10^{34} while the right-hand side is less than 10^{30}. In fact the right-hand side of Eq. (A1) can be neglected compared to the left-hand side until the k values are approximately 10^6 cm^{-1}. Thus in the vicinity of $H_i=\omega/\gamma$ Eq. (2a) is a valid approximation. For the acoustic mode near $H_i=\omega/\gamma$, Eq. (A1) yields approximately

$$(k^2-k_{03}^2)/k_{03}^2=a^2/k_{03}^2.$$

Since $|a^2/k_{03}^2|\approx 0.25\times10^{-2}$, it is permissible to let $k^2=k_{03}^2$ in the vicinity of $H_i=\omega/\gamma$. This gives Eq. (2b).

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Effects of Impurities on Spin Fluctuations in Almost Ferromagnetic Metals

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We investigate the effects of the mean free path on the spin susceptibility of almost ferromagnetic metals. Using these results, we calculate the low-temperature specific-heat contribution of the spin fluctuations. While the term linear in temperature T is unaffected by the mean free path, we find that the $T^3 \ln T$ contribution is rapidly modified by impurities, in contrast to the phonon case.

I. INTRODUCTION

SPIN fluctuations in almost ferromagnetic fermion systems have recently received very much attention. Such systems have an appreciably enhanced static spin susceptibility, indicating that the exchange interaction is important but not quite strong enough to lead to a ferromagnetic instability. For example, it was shown by Berk and Schrieffer[1] that in metals like Pd spin fluctuations lead to a change in the effective mass and can suppress superconductivity. Furthermore, Doniach and Engelsberg[2] made the important observation that in liquid He^3 spin fluctuations are responsible for the deviations of the temperature dependence of the specific heat from a Sommerfeld law. In Ref. 1 as well as in Ref. 2 it was furthermore shown that one can calculate the self-energy of the electrons due to exchange scattering in a simple fashion by formally introducing a propagator[3] for the paramagnon or spin fluctuations. There are several common features between the self-energy due to paramagnons and the one due to phonons.

However, as we show here, these similarities hold only in the limit of infinite mean free path, which is the only case for which exchange-enhanced spin fluctuations have been studied up to now. Since scattering centers are present in most physical situations, we want to study their influence on spin fluctuations in this communication. The physical consequences for such quantities as the electronic self-energy and the electronic specific heat are discussed.

Our aim is to study first how the paramagnon propagator changes if a mean free path due to scattering centers is introduced. This is done in the next section. It turns out that for momenta q such that $ql\ll1$, where l is the electronic mean free path, the paramagnon propagator changes drastically and becomes one which is of the diffusion type. The calculations are done by a standard vertex renormalization procedure. In Sec. III, we show how this change in the paramagnon propagator for $ql\ll1$ can influence various physical quantities by studying the specific heat and the effective mass due to spin fluctuations.

It is found that the effective mass at zero temperature and hence the term in the specific heat which is linear in T are practically independent of mean free path. The contribution to the specific heat which is of the form $T^3 \ln T$, however, is replaced by $T^3 \ln(T+T_{\mathrm{imp}})$, where T_{imp} is of the order of the impurity scattering rate times a reciprocal susceptibility enhancement factor. Thus, at low temperatures, this correction term goes over into a T^3 law, which has the same temperature dependence as many other contributions. We contrast this situation to the phonon case, where the $T^3 \ln T$ contribution to the electronic specific heat is not sensitive to mean free path. A magnonlike contribution of the form $T^{3/2}$ is found for temperatures below T_{imp}, whose coefficient is proportional to the three-halves power of

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[1] N. F. Berk and J. R. Schrieffer, Phys. Rev. Letter **17**, 433 (1966).

[2] S. Doniach and S. Engelsberg, Phys. Rev. Letters **17**, 750 (1966).

[3] We will use the terminology "propagator," despite the fact that paramagnons are not elementary excitations of the system, and note that this is permissible in the spirit of the RPA.

the impurity density. Our present calculations are consistent only to lowest order in the density and cannot resolve the existence of such high-order terms in the specific heat.

