Interfering Compound-Nucleus Resonances

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It is shown that the unitary form of the low-energy collision matrix based on R-matrix theory and Feshbach's unified theory of nuclear reactions can always be put into a well-known pole-resonance form for the elastic scattering, which contains only independent observable resonance parameters. The importance of the unitary resonance form of the collision matrix is discussed in connection with the statistical study of the fluctuations of the cross sections and the intermediate resonances.

1. INTRODUCTION

THE low-energy nuclear cross sections for the reactions which go through the formation of a compound nucleus are fitted, using the resonance-pole expansion form of the collision matrix. For isolated resonances this form, which can be obtained from any of the formal theories like the R-matrix theory[1] or Feshbach's unified theory of nuclear reactions[2,3] by assuming that the reaction passes through a well-defined compound-nucleus resonance, is just the Breit-Wigner amplitude. When the resonances start overlapping, the quantities which occur in the numerator of the resonance poles become complex, and certain relations can be found between these quantities and the complex resonance poles.[4] These relations ensure that the collision matrix remains unitary. However, in the above-mentioned formal reaction theories,[1,3] these relations are not simple, since they connect the resonance parameters of the collision matrix with the parameters which are defined in terms of the compound-nucleus Hamiltonian. The purpose of this paper is to express the unitary collision matrix based on these formal reaction theories[1,3] in the usual pole-resonance form, in which only the independent observable resonance parameters enter. The form which we obtain, starting from these formal reaction theories, has a direct relation to the compound-nucleus Hamiltonian, which is important in the study of Ericson's fluctuations[5] and intermediate resonances.[6] This point has also been emphasized recently by Feshbach.[3] This same unitary pole-resonance form can also be obtained in a simple way from a well-known result in scattering theory,[7] but then we do not have any connection of the resonance parameters of the collision matrix to the parameters of the compound-nucleus Hamiltonian. Our discussion here will be limited to purely elastic scattering. The multichannel problem will be considered elsewhere.

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2. UNITARY POLE-RESONANCE FORM OF COLLISION MATRIX U

Let us derive a common form of the collision matrix U from the formal reaction theories[1,3] when there are N interfering compound-nucleus resonances. In the R-matrix theory[1] the unitary collision matrix U for a single channel is given by

$$U=\Omega^2(1-L_0^*R_c)/(1-L_0R_c), \quad (1)$$

where Ω is a unit-modulus complex number, the logarithmic derivative L_0 of the outgoing-type wave function O in the external region is $L_0=[O'/O]_{r=a_c}-B$ (boundary-value constant), and the R_c function is given by

$$R_c=R_0+\sum_{\alpha=1}^{N}[\gamma_\alpha^2/(E_\alpha-E)]\equiv R_0+R, \quad (2)$$

where the slowly varying real parameters R_0, γ_α, and E_α are the constant background part of R_c, the width parameters, and the resonance parameters, respectively. From Eqs. (1) and (2) one obtains

$$U=u[1-2i\,\mathrm{Im}\eta\, R/(\eta R-1)], \quad (3)$$

where the new unit-modulus constant u and the parameter η are

$$u=\Omega^2(1-L_0R_0)^*/(1-L_0R_0) \text{ and } \eta=L_0/(1-L_0R_0). \quad (4)$$

With the redefinition of the parameters in Eq. (3), one finally puts U in the following form:

$$U=e^{2i\delta}\left[1-i\sum_{\alpha=1}^{N}\frac{\Gamma_\alpha}{E-E_\alpha}\times\left\{1+\tfrac{1}{2}(\omega+i)\sum_{\alpha=1}^{N}\frac{\Gamma_\alpha}{E-E_\alpha}\right\}^{-1}\right], \quad (5)$$

where

$$e^{2i\delta}=u, \quad \Gamma_\alpha=2\,\mathrm{Im}\eta, \quad \text{and} \quad \omega=\mathrm{Re}\eta/\mathrm{Im}\eta.$$

