havior at shorter times and smaller distances (in the kinetic stage). We believe it is extremely valuable, if not crucial in attempting to assess any theory, to translate its verbal assumptions into corresponding statements about the kinetic stage. Specifically it is necessary to state what the theory implies in terms of the "measurements" or correlation functions which are determined from molecular dynamics. Because of certain ambiguities, we have been unable to do this for the Rice-Allnatt theory. It is to be hoped that an improved self-consistent version of their theory will be pursued, and that other theories will also be cast in such testable forms.

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Tests of Born Approximations; Differential and Total Cross Sections for Elastic Scattering of 100- to 400-eV Electrons by Helium*

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The angular dependence of elastic scattering of 100- to 400-eV electrons by helium atoms has been measured for scattering angles of 5° to 30° with a high-resolution electron spectrometer. The observed relative differential cross sections were converted to an absolute basis by measuring their ratio to known cross sections for excitation to the 2^1P state. Integration over angles yielded total elastic scattering cross sections. Large deviations from the Born approximation are found, especially for small electron energies and small scattering angles. Experimental and theoretical results are analyzed and compared in some detail. Exchange and polarization are chiefly responsible for deviations from the Born approximation. The Ochkur (Bonham) approximation is shown to be almost equivalent to the first-order exchange approximation for electron energies of 350 and 500 eV. Analytical series expansions for differential and total Born cross sections are given. Our experimental results are in qualitative agreement with results of earlier experiments, but are quantitatively different and extend to smaller scattering angles.

I. INTRODUCTION

FOR the past 40 years many measurements and calculations on elastic scattering of electrons by helium atoms have been carried out. While earlier measurements of inelastic cross sections were in general severely limited by lack of energy resolution, this was not the case for elastic scattering since the ground state of an atom is well separated from the excited states. Therefore no special energy selection of the incident electron beam and no very good energy selection of the scattered electrons are required. Earlier experimental investigations most relevant to the present work are those of Hughes, McMillen, and Webb,[1] Werner,[2] and Westin.[3] Hughes *et al.*[1] and Werner[2] used conventional electron guns consisting of a cathode and some apertures to determine the geometry and energy of the electron beam. Hughes *et al.* measured the angular dependence of the elastically scattered electrons for various electron energies T between 25 and 700 eV with a 127° electrostatic energy analyzer. Their relative data were normalized on Born-approximation cross sections for $T=700$ eV. We reanalyzed the tabulated data of Ref. 1 and found that the ratio of tabulated and Born cross sections (of Sec. III) for $T=700$ eV varied from 1.15 for 7°, 1.13 for 9.5°, 0.98 for 12°, down to 0.84 for 37° and 47°, and up again to, for example, 1.8 for 117°. This suggests that the Born approximation is not yet valid at 700 eV and makes a normalization on Born cross sections inaccurate. In our opinion, the errors caused by this normalization may be 5–10%. Furthermore, Hughes *et al.* claim that their data for different energies of the incident electrons are comparable. This again is very dubious. Their electrostatic energy analyzer probably had a smaller transmission efficiency the smaller the energy of the incident electrons. Indeed, for 200 eV their[1] differential cross sections are only about 20% lower than our present cross sections, but they are lower by about 80–100% for 100 eV. Khare and Moiseiwitsch,[4] with rather sophisticated theoretical calculations, found reasonable agreement with the data of Ref. 1 for 700, 500, and 350 eV, but also significantly

* Research supported by the Advanced Research Projects Agency of the Department of Defense under Project DEFENDER.
† Guest worker from the University of Utrecht, The Netherlands, supported by NATO.
[1] A. L. Hughes, J. H. McMillen, and G. M. Webb, Phys. Rev. **41**, 154 (1932).
[2] S. Werner, Proc. Roy. Soc. (London) **A139**, 113 (1933).
[3] S. Westin, Kgl. Norske Videnskab. Selskabs, Skrifter **2**, 1 (1949).
[4] S. P. Khare and B. L. Moiseiwitsch, Proc. Phys. Soc. (London) **85**, 821 (1965).

higher cross sections for smaller T. Werner[2] used a "zonal" collection method with a ring-shaped Faraday cup and a retarding field to avoid measuring the inelastically scattered electrons. Werner directly measured absolute cross sections, however, only for scattering angles $\theta=45°$ and $90°$. Westin[3] used a magnetic electron spectrometer consisting of a magnetic monochromator and a magnetic analyzer. Westin performed absolute measurements for various angles and energies up to about 350 eV. Westin mentioned difficulties in determining the variation of effective scattering volume (path length) with scattering angle and gives no tabulation of cross sections as a function of scattering angle. Furthermore, Werner[2] and Westin[3] used a McLeod gauge for measuring the absolute pressure. At the time that their[2,3] measurements were performed, the McLeod-gauge pumping effect[5] was not yet known and their absolute cross sections may for this reason be subject to a small systematic error. Nevertheless, Werner and Westin's cross sections are for small T higher than those of Hughes *et al.*[1] and agree much better with our results.