II. EXCHANGE-ENHANCED SPIN SUSCEPTIBILITY FOR FINITE MEAN FREE PATH

In this section, we study the exchange-enhanced spin susceptibility in the presence of randomly distributed scattering centers. We will sometimes call this quantity the paramagnon propagator, keeping in mind that rigorously their analytic properties are slightly different. The calculations are done by applying a standard vertex renormalization procedure (see Ref. 4). We note that the spin susceptibility at zero temperature, $\chi(\mathbf{q}, \omega_0)$, can be written in terms of diagrams as shown in Fig. 1. Here solid lines denote the electron Green's function $G(\mathbf{p}, \omega)$, σ_3 denotes the third Pauli spin matrix, and $\Lambda(\mathbf{q}, \omega_0)$ is a renormalized vertex which in the case

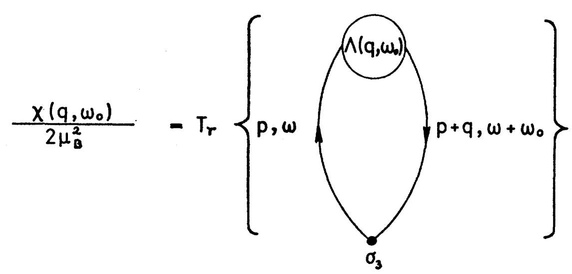

FIG. 1. Diagram for the calculation of the spin susceptibility. $\Lambda(\mathbf{q}, \omega_0)$ is the renormalized σ_3-vertex. Solid lines indicate electron Green's functions.

of a free electron gas is equal to σ_3. In the presence of a strong exchange interaction and randomly distributed scattering centers, the vertex $\Lambda(\mathbf{q}, \omega_0)$ is the solution of an integral equation, which in terms of diagrams is shown in Fig. 2. A solid line denotes now the electron propagator in the presence of exchange and impurity scattering. A dashed line indicates exchange scattering and a dotted line scattering by an impurity. Before we discuss this vertex equation we notice that in the absence of scattering centers, the vertex equation leads to the same sequence of diagrams for the susceptibility which has been discussed by Wolff[5] and others in a generalized random phase approximation (RPA) and similarly by Izuyama *et al.*[6] Since we want to concentrate on the effects of mean free path in this investigation we take a simple model for the exchange inter-

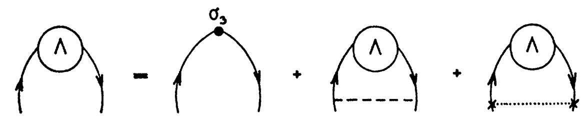

FIG. 2. Diagrammatic equation for the renormalized vertex $\Lambda(\mathbf{q}, \omega_0)$. A dashed line indicates the exchange interaction and a dotted line connects two impurity scattering events at the same impurity site.

action, namely, an s-wave scattering potential denoted by $\bar{V}_c$.

Furthermore, we do not consider the self-energy due to exchange interaction in the electron Green's function while calculating the susceptibility. That these are reasonable assumptions can be inferred from the work of Schrieffer and Berk.[7] Those authors showed that this model reproduces the long-wavelength static susceptibility which would be calculated from a more sophisticated approach, including the self-energy corrections. In writing down a vertex equation of the form shown in Fig. 2 we have neglected all overlapping diagrams of the type shown in Fig. 3. These can be left out because the important contributions to the corresponding integrals come when the intermediate states lie close to the Fermi surface. However, the overlapping diagrams of Fig. 3 require that over most of the range of integration the intermediate states lie far from the Fermi surface. This contribution is a factor $(p_F l)^{-1}$ smaller than the one from the corresponding diagrams without overlap. Thus we will discard them. The argument given here is the same which applies for neglecting overlapping impurity diagrams and overlapping impurity-phonon diagrams. However, it is clearly not valid if the exchange interaction is so strong that we are in the immediate vicinity of the ferromagnetic instability. Since the diagram [Fig. 3(a)] can be regarded as a renormalization of the exchange strength $\bar{V}_c$; namely, $\bar{V}_c' = \bar{V}_c[1+O((p_F l)^{-1})]$, we see that the present approach breaks down when $1-N(0)\bar{V}_c$ is of the order $(p_F l)^{-1}$. Here, $N(0)$ denotes the single spin density of states at the Fermi surface.

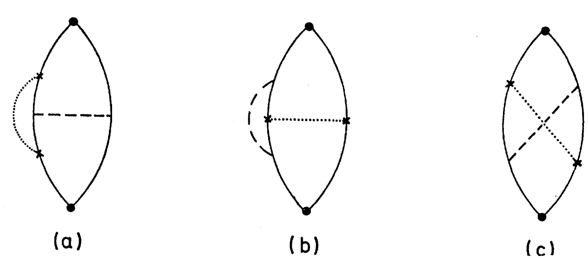

FIG. 3. Diagrams which have been left out because of their smallness in the calculation of the susceptibility.

