THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES VOL. 170, No. 2 10 JUNE 1968

Measurement of Positron Annihilation Line Shapes with a Ge(Li) Detector

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(Received 10 November 1967)

We observed that the annihilation radiation photopeak in a Ge(Li) detector is considerably broader than that of a γ ray of the same energy. It seems reasonable to assume that the increased width is the result of the Doppler broadening of the annihilation photopeak, i.e., the longitudinal Doppler shift of the radiations is measured, while the transverse shift is measured in the usual angular-correlation experiments. By using a computer stripping program to remove the distortion produced by the finite energy resolution of our detector, we obtain momentum distributions in agreement with those which have been published. Only one detector is necessary for these measurements, and all momentum channels are detected at once. However, the detector energy resolution severely limits the momentum resolution. Arguments are presented which indicate that no very large improvement in Ge(Li) detector resolution can be expected.

INTRODUCTION

WE and others[1-5] have observed the Doppler broadening of the annihilation radiation line due to the motion of the electron-positron system, using a Ge(Li) detector. Figure 1 shows a plot of the full width at half-maximum (FWHM) against energy for one of our detectors. As can be seen in the figure, the linewidth of the annihilation radiation is approximately double that of a γ ray of equivalent energy. Of course, while any energy-proportional detector will demonstrate the same effect, the broadening is in addition to the normal linewidth of the detector-amplifier combination. It follows, then, that only a very high resolution system will show annihilation broadening effects in measurable quantities. With our present resolution of 1.55 keV (FWHM) for a γ ray of 514 keV, we have attempted to extract some of the information about electron-momentum distributions which is implicit in this broadening.

In principle, the Doppler broadening of the annihilation photopeak gives the same information about the distribution of electrons in the substance in which the annihilation takes place as can be obtained by measurement of the angular correlation of the two quanta. Detector energy resolution limits the resolution of our measurement of the electron-momentum distribution, while finite solid angles of acceptance for the detectors limit the resolution using the angular-correlation technique. The electron-momentum space density function $\rho(\mathbf{k})$ is related to the pulse-height distribution of the counting rate, $I(n)$, in the same way that the coincidence counting rate is related to $\rho(\mathbf{k})$:

$$I(n) = \text{const}\times \iint \rho(\mathbf{k})\, dk_y dk_z,$$

where the x axis is chosen along the direction of emission of an annihilation photon and n is the channel number. By assuming the distribution in momentum space to be isotropic, the integral becomes

$$I(n) = \text{const}\times 2\pi \int_{k_x}^{\infty} k\rho(k)\, dk,$$

from which we have by differentiation

$$\rho(k) = \frac{dn/dk_x}{2\pi\times\text{const}} \frac{1}{k_x} \frac{dI(n)}{dn}.$$

This result is identical with that derived for the corresponding angular-correlation case by Stewart.[6]

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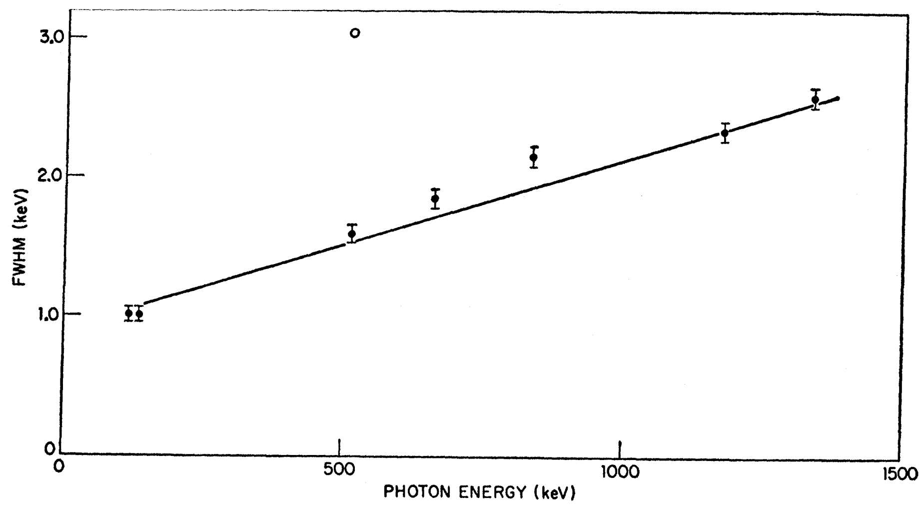

FIG. 1. Detector resolution as a function of photon energy. The open circle shows the FWHM for annihilation radiation.