II. PRESENT EXPERIMENT

In the present work we used the high-energy-resolution electron spectrometer designed by Kuyatt and Simpson[6] (see Fig. 7 of Ref. 6). Previous measurements with this apparatus were reported by Simpson, Menendez, and Mielczarek[7] and by Vriens, Simpson, and Mielczarek.[8] The procedure employed here for elastic scattering is very much the same as the procedure employed by Vriens *et al.*[8] for inelastic scattering. Again we operated the electron monochromator with an angular spread (full width at half-maximum) of the incident electron beam of about 0.75° and an energy spread of about 0.1 eV. Although this small energy spread is not required for measuring the angular dependence of elastic scattering, it enables us to measure the intensity of the 2^1P energy loss unaffected by contributions from nearby energy loss peaks, and therefore enables us to normalize our relative data using known cross sections for 2^1P excitation.[8] The narrow angular spread of the incident electron beam together with a correspondingly narrow angular acceptance of the energy analyzer enabled us to make accurate measurements at scattering angles down to 5°, while Hughes *et al.*,[1] for example, measured down to 11° at 200 eV and 14° at 100 eV. Small scattering angles appear to be most interesting since the greatest deviations from the Born approximation occur at small angles. This fact was qualitatively well known since the early work of Hughes *et al.*[1]

Our measurement procedure was as follows: The electron beam energy was set to its desired value and the apparatus was adjusted using the unscattered zero-angle beam. To ensure proper operation of the apparatus, the energy and angular profiles of the beam were measured and the apparatus was readjusted if required. We then measured the elastically scattered electron intensity $I_{\rm el}$ at several angles from 5° to 30° on opposite sides of the incident beam and averaged the readings from the two sides to reduce the effects of any asymmetries on the angular distribution. (The data for 30° were only taken on one side and are therefore slightly less accurate.) The ratios $I_{\rm el}/I_{2^1P}$ of elastically and 2^1P scattered electron currents were measured at angles of 5° and 10° on both sides of the incident beam and the measurements from the two sides were averaged. This series of measurements was carried out for incident electron energies of 100, 150, 200, 300, and 400 eV. Relative cross sections $\sigma(\Omega)$ per unit solid angle (steradian) were obtained by multiplying $I_{\rm el}$ by $\sin\theta$, where θ is the scattering angle, since the effective path length of the electrons in the scattering chamber is proportional to $1/\sin\theta$. The elastic to 2^1P ratios and the corresponding differential cross sections $\sigma_{2^1P}(\Omega)$ from Ref. 8 were used to put our relative $\sigma(\Omega)$ for elastic scattering on an absolute basis using the equation

$$\sigma(\Omega)=(I_{\rm el}/I_{2^1P})\sigma_{2^1P}(\Omega)\,. \qquad (1)$$

Differential cross sections obtained from the 5° ratio usually agreed with those obtained from the 10° ratio to better than 1%, with the maximum difference being 2%. The final cross sections were obtained by averaging the results of the two normalizations. The measured intensity ratios $I_{\rm el}/I_{2^1P}$ are given in Table I and the resulting differential cross sections are given in column 4 of Table II.

The accuracy of the cross sections is determined by three factors. (a) Normalization procedure: Combining the internal consistency of the procedure with the errors in the ratios given in Table I, we estimate a maximum uncertainty of 3%. (b) Accuracy of 2^1P cross sections: From Ref. 8 we estimate the uncertainties to be 3% for 400, 300, and 200 eV, 5% for 150 eV, and 8% for 100 eV. (c) Uncertainty in angular dependence of elastic scattering: Based on the possible error in individual readings and on the reproducibility of the readings, the

TABLE I. Ratios of experimental elastic and 2^1P scattered intensities and estimated uncertainties of these ratios.

T (eV)	$I_{\rm el}/I_{2^1P}$ $\theta=5°$	$\theta=10°$
400	0.596±0.015	3.53±0.08
300	0.547±0.020	2.44±0.08
200	0.585±0.015	1.92±0.08
150	0.696±0.030	1.83±0.08
100	1.05 ±0.02	2.03±0.04

[5] H. Ishii and K. Nakayama, Vacuum Symp. Trans. **8I**, 519 (1961); C. Meinke and G. Reich, Vakuum-Tech. **12**, 79 (1963); A. E. de Vries and P. K. Rol, Vacuum **15**, 135 (1965).
[6] C. E. Kuyatt and J. A. Simpson, Rev. Sci. Instr. **38**, 103 (1967).
[7] J. A. Simpson, M. G. Menendez, and S. R. Mielczarek, Phys. Rev. **150**, 76 (1966).
[8] L. Vriens, J. A. Simpson, and S. R. Mielczarek, Phys. Rev. **165**, 7 (1968).

uncertainty in the shape of the angular dependence has been estimated to vary from 2% at 5° to 6% at 30°.

