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Fermi-Liquid Transport Coefficients of Dilute Solutions of He^3 in He^4†

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A slightly modified form of the effective interaction between He^3 quasiparticles proposed by Bardeen, Baym, and Pines, used with variational solutions of the transport equation, yields results in satisfactory agreement with recent measurements of the spin diffusion and thermal-conductivity coefficients of two dilute solutions of He^3 in He^4 at very low temperatures.

INTRODUCTION

RECENT measurements[1] in the Fermi-liquid regime of the thermal conductivity of dilute solutions of He^3 in He^4 disagree considerably with the values calculated from the effective He^3 quasiparticle interaction of Bardeen, Baym, and Pines.[2] This interaction is determined from measurements[3] of the spin diffusion coefficient in the solutions. It is the purpose of this article to call into question the accuracy of the familiar expressions[4,5] used to relate the transport coefficients of a Fermi liquid to the quasiparticle interaction, and to suggest that agreement between theory and experiment can be substantially improved by using more precise solutions of the transport equation in determining the effective interaction.

The determinations of the thermal conductivity κ and diffusion coefficient D in terms of an effective interaction, as in Refs. 1 and 2, are based on the theoretical expressions given by Abrikosov and Khalatnikov[4] for κ and by Hone[5] for D. These expressions are derived by solving the linearized Boltzmann transport equation exactly, under the assumption that the excitation energies of the quasiparticles relevant to the transport are small compared with the temperature. Thus, factors of the form $g(p)=1+(\epsilon_p-\mu)^2/(\pi kT)^2$ are approximated by unity in the collision integral; ϵ_p is the energy of a quasiparticle of momentum p, μ the fermion chemical potential, T the temperature, and k Boltzmann's constant.

The errors introduced by this approximation can be examined by using the resulting Abrikosov-Khalatnikov and Hone solutions for the nonequilibrium quasiparticle distribution functions as trial functions in a standard variational calculation of κ and D. We give this calculation in Sec. I. In Sec. II, the resulting variational solutions are used in conjunction with the measurements[1,3] of κ and D to examine the consistency of the Bardeen, Baym, and Pines (BBP) theory of dilute He^3–He^4 solutions in this new context.

I. VARIATIONAL SOLUTIONS FOR κ AND D

The general procedure used here is given by Ziman[6]; in particular we follow fairly closely a derivation given by Rice[7] for κ and D in an interacting Fermi liquid, although we do not include Fermi-liquid effects, which are generally[2] 5–10% in dilute solutions of He^3 in He^4.

The two transport coefficients are given in terms of the irreversible entropy production by[7]

$$1/\kappa_{\text{var}}=T^2\dot{S}/j_\epsilon^2 \tag{1}$$

$$1/D_{\text{var}}=T\dot{S}/[2j_\sigma^2(\partial\mu_\sigma/\partial n_\sigma)], \tag{2}$$

where $\dot{S}$ is the rate of entropy production, j_ϵ the energy flux, j_σ the flux of quasiparticles of spin orientation σ, and $\partial\mu_\sigma/\partial n_\sigma$ the rate of change of the chemical potential for quasiparticles of spin orientation σ with respect to the number density of quasiparticles of the same spin. The subscript "var" denotes that we are finding variational solutions. If we write the quasiparticle distribution function as

$$f_{\mathbf{k}\sigma}=f_{\mathbf{k}\sigma}{}^0+\Phi_{\mathbf{k}\sigma}f_{\mathbf{k}\sigma}{}^0(1-f_{\mathbf{k}\sigma}{}^0)/kT, \tag{3}$$

where $f_{\mathbf{k}\sigma}{}^0$ is the equilibrium distribution function, then the energy and quasiparticle fluxes are[8]

$$\mathbf{j}_\epsilon=(2\pi)^{-3}\sum_\sigma\int d\mathbf{k}\mathbf{v}_\mathbf{k}(\epsilon_{\mathbf{k}\sigma}-\mu)\Phi_{\mathbf{k}\sigma}f_{\mathbf{k}\sigma}{}^0(1-f_{\mathbf{k}\sigma}{}^0)/kT, \tag{4}$$

$$\mathbf{j}_\sigma=(2\pi)^{-3}\int d\mathbf{k}\mathbf{v}_\mathbf{k}\Phi_{\mathbf{k}\sigma}f_{\mathbf{k}\sigma}{}^0(1-f_{\mathbf{k}\sigma}{}^0)/kT. \tag{5}$$

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[4] A. A. Abrikosov and I. M. Khalatnikov, Rept. Progr. Phys. **22**, 329 (1959).