For convenience, let us consider the case in which the electron-positron pair is moving with momentum p and, upon annihilation, one of the two quanta is emitted in the direction of p. Let this photon be detected by our Ge(Li) detector. With these assumptions we are considering only the component of momentum along the direction toward the detector; however, this component is the only one affecting the energy of the quanta, to first order. The detector receives a photon of energy $h\nu'$ emitted in the opposite direction. Conservation of energy and momentum along the direction of p require that

$$2mc^2 = h\nu + h\nu',$$

$$p = h\nu/c - h\nu'/c,$$

where m is the rest mass of an electron. To a very good approximation, the pair momentum may be expressed in terms of the difference in the observed energy $h\nu$ and the energy $h\nu_0(=mc^2)$ which would be observed for $p=0$:

$$p = 2(h\nu - h\nu_0)/c.$$

Thus the distribution of the number of annihilation events in photon energy is the same as the distribution of the number of electrons having a given momentum component parallel to the direction of emission of an annihilation quantum.

Note that even though the energy resolution of the detector is 1.55 keV FWHM, the broadening is the result of electron kinetic energies of eV order magnitude. Since positrons are thermalized before annihilation,[7] the momentum p is that of the electron. The electron kinetic energy is

$$E = p^2/2m$$
$$= 2(h\nu - h\nu_0)^2/mc^2.$$

For a channel shift equal to 1.5 keV of photon energy, the electron kinetic energy is 8.8 eV.

The finite resolution of the detector system results in photons of a single energy being counted as a distribution over a range of pulse-height channels. The annihilation decays resulting from each given momentum component p are spread into a distribution and the distribution of annihilation events is distorted by this broadening. The amount of broadening can be measured directly by observing the distribution of pulse heights of the annihilation γ-ray line spectrum. We take the observed line spectrum of 514.0 keV from a ^{85}Sr source as the response function of the detector for photons of annihilation quantum energy.

EXPERIMENTAL PROCEDURE

We have studied the annihilation processes in silver, polyethylene, and beryllium. Figure 2 shows the spectrum for the annihilation radiation in silver, and for comparison, the spectrum for the 514-keV line from a ^{85}Sr source. The annihilation spectra were observed by depositing a thin layer of ^{22}Na directly on the material in which annihilation is to take place, and covering the ^{22}Na with another piece of the same material. To observe the spectrum, the source was placed about 3.0 cm from the detector, with the distance being adjusted to prevent an excessive counting rate from broadening the observed spectrum. The count rate was adjusted so that the dead time of the analyzer would be less than 10%, with all photon energies being registered in the analyzer. The actual spectra were recorded using a post amplifier adjusted for a large bias cut and a moderate gain to provide the necessary spread in channels. No change in gain or bias cut was made between the observations of the annihilation spectra and the observations of the 514-keV line. An energy calibration was obtained by recording the 121.97- and

[7] S. M. Kim and A. T. Stewart, Phys. Rev. Letters 18, 385 (1967).