Combining these uncertainties as the square root of the sum of the squares, we find that the uncertainty in the differential cross sections varies from 5% at small angles and energies of 200–400 eV to 10% at large angles and small energies.

III. COMPARISON WITH THE BORN APPROXIMATION

In the first Born approximation, which applies at sufficiently high T, the differential cross section σ_K per unit momentum transfer is given by[9]

$$\sigma_K d(Ka_0)=\frac{8\pi a_0^2 R}{T(Ka_0)^3}|Z-F(K)|^2 d(Ka_0). \quad (2)$$

Here a_0 is the Bohr radius, $\mathbf{K}\hbar$ is the momentum transfer, Z is the atomic number, R is the Rydberg energy, and $F(K)$ is the atomic form factor defined by

$$F(K)=\sum_s(\Psi_0|\exp(i\mathbf{K}\cdot\mathbf{r}_s)|\Psi_0), \quad (3)$$

where Ψ_0 is the ground-state wave function. The summation in Eq. (3) extends over all the atomic electrons with position vectors $\mathbf{r}_s$. The differential cross section $\sigma(\Omega)$ per unit solid angle (steradian) is

$$\sigma(\Omega)d\Omega=\frac{4a_0^2}{(Ka_0)^4}|Z-F(K)|^2 d\Omega, \quad (4)$$

with $d\Omega=\sin\theta\, d\theta\, d\varphi$. For helium, Eq. (4) may be rewritten as

$$\sigma(\Omega)=\frac{16a_0^2}{(Ka_0)^4}|1-(\Psi_0|\exp(i\mathbf{K}\cdot\mathbf{r}_1)|\Psi_0)|^2$$

$$=\frac{16a_0^2}{(Ka_0)^4}\left|\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(Ka_0)^{2n}}{(2n)!}X^{(2n)}\right|^2, \quad (5)$$

with

$$X^{(2n)}=(\Psi_0|(\hat{K}\cdot\mathbf{r}_1)^{2n}|\Psi_0)/a_0^{2n}. \quad (6)$$

Here $\hat{K}$ is the unit vector in the direction of $\mathbf{K}$. From the above relations it follows that $\sigma(\Omega)$ is an even function of $\mathbf{K}$ and

$$\lim_{K\to\infty}\sigma(\Omega)=16a_0^2/(Ka_0)^4. \quad (7)$$

From Pekeris's[10] expectation value of r_1^2 it further follows, using $X^{(2)}=\frac{1}{3}(\Psi_0|r_1^2|\Psi_0)/a_0^2$ and Eq. (5), that

$$\lim_{K\to 0}\sigma(\Omega)=4a_0^2(X^{(2)})^2=0.63307a_0^2. \quad (8)$$

The most accurate theoretical Born-approximation calculations for helium have probably been made by Kim and Inokuti,[11] who used the 20-term Hylleraas wave function of Hart and Herzberg[12] to calculate, among other functions, values of $[Z-F(K)]/(Ka_0)^2$ for 21 values of Ka_0 between 0 and 10.

In the present case we compare experimental and Born cross sections for the energies and angles of Table II. Therefore, we have to interpolate between the Ka_0 values for which Kim and Inokuti calculated $[Z-F(K)]/(Ka_0)^2$. This interpolation is conveniently and accurately done in an analytic way. Lassettre[13] and Vriens[14] obtained suitable analytic representations of generalized oscillator strengths for inelastic scattering. Now we proceed in a similar way to get a suitable analytic representation of the Born $\sigma(\Omega)$ for elastic scattering. The ground-state wave function Ψ_0 is asymptotically proportional to $\exp(-\alpha_i r/a_0)$, with

Table II. Experimental and Born differential cross sections for elastic scattering of electrons by helium. The values within parenthesis given between the fourth and fifth columns are the cross sections calculated by Khare and Moiseiwitsch (Ref. 4) in the static-field approximation including polarization and exchange.

