2/(fission $\bar{\nu}$), respectively.

I. INTRODUCTION

In 1957 Lee and Yang[1] pointed out that the cross section for inverse β decay provides a further test of the two-component neutrino theory. More specifically, the cross section predicted by the two-component theory is twice as large as that predicted by the four-component theory with parity conservation. Direct measurements of the cross section for the reaction $p(\bar{\nu}_e,e^+)n$ were reported by Cowan *et al.*,[2] by Reines and Cowan,[3] and more recently by Nezrick and Reines.[4] References 3 and 4 conclude that the measured cross sections are consistent with the two-component neutrino description; however, the accuracy of the values reported in Refs. 2 and 3 is described as marginal[4] because of uncertainties in the energy spectrum of antineutrinos from the reactor. It has been suggested[4,5] that an independent experiment involving the reaction ^{3}He$(\bar{\nu}_e,e^+)^3$H would be very useful in removing some of the uncertainties in the conclusions of prior inverse β-decay experiments. The feasibility of measuring the cross section for the inverse β decay of ^{3}He is presently under study at the University of South Carolina. An important part of this study is the theoretical prediction of the cross section for this reaction which is the subject of this work.

Uncertainties in the measured cross section of the inverse β decay of ^{1}H and ^{3}He are partly due to the extremely small value of the cross section ($\sim10^{-43}$ cm^2) and partly to a lack of knowledge of the energy spectrum of incident antineutrinos. Thus, any comparison of the measured and predicted reaction rates is frustrated to the extent that the spectrum is uncertain.

[†] Work supported by the U. S. Atomic Energy Commission.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[2] C. L. Cowan, F. Reines, F. B. Harrison, H. W. Kruse, and A. D. McGuire, Science **124**, 103 (1956).

[3] Frederick Reines and Clyde L. Cowan, Jr., Phys. Rev. **113**, 273 (1959).

[4] F. A. Nezrick and F. Reines, Phys. Rev. **142**, 852 (1966).

[5] F. Reines and F. E. Kinard (private communication).

There are three methods which have been used to determine the antineutrino spectrum from fission product β decays. Muehlhause and Oleksa[6] and, independently, Carter *et al.*[7] derived antineutrino spectra from measured gross β spectra of U-235 fission products. The distribution of end-point energies was assumed to be Gaussian and parameters were adjusted to reproduce the measured β spectrum. The antineutrino spectrum was then directly calculated from the end-point distribution. In both experiments the β spectra were observed in plastic scintillators which led to uncertainties in the antineutrino spectrum for energies below about 1.8 MeV. While this would not affect the calculated cross section of the reaction $p(\bar{\nu}_e,e^+)n$ since the threshold for this reaction is 1.8 MeV, it would affect the reaction rate and cross section predicted for the ^{3}He inverse decay, because the threshold for this reaction is approximately 1.04 MeV.

Recently, Nezrick and Reines[4] reported a direct measurement of the antineutrino spectrum from a reactor. The spectrum was derived from a measurement of the energy spectrum of positrons from the reaction $p(\bar{\nu}_e,e^+)n$, where the target protons were in a liquid organic scintillator. This spectrum is in serious disagreement with the spectra of Refs. 6 and 7. In particular, the general shape has upward curvature (see Fig. 3) everywhere, whereas the curvature of those of Refs. 6 and 7 is generally downward. Furthermore, there is a significantly larger number of high-energy antineutrinos than previously reported, implying the presence of fission products with half-lives in the millisecond range.[4] Accuracy in this spectrum is not claimed for energies below 1.8 MeV, hence its use for predicting the reaction rate of the ^{3}He inverse β decay is not recommended.

The fission-antineutrino spectrum was calculated by Perkins and King[8] and was used to put limits on the average cross section of the proton inverse β decay.[9] The decay scheme information used in the calculations of Perkins and King[8] was that which was available to July 1957.

We have recalculated the fission-antineutrino spectrum for the following reasons: First, the uncertainties in the low-energy end of the experimentally derived spectra are important in the ^{3}He reaction; second, Wahl's charge distributions[10] of primary fission fragments, after fast neutron emission, appeared in the literature subsequent to the work of Perkins and King[8]; and third, a great deal more decay scheme information is available than was available at the time of the calculations of Ref. 8.