[4] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963).

[5] P. A. Wolff, Phys. Rev. **120**, 814 (1960); D. L. Mills and P. Lederer, Solid State Commun. **5**, 131 (1967).

[6] T. Izuyama, D. J. Kim, and R. Kubo, J. Phys. Soc. (Japan) **18**, 1025 (1963).

[7] J. R. Schrieffer and N. F. Berk, Phys. Letters **24A**, 604 (1967).

We write the vertex equation of Fig. 2 in the form

$$\Lambda(\mathbf{p},\omega;\mathbf{q},\omega_0)=G(\mathbf{p},\omega)G(\mathbf{p}+\mathbf{q},\omega+\omega_0)\times\left[\sigma_3+i\bar{V}_c\int\Lambda(\mathbf{p}',\omega';\mathbf{q},\omega_0)\frac{d^3p'd\omega'}{(2\pi)^4}+n_i\int|u(\mathbf{p}-\mathbf{p}')|^2\Lambda(\mathbf{p}',\omega;\mathbf{q},\omega_0)\frac{d^3p}{(2\pi)^3}\right].\quad(1)$$

Here n_i is the concentration of impurities and $u(\mathbf{p}-\mathbf{p}')$ is the impurity scattering potential. Since for small values of $\mathbf{q}$ we are dealing with the renormalization of an s-wave vertex, we note that only isotopic scattering by the impurities is important. We introduce the mean free time τ by

$$\tau^{-1}=\tfrac{1}{2}[n_iN(0)]\int|u(\theta)|^2d\Omega.\quad(2)$$

In order to solve Eq. (1), we make the following ansatz:

$$\Lambda(\mathbf{p},\omega;\mathbf{q},\omega_0)=\frac{G(\mathbf{p},\omega)G(\mathbf{p}+\mathbf{q},\omega+\omega_0)\Omega(\mathbf{q},\omega_0)}{1-\tau^{-1}J(\omega;\mathbf{q},\omega_0)},\quad(3)$$

where we have introduced the function

$$J(\omega;\mathbf{q},\omega_0)=[2\pi N(0)]^{-1}\times\int G(\mathbf{p},\omega)G(\mathbf{p}+\mathbf{q},\omega+\omega_0)\frac{d^3p}{(2\pi)^3}.\quad(4)$$

By inserting Eqs. (3) and (4) into Eq. (1) one obtains

$$\Omega(\mathbf{q},\omega_0)=\left\{1-N(0)\bar{V}_ci\int\frac{d\omega'\,J(\omega')}{1-\tau^{-1}J(\omega')}\right\}^{-1}.\quad(5)$$

If we insert this formal solution of the integral equation into Eq. (3) and calculate the spin susceptibility $\chi(\mathbf{q},\omega_0)$ with the renormalized vertex $\Lambda(\mathbf{p},\omega;\mathbf{q},\omega_0)$, we obtain

$$\chi(\mathbf{q},\omega_0)=2\mu_B{}^2N(0)\int\frac{d\omega'\,J(\omega')}{1-\tau^{-1}J(\omega')}\times\left[1-N(0)\bar{V}_ci\int\frac{d\omega'\,J(\omega')}{1-\tau^{-1}J(\omega')}\right]^{-1}.\quad(6)$$

From Eq. (4) and from the fact that $G(\mathbf{p},\omega)$ contains only the impurity self-energy, as discussed above, it follows that $J(\omega;\mathbf{q},\omega_0)$ is independent of $\bar{V}_c$. Thus, we may identify the numerator of Eq. (6) with

$$\chi_{\text{imp}}{}^{\text{norm}}(\mathbf{q},\omega_0)=i\int\frac{d\omega'\,J(\omega';\mathbf{q},\omega_0)}{1-\tau^{-1}J(\omega';\mathbf{q},\omega_0)},\quad(7)$$

where $\chi_{\text{imp}}{}^{\text{norm}}(\mathbf{q},\omega_0)$ denotes up to a factor $2\mu_B{}^2N(0)$ the spin susceptibility of an electron gas which contains scattering centers but otherwise is noninteracting. With this notation, Eq. (6) can be written in a more compact form as

$$\chi(\mathbf{q},\omega_0)=2\mu_B{}^2N(0)\frac{\chi_{\text{imp}}{}^{\text{norm}}(\mathbf{q},\omega_0)}{1-N(0)\bar{V}_c\chi_{\text{imp}}{}^{\text{norm}}(\mathbf{q},\omega_0)}.\quad(8)$$