T (eV)	θ (deg)	$(Ka_0)^2$	$\sigma(\Omega)/a_0^2$ (expt)		$\sigma(\Omega)/a_0^2$ (Born)	$\sigma(\Omega)_{ex}/\sigma(\Omega)_B$
400	5	0.224	1.15		0.588	1.95
	7.5	0.503	0.870		0.539	1.61
	10	0.893	0.688		0.479	1.44
	15	2.004	0.420		0.353	1.19
	20	3.546	0.262		0.246	1.07
	25	5.509	0.165		0.166	0.99
	30	7.878	0.105		0.112	0.93
300	5	0.168	1.39		0.599	2.33
	7.5	0.377	1.09		0.560	1.94
	10	0.670	0.863		0.512	1.69
	15	1.503	0.571		0.403	1.42
	20	2.660	0.368		0.300	1.23
	25	4.132	0.237		0.217	1.09
	30	5.908	0.146		0.155	0.95
200	5	0.112	2.04	(1.98)	0.610	3.34
	7.5	0.252	1.62	(1.82)	0.583	2.78
	10	0.447	1.28	(1.63)	0.548	2.34
	15	1.002	0.836	(1.22)	0.464	1.80
	20	1.773	0.561	(0.83)	0.375	1.50
	25	2.755	0.399	(0.54)	0.294	1.36
	30	3.939	0.275	(0.37)	0.226	1.22
150	5	0.0839	2.59		0.616	4.20
	7.5	0.189	2.16		0.595	3.63
	10	0.335	1.75		0.568	3.08
	15	0.751	1.18		0.499	2.35
	20	1.330	0.813		0.423	1.92
	25	2.066	0.574		0.348	1.65
	30	2.954	0.407		0.280	1.45
100	5	0.0559	3.60	(2.34)	0.622	5.79
	7.5	0.126	3.11	(2.22)	0.607	5.13
	10	0.223	2.66	(2.07)	0.589	4.52
	15	0.501	1.91	(1.71)	0.539	3.55
	20	0.887	1.39	(1.32)	0.480	2.89
	25	1.377	1.03	(1.00)	0.417	2.48
	30	1.969	0.765	(0.78)	0.356	2.15

[9] N. F. Mott, Proc. Roy. Soc. (London) **A127**, 658 (1930); H. Bethe, Ann. Physik **5**, 325 (1930).
[10] C. L. Pekeris, Phys. Rev. **115**, 1216 (1959).
[11] Y. K. Kim and M. Inokuti, Phys. Rev. **165**, 39 (1968).
[12] J. F. Hart and G. Herzberg, Phys. Rev. **106**, 79 (1957).
[13] E. N. Lassettre, J. Chem. Phys. **43**, 4479 (1965).
[14] L. Vriens, Phys. Rev. **160**, 100 (1967).

$\alpha_i = (I/R)^{1/2}$, where I is the ionization energy of helium. In analogy with Ref. 13, it follows that for complex finite values of K, $F(K)$ and $\sigma(\Omega)$ have poles only for $Ka_0 = \pm 2i\alpha_i$. As in Ref. 14, we introduce the variables $\alpha = 2\alpha_i$ and $x = (Ka_0/\alpha)^2$, with $\alpha^2 = 4I/R = 7.227$, and represent $\sigma(\Omega)$ by the series expansion

$$\sigma(\Omega) = \frac{0.63307a_0^2}{(1+x)^2}\left\{1+\sum_{\nu=1}^{\infty} c_\nu \left(\frac{x}{1+x}\right)^\nu\right\}, \quad (9)$$

with

$$\sum_{\nu=1}^{\infty} c_\nu = -0.5161. \quad (10)$$

Equation (9) is an even function of K, has poles only for $x=-1$, and thus for $Ka_0 = \pm i\alpha$, and satisfies Eqs. (7) and (8). The series expansion given by Eq. (9) is similar to previous[14] expansions for inelastic scattering. However, no analog of Eq. (10) could be given for inelastic transitions since Eq. (7) has no known analog for inelastic scattering. In fitting the c_ν coefficients of Eq. (9) via Eq. (4) to Kim and Inokuti's[11] data, we replaced Pekeris's[10] value 0.63307 in Eq. (9) by 0.632916 as obtained from the $X^{(2)}$ value of Ref. 11. We found $c_1 = -0.4033$ (uncertainty ± 0.0002); $c_2 = -0.0444$ (uncertainty ± 0.0008); $c_3 = c_4 = c_5 = -0.0071$; $c_6 = -0.0284$; $c_7 = -0.0114$; $c_{30} = -0.0172$; and all other $c_\nu = 0$. The c_ν values for $\nu \geq 3$ are not accurate. However, the above complete set of c_ν values gives $[Z-F(K)]/(Ka_0)^2$ values which agree within 0.005% with Kim and Inokuti's values for $(Ka_0)^2 < 10$, and within 0.06% for $10 < (Ka_0)^2 < 100$. In the following we now adopt the momentum transfer dependence of the Born cross sections obtained from Kim and Inokuti's data, i.e., we adopt the above c_ν values, but use the slightly more accurate multiplication factor 0.63307. The resulting Born cross sections calculated in this way with Eq. (9) are given in column 5 of Table II. Ratios of experimental and Born cross sections are given in column 6 of Table II. Born and experimental cross sections $\sigma(\Omega)$ are plotted against $(Ka_0)^2$ in Fig. 1. This way of plotting the results is particularly useful since the Born curve is independent of T and because the areas under the curves are proportional to the total elastic cross sections, as follows from Eqs. (2) and (4). Ratios of experimental and Born cross sections are plotted in Fig. 2, where we also give Werner's[2] data and some of Westin's[3] data. The Born cross sections are much too low for small scattering angles for all T, but more so for small T. The present data and those of Refs. 1–3 suggest that for larger angles and not too small T (e.g., $T \geq 300$ eV) the ratio of experimental and Born $\sigma(\Omega)$ decreases with increasing θ down to below unity and then increases again above unity for still larger θ. However, it is not entirely clear whether double scattering[15] may have had any appreciable effect in the very large angle measurements.[1–3]

