Conspiracy and Evasion in π and K Photoproduction*

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We attempt to fit π^+ and K^+ photoproduction data near the forward direction by Reggeized π and K exchange, respectively, allowing for possible conspiracies. We argue that the pion trajectory conspires, with a strength given by the nucleon pole term, while the kaon evades or conspires weakly. We show that the unequal-mass kinematics of K photoproduction in no way prevents or restricts the possibility of conspiracy. In the Appendix we discuss the choice of kinematics-free amplitudes, including the effect of gauge invariance.

I. INTRODUCTION

THE pion trajectory appears to be the best candidate for a conspiring Regge-pole trajectory.[1–3] A good test of this hypothesis is provided by photoproduction, since the contribution of higher trajectories vanishes near the forward direction. Also, there is a forward peak[4] of width $\Delta t \approx m_\pi^2$ in the π^+ photoproduction, indicating the influence of pion exchange. The peak also has the right energy dependence for pion exchange. Conspiracy has previously been suggested[5–7] as the explanation of this peak, although no quantitative fits were made.[8]

If the pion conspires, $SU(3)$ suggests that the kaon does likewise. However, the K^+ photoproduction data[9] shows a wider forward dip, rather than a peak. This, as well as other evidence, indicates that the K trajectory conspires very weakly, if at all. In the light of pion conspiracy, we are not able to explain the lack of K conspiracy. Two explanations are rejected. The relative sizes of the π and K mass seem to be contrary to what is needed for strong π conspiracy and weak K conspiracy, as discussed in Sec. III. In Sec. II we reject a second possible explanation (which has appeared in the literature for both this and other processes),[10,11] namely, that a Λ or Σ, with a mass different from that of a nucleon, is produced in K photoproduction, and that the unequal-mass kinematics modifies the conspiracy properties of the amplitude in a way which prevents or inhibits the same type of conspiracy as in the equal-mass case.

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The remainder of this paper can be summarized as follows: Section II contains a discussion of the application of the theory of conspiracy to photoproduction of π^+ and K^+. Section III contains the phenomenonological fit to the data, and a discussion of the results. The Appendix contains a discussion of the kinematics, concentrating on the effect of gauge invariance.

II. CONSPIRACY THEORY APPLIED TO PHOTOPRODUCTION

We first define the kinematics of the reactions $\gamma+p\to\pi^+n$ and $\gamma+p\to K^+\Lambda$ (or $K^+\Sigma$) as follows:

Particle	Mass	4-momentum
γ	0	K
π or K	μ	Q
p	M_1	P_1
n or Λ or Σ	M_2	P_2

Then $s=-(K+P_1)^2$, $t=-(K-Q)^2$, and $u=-(K-P_2)^2$. We also define the initial and final momenta in the t-channel center-of-mass system:

$$P=[t-(M_1+M_2)^2]^{1/2}[t-(M_1-M_2)^2]^{1/2}/2t^{1/2},$$
$$q=(t-\mu^2)/2t^{1/2}.$$

Conspiracy depends on the relation between definite-quantum-number exchange (in the t channel) and kinematics-free amplitudes. By kinematics-free we mean that (1) no amplitude has any kinematic singularity, and (2) no amplitude or linear combination of amplitudes has any kinematic zero. The choice of such amplitudes is discussed in the Appendix. These amplitudes, denoted by A_1, A_2 A_3, and A_4, are also defined in the Appendix. In the case $M_1=M_2$ they agree with the A amplitudes as defined by Ball.[12] As a single Regge trajectory has definite quantum numbers, we wish to relate the A's to amplitudes of definite quantum numbers in the t channel. These are the "parity conserving" amplitudes $\bar{f}_{\lambda_3\lambda_4,\lambda_1\lambda_2}{}^{\pm}$ introduced by Gell-Mann, Goldberger, Low, Marx, and Zachariasen.[13] In our case the subscripts take on the values 10, $\frac{1}{2}$ $\frac{1}{2}$ and 10, $\frac{1}{2}$ $-\frac{1}{2}$. (We note that $\bar{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$ and $\bar{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$ are mixed; that is, each one has an admixture of the quantum numbers of the other, but with a coefficient which

