the single-particle approximation we obtain

$$-\tfrac{1}{2}\alpha_{12}\mathbf{E}^2=-\frac{1}{2}\frac{\alpha\mathbf{E}^2}{\Delta W}(\tfrac{4}{5}\mu_N\mu_0\langle r^{-3}\rangle). \quad \text{(A3)}$$

Equation (A3) can be related to the experimentally observed magnetic-hyperfine-structure constant in the $p_{3/2}$ state by means of the well-known relation

$$ha_{3/2}=-\frac{\mu_0\mu_N}{I}\frac{16}{15}\langle r^{-3}\rangle \quad \text{(A4)}$$

to give

$$\alpha_{12}=-\tfrac{3}{4}\alpha hI(a_{3/2}/\Delta W). \quad \text{(A5)}$$

The quadrupole contribution can be evaluated in a very similar way. Equation (2.10) of Ref. 5 takes the form

$$-\tfrac{1}{2}\alpha_{02}\mathbf{E}^2=\frac{\langle s|-\boldsymbol{\mu}\cdot\mathbf{E}|p\rangle^2}{\Delta W^2}\langle p|\sum_i \tfrac{1}{2}\mathbf{Q}\cdot\sum_i\left(\frac{\mathbf{C}^2}{r^3}\right)_i|p\rangle, \quad \text{(A6)}$$

where the operators are the standard quadrupole operators. Simple angular momentum arguments immediately relate this to the quadrupole constant $b_{3/2}$ in the $p_{3/2}$ state and we obtain

$$\alpha_{02}=\tfrac{1}{2}\alpha(hb_{3/2}/\Delta W). \quad \text{(A7)}$$

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Annihilation of Positrons from Positronium Negative Ion $e^-e^+e^-$

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Annihilation of positrons from the positronium negative ion is calculated by a Schroeder six-parameter wave function. The calculation yields a lifetime against two- and three-photon annihilation of 5.02×10^{-10} and 5.66×10^{-7} sec, respectively. The first value is considerably better than the one given by Ferrell. It is found that the value of the lifetime against two-photon annihilation falls close to the values observed by Bell and Jørgensen in Cs and K, and by Weisberg and Berko in Rb, i.e., in low-density metals. This fact is considered as partial evidence that in such media the positronium ion may be formed. For the two-photon annihilation, a plot of calculated angular correlation of emitted photons and of the momentum distribution function is reported. The angular-correlation function shows the maximum at $\pi-\vartheta$, with $\vartheta=2.0\times10^{-4}$ rad, the departure from π being due to the presence of the spectator electron.

I. INTRODUCTION

In recent years positron annihilation experiments have been extensively used as a tool to obtain information about the electronic structure of solids. The experiments yielding the most information are (i) the measurement of the two-photon angular distribution and (ii) the measurement of the time distribution of two-photon annihilation events.[1]

Positrons injected into a solid from a radioactive source slow down rapidly to thermal energy, prior to annihilation with the electrons of the solid. Slowed-down positrons may annihilate from a free state or, in some cases, from bound states, such as, e.g., positronium e^-e^+. Each possible positron bound state gives, as a rule, a different lifetime against annihilation. In this connection it is useful to investigate theoretically all possible bound states to which the positron may give rise, with the aim of comparing the results with experimental data. As a matter of fact, many experimental data are available,[2] which could be interpreted on the basis of the assumption that positron bound states besides the well-known positronium are formed, such as H^-e^+, Cl^-e^+, Lie^+, and so on. A number of theoretical papers have been devoted to this topic.[3–8]

Here we are concerned with the system of two electrons bound to a positron, the so-called "positronium negative ion," which was first predicted theoretically by

[1] For early review and references, see P. R. Wallace, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1960), Vol. 10, p. 1. For up-to-date review articles, see *Proceedings of the Conference on Positron Annihilation* (Academic Press Inc., New York, 1966).

[2] See, e.g., R. E. Bell and M. H. Jørgensen, Can. J. Phys. **38**, 652 (1960); A. T. Stewart and R. H. March, Phys. Rev. **122**, 75 (1961); A. Bisi, A. Fiorentini, and L. Zappa, *ibid.* **131**, 1023 (1963).