II. ENERGY SPECTRUM OF ANTINEUTRINOS FROM URANIUM-235 FISSION FRAGMENTS

The number of antineutrinos emitted in the energy interval E to $E+dE$ as a result of the binary fission, by thermal neutrons, of a single U-235 nucleus can be written

$$N(E)=\sum_j y(ZA)b_jP_j(E)\,, \qquad (1)$$

where $y(ZA)$ is the yield of a primary fission fragment of mass A and charge Z which eventually, through one or more steps, β-decays via the jth branch with a branching ratio b_j. The quantity $P_j(E)$ is the proba-

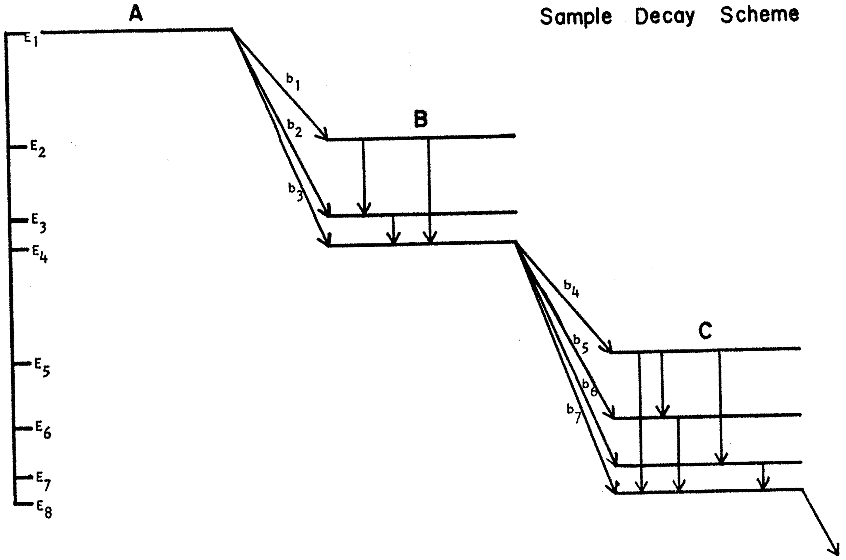

FIG. 1. Decay of hypothetical fission fragment A.

---

[6] C. O. Muelhause and S. Oleksa, Phys. Rev. **105**, 1332 (1957).
[7] R. E. Carter, F. Reines, J. J. Wagner, and M. E. Wyman, Phys. Rev. **113**, 280 (1959).
[8] J. F. Perkins and R. W. King, Nucl. Sci. Eng. **3**, 726 (1958).
[9] R. W. King and J. F. Perkins, Phys. Rev. **112**, 963 (1958).
[10] Arthur C. Wahl, J. Inorg. Nucl. Chem. **6**, 263 (1958).

bility that such a decay will result in the emission of an antineutrino in the energy interval E to $E+dE$.

Figure 1 shows two steps in the decay of a hypothetical primary fragment A. In the present work each primary fragment decay is followed to stability by reference to the *Nuclear Data Tables.*[11] The primary yields were calculated in this work, the branching ratios, when known, were taken from Ref. 11, and the probability factor $P_j(E)$ is the normalized antineutrino spectrum for each β decay, assuming an allowed Coulomb corrected shape. It should be noted that the contribution to $N(E)$ from nuclide B in Fig. 1 can be made in two ways. First, B could be a daughter of a primary fission product A. In this case, its contribution to the antineutrino spectrum at energy E' is given by

$$N(E')=y_A\{b_4P_4(E')+b_5P_5(E')+b_6P_6(E')\cdots\}.$$

Second, B might be a primary fission fragment itself, in which case its contribution is given by

$$N(E')=y_B\{b_4P_4(E')+b_5P_5(E')+b_6P_6(E')\cdots\},$$

where y_A and y_B are the primary yields of nuclides A and B, respectively. Each primary fragment was followed separately, in this fashion, from formation after fast neutron emission to stability. Since no cumulative yields were involved, the sum of all yields must be 200%. In cases where the details of each decay scheme are not known, the several unknown β-decay branches were replaced by one which decays to an assumed excited state in the daughter whose energy is decided by means of an averaging process discussed below.