12 J. Ball, Phys. Rev. **124**, 2014 (1961).
13 M. Gell-Mann *et al.*, Phys. Rev. **133**, B145 (1964).

asymptotically vanishes as $1/s$. This admixture cannot be removed by taking linear combinations of the amplitudes, but only by a partial-wave decomposition.)

The relations between the A's and the $\tilde{f}$'s is as follows[14]:

$$\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{+}=\frac{2^{1/2}(t-\mu^2)[t-(M_1-M_2)^2]^{1/2}}{t^{1/2}}\times[A_1-(M_1+M_2)A_4],$$

$$\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}=\frac{-2^{1/2}(t-\mu^2)[t-(M_1+M_2)^2]^{1/2}}{t^{1/2}}\times[A_1+(t-(M_1-M_2)^2)A_2+(M_1-M_2)A_3],\quad(1)$$

$$\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}=\frac{2^{1/2}(t-\mu^2)[t-(M_1-M_2)^2]^{1/2}}{t}\times[(M_1+M_2)A_1-tA_4],$$

$$\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}=\frac{2^{1/2}(t-\mu^2)[t-(M_1+M_2)^2]^{1/2}}{t}\times[(M_1-M_2)A_1+tA_3].$$

The differential cross section, for large s, is

$$\frac{d\sigma}{d\Omega}=\frac{1}{16\pi^2 s}\frac{[\frac{1}{2}(s-u)]^2}{4}\times\{|A_1|^2+t\,\mathrm{Re}A_1{}^*A_2+\tfrac{1}{2}t[t-(M_1-M_2)^2]|A_2|^2-\tfrac{1}{2}t|A_3|^2-\tfrac{1}{2}t|A_4|^2+t(M_1-M_2)\,\mathrm{Re}A_2{}^*A_3\}.\quad(2)$$

The only necessary property of the "parity conserving" amplitudes is that they correspond to definite quantum-number exchanges. This property remains if we multiply each amplitude by any kinematic function. Therefore, for purposes of discussion, we remove the kinematic factors multiplying the square brackets in Eqs. (1), and define $\tilde{f}$'s:

$$\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{+}=A_1-(M_1+M_2)A_4,\quad(3a)$$

$$\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}=A_1+[t-(M_1-M_2)^2]A_2+(M_1-M_2)A_3,\quad(3b)$$

$$\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}=(M_1+M_2)A_1-tA_4,\quad(3c)$$

$$\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}=(M_1-M_2)A_1+tA_3.\quad(3d)$$

The quantum numbers of the $\tilde{f}$'s can be found from their coupling to the $N\bar{N}$ system (in the t channel). Neglecting mixing, these couplings are

Spin 1, $J=l\pm1$ for $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{+}$ and $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$;
Spin 1, $J=l$ for $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$;
Spin 0, $(J=l)$ for $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$.

[14] These have been given previously by Ader *et al.* (Ref. 10).

The $N\bar{N}$ system has parity $P=(-)^{l+1}$ and a trajectory coupling to angular momentum J has signature $\tau=(-)^J$. Therefore $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{+}$ and $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$ have $\tau P=+$, while $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ and $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$ have $\tau P=-$. To the extent that $SU(2)$ is a good symmetry in π^+ photoproduction, and that $SU(3)$ is a good symmetry in K^+ photoproduction, $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ and $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$ have different G parity [or its $SU(3)$ generalization G_V as the K^+ is in a V-spin triplet]. The G parity of an $N\bar{N}$ state is $G=(-)^{l+S+I}$. Therefore $(-)^I GP=+$ for $S=1$, i.e., for $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{+}$, $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$, and $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$, while $(-)^I GP=-$ for $S=0$, i.e., for $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$. An example of a trajectory which contributes to $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$ and $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{+}$ is the ρ, and an example of a trajectory which contributes to $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ is the π. Since $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{+}$ and $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$ always have the same quantum numbers, we can take linear combinations of them without destroying their essential property of having definite quantum numbers. So we replace Eqs. (3a) and (3c) by