[3] S. Neamtan, G. Darewych, and G. Oczkowski, Phys. Rev. **126**, 193 (1962).

[4] V. I. Gol'danskii, A. V. Ivanova, and E. P. Prokop'ev, Zh. Eksperim. i Teor. Fiz. **47**, 659 (1964) [English transl.: Soviet Phys.—JETP **20**, 440 (1965)].

[5] V. I. Gol'danskii and E. P. Prokop'ev, Fiz. Tverd. Tela **6**, 3301 (1964). [English transl.: Soviet Phys.—Solid State **6**, 2641 (1965)].

[6] S. M. Neamtan and R. I. Verrall, Phys. Rev. **134**, A1254 (1964).

[7] W. Brandt, L. Eder, and S. Lundqvist, Phys. Rev. **142**, 165 (1966).

[8] Dinh Van Hoang, Zh. Eksperim. i Teor. Fiz. **49**, 630 (1965) [English transl.: Soviet Phys.—JETP **22**, 437 (1966)].

Wheeler.[9] It has been shown by various authors[9–12] that this system possesses dynamical stability and that its binding energy is 0.326 eV, against dissociation into positronium and a free electron and 7.129 eV against dissociation into three particles.[11] On the other hand, no attempt to evaluate the formation probability or the annihilation properties of this bound system, seems to be given in the literature,[13] except for some considerations in Ferrell's paper.[14]

The aim of the present paper is to obtain the lifetime against two- and three-photon annihilation and the angular-correlation function of the annihilation radiation for comparison with experiment. In a future paper we plan to treat other aspects of the problem.

II. ANNIHILATION PROCESS

The formulas which permit the calculation of the annihilation probability of positrons and electrons, including bound states, and of the angular distribution of emitted photons have been given by deBenedetti *et al.*[15] and by Ferrell[14] for the case when the electron-positron wave function is separable. The generalization of the problem to the case of an arbitrary configuration has been done by Lee.[16] For the present problem, however, we employ the scheme derived by Neamtan *et al.*[3] for the case of annihilation of positrons from H^-e^+, which is an extension of the procedure proposed by Jauch and Rohrlich for positronium annihilation.[17]

We consider the two- and three-photon annihilation of the positrons from the positronium negative ion $e^-e^+e^-$. The positronium ion has only a ground state.[10] The two electrons, therefore, must have antiparallel spins, the total spin of the system being equal to the positron spin $\mathbf{S}=\mathbf{s}_++\mathbf{s}_1+\mathbf{s}_2=\mathbf{s}_+$ (Fig. 1). As in the positronium ion there are spin-up and spin-down electrons, the positron may annihilate with an electron having spin either parallel or antiparallel to its own. In the first case, according to the general theory, the pair may annihilate only into an odd number of photons; in the second case, only into an even number. Furthermore, as the positronium ion is a bound three-body system, one-photon annihilation is not forbidden, though it is much less probable than the two- or the three-photon annihilation.

According to Jauch and Rohrlich,[17] for the transition

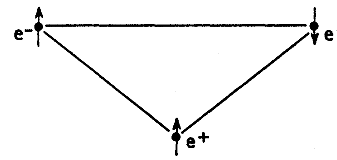

FIG. 1. Spin positions of particles in the positronium negative ion (s state).

probability amplitude for positronium-ion annihilation, we may take the free-pair annihilation amplitude integrated over the momentum distribution of the electrons and the positron in the positronium ion:

$$M=\left\{\sum_{m_+,m_1}\int M(-\mathbf{p}_1-\mathbf{p}_2,\,\mathbf{p}_1;\,m_+,\,m_1)\delta(\mathbf{p}+\mathbf{p}_2)+\sum_{m_+,m_2}\int M(-\mathbf{p}_1-\mathbf{p}_2,\,\mathbf{p}_2;\,m_+,\,m_2)\delta(\mathbf{p}+\mathbf{p}_1)\right\}\times\varphi(-\mathbf{p}_1-\mathbf{p}_2,\,\mathbf{p}_1,\,\mathbf{p}_2;\,m_+,\,m_1,\,m_2)d^3p_1d^3p_2,\quad(1)$$

where we have restricted ourselves to the center-of-momentum system ($\mathbf{P}=\mathbf{p}_++\mathbf{p}_1+\mathbf{p}_2=0$).