[5] D. Hone, Phys. Rev. **121**, 669 (1961).

[6] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, London, 1960), Chap. VII.

[7] M. J. Rice, Phys. Rev. **159**, 153 (1967).

[8] For convenience, we take $\hbar$ and V, the volume, equal to unity.

$\mathbf{v}_{\mathbf{k}}$ is the quasiparticle velocity ($\mathbf{v}_{\mathbf{k}}=\mathbf{k}/m^*$) and m^* is the quasiparticle effective mass. Furthermore, the entropy production may be written as[9]

$$\dot{S}=(4kT^2)^{-1}(2\pi)^{-12}\sum_{\sigma,\sigma'}\int d\mathbf{k}d\mathbf{k}'d\mathbf{k}''d\mathbf{k}'''$$
$$\times(\Phi_{\mathbf{k}\sigma}+\Phi_{\mathbf{k}'\sigma'}-\Phi_{\mathbf{k}''\sigma}-\Phi_{\mathbf{k}'''\sigma'})^2 f_{\mathbf{k}\sigma}{}^0 f_{\mathbf{k}'\sigma'}{}^0$$
$$\times(1-f_{\mathbf{k}''\sigma}{}^0)(1-f_{\mathbf{k}'''\sigma'}{}^0)\,\mathcal{Q}_{\mathbf{k}''\mathbf{k}'''\sigma\sigma'}{}^{\mathbf{k}\mathbf{k}'\sigma\sigma'},\quad(6)$$

where $\mathcal{Q}_{\mathbf{k}''\mathbf{k}'''\sigma\sigma'}{}^{\mathbf{k}\mathbf{k}'\sigma\sigma'}$ is the intrinsic scattering rate for the process

$$(\mathbf{k},\sigma)+(\mathbf{k}',\sigma')\rightarrow(\mathbf{k}'',\sigma)+(\mathbf{k}''',\sigma').\quad(7)$$

We do not allow for spin-flip because within the context of the BBP theory, which is all that is of interest to us here, it cannot occur. In terms of $V(q)$, the BBP effective quasiparticle interaction,

$$\mathcal{Q}_{\mathbf{k}''\mathbf{k}'''\uparrow\downarrow}{}^{\mathbf{k}\mathbf{k}'\uparrow\downarrow}=(2\pi)^4|V(\mathbf{k}''-\mathbf{k})|^2$$
$$\times\delta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}''}-\epsilon_{\mathbf{k}'''})\delta^3(\mathbf{k}+\mathbf{k}'-\mathbf{k}''-\mathbf{k}''')\quad(8)$$

and

$$\mathcal{Q}_{\mathbf{k}''\mathbf{k}'''\uparrow\uparrow}{}^{\mathbf{k}\mathbf{k}'\uparrow\uparrow}=\mathcal{Q}_{\mathbf{k}''\mathbf{k}'''\downarrow\downarrow}{}^{\mathbf{k}\mathbf{k}'\downarrow\downarrow}=(2\pi)^4$$
$$\times|V(\mathbf{k}''-\mathbf{k})-V(\mathbf{k}'''-\mathbf{k})|^2\delta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}''}-\epsilon_{\mathbf{k}'''})$$
$$\times\delta^3(\mathbf{k}+\mathbf{k}'-\mathbf{k}''-\mathbf{k}''').\quad(9)$$

In the latter case, a factor of $\frac{1}{2}$ should be included when summing final states in order to avoid double counting.

The solution of Hone for $\Phi_{\mathbf{k}\sigma}$ in the case of spin diffusion is

$$\Phi_{\mathbf{k}\uparrow}=\mathbf{k}\cdot\mathbf{v};\quad\Phi_{\mathbf{k}\downarrow}=-\mathbf{k}\cdot\mathbf{v},\quad(10)$$

while that of Abrikosov and Khalatnikov for thermal conduction is

$$\Phi_{\mathbf{k}\sigma}=(\epsilon_{\mathbf{k}}-\mu)\mathbf{k}\cdot\mathbf{v}/\mu.\quad(11)$$

Here $\mathbf{v}$ is some arbitrary velocity vector in the direction of the temperature or magnetization gradient. A factor of μ^{-1} has been included in Eq. (11) for dimensional convenience; over-all constant factors in $\Phi_{\mathbf{k}\sigma}$ are irrelevant.