$$\tilde{f}_a{}^{+}=A_1,\quad(3a')$$

$$\tilde{f}_b{}^{+}=A_4.\quad(3c')$$

We now invert Eqs. (3a′), (3b), (3c′), and (3d):

$$A_1=\tilde{f}_a{}^{+},\quad(4a)$$

$$A_2=\frac{t\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}-(M_1-M_2)\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}-[t-(M_1-M_2)^2]\tilde{f}_a{}^{+}}{t[t-(M_1-M_2)^2]},\quad(4b)$$

$$A_3=\frac{\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}-(M_1-M_2)\tilde{f}_a{}^{+}}{t},\quad(4c)$$

$$A_4=\tilde{f}_b{}^{+}.\quad(4d)$$

We first consider conspiracy and evasion for the case $M_1=M_2$. In this case, $f_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ and $f_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$ have different G parity. In order that A_3 be finite, $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$ must have a zero at $t=0$. In order that A_2 be finite, $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}-\tilde{f}_a{}^{+}$ must have a zero at $t=0$. This involves a combination of amplitudes with different quantum numbers and is therefore a conspiracy-evasion situation. The occurrence of zero can happen in two ways: the first, known as evasion, is that $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ and $\tilde{f}_a{}^{+}$ are separately zero at $t=0$. It then follows that $A_1(t=0)=0$, which is not required by kinematics. The second way, known as conspiracy, is that both are nonzero and $f_{10,\frac{1}{2}\frac{1}{2}}{}^{-}=f_a{}^{+}$ at $t=0$. This relation must hold for all values of s, and therefore a trajectory with a nonzero contribution to $\tilde{f}_a{}^{+}$ must be accompanied by a trajectory contributing to $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ at the same value of $\alpha(t=0)$ and with the same residue (defined appropriately).[15] [As usual, we neglect mixing. The $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ trajectory could conspire with the admixed part of $\tilde{f}_a{}^{+}$, but at an $\alpha(t=0)$ of the former equals $\alpha(t=0)$ of the former minus one.]

[15] Every trajectory is accompanied by a family of daughter trajectories spaced by integer values of α. In order to make the $s^{\alpha-1}$, $s^{\alpha-2}$, $\cdots$, dependence of $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}$ and $\tilde{f}_a{}^{+}$ the same, these daughters also take part in the conspiracy.

We now turn to the unequal-mass case. At $t=(M_1-M_2)^2$, $(M_1-M_2)\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}^{-}=\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}$ to make A_2 finite. Since the masses M_1 and M_2 are different, we cannot assume $SU(3)$ is good for the quantum numbers of these amplitudes. Consequently, a single trajectory (for example, the K trajectory) can contribute to both, and there is no reason for $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}^{-}$ and $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}$ to vanish separately. A condition between them only involves one trajectory and not a coincidence of two. At $t=0$, we find an evasion or conspiracy between $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}$ and $\tilde{f}a^{+}$. In order that A_2 and A_3 be finite, $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}^{-}=(M_1-M_2)\tilde{f}a^{+}$. Both sides of this may vanish separately (evasion), or both may be nonzero, and trajectories contributing to both sides have the same α (conspiracy). If $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}$ is roughly constant between $t=0$ and $t=(M_1-M_2)^2$, by combining the conditions at these two points we find, for the conspiracy case,

$$\tilde{f}a^{+}(t=0)\approx\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}^{-}[t=(M_1-M_2)^2].$$