A. Primary Fission Yields

The fission yields used in this calculation were computed by

$$Y(Z,A)=R(A)P(Z,A), \tag{2}$$

where $R(A)$ is the primary normalized mass yield for mass number A computed from the compilations of Zysin *et al.*[12] $P(Z,A)$ is Wahl's charge distribution[10] given by

$$P(Z,A)=\frac{1}{(0.94\pi)^{1/2}}\exp\left(-\frac{(Z-Zp)^2}{0.94}\right), \tag{3}$$

where Zp is the most probable charge for a primary fragment of mass number A. The Zp curves are also given in Ref. 10. Only nuclides with yields greater than 0.05% were included in the calculation.

B. End-Point Energies of the Unknown Nuclides

Equation (1) can be used directly only if the decay schemes are known. In such cases, the branching ratio and end-point energy of each decay is known and $P_j(E)$ can be determined. The sum of the primary yields of known nuclides is found to be 75% while that for unknown nuclides is 120%. The missing 5% is accounted for by the fission products with primary yields less than 0.05%. The contribution to the antineutrino spectrum from nuclides with unknown decay schemes was computed by assuming a β decay with a single branch and an end-point energy given by $E=\alpha Q$, where Q is the total energy available and $(1-\alpha)$ is the fraction which, on the average, is given off in γ rays. The parameter α is an average value taken from nuclei with known decay schemes and has been computed separately for even-even, odd-even, even-odd, and odd-odd nuclei. Equation (4) was used to compute α for each of the four groups of nuclei. The brackets indicate a yield-weighted average over all nuclides of the particular group.

$$\alpha=\Big(\sum_j b_jE_{0j}/Q\Big)_{\mathrm{av}}, \tag{4}$$

where E_{0j} is the β end-point energy and Q is the Q value of a particular decay. Q values for β decay, for each of the unknown nuclides, were taken from the tabulations of Seeger.[13] The values of α are 0.7988, 0.7281, 0.7932, and 0.8176 for even-even, even-odd, odd-even, and odd-odd nuclei, respectively.

The antineutrino spectrum for each fission product β decay is an independently calculated Coulomb, allowed spectrum for that end-point energy. The total spectrum is then calculated using Eq. (1). The resulting end-point energy distribution is shown in Fig. 2.

It might appear that the antineutrino spectrum would be very sensitive to uncertainties in the α's, since about 0.6 of the primary fission fragments have unknown decay schemes. This is not the case since most of the unknown primary fragments decay to nuclides with known decay schemes in one decay. It is found that more than 0.8 of all the antineutrinos result from β decays to known nuclear levels and with known branching ratios, and the spectrum is not found to be very sensitive to changes in the α's.

III. CROSS-SECTION CALCULATIONS

The average cross section in cm²/neutrino for inverse β decay by fission antineutrino is determined by

$$\bar{\sigma}=\int_T^\infty N(E)\sigma(E)dE\Big/\int_0^\infty N(E)dE, \tag{5}$$

where $N(E)$ is the number of antineutrinos of energy E per unit energy interval per fission and $\sigma(E)$ is the theoretical cross section of the inverse β decay for an incoming antineutrino energy E. The theoretical cross section can be expressed in terms of the comparative

---

[11] *Nuclear Data Tables*, edited by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.).

[12] Yu. A. Zysin, A. A. Lbov, and L. I. Selchenkov, *Fission Product Yields and Their Mass Distribution* (Consultants Bureau Enterprises, Inc., New York, 1964).

[13] P. A. Seeger, Nucl. Phys. 25, 1 (1961).