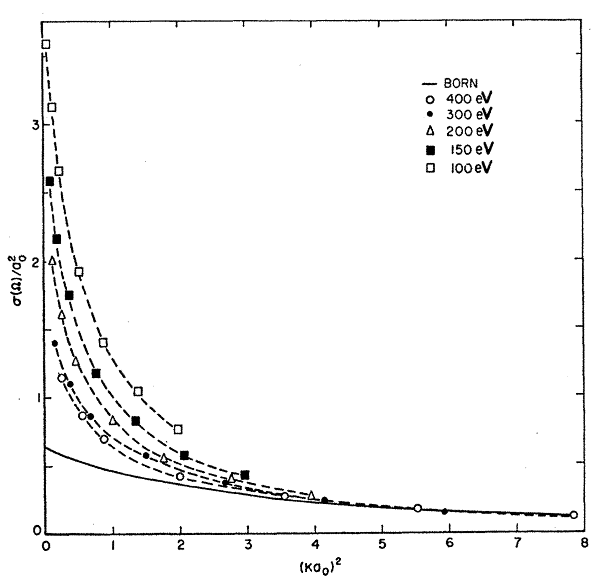

FIG. 1. Experimental and Born differential cross sections for elastic scattering of electrons by helium. The areas under the curves between $(Ka_0)^2=0$ and $(Ka_0)^2=4T/R$ are proportional to T times the total elastic scattering cross sections.

Integrating $\sigma(\Omega)$ over all angles (momentum transfers) gives for the total Born-approximation [Eq. (9)] elastic cross section

$$\sigma = 4.5752(R/T)\pi a_0^2\left\{1-(1+x_m)^{-1} + \sum_{\nu=1}^{\infty}\sum_{\mu=0}^{\infty}\frac{(-1)^\mu}{\mu+1}\binom{\nu}{\mu}[1-(1+x_m)^{-\mu-1}]c_\nu\right\}, \quad (11)$$

with $x_m = (Ka_0)_{\max}^2/\alpha^2 = 4T/\alpha^2 R$. Total Born cross sections calculated with Eq. (11) are given in Table III and values of $T\sigma/\pi a_0^2 R$ are plotted in Fig. 3. In the Born approximation, $T\sigma/\pi a_0^2 R$ tends to 3.537 for $T\to\infty$.

Total "experimental" cross sections estimated from the experimental $\sigma(\Omega)$ for $\theta \leq 30°$, the Born $\sigma(\Omega)$ for large θ, and (if necessary) smooth interpolation between, are also given in Table III and the corresponding values of $T\sigma/\pi a_0^2 R$ are plotted in Fig. 3. The uncertainties in the resulting "experimental" σ may be about 7% and even larger for $T=100$ eV, owing in part to possible experimental uncertainties and in part to insufficient knowledge of $\sigma(\Omega)$ for $\theta > 30°$.

Our experimental differential (Figs. 1 and 2) and total (Fig. 3) cross sections suggest that the Born limit is approached only very slowly for large T.

IV. COMPARISON WITH OTHER THEORIES

Recent theoretical calculations, more sophisticated than the first Born approximation, have been made by

[15] G. E. Chamberlain, J. A. Simpson, S. R. Mielczarek, and C. E. Kuyatt, J. Chem. Phys. 47, 4266 (1967).