This condition is just the conspiracy condition for equal masses. Therefore the equal-mass limit is smooth. To complete this discussion of the equal-mass limit, we must study the effect of a small, approximately constant $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}$, which serves to carry the conspiracy from the point $t=0$ (where it actually occurs, i.e., where 2 trajectories coincide) to the point $t=(M_1-M_2)^2$. For this purpose we write the differential cross section in terms of the $\tilde{f}$'s

$$\frac{d\sigma}{d\Omega}=\frac{1}{16\pi^2 s}\frac{[\frac{1}{2}(s-u)]^2}{8}\Bigg[|\tilde{f}a^{+}|^2-t|\tilde{f}b^{+}|^2+\frac{t}{t-(M_1-M_2)^2}|\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}^{-}|^2-\frac{1}{t-(M_1-M_2)^2}|\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}|^2\Bigg].\quad(5)$$

The conspiracy and pseudothreshold conditions give $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}\approx(M_1-M_2)\tilde{f}a^{+}\approx(M_1-M_2)\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}^{-}$. Therefore the ratio of the contribution to the cross section from the term involving $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}$ to that involving $\tilde{f}_a^{+}$ is $(M_1-M_2)^2/[t-(M_1-M_2)^2]$. For t far from $t=0$ [compared to $(M_1-M_2)^2$], the contribution of $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}^{-}$ is small. But in this region the mass difference can be ignored, $SU(3)$ is good, G_V is a conserved quantum number, and therefore a trajectory with $(-)^V G_V P=-$ should have an unimportant contribution to $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}$.[16]

In summary, the role of conspiracies versus evasion is essentially the same in the equal-mass and unequal-mass cases. We venture to guess that this result generalizes to other processes, and any equal-mass conspiracy is the limit of some unequal-mass conspiracy.

III. QUANTITATIVE COMPARISON WITH EXPERIMENT

At small angles, the contribution of known trajectories higher than the π (or K) vanish. These higher trajectories ρ, ω, A_2, etc., $K^*(890)$, $K(1420)$ have $\tau P=+$, and therefore contribute only to $\tilde{f}_a^{+}$ and $\tilde{f}_b^{+}$. It is very unlikely that they conspire; hence their contribution to A_1 vanishes ($\sim t$) at $t=0$ [see Eqs. (3b) and (3d)]. Therefore their contribution to $d\sigma/d\Omega$ vanishes ($\sim t|A_4|^2$) at $t=0$. The small angle cross section, if any, is due therefore to π and K trajectories and their coconspirators. We write the contribution of a Regge pole to an amplitude $\tilde{f}_x$ (where x refers to any of the subscripts and superscripts with the quantum numbers of the pole) as

$$\tilde{f}_x=\frac{1+\tau e^{-i\pi\alpha(t)}}{[2+2\tau\cos\pi\alpha(t)]^{1/2}}\left(\frac{s-u}{2s_0}\right)^{\alpha(t)-1}\frac{1}{s_0}\tilde{b}_x(t).\quad(6)$$

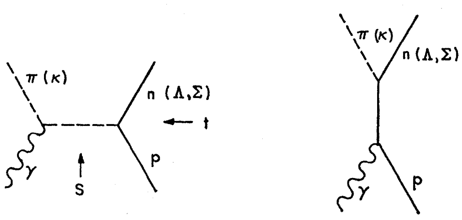
The residue function $\tilde{b}_x(t)$ includes the particle poles for α passing through non-negative integers of the correct signature. We have chosen $\tilde{b}_x(t)$ in this rather unconventional way so that it is closely related to the amplitude obtained by the Born approximation for the exchange of a spin-zero particle. The other factors are the phase, given by the signature, and the Regge energy dependence. For the π or K trajectory $\tilde{b}_{10,\frac{1}{2}\frac{1}{2}}^{-}$ must have the π or K pole at $t=\mu^2$.