Inserting Eqs. (8)–(11) into (6), we find that for thermal conduction,

$$\dot{S}_\kappa=2(kT^2\mu^2)^{-1}(2\pi)^{-8}\int d\mathbf{k}d\mathbf{k}'d\mathbf{k}''d\mathbf{k}'''$$
$$\times([\epsilon_{\mathbf{k}}\mathbf{k}+\epsilon_{\mathbf{k}'}\mathbf{k}'-\epsilon_{\mathbf{k}''}\mathbf{k}''-\epsilon_{\mathbf{k}'''}\mathbf{k}''']\cdot\mathbf{v})(\epsilon_{\mathbf{k}}-\mu)(\mathbf{k}\cdot\mathbf{v})$$
$$\times\delta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}''}-\epsilon_{\mathbf{k}'''})\delta^3(\mathbf{k}+\mathbf{k}'-\mathbf{k}''-\mathbf{k}''')$$
$$\times(|V(\mathbf{k}''-\mathbf{k})|^2+\tfrac{1}{2}|V(\mathbf{k}''-\mathbf{k})-V(\mathbf{k}'''-\mathbf{k})|^2)$$
$$\times f_{\mathbf{k}}{}^0 f_{\mathbf{k}'}{}^0(1-f_{\mathbf{k}''}{}^0)(1-f_{\mathbf{k}'''}{}^0),\quad(12)$$

while, for spin diffusion,

$$\dot{S}_D=2(kT^2)^{-1}(2\pi)^{-8}\int d\mathbf{k}d\mathbf{k}'d\mathbf{k}''d\mathbf{k}'''$$
$$\times\tfrac{1}{4}[(\mathbf{k}-\mathbf{k}'-\mathbf{k}''+\mathbf{k}''')\cdot\mathbf{v}]^2\delta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}''}-\epsilon_{\mathbf{k}'''})$$
$$\times\delta^3(\mathbf{k}+\mathbf{k}'-\mathbf{k}''-\mathbf{k}''')|V(k''-k)|^2 f_{\mathbf{k}}{}^0 f_{\mathbf{k}'}{}^0$$
$$\times(1-f_{\mathbf{k}''}{}^0)(1-f_{\mathbf{k}'''}{}^0).\quad(13)$$

In Eq. (12), we have made use of the symmetry of the scattering rate with respect to initial and final states to write the factor $\frac{1}{4}(\Phi_{\mathbf{k}\sigma}+\Phi_{\mathbf{k}'\sigma'}-\Phi_{\mathbf{k}''\sigma}-\Phi_{\mathbf{k}'''\sigma'})^2$ as $\Phi_{\mathbf{k}\sigma}\times(\Phi_{\mathbf{k}\sigma}+\Phi_{\mathbf{k}'\sigma'}-\Phi_{\mathbf{k}'''\sigma}-\Phi_{\mathbf{k}'''\sigma'})$.

We now resolve the integration so as to obtain the lead term in T at low temperatures. Separating angle and energy integration in the usual way,[10] we are able to write

$$\dot{S}_\kappa=[v^2p_F{}^3(m^*)^4/96\pi^7\mu^2]\int\sin\theta d\theta d\varphi$$
$$\times(\sin^2(\tfrac{1}{2}\theta)/\cos(\tfrac{1}{2}\theta))W(\theta,\varphi)\int d\epsilon d\epsilon' d\epsilon'' d\epsilon'''(\epsilon-\mu)$$
$$\times[(\epsilon-\epsilon')+\cos\varphi(\epsilon'''-\epsilon'')]f(\epsilon)f(\epsilon')(1-f(\epsilon''))$$
$$\times(1-f(\epsilon'''))\delta(\epsilon+\epsilon'-\epsilon''-\epsilon''')\quad(14)$$

and

$$\dot{S}_D=[v^2p_F{}^3(m^*)^4/96\pi^7]\int\sin\theta d\theta d\varphi(\sin^2(\tfrac{1}{2}\theta)/\cos(\tfrac{1}{2}\theta))$$
$$\times(1-\cos\varphi)W_{\uparrow\downarrow}(\theta,\varphi)\int d\epsilon d\epsilon' d\epsilon'' d\epsilon'''\delta(\epsilon+\epsilon'-\epsilon''-\epsilon''')$$
$$\times f(\epsilon)f(\epsilon')(1-f(\epsilon''))(1-f(\epsilon''')),\quad(15)$$