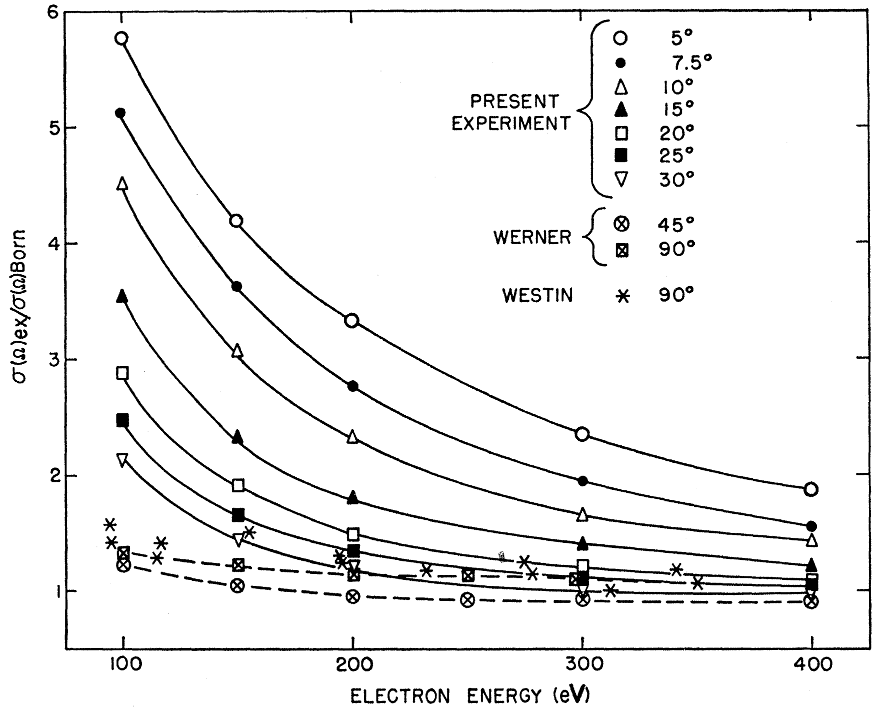

FIG. 2. Ratio of experimental and Born differential cross sections as a function of the incident energy for different scattering angles.

Bell, Moiseiwitsch, and Shields[16] and by Khare and Moiseiwitsch.[4,17] In Ref. 16, total elastic cross sections were calculated in the first Born ($B1$), second Born ($B2$), static field ($N1$), and first-order exchange ($E1$) approximation. These[16] results are also plotted in Fig. 3; $E1B2$ corresponds[16] to the first-order exchange approximation with the second Born correction for distortion; "best theory" corresponds to the sum of the partial-wave cross sections σ_l (l is azimuthal quantum number), where σ_0 and σ_1 were taken from Morse and Allis,[18] and σ_l for $l>2$ from the $E1B2$ approximation including[16] the polarization effect due to the 2^1P state. Note that the zero in Fig. 3 has been displaced. In Ref. 16 a very simple approximate wave function was used for the ground state. Figure 3 shows that the present first Born cross sections and those of Ref. 16 are nearly the same. This agrees with the observation of Huzinaga,[19] Inokuti,[20] and Kim and Inokuti[11] that different approxi-

TABLE III. Experimental and Born total cross sections for elastic scattering of electrons by helium.

T (eV)	$\sigma/\pi a_0^2$ (expt)	$\sigma/\pi a_0^2$ (Born)	σ_{ex}/σ_B
400	0.142	0.1153	1.23
300	0.190	0.1514	1.25
200	0.308	0.2198	1.40
150	0.443	0.283	1.56
100	0.762	0.395	1.93

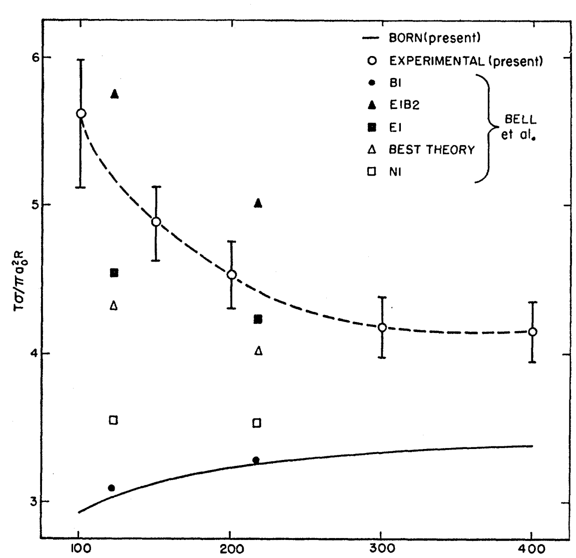

FIG. 3. Experimental and theoretical total elastic cross sections times $T/\pi a_0^2 R$ as a function of the electron energy. Dashed curve and open circles are present experimental results, and the solid curve gives the Born cross sections obtained from Kim and Inokuti's amplitudes (Ref. 11). The other results are from Bell *et al.* (Ref. 16) (see text).

[16] K. L. Bell, B. L. Moiseiwitsch, and D. B. Shields, in *Proceedings of the Third International Conference on Electronic and Atomic Collisions*, edited by M. R. C. McDowell (North-Holland Publishing Co., Amsterdam, 1964), p. 40.