As the conspiracy must hold for all values of s and for both the real and imaginary parts of the amplitude, conspiring trajectories must have the same spin and signature, as well as the same internal quantum numbers. If the pion conspires, it conspires with a trajectory contributing to $\tilde{b}_a^{+}$ which has $\tau P=+$. The conspiring trajectory therefore has $\tau=+$, $P=+$, $I=1$, and $G=-$ [since $\tilde{f}_a^{+}$ has $(-)^I GP=+$]. As there is no known low-mass $J^P=0^+$, $I^G=1^-$ particle, $\tilde{b}_a^{+}$ will not have a pole. This can happen either because the conspiring trajectory is almost flat, or because the residue vanishes when $\alpha=0$. (In order that there be no particle, the residue must vanish for every process.) If the trajectory has negative slope, the particle would have imaginary mass, and therefore must be a ghost, and the residue must vanish. If the K conspires, its co-conspirator has similar properties.

In order to evaluate the residue of the π or K pole in $\tilde{b}_{10,\frac{1}{2}\frac{1}{2}}^{-}$, we must compute the Born approximation. Gauge invariance requires that we include the part of the direct nucleon pole [Fig. 1(b)] coming from the charge of the nucleon, along with π or K exchange [Fig. 1(a)]. We find the sum of these to contribute

$$A_1=-\frac{eg}{(s-M_1^2)},\quad A_2=\frac{2eg}{(s-M_1^2)(t-\mu^2)}.\quad(7)$$

[16] If $\tilde{f}_{10,\frac{1}{2}\frac{1}{2}}$ is not roughly constant, but is very small at $t=(M_1-M_2)^2$, we get a conspiracy between the $(-)^V G_V P=+$, $\tau P=-$, and the f_a^{+} amplitudes. If we take the equal-mass limit, assuming that the t derivative of the residue is bounded, this possible conspiracy vanishes. We ignore this conspiracy in what follows.



(a) (b)

FIG. 1. t- and s-channel Born-approximation diagrams.

In these expressions, g is to be replaced by $g_{\pi pn}$, $g_{Kp\Lambda}$, or $g_{Kp\Sigma}$ as appropriate. The normalization of e and g is

$$\frac{e^2}{4\pi}=\frac{1}{137},\quad \frac{g_{\pi pn}^{\;2}}{4\pi}=\frac{(2^{1/2}g_{\pi NN})^2}{4\pi}=2\times 14.7.$$

Gauge-invariant Reggeized π or K exchange must include the effect of the nucleon pole as well as π or K exchange. The expressions for A_1, A_2 given by Eq. (7) are to be used to calculate the Regge residues $\tilde{b}$ rather than the amplitudes $\tilde{f}$ as they do not include the s^α dependence or the phase. As we are interested only in a small region in t, near $\alpha(t)=0$, the s^α dependence and the phase can be neglected for π photoproduction. In the equal-mass case $s-M_1^2$ and $\frac{1}{2}(s-u)$ are the same at $t=\mu^2$. In the unequal-mass case, daughter trajectories exist, and the difference between these quantities is included in the contribution of the daughters. Therefore, from Eq. (3), the Born approximation [Figs. 1(a) and 1(b)] gives

$$\tilde{b}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}=eg\frac{t+\mu^2-2(M_1-M_2)^2}{t-\mu^2},\tag{8a}$$

$$\tilde{b}_a{}^{+}=-eg,\tag{8b}$$

$$\tilde{b}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}=eg(M_2-M_1).\tag{8c}$$