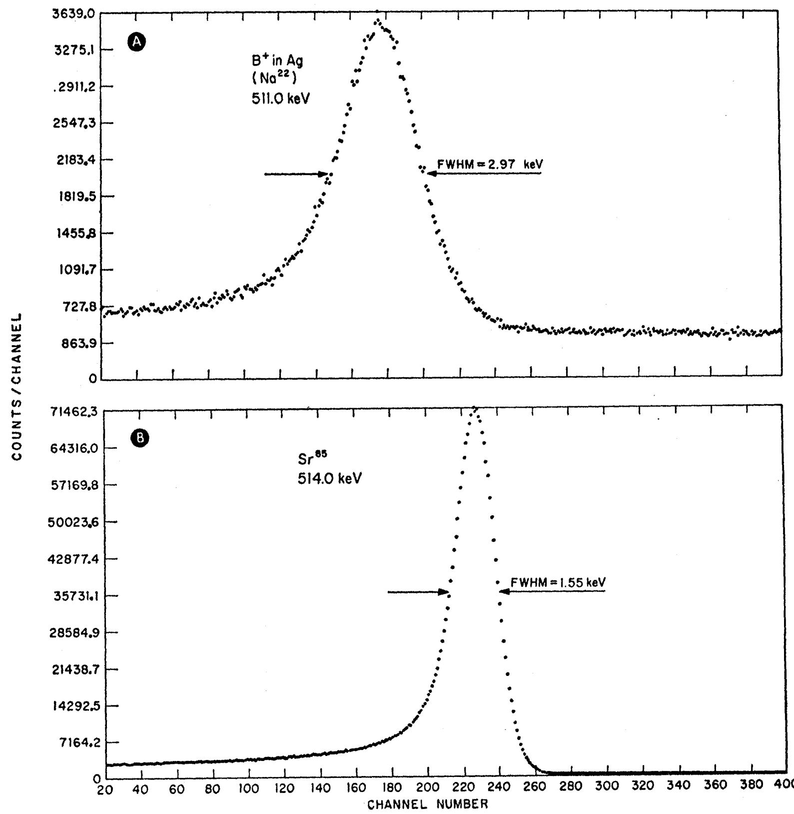

FIG. 2. (A) Pulse-height distribution of annihilation radiation from annihilation events in silver; (B) pulse-height distribution of the 514-keV line from a ^{85}Sr source.

the 136.33-keV line from a ^{57}Co source, using a smaller bias cut but with the same gain settings.

The detector used for these measurements was prepared especially for them by cutting a 0.4-cm slice off the side of a piece of previously lithium-drifted germanium.[8] The device measured 0.4×1.0 cm in area and was drifted to a depth of 0.8 cm. The capacitance was measured to be 0.7 pF. A TMC Model No. 327A preamplifier, with a selected FET, was used to deliver signals to a Tennelec Model No. TC 200 amplifier, while the biased amplifier was a Tennelec Model No. TC 250. The spectra were recorded in a Nuclear Data Data Model No. 181 pulse-height analyzer. We found that there was less than one channel shift in peak position over a month-long period with this system, and the gains could be reset to within two channels. The gain of the system was adjusted so that one channel represented 58.6 eV of photon energy.

Since the counts associated with each annihilation photon energy are distributed over many channels, in a manner given by the detector response function, the annihilation spectrum in Fig. 2 is a distorted-momentum distribution. We removed the distortion by stripping out the detector response function with a computer program, discussed in the next section. The resulting count-rate distributions were then numerically differentiated to convert them to electron space momentum density functions. These distributions for silver, polyethylene, and beryllium are shown in Fig. 3. The smooth curve for silver is taken from the experimental data of Stewart.[6] The other two curves are smooth fits to our data.

[8] Studies of parameters affecting the resolution of this detector are discussed in Ref. 9.

DATA ANALYSIS

We were unsuccessful in our attempts to use a standard least-squares fitting procedure to remove the broadening effect of the finite energy resolution of our detector. Negative contributions to the momentum spectrum resulted. Since it is not possible to incorporate a constraint to positive values into a least-squares technique, we resorted to an iterative computational technique.

If one had the undistorted electron-momentum distribution (ideal spectrum), the observed spectrum could be calculated by distributing a fraction of the counts at each point of the ideal spectrum among all the channels in the amounts dictated by the response function of the detector. Thus for any assumed ideal spectrum, a synthetic observed spectrum may be calculated. This may be expressed as a matrix equation