[17] S. P. Khare and B. L. Moiseiwitsch, in *Proceedings of the Third International Conference on Electronic and Atomic Collisions*, edited by M. R. C. McDowell (North-Holland Publishing Co., Amsterdam, 1964), p. 49.

[18] P. M. Morse and W. P. Allis, Phys. Rev. **44**, 269 (1933).

[19] S. Huzinaga, Progr. Theoret. Phys. (Kyoto) **23**, 562 (1960).

[20] M. Inokuti, Progr. Theoret. Phys. (Kyoto) **25**, 717 (1961).

mate wave functions give nearly the same Born cross sections for large scattering angles. For small angles simple wave functions may lead to inaccuracies of 10–20%.

Khare and Moiseiwitsch[4,17] calculated differential cross sections (some of which are given in Table II) and made allowance for electron exchange, polarization, and the static field of the helium atom. Their results are very illustrative and show clearly that electron exchange and especially polarization are mainly responsible for the sharp increase in the differential cross sections towards small scattering angles. Nevertheless, our experimental cross sections still increase faster towards small scattering angles than the theoretical cross sections of Ref. 4 (see Table II). Further, for 200 eV the agreement between theory[4] and experiment for the larger scattering angles is not very good.

Other theoretical calculations in which exchange and polarization are included have been made by LaBahn and Callaway.[21] However, these calculations extend only up to 50 eV.

V. EXCHANGE APPROXIMATIONS

Total elastic cross sections for large T are proportional to $T^{-1}[b_1+b_2T^{-1}+O(T^{-2})]$ just as for nondipole excitation[14] in which the direct scattering amplitude is not identical to zero. The total cross sections for dipole transitions are, for large T, proportional to $T^{-1}[\ln T+b_1+b_2T^{-1}+O(T^{-2})]$.[14] From our total elastic scattering cross sections (Fig. 3) we see that the coefficient b_2 is positive, whereas for inelastic scattering[14,22,23] this coefficient is negative. The first Born contribution to b_2 is negative (Fig. 3, solid curve), just as for nondipole excitations.[14] The exchange contribution to b_2 is positive[4,16,17] (in contrast to inelastic scattering, where the exchange contribution to b_2 is negative), and the polarization contribution to b_2, which is generally understood to be small for inelastic scattering, is positive and very significant[4,16,17] for elastic scattering.

In a generalization of the Ochkur (Bonham) approximation[24,25] to elastic scattering by helium, the cross section per unit solid angle is

$$\sigma(\Omega)=4a_0^2\left[\frac{Z}{(Ka_0)^2}-\frac{F(K)}{(Ka_0)^2}+\frac{RF(K)}{2T}\right]^2. \quad (12)$$

[21] R. W. LaBahn and J. Callaway, Phys. Rev. 135, A1539 (1964); 147, 28 (1966).

[22] H. R. Moustafa, F. J. deHeer, and J. Schutten, Physica (to be published).

[23] B. L. Schram, F. J. deHeer, M. J. Van der Wiel, and J. Kistemaker, Physica 31, 94 (1965); B. L. Schram, H. R. Moustafa, J. Schutten, and F. J. deHeer, *ibid.* 32, 734 (1966); B. L. Schram, *ibid.* 32, 197 (1966).

[24] V. I. Ochkur, Zh. Eksperim. i Teor. Fiz. 45, 734 (1963) [English transl.: Soviet Phys.—JETP 18, 503 (1964)]; R. A. Bonham, J. Chem. Phys. 36, 3260 (1962); M. R. H. Rudge, Proc. Phys. Soc. (London) 85, 607 (1965); see also Ref. 14.

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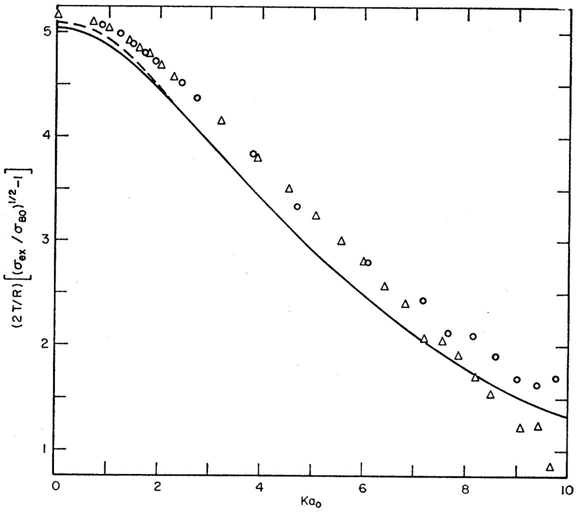

FIG. 4. The function $(2T/R)[(\sigma_{ex}/\sigma_B)^{1/2}-1]$ plotted against Ka_0. The solid and dashed curve are obtained from the generalization of the Ochkur (Bonham) approximation, using scattering amplitudes of Kim and Inokuti (Ref. 11) (solid curve) and Born cross sections of Khare and Moiseiwitsch (Ref. 4) (dashed curve). The circles and triangles are obtained from the first-order exchange and the first Born cross sections of Khare and Moiseiwitsch for 500-eV (circles) and 350-eV (triangles) electrons.