where

$$f(\epsilon)=\{\exp[\beta(\epsilon-\mu)]+1\}^{-1},\quad(16)$$

p_F is the Fermi momentum, θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$, and φ is the angle by which the plane containing the momenta $\mathbf{k}''$ and $\mathbf{k}'''$ is rotated relative to that containing $\mathbf{k}$ and $\mathbf{k}'$; also,

$$W(\theta,\varphi)=\pi\{|V(\mathbf{k}''-\mathbf{k})|^2+\tfrac{1}{2}|V(\mathbf{k}''-\mathbf{k})-V(\mathbf{k}'''-\mathbf{k})|^2\}$$
$$=\pi\{|V(2p_F\sin(\tfrac{1}{2}\theta)\sin(\tfrac{1}{2}\varphi))|^2$$
$$+\tfrac{1}{2}|V(2p_F\sin(\tfrac{1}{2}\theta)\sin(\tfrac{1}{2}\varphi))$$
$$-V(2p_F\sin(\tfrac{1}{2}\theta)\cos(\tfrac{1}{2}\varphi))|^2\},\quad(17)$$

while

$$W_{\uparrow\downarrow}(\theta,\varphi)=2\pi|V(2p_F\sin(\tfrac{1}{2}\theta)\sin(\tfrac{1}{2}\varphi))|^2.\quad(18)$$

The energy integration may now be completed; in Eq. (14), the coefficient of $\cos\varphi$ integrates to zero, leaving

$$\int d\epsilon d\epsilon' d\epsilon'' d\epsilon'''(\epsilon-\mu)(\epsilon-\epsilon')f(\epsilon)f(\epsilon')(1-f(\epsilon''))$$
$$\times(1-f(\epsilon'''))\delta(\epsilon+\epsilon'-\epsilon''-\epsilon''')=(8\pi^4/15)(kT)^5.\quad(19)$$

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[9] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, London, 1960), pp. 277 and 282.

[10] E.g., D. Pines and P. Nozières, *The Theory of Quantum Liquids* (W. A. Benjamin, Inc., New York, 1966), Vol. I, p. 60.

Similarly,

$$\int d\epsilon d\epsilon' d\epsilon'' d\epsilon''' f(\epsilon) f(\epsilon')(1-f(\epsilon''))(1-f(\epsilon''')) \times\delta(\epsilon+\epsilon'-\epsilon''-\epsilon''')=\tfrac{2}{3}\pi^2(kT)^3. \quad (20)$$

Thus we finally find

$$\dot{S}_\kappa=(45\pi^2\mu^2)^{-1}k^4T^3v^2p_F{}^3(m^*)^4 \times\langle W(\theta,\varphi)\sin^2(\tfrac{1}{2}\theta)/\cos(\tfrac{1}{2}\theta)\rangle \quad (21)$$

and

$$\dot{S}_D=(36\pi^4)^{-1}k^2T^2v^2p_F{}^3(m^*)^4 \times\langle W(\theta,\varphi)(1-\cos\varphi)\sin^2(\tfrac{1}{2}\theta)/\cos(\tfrac{1}{2}\theta)\rangle, \quad (22)$$

where

$$\langle f(\theta,\varphi)\rangle\equiv(4\pi)^{-1}\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta\, f(\theta,\varphi). \quad (23)$$

Equations (4) and (5) for the energy and particle currents are much more easily evaluated:

$$\mathbf{j}_\epsilon=(4\pi^3\mu m^*)^{-1}\int d\mathbf{k}\,\mathbf{k}(\epsilon_k-\mu)^2(\mathbf{k}\cdot\mathbf{v})f_{\mathbf{k}}{}^0(1-f_{\mathbf{k}}{}^0)/kT$$

$$=p_F{}^3\mathbf{v}(kT/\mu)(\tfrac{1}{9}kT) \quad (24)$$

and

$$\mathbf{j}_\uparrow=-\mathbf{j}_\downarrow=(8\pi^3m^*)^{-1}\int d\mathbf{k}\,\mathbf{k}(\mathbf{k}\cdot\mathbf{v})f_{\mathbf{k}}{}^0(1-f_{\mathbf{k}}{}^0)/kT$$