In the above, $M(\mathbf{p}_i,\mathbf{p}_j;\,m_i,m_j)$ in the lowest order stands for the matrix element of the free-pair annihilation (i) into two photons in the case $e^-e^+e^-\rightarrow\gamma+\gamma+e^-$ and (ii) into three photons in the case $e^-e^+e^-\rightarrow\gamma+\gamma+\gamma+e^-$; here $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_+$, and m_1, m_2, m_+ are the momenta and spin orientations of the two electrons and the positron, respectively. $\mathbf{p}=\mathbf{k}_1+\mathbf{k}_2$ (or $\mathbf{p}=\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3$) is the total momentum of the emitted photons. $\varphi(-\mathbf{p}_1-\mathbf{p}_2,\,\mathbf{p}_1,\,\mathbf{p}_2;\,m_+,m_1,m_2)$ represents the Schrödinger wave function of the positronium ion in the momentum representation[18]:

$$\varphi(\mathbf{p}_+,\mathbf{p}_1,\mathbf{p}_2;\,m_+,m_1,m_2)=N(2\pi)^{-9/2}\int\varphi(\mathbf{x}_+,\mathbf{x}_1,\mathbf{x}_2;\,m_+,m_1,m_2)\times\exp[-i(\mathbf{p}_+\cdot\mathbf{x}_++\mathbf{p}_1\cdot\mathbf{x}_1+\mathbf{p}_2\cdot\mathbf{x}_2)]d^3x_+d^3x_1d^3x_2,$$

where N is the normalization constant.

Since the particle momenta involved are small compared to the photon momenta, one may expand $M(\mathbf{p}_i,\mathbf{p}_j;\,m_im_j)$ in powers of p_i/m and p_j/m, where m is the electron mass, and retain only the leading zero-momentum term. Therefore, we can write

$$M=\left\{\sum_{m_+,m_1}M_{m_+,m_1}(0,0)\int\delta(\mathbf{p}+\mathbf{p}_2)+\sum_{m_+,m_2}M_{m_+,m_2}(0,0)\int\delta(\mathbf{p}+\mathbf{p}_1)\right\}\times\varphi(-\mathbf{p}_1-\mathbf{p}_2;\,\mathbf{p}_1,\,\mathbf{p}_2;\,m_+,\,m_1,\,m_2)d^3p_1d^3p_2.\quad(2)$$

[9] J. A. Wheeler, Ann. N. Y. Acad. Sci. **48**, 219 (1946).
[10] E. A. Hylleraas, Phys. Rev. **71**, 491 (1947).
[11] W. Kolos, C. C. J. Roothaan, and R. A. Sack, Rev. Mod. Phys. **32**, 178 (1960).
[12] U. Schroeder, Z. Physik **173**, 221 (1963).
[13] Calculations on some properties of $e^-e^+e^-$ can be found in the author's M. Sc. thesis, Moscow State University (unpublished).
[14] R. A. Ferrell, Rev. Mod. Phys. **28**, 308 (1956).
[15] S. deBenedetti, C. E. Cowan, W. R. Konneker, and H. Primakoff, Phys. Rev. **77**, 205 (1950).
[16] Chang Lee, Zh. Eksperim. i Teor. Fiz. **33**, 365 (1957) [English transl.: Soviet Phys.—JETP **6**, 281 (1958)].
[17] J. M. Jauch and R. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1959).

[18] We set $\hbar=1$.

By proceeding in a similar way to that of Neamtan *et al.*,[3] after a certain amount of manipulation, we are led to the expression

$$M=NA(m_+,m)\int d^3x_1\,d^3x_2[\varphi(\mathbf{x}_2,\mathbf{x}_2,\mathbf{x}_1)+\varphi(\mathbf{x}_2,\mathbf{x}_1,\mathbf{x}_2)]$$

$$\times\exp(-i\mathbf{p}\cdot\mathbf{x}_2)[(2\pi)^{-3/2}\exp(i\mathbf{p}'\cdot\mathbf{x}_1)\chi_{(1/2)m_1}(m_1)], \quad (3)$$

where $A(m_+,m)=\sum_{m+,m} M_{m_+,m}(0,0)\chi_{00}(m_+,m)$. $\chi_{sm}(m)$ stands for the spin function. The momenta are connected by the relation $\mathbf{P}=\mathbf{p}+\mathbf{p}'=0$.