Let us sketch the derivation of the corresponding expression for Feshbach's theory from Ref. 3. There it is shown that for the case of single channel, the S matrix (which we have called U) can be put in the form

$$U=e^{2i\delta}\frac{1-\pi i\langle\psi_i^\dagger|H_{PR}(E-\mathcal{H}_{RR})^{-1}H_{RP}|\psi_i^\dagger\rangle}{1+\pi i\langle\psi_i^\dagger|H_{PR}(E-\mathcal{H}_{RR})^{-1}H_{RP}|\psi_i^\dagger\rangle}. \quad (6)$$

In Eq. (6) we have used the same notations as those used in Ref. 3. The operator $\mathcal{H}_{RR}$ has real eigenvalues E_α and the corresponding eigenfunctions χ_α. For the case of N resonances, Eq. (6) can be rewritten in the following form:

$$U=e^{2i\delta}\left[1-i\sum_{\alpha=1}^{N}\frac{\Gamma_\alpha}{E-E_\alpha}\left\{1+\tfrac{1}{2}i\sum_{\alpha=1}^{N}\frac{\Gamma_\alpha}{E-E_\alpha}\right\}^{-1}\right], \quad (7)$$

where δ is the phase shift associated with the potential scattering and the width parameter

$$\Gamma_\alpha=2\pi|\langle\chi_\alpha|H_{RP}|\psi_i^\dagger\rangle|^2.$$

From Eqs. (5) and (6) we observe that U has the same form in the R-matrix theory as well as in Feshbach's theory, except that in the latter case $\omega=0$.

The second term in the square bracket of expression (5) can be rewritten as

$$\frac{\sum_\alpha \Gamma_\alpha \prod_{\beta\neq\alpha}(E-E_\beta)}{\prod_\beta(E-E_\beta)+\frac{1}{2}(\omega+i)\sum_\alpha \Gamma_\alpha \prod_{\beta\neq\alpha}(E-E_\beta)}. \quad (8)$$

Let us write the denominator in expression (8) as

$$\prod_{\alpha=1}^{N}(E-Z_\alpha), \quad (9)$$

where $Z_\alpha=\mathcal{E}_\alpha-\frac{1}{2}iO_\alpha$ are the complex poles of the unitary collision matrix. The connection between Z_α and E_α, Γ_α, is given by

$$\prod_\alpha(E-Z_\alpha)=\prod_\alpha(E-E_\alpha)+\tfrac{1}{2}(\omega+i)\sum_\alpha \Gamma_\alpha \prod_{\beta\neq\alpha}(E-E_\beta). \quad (10)$$

Using expressions (8)–(10), we finally arrive at the desired result, which is given by

$$U=e^{2i\delta}\left[1-i\sum_{\alpha=1}^{N}\frac{G_\alpha}{E-Z_\alpha}\right], \quad (11)$$

where

$$G_\alpha=[2\,\mathrm{Im}Z_\alpha^*]\prod_{\beta\neq\alpha}^{N}(Z_\alpha-Z_\beta^*)(Z_\alpha-Z_\beta)^{-1}. \quad (12)$$

3. CONCLUSION

We would now like to point out that it is the statistical distribution of the parameters of U which are needed when we calculate the fluctuations in the cross sections[5] and the widths of the intermediate resonances.[6] Since the quantities G_α are functions of Z_β, it is sufficient to study the distribution of the complex poles Z_β using the relations given by expression (10). The distribution of the real parameters E_α, Γ_α of the compound nucleus occurring in (10) have been very well studied[8] in the past, using R-matrix theory. The statistical distribution of the parameters Z_β have now been worked out,[9] using the relations given by (10) and the known statistical distributions of E_α, Γ_α. Since relations in (10) will depend on the choice of a particular formal reaction theory, it will be interesting to see which of the formal reaction theories gives a better agreement with the analysis of the fluctuations in the elastic scattering cross sections.

We remark here that for the multichannel case, the data on cross sections have been analyzed by a number of persons, using the resonance form of the collision matrix which contains more parameters than the unitarity will allow us to have. A unitary pole-resonance form of the collision matrix for the multichannel case, similar to that given by expressions (11) and (12), is needed for the analysis of the reaction data.

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