$$=p_F{}^3\mathbf{v}/6\pi^2. \quad (25)$$

Combining Eqs. (21), (22), (24), and (25) in Eqs. (1) and (2), we find for the variational transport coefficients,

$$\kappa_{\text{var}}=\left(\frac{5\pi^2}{9}\right)\frac{v_F{}^3}{m^*T\langle W(\theta,\varphi)\sin^2(\frac{1}{2}\theta)/\cos(\frac{1}{2}\theta)\rangle} \quad (26)$$

and

$$D_{\text{var}}=4\pi^2 \times\frac{v_F{}^2}{(m^*)^3k^2T^2\langle W_{\uparrow\downarrow}(\theta,\varphi)\sin^2(\frac{1}{2}\theta)(1-\cos\varphi)/\cos(\frac{1}{2}\theta)\rangle}, \quad (27)$$

or, aside from corrections involving Fermi-liquid effects,

$$D_{\text{var}}=\tfrac{3}{4}D_{\text{H}} \quad (28a)$$

and

$$\kappa_{\text{var}}=(5/12)\kappa_{\text{A-K}}. \quad (28b)$$

In these equations, $v_F=p_F/m^*$ and the subscripts H and A–K denote the solutions of Hone and of Abrikosov and Khalatnikov, respectively.

Actually, the only Fermi-liquid correction is a factor $1+F_0{}^a$ in the spin diffusion coefficient. The Landau parameter $F_0{}^a$ measures the exchange correction to the spin susceptibility and is less than one-tenth in dilute He^3-He^4 solutions.[2] We have included this correction in Eq. (27) for use in the calculations of the following section, although it really is of no importance to the eventual results.

If the expressions (10) and (11) happened to be exact solutions of the transport equation, then we would have found, instead of Eq. (28), $D_{\text{var}}=D_{\text{H}}$ and $\kappa_{\text{var}}=\kappa_{\text{A-K}}$. The deviation from this equality is in some sense a measure of the accuracy of the usual solutions. Because D_{var} and κ_{var} are derived from a variational principle, they are, on the face of it, at least as accurate, if not more so, than the usual solutions. It should also be noted that the variational solutions place lower bounds on the exact transport coefficients (for a given scattering amplitude), since they are based on the principle of maximizing the entropy production. On the other hand, the usual solutions overestimate the transport coefficients, since they are derived by effectively neglecting some of the scattering.

The difference between D_{var} and D_{H} is relatively small. However, we see that $\kappa_{\text{A-K}}$ could be more than a factor of two greater than the exact κ. The reason is that in thermal conductivity the quantity being transported is essentially $\epsilon_p-\mu$ so that particles far from the Fermi surface play an important role, and $(\epsilon_p-\mu)^2$ cannot generally be assumed to be $\ll(\pi kT)^2$. A similar calculation for the viscosity η yields

$$\eta_{\text{var}}=\tfrac{3}{4}\eta_{\text{A-K}}; \quad (29)$$

the coefficient of proportionality is the same as for the spin diffusion coefficient because the momentum and particle fluxes involve, to lowest order, the same powers of $\epsilon_p-\mu$.

II. APPLICATION TO DILUTE SOLUTIONS OF He^3 IN He^4

Turning to the dilute solutions of He^3 in He^4, let us suppose, for the moment, that the variational results D_{var} and κ_{var} are accurate. Then the BBP effective interaction $V_{\text{BBP}}(k)$, constructed to fit the spin diffusion coefficients at 1.3 and 5% He concentration using D_{H}, will no longer reproduce the data; rather, since D_{var}, as well as κ_{var}, scale inversely as the square of the interaction, one must replace the BBP interaction by

$$V_1(k)=(\tfrac{3}{4})^{1/2}V_{\text{BBP}}(k) \quad (30)$$

to fit the spin diffusion data. The prediction for the thermal conductivity using V_1 in the variational solution is then

$$\kappa=\frac{(5/12)}{(3/4)}\kappa_{\text{BBP}}=\frac{5}{9}\kappa_{\text{BBP}}\simeq0.56\kappa_{\text{BBP}}, \quad (31)$$

where κ_{BBP} is the prediction based on the BBP interaction and the Abrikosov-Khalatnikov formula for κ. Now the recent measurement of Abel *et al.*[1] gives

