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Mass Spectrum and Strong Decays of 1^+ Mesons in $O(4,2)$ Theory*

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A mass spectrum for mesons with explicit dependence on the tilting parameters has been obtained in the dynamical $O(4,2)$ theory. The spin dependence of the masses has been assumed. The assignments of n, j^P, and isospin of many new meson states are proposed. As an example, the partial (strong) decay rates of mesons $1^+ \to 1^- + 0^-$ have been computed in good agreement with experiment.

THE discovery of numerous high-spin baryon and meson resonances[1] has posed a great problem for theoreticians in formulating a systematic and disciplined theory to understand the particle interactions. The recent dynamical $O(4,2)$ scheme put forth by Barut and his collaborators seems to be highly promising in this regard.[2–6] The theory has been successfully applied to form factors, magnetic moments,[2,3] strong decay rates,[4] and mass spectrum of baryons[2] and p-p scattering at fixed angle.[5] The strong decay rates of 2^+ and 1^- mesons have also been computed, in good agreement with experiment.[6]

In this paper, we obtain a mass spectrum for mesons, similar to the case of baryons,[2] from current conservation [the current being most general and linear in the $O(4,2)$ algebra and linear in momenta] and universality of the charge. In addition, we assume the $j(j+1)$ form of spin dependence of the masses.[7] Unlike Ref. 3, we retain the effect of "tilting" in the mass spectrum.[8] We then assign n (the relativistic principal quantum number), spin-parity, and isospin to many new mesons whose spin-parities are yet to be determined experimentally. With these assignments, we have computed the partial (strong) decay rates of 1^+ mesons. As usual, we incorporate the internal spin group $SU(3)$ in the rest frame of the particles and a definite symmetry breaking is introduced in an arbitrary frame.[4,6]

The states are labeled by $|\alpha,a\rangle \equiv |n,j^P,j_3;\, I,I_3,Y,N\rangle$, where n, j, j_3, I, I_3, and Y have their usual meaning and N denotes the dimension of $SU(3)$. The Lie algebra of $O(4,2)$ contains L_{ij} (i, $j=1$, 2, 3) as angular momentum operators; L_{i4}, the analog of the Lenz vector; $M_i = L_{i5}$, the generators of the pure Lorentz transformations; $T = L_{45}$, the tilt; $\Gamma_4 = L_{46}$, the Lorentz scalar; and $\Gamma_\mu = (L_{56} L_{i6})$, the algebraic current operator. For the sake of completeness, we will repeat a few of the steps reported earlier.[2,3]

The most general linear conserved-current operator in $O(4,2)$ theory is given by

$$\mathcal{J}_\mu = a\Gamma_\mu + bP_\mu + cP_\mu\Gamma_4. \tag{1}$$

By demanding the constancy of charge between the tilted states

$$|\tilde{n}jm\rangle = e^{i\theta_n L_{45}}|njm\rangle$$

and current conservation, one obtains the following expressions for the charge and mass of the particles[2]:

$$q = an\cosh\theta_n + 2M_n b + 2M_n c\sinh\theta_n, \tag{2}$$

$$M_n^2 = [2(c^2+b^2/n^2)]^{-1}\{a^2+2\beta c+2\gamma b/n^2 + [(a^2+2\beta c+2\gamma b/n)^2 - 4(\beta^2+\gamma^2/n^2)(c^2+b^2/n^2)]\}, \tag{3}$$

where

$$\beta = aM_n\tanh\theta_n + cM_n^2,$$
$$\gamma = aM_n n(\cosh\theta_n)^{-1} + bM_n^2,$$

and

$$\sinh\theta_n = n(\beta - cM_n^2)(\gamma - bM_n^2). \tag{4}$$

Now, we make some simplifying assumptions, namely that $\beta \approx \gamma$, both very small, and that c is small also. Then from (3), we obtain

$$M_n^2 \approx \frac{n^2(a/b)^2 + 2(\gamma/b)}{1+n^2(c/b)^2}. \tag{5}$$

If we assume further the usual $j(j+1)$ form for the spin dependence of the masses,[7] we get

$$M_n^2 = \frac{[n^2+j(j+1)](a/b)^2 + 2(\gamma/b)}{1+[n^2+j(j+1)](c/b)^2}. \tag{6}$$