$$S=XR,$$

where S is the synthetic spectrum, X is the ideal

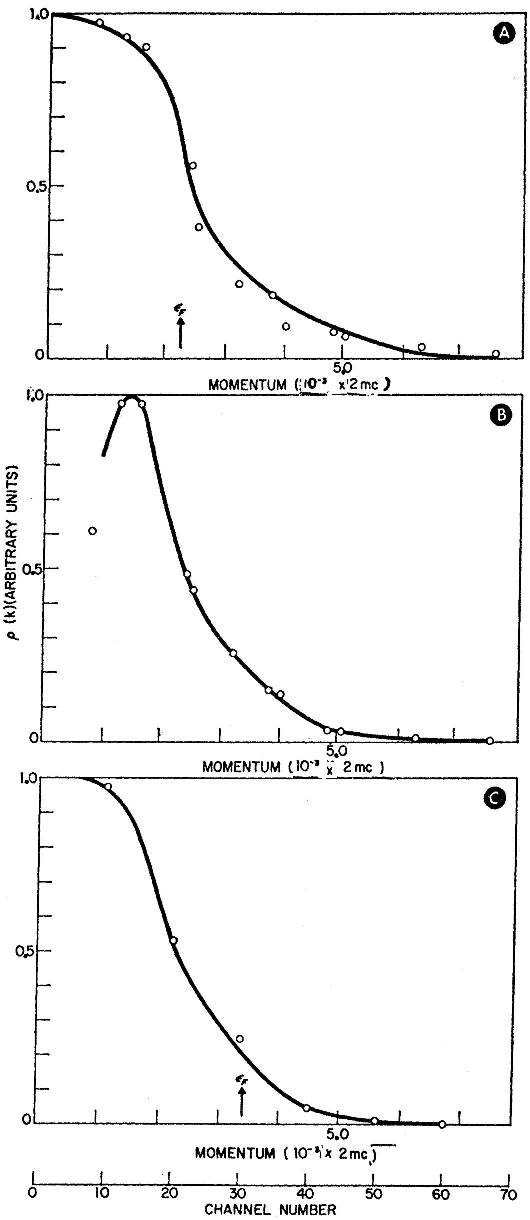

FIG. 3. Electron momentum space densities derived from positron annihilation in (A) silver, (B) polyethylene, and (C) beryllium. The circles represent our data. The smooth curve for silver in graph A is taken from Ref. 6; smooth curves in graphs B and C are drawn through the data points.

spectrum, and R is a rectangular matrix in which each row is a detector response function appropriate to the corresponding component of X. Each row in R can be constructed from the observed distribution for the 514-keV line from the ^{85}Sr source by shifting the channel numbers of this distribution, so that the peak channel would correspond in turn to the channel position of each point on the ideal distribution.

If some distribution is assumed for X and the resulting S calculated, then S may be tested for goodness of fit by finding χ^2 for the fit between S and the observed spectrum A by

$$\chi^2 = \sum (S_i - A_i)^2 / A_i,$$

where i is the channel number and the sum is taken over all channels. Initially the components of X may be chosen arbitrarily, but restricted to be positive; the value of χ^2 calculated and the components of X are adjusted to make each S_i closer to the same A_i. The new χ^2 is then found. If the new χ^2 is smaller than the old, the process is repeated until the X is found for which χ^2 has a minimum value.

We begin the iterative procedure by taking for the X_i the values of A_j for the same channel numbers. Only 15 or 20 components are assumed for X so that these points are spaced 6–15 channels apart. The X_i are then adjusted by multiplying each component in X by the ratio of the sum of the number of counts in the five channels of the observed spectrum distribution A about the center channel point corresponding to this component, to the sum of the same five points in S. Since a ratio correction is used, no negative points can be generated, but, of course, we do not obtain a true best fit by this procedure. The procedure does not lend itself to a calculation of the errors of each point in the X distribution, and the value χ^2 provides only a rough estimate of the total error of the fit. For this reason no error bars are shown on the points plotted in Fig. 3. Self-consistency was checked by using the computer program to generate the distortion-corrected distributions with different channel spacings; the resulting distributions are almost indistinguishable.

DETECTOR RESOLUTION

Since the measurement of electron-momentum distributions by our technique is limited by the finite resolution of the Ge(Li) detector, we have taken great pains[9] to make the FWHM as small as possible. Two major contributions to the linewidth are amplifier noise and charge-collection statistics. These factors are independent of each other and should add in quadrature.

The statistical effects of charge collection have been treated by Fano[10] for the case of a gaseous ionization chamber, but the Ge(Li) detector is quite similar, being, in fact, a solid-state ionization chamber. If the production of charge pairs were statistically independent, the standard deviation from the center of a photopeak would be given by

$$\sigma = (E\epsilon)^{1/2},$$

where σ is this standard deviation of the photopeak in

[9] J. P. Hurley, J. M. Mathiesen, and V. L. DaGragnano, USNRDL-TR-55, 1967 (unpublished); Nucl. Instr. Methods (to be published).
[10] U. Fano, Phys. Rev. **72**, 26 (1947).