Only the pole $eg\{2[\mu^2-(M_1-M_2)^2]/(t-\mu^2)\}$ in $\tilde{b}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ is rigorously given by the Born approximation. This pole comes exclusively from A_2. The usual assumption is that the pole term alone gives a good approximation to the residue. However, the numerator in Eq. (8a) is rapidly varying, suggesting that the pole alone might not be a good approximation. For a phenomenological fit to π photoproduction data we use A_1, A_2, and A_3 in the form

$$A_1=-K\frac{eg}{s-M_1^2},\tag{9a}$$

$$A_2=\frac{2eg}{(s-M_1^2)(t-\mu^2)},\tag{9b}$$

$$A_3=0.\tag{9c}$$

These are the final forms for the A's and are properly Reggeized. In these equations A_3 is determined if the π (or K) and its conspirator are the only trajectories considered by the requirement that $\tilde{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$ be unimportant for $-t\gg(M_1-M_2)^2$. A_4 is not included in Eqs. (9), since by Eq. (3a) it is expected to be of order $1/(M_1+M_2)$ relative to A_1. It appears in the cross section [Eq. (2)] multiplied by t. Therefore it contributes $t/(M_1+M_2)^2$ relative to A_1, and can be neglected for small t.

If the magnetic moments had been kept in the nucleon poles [Figs. 1(b) and 2] they would have contributed only to A_3 and A_4. As A_3 is already determined and A_4 is unimportant, we ignore the magnetic moments.

In Eqs. (9), K is kept as a free parameter. It is a measure of the strength of the conspiracy. The no-conspiracy solution is given by $K=0$, and the assumption $\tilde{b}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}=\text{constant}/(t-\mu^2)$ is given by $K=2$. In Fig. 3 we compare the π^+ data of Boyarski *et al.*[4] at photon momenta of 5 BeV/c and 16 BeV/c with the cross section from Eqs. (9) and (2). We have disregarded the variation of α with t, as we are interested only in small t values. The use of two different energies is a test of our energy dependence. $K=1$ gives a good fit to both sets of data. This is just the Born approximation, Eq. (7), with A_3 determined by keeping only two trajectories. The numerator of $\tilde{b}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ is not constant, but has a zero at $t=-\mu^2$. The strength of the conspiracy is one-half what it would be if $\tilde{b}_{10,\frac{1}{2}\frac{1}{2}}{}^{-}$ had just a pole, and no zero.

If $SU(3)$ held exactly, and the π conspired, the K also would conspire. In the real, broken $SU(3)$ world, it would be reasonable for this situation to still hold. Since we find a conspiracy solution for the π, one would expect a conspiracy for the K. We try a solution for K photoproduction similar to Eqs. (9). With the K, however, we must keep the variation of α with t. We multiply Eqs. (9) by $[(s-u)/2s_0]^{\alpha(t)}$ and by the phase given by the signature factor. We use $\alpha'(t)=1$ BeV^{-2}, $s_0=1$ BeV^2 for both the K and its co-conspirator (if any). Our results remain qualitatively the same if we take a different slope for the co-conspirator trajectory or if we take s_0 in the range 0.5 BeV$^2<s_0<1.5$ BeV^2. We use the results of Kim[17] for $g_{N\Lambda K}$ and $g_{N\Sigma K}$. These values are $g_{N\Lambda K}/4\pi=16$, $g_{N\Sigma K}^2/4\pi\approx 0$. We add the cross sections for $\gamma p\to K^+\Lambda$ and $\gamma p\to K^+\Sigma^0$ and compare the result in Fig. 4 with the data of Boyarski *et al.*[9]

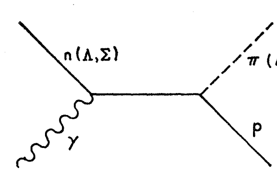


FIG. 2. u-channel Born-approximation diagram.

[17] J. Kim, Phys. Rev. Letters 19, 1079 (1967).