In a previous study of the radiative decay of bosons, $\sinh\theta_n \approx n(c/b)$ has been fixed.[9] In Fig. 1, we plot M_n^2 versus $n^2+j(j+1)$. The π tower contains $A_1(n=2, j^P=1^+)$, $B(n=3, j^P=0^-)$, and $\pi_{1640}(n=4, j^P=1^+)$ and the K tower contains $K_V(n=2, j^P=0^-)$, $K_A(n=2,$

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[3] A. O. Barut, Nucl. Phys. (to be published).
[4] A. O. Barut and K. C. Tripathy, Phys. Rev. Letters **19**, 1081 (1967).
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[8] Our assignments (see Fig. 1) differ from Ref. (3). We consider, for example, two different representations for $I=1$, one with 0^-, 1^+, 2^-, $\cdots$ (π tower) and the other with 1^-, 2^+, 3^-, $\cdots$ (ρ tower), and similar considerations for the $I=\frac{1}{2}$ case. Besides, we have incorporated the tilting-parameter effect on the mass spectrum unlike Ref. 3 where $\theta_n \approx 0$ has been assumed.
[9] A. O. Barut and K. C. Tripathy (to be published).

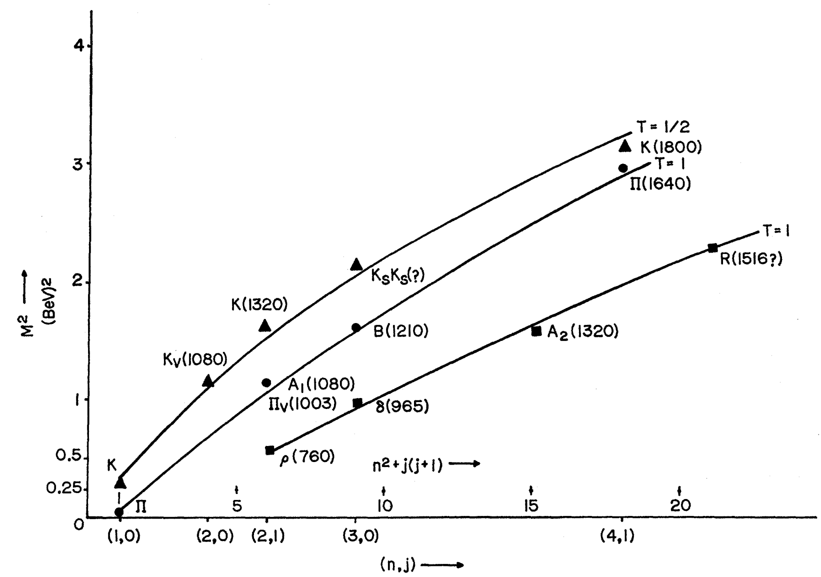

FIG. 1. The assignment of $I=1$, $I=\frac{1}{2}$ mesons in the $O(4,2)$ representation.

$j=1^+$), $K_{1800}(n=4,\ j=1^+)$, and so on. We have assigned $A_1(1080)$, $K_A(1320)$, $D(1285)$, $E(1420)$ to an axial-vector octet and $\pi(1640)$, $K_A(1800)$, $D'(?)$, $E'(?)$ to another axial-vector octet. The partial (strong) decay widths of these octets involving a vector meson and a pseudoscalar meson in the final state have been computed with a single parameter characterizing the $O(4,2)$ reduced matrix element. The calculated rates are in good agreement with the present experimental data[10] within errors which fairly confirm our assignments and assumptions.