By inspecting Eq. (7) together with the defining Eq. (4), one notices that for infinite mean free path Eq. (8) reduces to the usual exchange enhanced susceptibility. Furthermore, in the limit $\bar{V}_c=0$ and $\omega_0=0$, Eq. (8) is identical with the static spin susceptibility of impure metals, which was first calculated by de Gennes.[8] For the purpose of this work, we restrict ourselves to the small frequency and small-wavelength case ($\omega_0/\epsilon_F\ll1$, $q/p_F\ll1$) but do not impose any limitations on $\omega_0\tau$ and ql. In evaluating Eq. (7) we make use of the fact that the function $J(\omega;\mathbf{q},\omega_0)$ has been discussed in detail in Ref. 8 for $\omega_0=0$, the static limit. Generalizing these results to finite ω_0 one obtains

$$J(\omega;\mathbf{q},\omega_0)=\frac{im^2}{4\pi^2N(0)}\frac{1}{q}\ln\left(\frac{K(\omega)+K(\omega+\omega_0)+q}{K(\omega)+K(\omega+\omega_0)-q}\right),\quad(9)$$

where the function $K(\omega)$ is defined by

$$K(\omega)=(i/2l)+\operatorname{sgn}\omega(p_F{}^2+2m\omega)^{1/2}.\quad(10)$$

Because of the sgn function in Eq. (10) the integration of $J(\omega;q,\omega_0)$ with respect to ω separates into three distinct regions. For positive ω_0 these regions are given by (1) $\omega<-\omega_0$, (2) $-\omega_0<\omega<0$, and (3) $\omega>0$. In region (2) the terms $K(\omega)$ and $K(\omega+\omega_0)$ almost cancel, so that the logarithm is a sensitive function of ω_0. However, in regions (1) and (3) no such cancellation occurs and the ω_0 dependence may be neglected since $\omega_0/\epsilon_F\ll1$. Thus in these regions $J(\omega;\mathbf{q},\omega_0)$ can be replaced by $J(\omega;\mathbf{q},0)$. Furthermore, in the integral over $J(\omega;\mathbf{q},0)$ we may ignore the restriction that ω has to be outside the interval $-\omega_0$ to 0, so that we are left with

$$\chi_{\text{imp}}{}^{\text{norm}}(\mathbf{q},\omega_0)=i\int_{-\omega_0}^{0}\frac{d\omega\,J(\omega;\mathbf{q},\omega_0)}{1-\tau^{-1}J(\omega;\mathbf{q},\omega_0)}+i\int_{-\infty}^{+\infty}\frac{d\omega\,J(\omega;\mathbf{q},0)}{1-\tau^{-1}J(\omega;\mathbf{q},0)}.\quad(11)$$

The second term in Eq. (11) is readily identified as the integral for the static susceptibility in the presence of impurities which was calculated by de Gennes.[8] Since in the small $\mathbf{q}$ range under consideration there is essentially no difference between this susceptibility and the one for a free electron gas, we may replace this term by (1). In the first term we use the fact that $\omega/\epsilon_F\ll1$ in that interval so that $J(\omega;\mathbf{q},\omega_0)$ can be written as

$$J(\omega;\mathbf{q},\omega_0)\simeq\frac{im^2}{4\pi N(0)}\frac{1}{q}\ln\frac{i/l+q+\omega_0/v_F}{i/l-q+\omega_0/v_F}.\quad(12)$$

With these simplifications the susceptibility given by Eq. (11) can be written as

$$\chi_{\text{imp}}{}^{\text{norm}}(\mathbf{q},\omega_0)=\left[1-\tfrac{1}{2}(u+iu_0)\ln\frac{u+iu_0+1}{u+iu_0-1}\right]\times\left[1-\tfrac{1}{2}(iu_0)\ln\frac{u+iu_0+1}{u+iu_0-1}\right]^{-1},\quad(13)$$

[8] P. G. de Gennes, J. Phys. Radium **23**, 630 (1962).

