Heat Torques and Circulation in Superfluid Helium

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(Received 15 January 1968)

The small torques experienced by heated cylinders immersed in rotating superfluid helium have been measured and the results are compared with the predictions made for "heat-exchange" torques by Penney and Overhauser. They have pointed out that whenever there is a net superfluid circulation about a heated object, the object should experience a torque proportional to the circulation and power input and inversely proportional to the absolute temperature. The measured torques show this expected dependence on power and temperature. The existence of such torques is inherent in the two-fluid thermohydrodynamics of He II and is a result of the transfer of superfluid angular momentum to the object as superfluid is converted to normal fluid at the heated surface. Torques of order 10^{-6} to 10^{-7} dyn cm are observed at 1.30°K for power inputs of a few milliwatts and for cylinders 2 mm in diameter and 1 cm long. If the superfluid circulation is quantized, as first suggested by Onsager and Feynman, the torque should also be quantized. The torque values measured in the present experiment correspond to those expected for a few quanta of circulation about the cylinder, but the present experimental noise levels do not permit clear resolution of individual quantum levels.

I. INTRODUCTION

It has been pointed out recently by Penney[1] that a heated object immersed in flowing superfluid helium will experience forces even in pure potential flow. The existence of such forces, which would not be expected for pure potential flow of a *classical* nonviscous fluid, is entirely a consequence of the "two-fluid" thermohydrodynamics of He II and results from the transfer of superfluid momentum to the object as superfluid is converted to normal fluid at the heated surface. A case of particular interest is that of a superfluid flow pattern which gives a net "circulation" ($\oint \mathbf{v}_s \cdot d\mathbf{l} \neq 0$) about the heated object. In this situation, the "heat-exchange" forces should produce upon the object a net torque which can serve to give a direct measure of the circulation.[2]

In the present experiments, we have measured torques exerted on small heated cylinders suspended in rotating superfluid helium. The observed dependence of these torques on the heater power and bath temperature is consistent with the predictions of the heat-exchange analysis.[2] The observed torque magnitudes correspond typically to net circulations of the order of a few quanta,[3] lending support to the speculation[2] that the approach might be useful for investigating the predicted quantization of circulation in superfluid helium.

II. ANALYSIS OF THE HEAT-EXCHANGE TORQUE PROBLEM

The appearance of forces on heated surfaces immersed in flowing superfluid helium is a direct consequence of the peculiarities of the two-fluid hydrodynamics which characterizes the flow of He II. In order to understand the physical origin of the forces we must consider the boundary conditions for the normal and superfluid velocity components. Since the normal fluid is assumed to be a classical viscous fluid, the tangential velocity component $v_{\parallel}^{n}$ must vanish at a solid surface. The superfluid is assumed to be inviscid and to undergo potential flow and thus *may* have a nonzero tangential velocity at the surface. In the two-fluid model, the condition that there be no mass flow across a solid boundary places restrictions not on the separate velocity fields $v_{\perp}^{s}$ and $v_{\perp}^{n}$ directly, but on the combined mass flow, requiring only that $\rho_s v_{\perp}^{s} + \rho_n v_{\perp}^{n} = 0$ at a solid surface. At a heated surface a counterflow of normal and superfluid is established with the normal fluid streaming away from the surface. The boundary conditions then require that the superfluid have a velocity component perpendicular to the heated surface which is sufficient to satisfy the condition on the combined mass flow.

Consider now the situation in which the heated surface is immersed in a steady superfluid flow. The superfluid which moves to the surface and is converted to normal fluid must give up all of its tangential momentum to the boundary as it arrives. If the heat is supplied to the surface at a rate $\dot{Q}$ then the total mass of superfluid converted to normal fluid per unit time will be simply $\dot{Q}/S_nT$, where S_n is the specific entropy of the normal fluid and T is the temperature of the bath. In the temperature range of interest in the present experiments, from about 1.2°K to T_λ, S_n is nearly independent of temperature. Thus the force transmitted to the boundary will be just $F = \dot{Q}v^s/S_nT$.