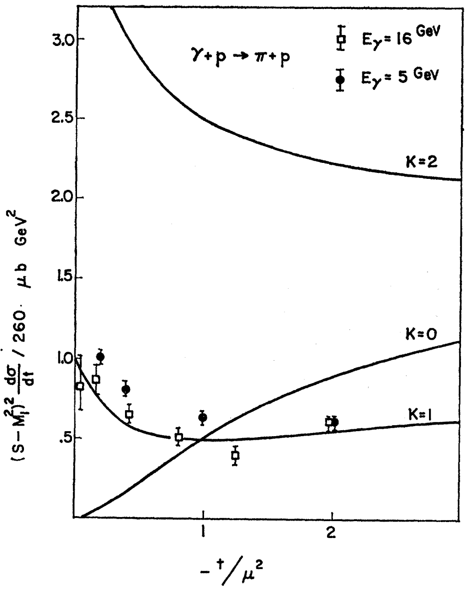


FIG. 3. π^+ differential cross-section data of Boyarski *et al.* (Ref. 4) compared with the conspiracy model for three strengths of the conspiracy.

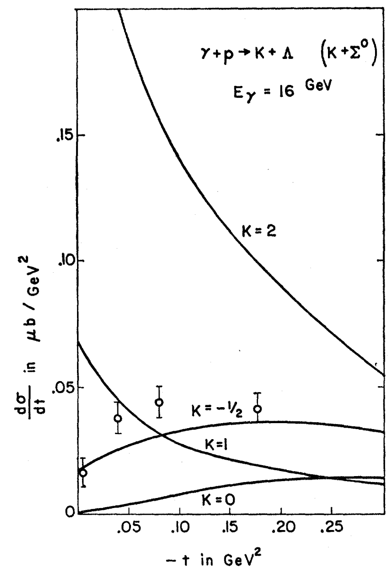


FIG. 4. The K^+ differential cross-section data of Boyarski *et al.* (Ref. 9) compared with the conspiracy model.

We see that the $K=0$ solution fits the data.[18] Therefore the K conspires very little, if at all, unlike the π. As a consequence, K exchange is unimportant in K photoproduction. At higher $|t|$ values, K^* exchange becomes important. If K exchange were important, very few Σ's would be produced, since $g_{N\Lambda K} \gg g_{N\Sigma K}$. However, Boyarski *et al.*[9] find approximately equal numbers of Λ's and Σ's. The results of Sec. II show that the lack of K conspiracy is not caused by the unequal-mass kinematics. It must therefore be attributed to something else, perhaps the dynamics of $SU(3)$ breaking. We are able, however, to argue that this does not happen via the π-K mass difference, or equivalently by the $\alpha(t=0)$ difference. For the π, $\alpha(0)\approx -0.02$ while for the K, $\alpha(0)\approx -0.25$. A conspiracy strength proportional to $1/\alpha$ would be consistent with our results. However, a conspiracy strength proportional to α seems more reasonable. This is because the residue at $t=0$ for daughter trajectories has a $1/\alpha$ factor relative to the residue of the conspiring parent. [This is a result of the $O(4)$ theory.[19]] For the daughters, which are far from $\alpha=0$, $SU(3)$ should be better, and the π daughter should have about the same residue as the K daughter. Then the π would have a residue which is a factor $\alpha_\pi(0)/\alpha_K(0)$ smaller than the K at $t=0$. We are therefore unable to find an explanation of the lack of K conspiracy.

To summarize the results of this paper: The π trajectory conspires with a trajectory which is the parity doublet of the pion. The 0^+ particle on this trajectory does not occur at low energy, perhaps because this trajectory is almost flat or has negative slope. The strength of the conspiracy in π^+ photoproduction is given by the s-channel nucleon pole. The pion-trajectory residue (assuming linear extrapolation) has a zero near $t=-\mu^2$. The strength of the conspiracy is one-half what it would be if this zero did not occur. The same type of conspiracy can occur in K^+ photoproduction for the K trajectory. *A priori*, it would be expected to be a stronger conspiracy than the π. However, the K photoproduction data are consistent with evasion or very small conspiracy.