where we have introduced the quantities $u=\omega_0/qv_F$, $u_0=(lq)^{-1}$. We note at this point that the numerator of Eq. (13) comes from the evaluation of the particle-hole "bubble" in the presence of a mean free path, while the denominator is the result of the vertex renormalization, that is, the sum over the impurity ladder diagrams. For $\omega_0=0$, the renormalization factor takes the familiar form $1-(ql)^{-1}\arctan(ql)$. By substituting this result into Eq. (8) we find for the exchange-enhanced spin susceptibility in the presence of impurities

$$\chi(\mathbf{q},\omega_0)=2\mu_B^2N(0)\alpha(\mathbf{q},\omega_0)\times\left[1-\tfrac{1}{2}(iu_0)\ln\frac{u+iu_0+1}{u+iu_0-1}-N(0)\bar{V}_c\alpha(\mathbf{q},\omega_0)\right]^{-1},\quad(14)$$

where $\alpha(\mathbf{q},\omega_0)$ is given by

$$\alpha(\mathbf{q},\omega_0)=1-\tfrac{1}{2}(u+iu_0)\ln\frac{u+iu_0+1}{u+iu_0-1}.\quad(15)$$

We now discuss Eq. (14) in certain limiting cases.

(a) $ql\gg1$ and $qv_F\gg\omega_0$. In this case we expect that the paramagnon propagator will strongly resemble the propagator without impurities. Expanding Eq. (14) with respect to u, u_0 we obtain

$$\chi(\mathbf{q},\omega_0)=2\mu_B^2N(0)[1-\tfrac{1}{2}\pi u_0+i(\tfrac{1}{2}\pi u-2uu_0)]\times[1-N(0)\bar{V}_c(1-\tfrac{1}{2}\pi u_0)-i(\tfrac{1}{2}\pi u-2uu_0)]^{-1}.\quad(16)$$

One readily finds that $\mathrm{Im}\chi(\mathbf{q},\omega_0)$ has its maximum value at

$$u=(2/\pi)(1-N(0)\bar{V}_c)(1-\tfrac{1}{2}\pi u_0+(2/\pi)u_0),\quad(17)$$

and is given by

$$\mathrm{Max}\{\mathrm{Im}\chi(\mathbf{q},\omega_0)\}=\mu_B^2N(0)/[1-N(0)\bar{V}_c].\quad(18)$$

Thus one finds that a finite mean free path leads to a shift and a reduction (or additional broadening) of the peak in the imaginary part of the spin susceptibility. The positions of the peak of $\mathrm{Im}\chi(\mathbf{q},\omega_0)$ are indicated in region II of Fig. (4) by a straight line.

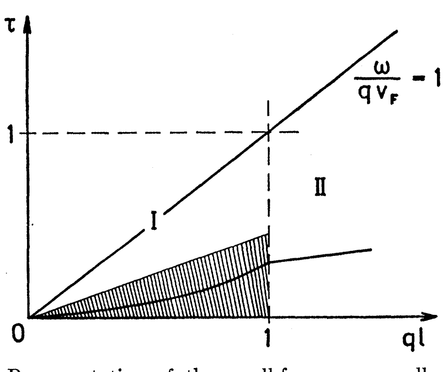

FIG. 4. Representation of the small-frequency–small-momentum region of the spin susceptibility, including mean free path. The lower solid line indicates the maximum of the spectral function. Note in particular that the linear (paramagnon) relationship of region II is changed to quadratic in region I: The shaded portion of region I is referred to as a diffusion region in the text. For convenience the scale has been greatly changed.

(b) $ql\ll1$. In the long-wavelength limit we find by expansion

$$\chi(\mathbf{q},\omega_0)=2\mu_B^2N(0)(ql)^2\times[(1-N\bar{V}_c)(ql)^2-3(\omega_0\tau)^2-3i\omega_0\tau]^{-1}.\quad(19)$$

If considered as function of the complex variable ω_0, $\chi(\mathbf{q},\omega_0)$ has two poles on the negative imaginary ω_0 axis. For $\omega_0\ll qv_F$ only the pole close to the real axis is important and the other pole, which is far from the real axis, can be neglected. This leads to a diffusion type of equation for $\chi(\mathbf{q},\omega_0)$ of the form

$$\chi(\mathbf{q},\omega_0)=\frac{2\mu_B^2N(0)}{1-N(0)\bar{V}_c}\frac{Dq^2}{Dq^2-i\omega_0},\qquad\frac{\omega_0}{qv_F}\ll1\quad(20)$$

with the "effective" diffusion constant

$$D=\tfrac{1}{3}(\tau v_F^2)(1-N\bar{V}_c).$$

The peaks in $\mathrm{Im}\chi(\mathbf{q},\omega_0)$ occur at $\omega_0=Dq^2$. The quadratic relationship between ω_0 and q is indicated in region I of Fig. (4).