The flow pattern of particular interest here is the vortex pattern associated with a persistent net circulation $\Gamma \equiv \oint \mathbf{v}_s \cdot d\mathbf{l}$ about an object immersed in He II. If the object is a heated cylinder of radius a then the angular momentum transmitted to the cylinder per unit mass of converted superfluid will be just

$$av^s(r=a) = \Gamma/2\pi\,, \quad (1)$$

[1] R. Penney, Phys. Fluids **10**, 2147 (1967).
[2] R. Penney and A. W. Overhauser, Phys. Rev. **164**, 268 (1967).
[3] L. Onsager, Nuovo Cimento **6**, Suppl. 2, 249 (1949); R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1955), Vol. I, Chap. II.

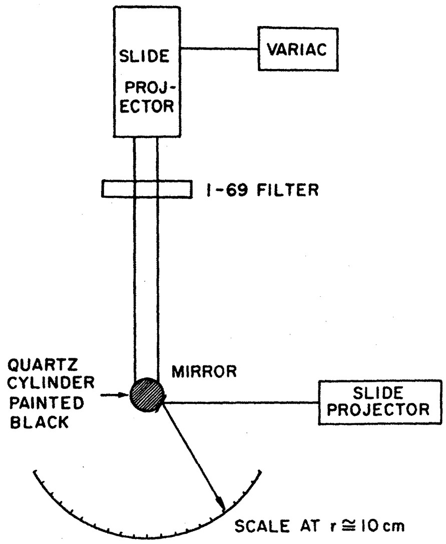

FIG. 1. Block diagram showing the relationship between the light beam used to heat the cylinder and the system used for angular deflection measurements.

and the resulting torque will be

$$\tau=\frac{\dot{Q}\Gamma}{S_n T 2\pi}. \tag{2}$$

Strong evidence exists[4] that vortex rings formed by ions accelerated in liquid He II carry circulation of one quantum h/M, where h is Planck's constant and M is the mass of a helium atom. Although a number of ingenious attempts have been made to investigate the circulation in free vortex lines and in vortices trapped on solid objects, multiple quantization of circulation in He II has not yet been clearly demonstrated.[5–8] It is, nevertheless, generally expected that $\Gamma=n(h/M)$, where n is an integer. If the circulation is quantized, it is clear that one must also expect quantization of the heat-exchange torque given by Eq. (2).[2]

III. EXPERIMENTAL APPROACH

In the present experiments, heat-exchange forces were investigated by measuring the torque on a small heated glass cylinder suspended by a sensitive quartz torsion fiber in rotating superfluid helium. The probe cylinders were painted black, and heat was supplied to them by illuminating them with a calibrated projection lamp as shown in Fig. 1. A small mirror mounted on the cylinder permitted measurement of the angular deflection by the usual galvanometer technique. A low-intensity light source was used for the read-out beam in order to reduce extraneous heating of the cylinder. The light beam used to heat the cylinder is first passed through a Corning 1-69 filter which restricts the power to the visible spectrum. A simple calibration of the total light power incident on the Dewar system can then be carried out using an E.G. and G. "Lite-Mike"[9] which reads directly the total light power in this range. Furthermore, with the infrared thus eliminated, no problems arise in calibrating the total heating power as a function of the easily adjusted projection lamp voltage. Losses at the many glass surfaces in the Dewar system can be calculated rather well and the absorption of the black paint in the visible can also be known reasonably well. In these experiments, the over-all power calibration is probably good to within 30% while the relative power levels probably have a precision of about 5% and agree well with the published data for such lamps.