The vector transitional amplitude between the state $|n,j^\pm,m\rangle$ and the ground state $|1,0^-,0\rangle$ is given by

$$\begin{aligned}\mathcal{A}[(nj^\pm m)\to(1,0^-,0)] &= \langle \bar{n},j^\pm,m|\Gamma_\mu|\bar{1},0^-,0;\,p\rangle \\ &= \sum_{n''}\langle nj^\pm m|\Gamma_\mu'|n''j''0\rangle\langle n''j''0|f(\xi)|1,0^-,0\rangle \\ &= \sum_{n''}\langle nj^\pm m|\Gamma_\mu'|n''\rangle f_{n''1}(\xi),\end{aligned}\tag{7}$$

where

$$\Gamma_\mu' = e^{-i\theta_n L_{45}}\Gamma_\mu e^{i\theta_n L_{45}}\tag{8}$$

and

$$\begin{aligned}f_{n''1}(\zeta) &= \langle n''j''0|e^{-i\theta_n L_{45}}e^{-i\xi L_{35}}e^{i\theta_1 L_{45}}|1,0,0\rangle \\ &\equiv \langle n''j''0|e^{-i\alpha L_{34}}e^{-i\beta L_{45}}e^{-i\gamma L_{34}}|1,0,0\rangle.\end{aligned}\tag{9}$$

In (9), we have brought $f(\xi)$ to the Eulerian form, the variables being given by

$$\begin{aligned}\cosh\tfrac{1}{2}\theta\,\cosh\tfrac{1}{2}\xi &= \cos\tfrac{1}{2}(\alpha+\gamma)\,\cosh\tfrac{1}{2}\beta, \\ \sinh\tfrac{1}{2}\theta\,\sinh\tfrac{1}{2}\xi &= -\cos\tfrac{1}{2}(\alpha-\gamma)\,\sinh\tfrac{1}{2}\beta, \\ \sinh\tfrac{1}{2}(\theta_1+\theta_n)\,\sinh\tfrac{1}{2}\xi &= \sin\tfrac{1}{2}(\alpha+\gamma)\,\cosh\tfrac{1}{2}\beta, \\ \cosh\tfrac{1}{2}(\theta_1+\theta_n)\,\sinh\tfrac{1}{2}\xi &= -\sin\tfrac{1}{2}(\alpha-\gamma)\,\sinh\tfrac{1}{2}\beta,\end{aligned}\tag{10}$$

[10] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

and

$$\theta=\theta_1-\theta_n.$$

Knowing θ and ξ, the parameters α, β, and γ can easily be evaluated. Now,

$$\begin{aligned}\mathcal{A}[(2,1^+,1)\to(1,0^-,0)] &= \sum_n\langle 2,\,1^+,\,-1|\Gamma_+|n\rangle\langle n|f(\xi)|1,0^-,0\rangle \\ &= -\frac{i}{\sqrt{2}}\sum_n\langle 2,\,1,\,-1|(L_{16}+iL_{26})|n\rangle\langle n|f(\xi)|1,0,0\rangle \\ &= -\frac{i}{\sqrt{2}}\left[\frac{1}{\cosh^2(\frac{1}{2}\beta)}-\frac{\sinh^2(\frac{1}{2}\beta)}{\cosh^4(\frac{1}{2}\beta)}\right].\end{aligned}\tag{11a}$$

Similarly,

$$\begin{aligned}\mathcal{A}[(2,1^+,0)\to(1,0^-,0)] &= \sum_n\langle 2,1,0|\Gamma_3|n\rangle\langle n|f(\xi)|1,0,0\rangle \\ &= -\frac{i}{\sqrt{2}}\left[\frac{1}{\cosh^2(\frac{1}{2}\beta)}-\frac{\sinh^2(\frac{1}{2}\beta)}{\cosh^4(\frac{1}{2}\beta)}\right. \\ &\qquad\left.-2\cosh2\alpha\frac{\sinh^2(\frac{1}{2}\beta)}{\cosh^4(\frac{1}{2}\beta)}\right],\end{aligned}\tag{11b}$$