It is shown in the next section that the drastic changes seen in Eq. (20) as compared with Eq. (16) have important consequences for physical quantities such as the electronic specific heat and self-energy.

III. APPLICATION TO THE SPECIFIC HEAT

One of the interesting results of the spin-fluctuation theory is that the low-temperature specific heat of almost ferromagnetic Fermi systems behaves as

$$C_v(T)=\tfrac{1}{3}m^*p_FT+\eta T^3\ln(T/\Theta_1)+\cdots,\quad(21)$$

where η is large because of the strong exchange enhancement. We now investigate the influence of mean free path on this result. We will find that the term linear in T is unaffected by mean free path while the term $T^3\ln T$ disappears at low temperatures, that is, for $T\lesssim(\tau)^{-1}(1-N(0)\bar{V}_c)$.

The situation is different in the ordinary electron-phonon system. Although the low-temperature specific heat takes the same form[9] as Eq. (21), we show that it is unaffected by mean free path.

There is a simple physical reason for the difference in these two systems. The peculiar logarithmic term in C_v arises because of the special momentum dependence of the "boson" spectral function. For phonons and paramagnons in pure metals this function depends only on the ratio ω/k in the small frequency, small momentum regime which is important for the low-temperature specific heat. When impurities are introduced the phonon spectral function still retains this dependence, but as discussed in the previous section the paramagnon propagator changes to an ω/k^2 dependence. This all-important change leads to the disappearance of the logarithmic term for paramagnons at low temperature.

[9] G. M. Eliasberg, Zh. Eksperim. i Teor. Fiz. **43**, 1005 (1962) [English transl.: Soviet Phys.—JETP **16**, 780 (1963)].

We prove now that m^* and hence the Sommerfeld term in C_v are unchanged by mean free path. For this purpose we have to calculate the self-energy $\Sigma(\mathbf{p}, p_0)$ at $T=0$ and use the connection

$$m^*/m \simeq 1-\mathrm{Re}(\partial\Sigma/\partial p_0)\,|_{p_0=0}.$$

Such a calculation can be performed without difficulty since diagrams with overlapping impurity and exchange lines may be neglected, as discussed in the preceding section.

Since we want to consider finite temperatures, we write the self-energy in terms of temperature Green's functions[4] as follows:

$$\Sigma(\mathbf{p}, i\omega_n)=i\bar{V}_c^2 T\sum_n\int\frac{d^3p}{(2\pi)^3}G(\mathbf{p}', i\omega_n)\times\chi(\mathbf{p}-\mathbf{p}', i(\omega_n-\omega_m)).\quad(22)$$

Here $G(\mathbf{p}, i\omega_n)$ is the electron Green's function in the presence of impurities and $\chi(\mathbf{p}, i\omega_n)$ is the paramagnon propagator which is obtained from the causal susceptibility with exchange and mean free path. We show next that in Eq. (22) $G(\mathbf{p}, i\omega_n)$ may be replaced by the Green's function in the absence of impurities. In order to see this one has to write Eq. (22) after proper analytical continuation in terms of spectral functions [see, for example, Ref. 4, Eq. (21.26), where the analog has been done for the phonon case]. Then one notices that there appears always an integral of the electron spectral function $\mathrm{Im}G(\mathbf{p}, \omega)$ over the energy ϵ_p, which can be carried out alone. Since the energy integral of the spectral function is unity, regardless of broadening, it is seen that the mean free path broadening of the intermediate-state electron Green's function leads to no effects on the self-energy and can be neglected. Thus, all the mean free path dependence of $\Sigma(\mathbf{p}, p_0)$ must come from the mean free path dependence of the "boson" propagator. From this we can immediately conclude that there will be no effect on $\Sigma(\mathbf{p}, p_0)$ and hence on the specific heat in the phonon case since the phonon propagator is essentially unaffected by mean free path.

With these simplifications for $G(\mathbf{p}, \omega)$ we may write in the standard manner

$$\frac{\partial\Sigma(p_F, p_0)}{\partial p_0}=\frac{N(0)\bar{V}_c^2}{4p_F^2}\int_0^{p_1}p'dp'\times\int_{-\infty}^{+\infty}dp_0'\left(-\frac{\partial f(p_0')}{\partial p_0'}\right)\chi(\mathbf{p}', p_0'-p_0),$$

$$\left.\frac{\partial\Sigma(p_F, p_0)}{\partial p_0}\right|_{p_0=0}=\frac{N(0)\bar{V}_c^2}{4p_F^2}\int_0^{p_1}p'dp'\,\chi(\mathbf{p}', 0)+O(T^2).\quad(23)$$

Here p_1 is a cutoff momentum of order p_F. Equation (23) shows that in the calculation of m^* only the static susceptibility enters. But as pointed out before this quantity is essentially independent of mean free path[8] for $p\leq 2p_F$. Thus m^* itself at $T=0$ is independent of mean free path and the Sommerfeld term in the specific heat is unchanged.