It is now necessary to integrate the square of the matrix element over all the final states and sum over the spin positions of the remaining electron. As a result we obtain

$$J(\mathbf{p})=\sum_{m_1}\int|M|^2d^3p'=N^2|A(m_+,m)|^2$$

$$\times\int d^3x_1d^3x_2d^3x_1'd^3x_2'\exp[i\mathbf{p}\cdot(\mathbf{x}_2'-\mathbf{x}_2)]$$

$$\times[\varphi(\mathbf{x}_2'\mathbf{x}_2'\mathbf{x}_1')+\varphi(\mathbf{x}_2'\mathbf{x}_1'\mathbf{x}_2')]^*$$

$$\times[\varphi(\mathbf{x}_2\mathbf{x}_2\mathbf{x}_1)+\varphi(\mathbf{x}_2\mathbf{x}_1\mathbf{x}_2)]\delta(\mathbf{x}_1'-\mathbf{x}_1)\delta_{m_1m_1'}.$$

Integration over $\mathbf{x}_1'$ and replacement of $\mathbf{x}_2'$ by $\mathbf{x}_3$ gives

$$J(\mathbf{p})=N^2|A(m_+,m)|^2\int d^3x_1d^3x_2d^3x_3$$

$$\times\exp[i\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_2)][\varphi(\mathbf{x}_3\mathbf{x}_3\mathbf{x}_1)+\varphi(\mathbf{x}_3\mathbf{x}_1\mathbf{x}_3)]^*$$

$$\times[\varphi(\mathbf{x}_2\mathbf{x}_2\mathbf{x}_1)+\varphi(\mathbf{x}_1\mathbf{x}_2\mathbf{x}_2)]. \quad (4)$$

III. ANNIHILATION RATE

A. Two-Photon Annihilation

The two-photon annihilation rate follows from (4) if we integrate over all values of $\mathbf{p}$:

$$\Gamma=\int J(\mathbf{p})d^3p=2^{-1}\Gamma_{\mathrm{pos}}\frac{\rho(e^-e^+e^-)}{\rho(e^-e^+)},$$

where

$$\Gamma_{\mathrm{pos}}=(2\pi)^3|A'(m_+,m)|^2\rho(e^-e^+)$$

is the known two-photon annihilation rate for positronium from the ground state; we have set $A(m_+,m)=(1/\sqrt{2})A'(m_+,m)$ in order to obtain the proper expression for Γ_{pos}. The quantity $\rho(e^-e^+)=0.0398a_0^{-3}$ is the density of the electron at the position of the positron in the ground state of positronium, a_0 being the Bohr radius. The corresponding electron density in the positronium ion is

$$\rho(e^-e^+e^-)=N^2\int d^3x_1d^3x_2|\varphi(\mathbf{x}_2\mathbf{x}_2\mathbf{x}_1)+\varphi(\mathbf{x}_2\mathbf{x}_1\mathbf{x}_2)|^2.$$

To calculate $\rho(e^-e^+e^-)$ and N we use the Schroeder six-parameter variational wave function[12]

$$\varphi(\mathbf{x}_+,\mathbf{x}_1,\mathbf{x}_2)=f(|\mathbf{x}_1-\mathbf{x}_2|/a_0)g(|\mathbf{x}_1-\mathbf{x}_+|/a_0,|\mathbf{x}_2-\mathbf{x}_+|/a_0), \quad (5)$$

with

$$f(x)=x^ne^{-\alpha x}+be^{-\eta x},$$

$$x=|\mathbf{x}_1-\mathbf{x}_2|/a_0,$$

$$g(y,z)=e^{-1/2\epsilon_0(y+z)}\{e^{-1/2\beta(y-z)}+e^{1/2\beta(y-z)}\},$$

where

$$y=|\mathbf{x}_1-\mathbf{x}_+|/a_0,\quad z=|\mathbf{x}_2-\mathbf{x}_+|/a_0,$$

and

$$n=2.5,\quad \alpha=0.3817,\quad b=12.22,\quad \eta=0.0028,$$
$$\epsilon_0=0.6693,\quad \beta=0.3734.$$

After a straightforward but rather tedious integration to calculate $\rho(e^-e^+e^-)$ and N and using $\tau_{\mathrm{pos}}=1.25\times10^{-10}$ sec, we find, for two-photon annihilation, $\tau=5.02\times10^{-10}$ sec and $\Gamma=1.99\times10^9$ sec^{-1}.