$$\kappa_{\text{expt}}(1.3\%)=0.71\kappa_{\text{BBP}}(1.3\%)$$

and

$$\kappa_{\text{expt}}(5.0\%)=0.46\kappa_{\text{BBP}}(5.0\%) \quad (32)$$

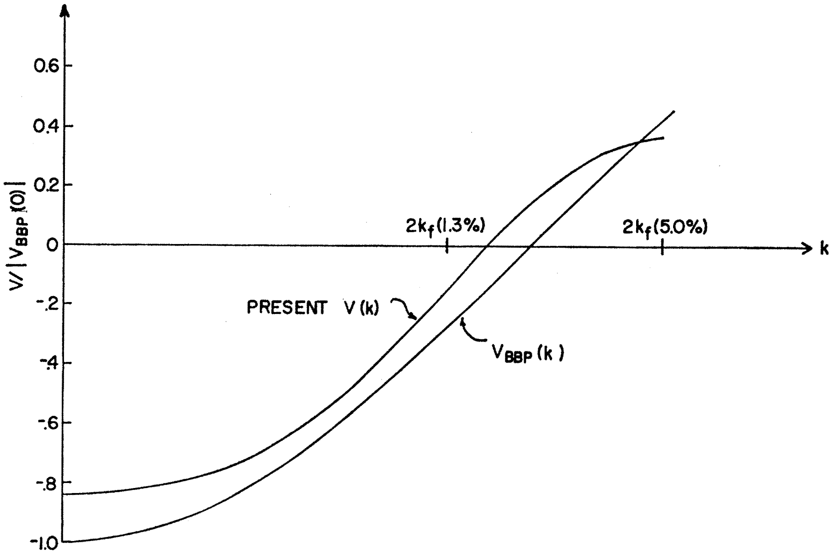

FIG. 1. Effective interaction $V(k)$, as defined in Eq. (33) of the text, plotted in units of $|V_{BBP}(0)|=0.075m_4s^2/n_4$, and compared with $V_{BBP}(k)$.

in the Fermi-liquid regime; comparing with Eq. (31) we see that the corrections suggested by the variational solutions greatly improve the consistency of the BBP theory with experiment.

Instead of determining $V(k)$ from the experimental D and then using this result to predict κ, one could use all the measurements of D and κ to obtain a more definitive form for $V(k)$. This was tried by Abel *et al.*[1]; it is noteworthy, though, that they were unable to construct a reasonable form for $V(k)$ that would give, via the formulas D_H and κ_{A-K}, good quantitative fits to the measured D and κ for 1.3 and 5% solutions. We have carried out a similar search[11] for a $V(k)$ by machine, using both the usual formulas D_H and κ_{A-K} and the variational formulas for D and κ. We find that it is impossible to improve materially on the results of Ref. 1 with any reasonably slowly varying $V(k)$ when using the former set of formulas. With the variational formulas, however, we were able to do considerably better; one of our best results is a power-series fit:

$$V(k)=V_0[1-1.15y-4.16y^2+6.21y^3-2.32y^4], \quad (33)$$

where $y=(k/2k_0)^2$, k_0 is the Fermi momentum of a 5.0% solution, $k_0/\hbar=0.318$ Å^{-1}, and

$$V_0=-0.064m_4s^2/n_4, \quad (34)$$

with m_4 the He^4 atomic mass and s and n_4 the first-sound velocity and number density of pure He^4 at $T=0$. The interaction (33) is shown in Fig. 1; $V_{BBP}(k)$ is included for comparison. The magnitude of $V(k)$ is generally smaller than that of $V_{BBP}(k)$, largely as a result of the factor $\frac{3}{4}$ in Eq. (28a). We note that $V(k)$ begins to flatten off at high k, an alteration in qualitative agreement with the work of Campbell.[12] As can be seen in Table I, the interaction $V(k)$ gives generally a 10% fit to the experiments, which is within the published experimental uncertainty, and a substantial improvement over the best fits one can obtain using the Hone and Abrikosov-Khalatnikov solutions for the transport coefficients. The Landau coefficients calculated from this new $V(k)$ are not markedly different from those calculated from V_{BBP}.

It is significant that no physically reasonable form for the effective interaction could be chosen to give an exact fit to the transport data. Beyond experimental uncertainty, this must be attributed to the inherent inaccuracy of the variational solutions for the transport coefficients, combined with the errors entailed in the approximation of the exact scattering amplitude by a simple velocity- and concentration-independent effective interaction. The relative magnitude of the velocity and concentration dependence of the scattering amplitudes should be on the order of the concentration, and hence could produce corrections on the order of 10% in the transport coefficients. These corrections are small, however, compared with the improvements made by using the variational solutions to the transport equation instead of the usual solutions.