and

$$\begin{aligned}\mathcal{A}[(4,1^+,1)\to(1,0^-,0)] &= -(2i/\sqrt{5})\frac{\sinh^2(\frac{1}{2}\beta)}{\cosh^4(\frac{1}{2}\beta)}[1-\tanh^2(\tfrac{1}{2}\beta)] \\ &\qquad\times(1-\tfrac{3}{2}\cos2\alpha),\end{aligned}\tag{12a}$$

$$\begin{aligned}\mathcal{A}[(4,1^+,0)\to(1,0^-,0)] &= -(i/\sqrt{5})\frac{\sinh^2(\frac{1}{2}\beta)}{\cosh^4(\frac{1}{2}\beta)} \\ &\qquad\times[1-4\cos2\alpha+\tanh^2(\tfrac{1}{2}\beta)(1-6\cos4\alpha)].\end{aligned}\tag{12b}$$

TABLE I. Comparison between calculated and observed partial decay rates of 1^+ mesons.

Decay modes	Γ(theory) (MeV)	Γ(experiment) (MeV)
1. $A_1 \to \rho\pi$	**130**(input)	30–130
2. $K_A \to K^*\pi$	50.7	dominant modes
3. $K_A \to \rho K$	15.3	
4. $K_A \to \omega K$	5.4	$<2.1\pm1.4$
5. $E \to \bar{K}^*K+K^*\bar{K}$	34.2	39.2 ± 7.9
6. $\pi(1640) \to \rho\pi$	30.05	$<40\pm8$
7. $K_A(1800) \to K^*\pi$	14.3	28 ± 10
8. $K_A(1800) \to \rho K$	3.89	5.6 ± 2
9. $K_A(1800) \to \omega K$	3.54	8 ± 2

In the evaluation of the above matrix elements, we have followed the procedure outlined in Ref. 4.

The calculated amplitudes (11) and (12) are relativistically invariant. To obtain the partial decay rates, we square the amplitude and sum over the initial spin state and then multiply with the invariant phase space and the square of the $SU(3)$ isoscalar factor. The calculated rates and the corresponding observed rates[10] have been displayed in Table I. The agreement seems to be good. The crucial test of our calculation depends, however, on clear experimental information of the spin and parity assignments of these particles.

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Soft-Photon Theory of Nucleon-Nucleon Bremsstrahlung*

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As a consequence of a soft-photon theorem, part of the amplitude for nucleon-nucleon bremsstrahlung depends only on phase parameters measurable in elastic N-N scattering and on the static electromagnetic properties of the nucleon. The corresponding cross section is computed in a fully covariant and gauge-independent fashion, and is found to be in quantitative agreement with most of the existing experimental data.

I. INTRODUCTION

In recent years, several measurements of nucleon-nucleon bremsstrahlung have been performed at low and intermediate energies.[1–5] These experiments are of importance to the theory of nuclear forces, since the electromagnetic field can probe the interaction in areas not available in elastic scattering. The electromagnetic interaction is weak and can normally be treated in lowest order of perturbation theory, which facilitates the theoretical interpretation of the results. Along with these advantages comes the fact that the electromagnetic current is conserved, which leads to soft-photon theorems.[6,7] In this paper, we shall apply the soft-photon technique to proton-proton and neutron-proton bremsstrahlung in order to determine to what extent the interesting aspects are masked by the low-energy behavior of the radiation.

II. SOFT-PHOTON THEOREM

We consider an expansion of the bremsstrahlung amplitude in powers of the frequency ω of the radiated photon and shall demonstrate that the two leading terms depend only on the elastic (mass-shell) properties of the N-N interaction. This was first done by Low.[6] It is necessary to repeat the proof in order to obtain the model-independent expression for fermion-fermion scattering. The model-independent amplitude has previously been given for p-p bremsstrahlung without derivation.[8]

The manipulations necessary to obtain the low-frequency limit of the bremsstrahlung amplitude are

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