The third amplitude within the square brackets of Eq. (12) is the exchange amplitude. We now denote the $\sigma(\Omega)$ obtained from a theoretical approximation in which exchange is taken into account by σ_{ex}, and the Born $\sigma(\Omega)$ by σ_B, and introduce the function

$$D(K)=(2T/R)[(\sigma_{ex}/\sigma_B)^{1/2}-1]. \quad (13)$$

In the generalization of the Ochkur approximation, $D(K)$ can be expressed as

$$D(K)=F(K)(Ka_0)^2[Z-F(K)]^{-1} =2Za_0(\sigma_B)^{-1/2}-(Ka_0)^2 \quad (14)$$

and $D(K)$ is a very convenient function, since it depends only on K and not on T. We calculated $D(K)$ using Kim and Inokuti's[11] values of $[Z-F(K)]/(Ka_0)^2$ and $F(K)$. The result is reproduced in Fig. 4 by the solid curve. For $T=350$ and 500 eV, Khare and Moiseiwitsch[4] calculated differential first Born and first-order exchange cross sections for elastic scattering of electrons by helium. Their σ_B inserted in Eq. (14) give the dashed curve of Fig. 4. The dashed and solid curves coincide for $Ka_0>2.5$. The small discrepancy between the dashed and solid curves for small K is due to the approximate nature of the wave function used in Ref. 4. The first-order exchange σ_{ex} and the σ_B of Ref. 4 inserted in Eq. (13) give the circles (500 eV) and triangles (350 eV) in Fig. 4. The agreement between the generalization of the Ochkur approximation and the first-order exchange approximation is very good. The coincidence of the first-order exchange curves for 350 and 500 eV for $Ka_0<6$ suggests that Eq. (12) is also a good approximation to the first-order exchange approximation for smaller T. The scatter of the circles and triangles in Fig. 1 for $Ka_0 > 7$ comes from the fact that Khare and Moiseiwitsch[4] gave their cross sections for large K only to three significant figures. Note that the zero in Fig. 4 is displaced.

Recently, Vriens *et al.*[8] measured differential cross sections for 2^3S excitation of helium and found large discrepancies between experimental and Ochkur (theory) cross sections. Miller and Krauss[26] thereupon calculated differential Born-Oppenheimer and Ochkur 2^3S cross sections, and found these two approximations to be in excellent agreement above 100 eV.

Since the Ochkur approximation is a good approximation to the Born-Oppenheimer approximation for such entirely different processes as elastic scattering and 2^3S excitation, the same may be true for other transitions. The experiment of Vriens *et al.*,[8] however, casts serious doubt upon the validity of exchange approximations in which only first-order terms are included, for electron energies of a few hundred eV. One must keep in mind, however, the possibility that these exchange approximations may become valid at much higher electron energies, since it is known that cross sections for 2^1S excitation in helium are only in good agreement with the Born approximation for electron energies above about 1500 eV.

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Detachment of Electrons from H^- and O^- Negative Ions by Electron Impact*

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The energy range for the electron detachment cross section for H^- has been extended down to 8.4 eV. The absolute cross section for detachment of electrons from atomic-oxygen negative ions has been measured in the energy range 7.1 to 487.1 eV. Results of these measurements are compared with Bethe-Born calculations of the cross section. This calculation with the semiclassical Coulomb correction is in qualitative agreement with the experimental results above 20-eV electron energy for both H^- and O^-, although the energy dependence of the H^- cross section from 100 to 500 eV is not consistent with the slope predicted theoretically for the high-energy limit.

I. INTRODUCTION

The cross section for detachment of electrons of the negative hydrogen ion by electron impact has been the subject of a large number of theoretical predictions,[1–5] with discordant results, and two experiments.[6,7] The Bethe-Born (BB) approximation as used by McDowell and Williamson[2] with the semiclassical Coulomb repulsion of Geltman[1] was found to be in best agreement with our experiment.[6] This previous measurement for H^- did not extend to low enough energies to ascertain the validity of the Coulomb correction. The energy range of the H^- cross section has been extended in the work reported here down to 8.4 eV, where a comparison of the experiment and the BB calculation with the Coulomb correction can be made. Since the cross section for electron detachment from negative ions may be expected to depend strongly on electron affinity and the negative-ion structure, we have also made a study of the detachment of electrons

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