In summary, we wish to emphasize that the generally accepted solutions to the transport equation in the Landau Fermi-liquid theory may contain considerable inaccuracies, and that the present situation provides the

TABLE I. Values of κT (erg/sec cm °K) and DT^2 [cm^2(°K)^2/sec] for 1.3 and 5.0% He^3 concentrations, as calculated from the effective interaction $V(k)$ in Eq. (7), and from experiment.[a]

	$\kappa T(1.3\%)$	$\kappa T(5.0\%)$	$DT^2(1.3\%)$	$DT^2(5.0\%)$
Experiment	11	24	17.2×10^{-6}	90×10^{-6}
Calculated	10	26	18.6×10^{-6}	82×10^{-6}

[a] See Refs. 1 and 3.

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[11] The possibility of nonmonotonic behavior of $V(k)$ was allowed for.

[12] L. J. Campbell, Phys. Rev. Letters **19**, 156 (1967).

first experimental test of these inaccuracies. Within the context of the variational calculation, and within recognized theoretical limitations, the use of an effective interaction, as proposed by BBP, is consistent with recent measurements of the thermal conductivity and spin diffusion coefficients of dilute solutions of He^3 in He^4. At the same time, improved solutions to the transport equation are evidently desirable in general. It is straightforward to arrive at these within the variational formulation by using higher Legendre polynomials $P_l(\mathbf{k}\cdot\mathbf{v}/kv)$ with variable coefficients in the trial functions $\Phi_{\mathbf{k}\sigma}$. These coefficients should be chosen to maximize the transport coefficients. The values of the variable coefficients will depend on the shape of the quasiparticle scattering amplitudes; thus, one is in the position of having to determine both the detailed solution of the transport equation and the quasiparticle scattering amplitude from experiment. It is our feeling that there is little to be gained from this calculation for the particular case of the dilute He^3-He^4 solutions at the present time, especially in view of the consistency with experiment of the simpler calculation presented here.

Erratum

Alignment of the H_2^+ Molecular Ion by Selective Photodissociation. II. Experiments on the Radio-Frequency Spectrum, C. B. Richardson, K. B. Jefferts, and H. G. Dehmelt [Phys. Rev. **165**, 80 (1968)]. In the first paragraph the third sentence should read: "Interesting small effects such as unknown electron-proton interactions, radiative corrections, centrifugal stretching, vibrational effects, rotational mixing of excited states, and electron slippage should be revealed." The sentence including Eq. (1) should read: "The proper field is derived from a potential of the form

$$\phi=\phi_0 \cos\Omega t[(z^2-\tfrac{1}{2}r^2)/z_0^2] \qquad (1)$$

resulting from the application of a cosine emf of frequency Ω between a hyperbolic electrode of the form $r^2=2z^2+r_0^2$ and a pair of the form $r^2=2(z^2-z_0^2)$." The proper expression for the well depth is $eD_z=e\phi_0\omega_z/\sqrt{2}\,\Omega$. Equation (11) should read:

$$3I_{z'}S_{z'}=\mathbf{I}\cdot\mathbf{S}-[2/(2K-1)(2K+3)]\sum_{ij}\tfrac{3}{2}I_iS_j(K_iK_j+K_jK_i-\delta_{ij}K^2).$$

Equation (12) should read:

$$\begin{aligned}6I_{z'}S_{z'}=[2I_zS_z+I^-S^++I^+S^-]-[&4I_zS_z(3K_z^2-K^2)-(I^+S^-+I^-S^+)(3K_z^2-K^2)\\&+3(I_zS^-+I^-S_z)(K_zK^++K^+K_z)+3(I_zS^++I^+S_z)(K_zK^-+K^-K_z)\\&+3I^+S^+K^-K^-+3I^-S^-K^+K^+]/(2K-1)(2K+3).\end{aligned}$$

The following paragraph was omitted: "ACKNOWLEDGMENTS: We are indebted to Philip Ekstrom for his help with the design of the digital processing equipment, to Jacob Jonson for the glassblowing on several ion-storage tubes, and to Klaus Zieher for proofreading parts of the manuscript."