We first note that the value of the annihilation rate for the positronium ion is nearly equal to that of the spin-averaged neutral positronium rate ($\Gamma_{\mathrm{pos}}=2.00\times10^9$ sec^{-1}). Second, we note that the value of the lifetime lies below but near the experimental values observed by Bell and Jørgensen[2] for Cs and K (4.3 ± 0.2 and $4.0\pm0.2\times10^{-10}$ sec) and recently by Weisberg and Berko[19] for Rb ($4.06\pm0.1\times10^{-10}$ sec), i.e., for low-electron-density metals.[20] To date, the theoretical calculations do not fit the experimental values for the above metals. We believe, although caution is called for, that these metals may provide media in which the positronium negative ion may be formed.

Further, up to the appearance of Weisberg and Berko's work, we were tempted to assign to the positronium-ion decay the anomalously long lifetime (of about 5.0×10^{-10} sec) corresponding to 5% of the annihilation events observed by Bell and Jørgensen in the metals investigated. In fact this value almost precisely coincides with the one calculated in the present work, and seems to be independent of the medium. But in their work, Weisberg and Berko assert that this second lifetime is an artifact of the sample preparation.[21] Clearly, further experimental and theoretical work is needed to explain this anomaly in a conclusive way. For the time being, we confine ourselves to our first suggestion, namely, that in such media as Cs, Rb, and K the positronium negative ion may be formed.

In order to check our result, we have recalculated the

[19] H. Weisberg and S. Berko, Phys. Rev. **154**, 249 (1967).

[20] To characterize the metal electron density, one commonly uses the dimensionless parameter r_s, giving the radius of a sphere, in units of the Bohr radius, containing on the average one valence electron. In real metals one has $1.8\lesssim r_s\lesssim5.6$. For K, Rb, and Cs, r_s ranges approximately from 4.9 to 5.6.

[21] On this point, Professor Bell is of the opinion that at this stage the long lifetime has not been completely ruled out, although he personally hopes that it will disappear.

lifetime of the positronium ion using the same Hylleraas two-parameter wave function[10], used by Ferrell, and have obtained $\tau=5.12\times10^{-10}$ sec and $\Gamma=1.95\times10^{9}$ sec^{-1}, which almost precisely coincides with the values found with the more refined wave function used in the present work. The results given by Ferrell ($\tau=3.27\times10^{-10}$ sec and $\Gamma=3.06\times10^{9}$ sec^{-1}) are recovered by dropping the c_2^2 terms in the expression to which the Hylleraas wave function leads:

$$\tau=1/\Gamma=(2\times8\times3.10)^{-1}\times10^{-9}$$
$$\times\left\{(4+48c_2+576c_2^2)\bigg/\left[\frac{11+156c_2+1992c_2^2}{2(8+96c_2+1920c_2^2)}\right]^3\right\},$$

where $c_2=0.05$.

Thus, it is not legitimate to neglect the quadratic terms in the variational parameter, as that leads to an error of about 50% in the results. At this point it is interesting to note that the electron density at the positron in the negative ion turns out to be about the same as for the neutral atom [$0.99\rho(e^-e^+)$ in an exact calculation, compared with $1.53\rho(e^-e^+)$ in Ferrell's estimate], a result which is surprising at first sight. It turns out that in the positronium negative ion the effect of screening is such as to just cancel out the additional density that one would expect from the second electron. Ferrell[22] has pointed out that the cancellation is not entirely accidental, and that there is a good physical reason for expecting it to happen. He explains that this can be seen from the dual-screening form of the trial wave function that Hylleraas first used to represent radial correlation; i.e., the variational function is taken to be the symmetrized product of two hydrogenic ground-state wave functions, one bound strongly and the other weakly. Computation then yields (i) an electron density quite accurately equal to that which would correspond to the more tightly bound inner electron alone, and (ii) a binding of the inner electron about the same as in the neutral atom.