Fused quartz fibers with torsion constants of order 10^{-5} dyn cm/rad for a 10-cm length were produced by standard methods, and these give sufficient basic sensitivity for the detection of circulation at the single-quantum level. The fiber torsion constants were determined by measuring the period of small torsional oscillations of a cylinder with known moment of inertia. The glass cylinders used for calibration and in the experiments ranged from 1.6 to 2.5 mm diam and were about 1 cm long. The adjustable suspension system used for the cylinder-fiber assembly has been described

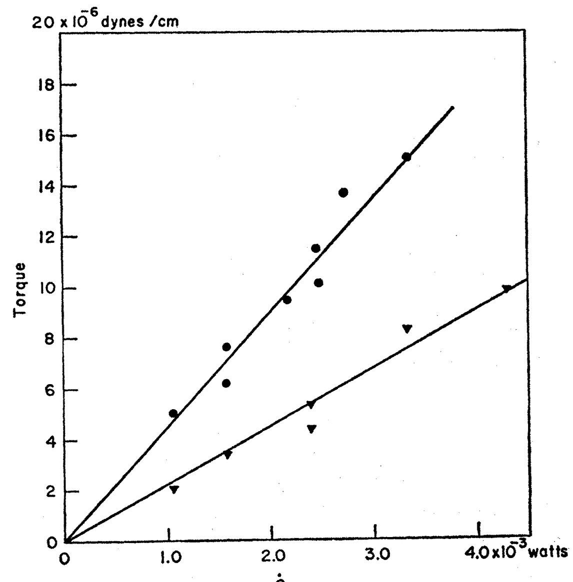

FIG. 2. Examples of the torque on a heated cylinder (2 mm diam, 1 cm long) immersed in rotating superfluid helium as a function of the heater power. The agreement with the linear dependence predicted by the analysis is within the limits of experimental error.

[4] G. W. Rayfield and F. Reif, Phys. Rev. 136, A1194 (1964).
[5] W. F. Vinen, Proc. Roy. Soc. (London) A260, 218 (1961).
[6] S. C. Whitmore and W. Zimmerman, Jr., Phys. Rev. Letters 15, 389 (1965).
[7] W. A. Steyert, R. D. Taylor, and T. A. Kitchens, Phys. Rev. Letters 15, 546 (1965).
[8] G. B. Hess and W. M. Fairbank, Phys. Rev. Letters 19, 216 (1967).

[9] Edgerton, Germeshausen, and Grier, Inc., Bedford, Mass., Model 560.

previously.[10] As a result of the asymmetry of the cylinder due to the small attached mirror, it was also necessary to shield the suspended system from stray electric fields, and this was accomplished by surrounding the Dewar system with a cylindrical brass screen. This slight asymmetry also causes the cylinder to respond to the usual Bernoulli forces associated with the flow of either normal fluid or superfluid. In the presence of a persistent superfluid circulation this will merely cause a small steady zero offset from which the deflection due to the heat-exchange torque can then be measured. It should be noted that the suspended cylinders used in the present work do *not* render the bath doubly connected.

In a typical run, the helium was brought into rotation just above the λ point by crudely stirring it at 30 rpm for periods of 1 h or more with an impeller mounted on the fiber suspension shaft. The bath was then slowly pumped below T_λ, the stirring stopped, and the normal-fluid motion permitted to decay for another hour before the heat-exchange torque measurements were made. The equilibrium position with the heater off was established first and changes in this position determined as a function of the heater power and the bath temperature. The sign of the circulation deduced from the initial torque measurements agreed with that expected from the sign of the driven rotation. On a few occasions, the larger circulations proved to be unstable and were observed to decay and in some cases even to