In order to study mean free path effects on the $T^3\ln T$ term in the specific heat, one would have to calculate the terms of higher order in p_0 and T of the self-energy. In these calculations the dynamical susceptibility enters and big effects are expected. However, there is a simple way in which one can see directly the effect of mean free path on the $T^3\ln T$ term. For this purpose we generalize the considerations by Brenig *et al.*[10] concerning the free energy corrections due to spin fluctuations to our situation and write

$$\Delta F=-\frac{6V}{(2\pi)^4}\int d^3k\int_0^\infty d\omega\int_0^{N\bar{V}_c}d\lambda\times\mathrm{Im}\left\{\frac{\chi_{\mathrm{imp}}{}^{\mathrm{norm}}(\mathbf{k}, \omega+i\eta)}{1-\lambda\chi_{\mathrm{imp}}{}^{\mathrm{norm}}(\mathbf{k}, \omega+i\eta)}-\chi_{\mathrm{imp}}{}^{\mathrm{norm}}(\mathbf{k}, \omega+i\eta)\right\}\times\{\tfrac{1}{2}+(e^{\beta\omega}-1)^{-1}\}.\quad(24)$$

In general, one has to insert the temperature-dependent susceptibility $\chi_{\mathrm{imp}}{}^{\mathrm{norm}}(\mathbf{k}, \omega+i\eta)$ into Eq. (24). However, if one is interested only in a possible $T^4\ln T$ term in ΔF, then the temperature dependence of $\chi_{\mathrm{imp}}{}^{\mathrm{norm}}(\mathbf{k}, \omega+i\eta)$ can be neglected.[10] The same is true for the term $\frac{1}{2}$ in the last bracket of the integrand of Eq. (24). In performing the integrations in Eq. (24) one has to distinguish between the regions I and II of Fig. 4. In region I we replace $\chi_{\mathrm{imp}}{}^{\mathrm{norm}}(\mathbf{k}, \omega)$ by an expression similar to Eq. (19). The appropriate integral to evaluate in that region is

$$\Delta F_1=-\frac{3VD_0}{2\pi^3}\int_0^{N\bar{V}_c}d\lambda\int_{\text{region I}}dk\int d\omega\frac{\omega k^4}{e^{\beta\omega}-1}\times[D_0^2(1-\lambda)^2k^4+\omega^2]^{-1},\quad(25)$$

where we have left out contributions which do not depend on $\bar{V}_c$. Here $D_0=\frac{1}{3}V_Fl\neq 3$. After performing the integration, we calculate the corresponding contribution to the specific heat ΔC_1. We find

$$\Delta C_1=(0.5)P_F^3(T/T_F)^{3/2}[P_Fl(1-N(0)\bar{V}_c)]^{-3/2}+0(T^3).$$

The magnonlike $T^{3/2}$ term in this result represents a correction to the free energy which is proportional to $n_i^{3/2}$. However, our diagrams, while correct in the RPA, have left out other contributions which are at similarly high order in n_i, under the assumption

$$P_Fl(1-N(0)\bar{V}_c)\gg 1.$$

Thus we believe this contribution should be dropped in the framework of the RPA.

In region II we approximate $\chi_{\mathrm{imp}}{}^{\mathrm{norm}}(\mathbf{q}, \omega_0)$ by an expression similar to Eq. (16). In that case, the important

[10] W. Brenig, H. J. Mikeska, and E. Riedel, Z. Physik **206**, 439 (1967).

integral to investigate is

$$\Delta F_2 = -\frac{3V}{2\pi^3}\int_0^{N\bar{V}_c} d\lambda \int_{\text{region II}} dk \int d\omega \frac{1}{e^{\beta\omega}-1} \times \frac{\alpha\omega k^3}{k^2+\alpha^2\omega^2}, \quad (26)$$

where

$$\alpha V_F = \pi\lambda(1-\lambda)^{-1}.$$

In the absence of scattering centers ($\tau^{-1}=0$) this integral gives the $T^4 \ln T$ contribution to the free energy. In our case, however, such a term does not appear in ΔF_2. The reason for this is that we now have a cutoff at low momenta. More specifically, the $T^4 \ln T$ term is changed now into a term of the form $T^4 \ln(T+T_{\text{imp}})$, where T_{imp} is equal to $\epsilon_F(1-N(0)\bar{V}_c)(p_F l)^{-1}$. Thus for $T \lesssim T_{\text{imp}}$ there will be only a T^3 contribution of the spin fluctuations to the electronic specific heat in RPA.