B. Three-Photon Annihilation

Proceeding in a similar way to that for two-photon annihilation, and using $\tau_{\text{pos}}=1.4\times10^{-7}$ sec and $\Gamma_{\text{pos}}=0.71\times10^{7}$ sec^{-1}, for the three-photon annihilation we find $\Gamma=1.76\times10^{7}$ sec^{-1} and $\tau=5.66\times10^{-7}$ sec.

IV. TWO-PHOTON ANGULAR CORRELATION

For the calculation of the angular correlation between the two emerging annihilation photons we need to integrate (4) over $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$. As a result we obtain an expression which depends on $\mathbf{p}=\mathbf{k}_1+\mathbf{k}_2$, the total momentum of the two photons. Since the wave function (5) is invariant under rotation, the expression (4) will be independent of the direction of $\mathbf{p}$ and can be replaced by its integral over solid angle in the momentum space divided by 4π. By omitting unimportant multiplicative factors we have

$$\gamma(p)=\frac{1}{4\pi}\int J(p)d\Omega$$
$$=\int d^3x_1d^3x_2d^3x_3\frac{\sin p|\mathbf{x}_2-\mathbf{x}_3|}{p|\mathbf{x}_2-\mathbf{x}_3|}$$
$$\times[\varphi(\mathbf{x}_3\mathbf{x}_3\mathbf{x}_1)+\varphi(\mathbf{x}_3\mathbf{x}_1\mathbf{x}_3)]^*$$
$$\times[\varphi(\mathbf{x}_2\mathbf{x}_2\mathbf{x}_1)+\varphi(\mathbf{x}_2\mathbf{x}_1\mathbf{x}_2)]. \quad (6)$$

On substituting the wave function (5) into Eq. (6) and carrying out the integration, we are led to the expression

$$\gamma(p)=\left\{\frac{\Gamma(4.5)}{p(L^2+p^2)^{2.25}}\sin[(4.5)\arctan(p/L)]\right.$$
$$+\frac{\Gamma(4.5)}{p(M^2+p^2)^{2.25}}\sin[(4.5)\arctan(p/M)]$$
$$\left.+\frac{2bN}{(p^2+N^2)^2}+\frac{2bP}{(p^2+P^2)^2}\right\}^2, \quad (7)$$

where

$$L=\alpha+\tfrac{1}{2}\epsilon_0+\tfrac{1}{2}\beta=0.90305,$$
$$M=\alpha+\tfrac{1}{2}\epsilon_0-\tfrac{1}{2}\beta=0.52965,$$
$$N=\eta+\tfrac{1}{2}\epsilon_0+\tfrac{1}{2}\beta=0.52415,$$
$$P=\eta+\tfrac{1}{2}\epsilon_0-\tfrac{1}{2}\beta=0.15075,$$

and $\Gamma(x)$ is the gamma function.

Figure 2(a) shows a plot of the angular-correlation function of the emitted photons (7) versus $(p/mc)\times10^{-3}=\vartheta$ (mrad), and Fig. 2(b) shows a plot of the momentum distribution function $N(p)=4\pi p^2\gamma(p)$.

V. CONCLUSIONS

In metals with sufficiently low electron density a thermalized positron may correlate with more than one electron; we think that in such cases there is a real possibility for a positron to form a positronium negative ion before annihilating. On going to metals with greater electron density, this possibility vanishes, because the presence of more electrons increases the probability of collision by positrons, hampering the formation of the bound state. We may state this in another way. We recall that the positronium ion has relatively large spatial dimensions. The average interparticle distances in this system are[11]

$$\langle|\mathbf{x}_1-\mathbf{x}_2|\rangle=8.58a_0, \quad \langle|\mathbf{x}_1-\mathbf{x}_+|\rangle=\langle|\mathbf{x}_2-\mathbf{x}_+|\rangle=5.506a_0.$$

This implies that in a given medium, in order that the

[22] R. A. Ferrell (private communication).