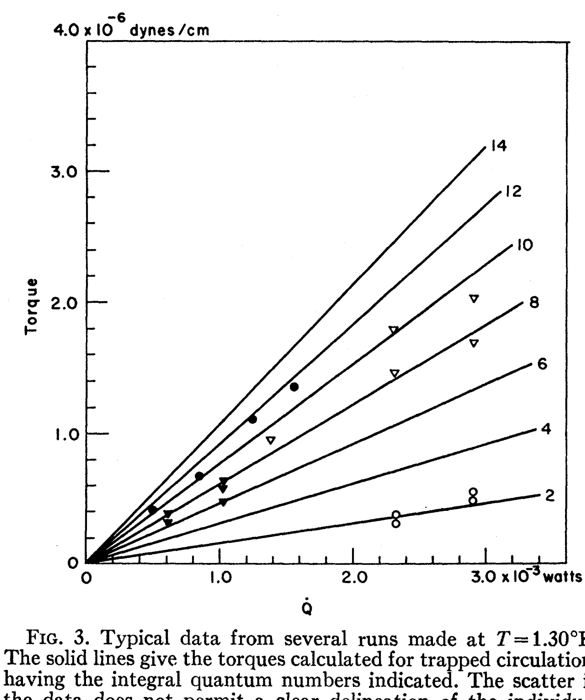

FIG. 3. Typical data from several runs made at $T=1.30°K$. The solid lines give the torques calculated for trapped circulations having the integral quantum numbers indicated. The scatter in the data does not permit a clear delineation of the individual quantum levels.

[10] J. E. Mercereau and T. K. Hunt, Phys. Rev. Letters 8, 243 (1962).

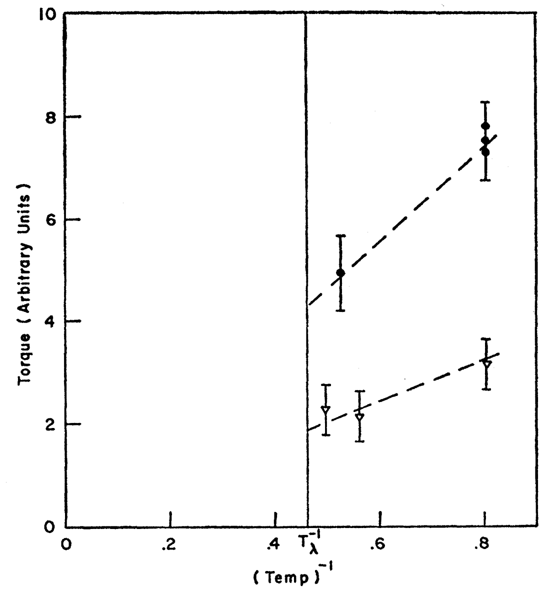

FIG. 4. Data showing the effect of temperature on the torque observed at a power level of 1.5 mW for two values of the apparent circulation. Within the rather large experimental uncertainties, the results are consistent with the T^{-1} temperature dependence predicted by the analysis.

change sign during the early part of a run.[5,6] Data from runs for which the initial readings could not later be repeated were discarded. Measurements made above T_λ showed a small "radiometer effect,"[11] whose magnitude and sign were, as expected, a function of the relative orientation of the cylinder mirror and light beam. In each case, the sign of the small torque *above* T_λ was that expected for the radiometer effect in a normal liquid. Below the λ point, the torques measured were independent of the relative orientation of cylinder and light beam and were commonly larger than the simple radiometer torque seen above T_λ.

Torques were also observed when no deliberate stirring was used during pump down.[5,6] Measurements made during such runs tended toward smaller deduced values of the circulation and exhibited no preferred sign of rotation from run to run. We believe that the turbulence of the boiling helium above T_λ can easily account for a small net circulation of either sense becoming trapped about the probe cylinder.