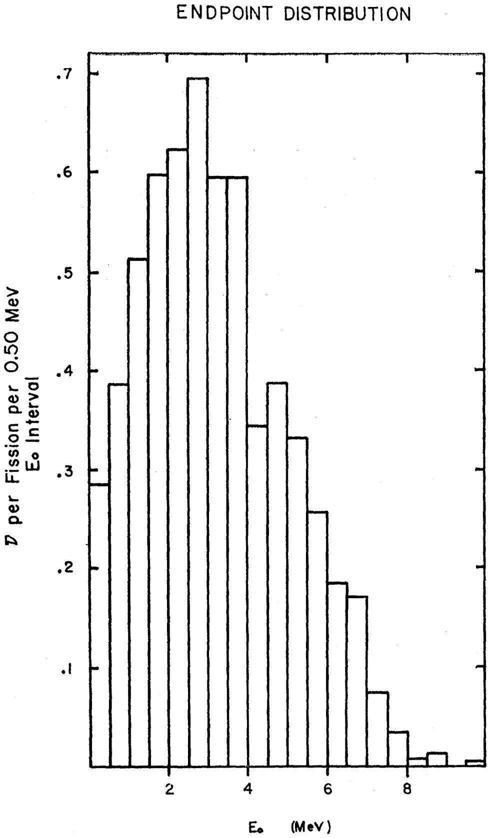

FIG. 2. β end-point energy distribution.

half-life of the associated β decay as follows[14]:

$$\sigma(E)=(2\pi^2\hbar^3/m^5c^8)(\ln 2/ft)F(ZE) \times(E-T)\{(E-T)^2-m^2c^4\}^{1/2}, \quad (6)$$

where T is threshold for the inverse decay.

The denominator of Eq. (6) is the total number of antineutrinos per fission and is 6.06 for the $N(E)$ calculated in this work. The results of this calculation yield cross sections of 1.09×10^{-43} cm$^2/\bar{\nu}$ for the reaction $p(\bar{\nu}_e,e^+)n$ and 2.46×10^{-43} cm$^2/\bar{\nu}$ for the reaction $^3\mathrm{He}(\bar{\nu}_e,e^+)^3\mathrm{H}$.

IV. DISCUSSION

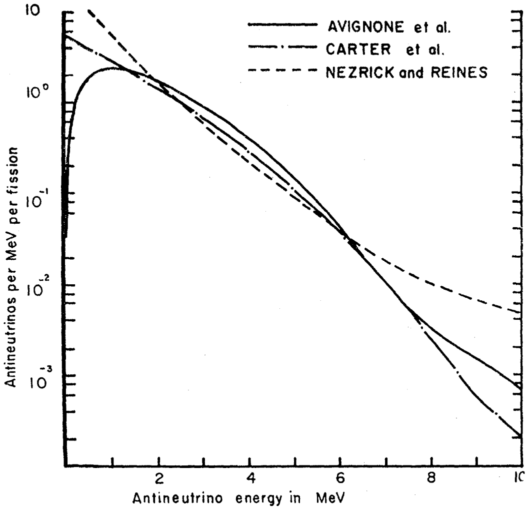

FIG. 3. Energy spectrum of antineutrinos from U-235 fission products in secular equilibrium.

It can be seen from Fig. 3 that the present calculation does not completely account for the high-energy end of the experimental fission-antineutrino spectrum of Nezrick and Reines.[4] The authors of Ref. 4 conclude that the high-energy tail implies the presence of β emitters with half-lives in the millisecond region. We have attempted to account for this tail by generating primary yields with Wahl's fission charge distributions. Although these yields have caused us to include forty-five short-lived β unstable primary fission fragments not included in the list of yields used by King and Perkins[9] we have not predicted the tail of Ref. 4. It is also found that a large increase in the α parameters used in this work does not give a high-energy tail like that of Ref. 4, but rather shifts the centroid of the spectrum slightly. The small effect of α on the spectrum is explained by the fact that nuclei of unknown decay schemes decay to nuclei with known decay schemes usually in one step. We find that the nuclei with yields less than 0.05% which account for 0.0025 of the total yield, which were not included in this calculation, have neither yields nor Q values large enough to explain the tail.

The cross section predicted in this work for the proton reaction is in good agreement with the predictions of Refs. 4, 7, and 9 and also with the measured cross section reported in Refs. 3 and 4. The value of 6.06 antineutrinos per fission is also in good agreement with the commonly accepted value of 6.1 antineutrinos per fission.

---

[14] C. S. Wu and S. A. Moskowski, *Beta Decay* (Interscience Publishers, Inc., New York, 1966).