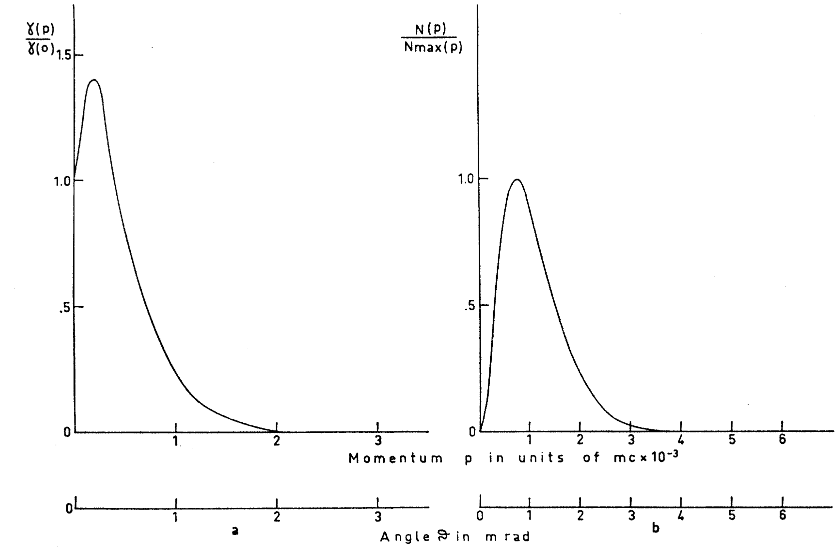

FIG. 2. (a) Curve for the angular correlation of the pair of the emitted photons. In the case of positronium we have a delta function $\delta(\mathbf{p})=\delta(\mathbf{k}_1+\mathbf{k}_2)$, corresponding to the fact that the angle between the directions of the emitted photons is π. The presence of a spectator electron in $e^-e^+e^-$, which may carry off part of the total momentum, causes the most probable angle between annihilation photons to be different from π by an angle ϑ of about 2×10^{-4} rad. Thus ϑ measures the departure from π of the angle between the directions of the two annihilation photons. (b) Momentum distribution curve.

positronium ion can exist as a separate, well-defined entity, available space comparable to the positronium-ion dimensions is needed. The gases, for example, provide such media. Among the real metals, probably, Cs, Rb, and K also have room enough to form the positronium ion.

ACKNOWLEDGMENTS

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L_2-Subshell Yield Measurements in Pu^{240}, U^{236}, and U^{234}

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The fluorescence, Auger, and Coster-Kronig yields of the L_2 subshells in Pu^{240}, U^{236}, and U^{234} have been determined from measurments on L x-rays emitted following α decay of Cm^{244}, Pu^{240}, and Pu^{238}. The techniques employed include α–L x-ray coincidence counting using a silicon detector and NaI(Tl) crystal, proportional-counter spectrometry and, in the case of Pu^{238}, the use of a curved-crystal spectrograph. The following results were obtained: Pu^{240}, $\omega_2=0.466\pm0.023$, $a_2=0.11\pm0.08$, $f_{23}=0.42\pm0.08$; U^{236}, $\omega_2=0.535\pm0.042$, $a_2=0.09\pm0.11$, $f_{23}=0.37\pm0.07$; U^{234}, $\omega_2=0.497\pm0.035$, $a_2=0.07\pm0.07$, $f_{23}=0.43\pm0.06$. The measured fluorescence yields show close agreement with those calculated on a semiempirical basis by Listengarten, but the Auger and Coster-Kronig yields are, respectively, much smaller and much larger than predicted.

1. INTRODUCTION

ALTHOUGH the Auger effect plays a fundamental role in the reorganization of the atomic electrons following inner-shell ionization of the atom, it was for many years neglected as a subject for quantitative study. Interest in the Auger effect has however been stimulated by the increasing study of processes, such as electron capture and internal conversion, which involve the interaction of the nucleus with the orbital electrons. Since these processes lead to inner-shell ionization, measurements of the subsequent x-ray or Auger electron emission may yield important information about the initial nuclear process.

One important measure of the mechanism of de-excitation of the ith atomic shell is the fluorescence yield ω_i, which is defined as the number of characteristic x rays emitted per primary vacancy in the shell. The corresponding Auger yield a_i is defined as the

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