IV. EXPERIMENTAL RESULTS

The torque on the cylinder was measured as a function of the heat-input power $\dot{Q}$ by varying the voltage on the illuminating projector and observing the resulting changes in the angular deflection of the cylinder. It was necessary in general to read the extremes of the omnipresent residual fluctuations in the position of the light spot in order to obtain averages for determining the equilibrium position. Typical results for the

[11] P. G. Strelkov, J. Phys. (USSR) 3, 53 (1940).

torque-$\dot{Q}$ relation in a number of runs are shown in Figs. 2 and 3. Each of the points shown is the average midpoint of about 5–10 swings. Within the experimental uncertainties, the torque is linearly proportional to the power input in agreement with the predictions made earlier.[2] The straight lines in Fig. 3 show the expected dependence of the torque on power for the various circulation quantum levels indicated. We include these lines in order to illustrate the torque magnitudes and the sensitivities involved and do not suggest that our results confirm multiple quantization of circulation in superfluid helium. The present residual noise levels preclude a clear cut delineation of the individual quantum levels.

Data for two runs on the temperature dependence of the torque at constant heat-input level are shown in Fig. 4. Within the rather large experimental uncertainties for runs of this type, the results are consistent with the T^{-1} dependence of Eq. (2). The maximum fractional change in the torque with temperature is rather small since one is limited to temperatures from T_λ down to about 1.1°K by the requirement that the power level be kept in the range for which the thermohydrodynamics of He II remains linear. This power level is reached more quickly both at very low temperatures and near T_λ. For power levels outside this range, violent instabilities occur and meaningful torque measurements are not possible.

V. HEAT TORQUES AND QUANTIZATION OF SUPERFLUID CIRCULATION

In addition to a further demonstration of the utility of the two-fluid model, the heat-exchange torque phenomenon provides another approach for the study of the predicted quantization of superfluid circulation about solid objects in He II. In this connection, a number of general features of the method should be emphasized. The heat-exchange torque is insensitive to motion of the normal fluid so that no assumptions about the normal-fluid velocity need be made when interpreting the results. While it is necessary to use a relatively large probe to permit large heat inputs within the linear range of the He II thermohydrodynamics, this should at the same time permit the observation of larger stable trapped circulations than with the far smaller probes required by the other methods previously attempted.[12] The difficulties encountered in previous attempts to produce known initial circulations while cooling through T_λ remain and will probably be somewhat aggravated due to the very small angular velocities associated with small quantum-number circulations in the larger systems used for heat-torque measurements.

There are several obvious refinements of the present system which should improve its utility for study of the predicted quantization phenomenon. If the problems which were attributed to partially attached vortices in the vibrating-wire experiments[5,6] are present, they may be aggravated in the present arrangement by the fact that the simple suspended system leaves the helium simply connected. This can be corrected by fixing the probe cylinder with fiber suspensions at both top and bottom. Such an arrangement would also make feasible the use of resistance heating which would permit a better calibration of the heat input to the probe cylinder.

Though the basic torque sensitivity of the present system is sufficient to permit the detection of a single quantum of circulation, the average noise level would not permit a clear delineation of individual quantum levels. By switching the heater on and off and thus driving the suspended system in resonance, an immediate gain of $2q/\pi$ in signal-to-noise ratio may be obtained.[13] Here, q is the quality factor for the torsional oscillations of the system. For the systems used thus far, this would yield a gain of the order of 10 and should permit individual quantum levels to be resolved.

VI. CONCLUSION

We have observed the appearance of torques exerted upon small cylinders suspended in rotating superfluid helium when they are heated by an external light source. The power dependence and temperature dependence of these torques are consistent with the predictions made for heat-exchange torques by Penney. This result may be regarded as another demonstration of the usefulness of the two-fluid model for analyzing the thermohydrodynamics of liquid He II. The heat-exchange torque phenomenon can also provide another approach for the study of quantization of superfluid circulation about solid objects in He II.

ACKNOWLEDGMENTS

The author would like to thank R. Penney and A. W. Overhauser for their interest in this experiment and for many helpful discussions. He also wishes to thank the staff of the Phoenix Memorial Laboratory at the University of Michigan for their hospitality during a portion of this work.

[12] D. J. Griffiths, Proc. Roy. Soc. (London) **A277**, 214 (1964).

[13] F. V. Hunt, Rev. Sci. Instr. **34**, 1254 (1963).