We now return to the self-energy approach and examine in detail the mean free path corrections to the higher-order frequency terms contained in Eq. (23). We divide the momentum integration again into the regions $(lq) \gtrless 1$ and use the approximate Eqs. (16) and (18) for the susceptibility in those regions. The result is found to be

$$\sum(p_F, p_0) = A(p_0^3+\pi^2 p_0 T^2) \ln \frac{(p_0+T_{\text{imp}})\pi}{2p_1 v_F[1-N(0)\bar{V}_c]},$$

with

$$A = \frac{\pi^2[N(0)\bar{V}_c]^3}{64\epsilon_F^2[1-N(0)\bar{V}_c]^3}, \quad (27)$$

where we have dropped the term linear in p_0 since we have discussed it before. We see by inspection that in the limit of infinite mean free path ($\tau v_F \to \infty$) there are terms in Eq. (27) which are logarithmic in p_0. As shown in Ref. 2 (see also Ref. 9 for the phonon case) they lead to $T^3 \ln T$ contribution to the specific heat. However, for finite τv_F the logarithmic p_0 dependence disappears for p_0 $T_{\text{imp}} < 1$, and hence there can be no logarithmic temperature contribution to the specific heat either. By using the standard relation[4] between $\text{Re}\Sigma(p_F, \omega)$ and $C_v(T)$ we find, approximately,

$$C_v(T) \simeq \tfrac{1}{3} m^* p_F T + \tfrac{1}{5} 6\pi (p_1^3/\Theta^3) T^3 \ln[(T+T_{\text{imp}})/\Theta], \quad (28)$$

where we have set $\Theta = 2v_F p_1[1-N(0)\bar{V}_c]/\pi$. Riedel[11] has shown for the pure case that the factor in front of the $T^3 \ln T$ term has to be changed by $\frac{1}{3}$ because of additional "boson" contributions which are not yet obtained in a treatment of the above form. They result from bosonlike thermal excitations of the paramagnon spectrum. The same is found true in the present case if Riedel's analysis is used. Thus the over-all factor of the term $T^3 \ln(T+T_{\text{imp}})$, $\frac{6}{5}$, should become $\frac{2}{5}$. The result obtained this way is identical with the one which is obtained from Eq. (24).

IV. CONCLUSIONS

The purpose of this paper has been a study of the mean free path effects on the spin susceptibility of strongly exchange-enhanced Fermi systems. An application of our findings to the low-temperature specific heat showed no effects for the Sommerfeld term but altered the next higher-order temperature term; namely, from a $T^3 \ln T$ to a $T^3 \ln(T+T_{\text{imp}})$ behavior [see Eq. (28)]. The ratio T_{imp}/Θ is of the order $(p_F l)^{-1}$ and thus much smaller than 1. As a result only the low-temperature contributions of the spin fluctuations are effected by impurities ($T \leq T_{\text{imp}} \ll \Theta$). For $p_F l \simeq 300$ and $N(0)\bar{V}_c \simeq 0.95$, we estimate T_{imp} to be 2–5°K.

Thus in contrast to the phonon case where we showed that the $T^3 \ln T$ contribution is essentially unaffected by mean free path we find that impurities strongly modify the paramagnon contribution to the $T^3 \ln T$ term. For this reason, we are pessimistic about the possibility of observing such a term in disordered alloys. We suggest that very pure systems with large exchange enhancement may exhibit a $T^3 \ln T$ term due to spin fluctuations, but that this term is very sensitive to the addition of impurities. We note that at $T < T_{\text{imp}}$ other temperature corrections to the specific heat, such as the magnonlike $T^{3/2}$, can occur. These corrections, however, are all of higher order in the impurity concentration, and are not necessarily correct in the RPA. Calculations to higher order do not appear feasible at present.

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[11] E. Riedel, 8th Scottish Universities' Summer School in Physics, University of St. Andrews, Scotland, 1967 (unpublished); and Z. Physik **210**, 503 (1968).