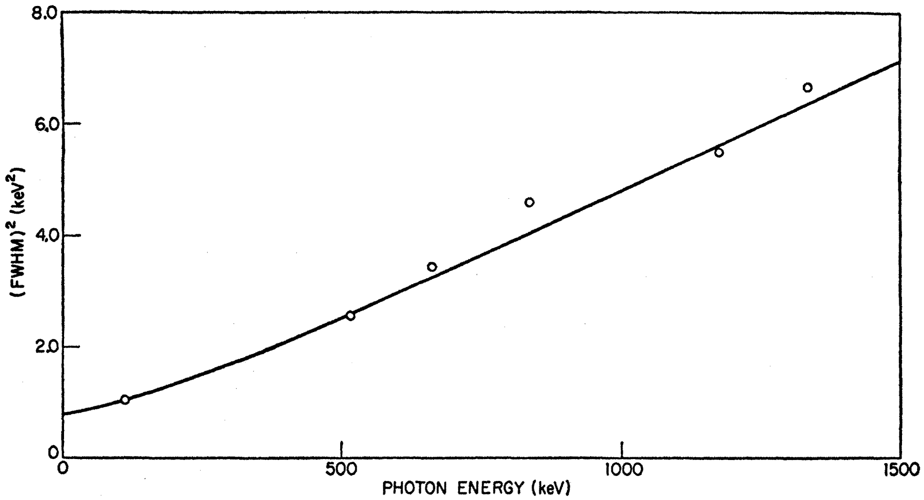

FIG. 4. $FWHM^2$ as a function of photon energy.

energy units, E is the photon energy, and ϵ is the energy expended per charge pair produced. Since the charge pairs are not produced independently, the peak width should be statistically determined by the number of independent events, which is less than the number of charge pairs by the Fano factor F. Thus, relating σ to the FWHM, we have

$$(\mathrm{FWHM})_s = 2.36\sigma = 2.36(EF\epsilon)^{1/2},$$

where $(\mathrm{FWHM})_s$ is the statistical contribution to the FWHM. Letting $(\mathrm{FWHM})_e$ be the electronic or other independent contribution to the width,

$$(\mathrm{FWHM})^2 = (\mathrm{FWHM})_e^2 + (2.36)^2 \epsilon F E,$$

and a plot of $(\mathrm{FWHM})^2$ versus energy should have a slope proportional to the Fano factor. Various values of the Fano factor have been quoted[11] with recent measurements indicating a value close to 0.12. Using this value for F and a value of 2.9 eV for ϵ, one calculates a value of 0.44 keV at 100 keV. Thus reducing amplifier noise to zero can be expected to improve the resolution by only a factor of about 2.

The curvature of the plot of $(\mathrm{FWHM})^2$ versus energy in Fig. 4 indicates that the energy resolution of the detector is determined by additional factors other than charge-collection statistics which are not statistically independent, or are dependent on energy in a nonlinear fashion. While amplifier noise does provide a contribution to the linewidth, this should be independent of photon energy. An alternative possibility is that the Fano factor may be a function of energy, with the change in slope shown in Fig. 4 displaying this variation in Fano factor. The data in Fig. 4 could then be interpreted as being due to the variation of the Fano factor from a value of 0.16 at about 100 keV to a value of about 0.21 at higher energies. Murray's data[3] would also display a similar curvature of a plot of $(\mathrm{FWHM})^2$ versus energy.

[11] The value of 0.13 for the Fano factor was given by Day, Dearnaly, and Palms at the Thirteenth Nuclear Science Symposium, Boston, 1966 (unpublished).

CONCLUSION

The electron-momentum distributions shown in Fig. 3 demonstrate that one can obtain these distributions by measuring the Doppler broadening of the annihilation radiation. It was not found possible to obtain distribution points much closer together than those shown in the figure with our stripping program. Our energy resolution is thus equivalent to a measurement of momentum by means of angular correlation with an acceptance angle of approximately 2×10^{-3} rad. The technique does, however, offer the considerable advantage of requiring only one detector, and thus it might be useful in an experiment in which momentum distribution is measured as a function of positron lifetime.

ACKNOWLEDGMENTS

The authors wish to thank N. E. Scofield for helpful discussions of the computer stripping program and V. L. DaGragnano for help with the electronic apparatus. One of us (H.P.H.) wishes to thank the U.S. Naval Radiological Defense Laboratory for its hospitality during his year as Scientist-in-Residence at the Laboratory.