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APPENDIX: CHOICE OF KINEMATICS-FREE AMPLITUDES

There are several methods for finding the kinematics-free amplitudes. We will discuss three: (1) Some singularities of helicity amplitudes are simple in each channel. The kinematics-free amplitudes are found by

[18] A K conspiracy with $|K|\lesssim\frac{1}{2}$ added to K^* exchange will fit the data. Ball, Frazer, and Jacob (Ref. 8) suggest a conspiring K. Their solution corresponds to $K\approx+\frac{1}{2}$. The contribution of the K then has a forward peak, so that, in order to reproduce the experimental forward dip, the K^* residue must be rather rapidly varying, which is not entirely satisfactory. If $K\approx-\frac{1}{2}$, the K contribution has a forward dip, and the K^* contribution enters more gradually as $-t$ increases away from $t=0$. In any case, a small, self-conspiring, K^*-Pomeranchon cut contributes to the A_1 amplitude at $t=0$. It is possible for this cut to provide the entire forward cross section. An experimental determination of the Λ/Σ ratio would provide a definite test of the K conspiracy.

[19] D. Freedman and J. Wang, Phys. Rev. **160**, 1560 (1967).

studying the crossing matrix between the channels.[20] (2) Use of invariant amplitudes. (3) Finding threshold and pseudothreshold factors from the values of orbital angular momentum. Although each one of these is difficult to apply in general, their application to individual, simple processes is straightforward. For processes without a zero-mass particle these methods agree. In the zero-mass case these methods do not agree. This was found by Frautschi and Jones[7] in comparing methods (1) and (3) for pion photoproduction. In some amplitudes, at thresholds involving the photon, there is a disagreement by one power of the momentum appropriate to that threshold. Therefore we concentrate on these thresholds, and the effect that gauge invariance has on them. All other singularities and zeros are the same in all methods, and we use the invariant amplitudes to find them. In particular, the point $t=0$, at which conspiracy occurs, does not involve any complications.

We first consider the invariant-amplitude approach. For the equal-mass case, $M_1=M_2$, this has been worked out by Ball.[12] However, since we wish to generalize to the unequal-mass case, and since there has been some confusion about the role of gauge invariance in the choice of these amplitudes, we repeat the final states of his derivation. The amplitude is $\bar{U}(2)TU(1)$, where

$$T=\sum_{i=1}^{8} B_i(s,t)N_i\,,$$

and

$$N_1=i\gamma_5\gamma\cdot\epsilon\gamma\cdot K\,,\qquad N_5=\gamma_5\gamma\cdot\epsilon\,,$$
$$N_2=i\gamma_5(P_1+P_2)\cdot\epsilon\,,\quad N_6=\tfrac{1}{2}\gamma_5\gamma\cdot K(P_1+P_2)\cdot\epsilon\,,$$
$$N_3=2i\gamma_5Q\cdot\epsilon\,,\qquad N_7=\gamma_5\gamma\cdot KK\cdot\epsilon\,,$$
$$N_4=2i\gamma_5K\cdot\epsilon\,,\qquad N_8=\gamma_5\gamma\cdot KQ\cdot\epsilon\,,$$

and ϵ is the photon polarization. N_4 and N_7 are identically zero (for on-mass-shell photons) and therefore the amplitudes B_4 and B_7 are without content. Ball shows that the other six amplitudes are the proper invariant amplitudes *if gauge invariance is not imposed as a kinematical constraint*.[21] However, we argue, in what follows, that gauge invariance properly belongs to the realm of kinematics, and must therefore be imposed in defining the invariant amplitudes. Gauge invariance imposes the following two conditions on the B's:

$$(s-M_1^2)B_2=(t-\mu^2)(B_3-\tfrac{1}{2}B_2)\,,\qquad\text{(A1)}$$

$$2B_5-B_6(s-M_1^2)+(t-\mu^2)(B_8-\tfrac{1}{2}B_6)=0\,.\quad\text{(A2)}$$

Therefore, the number of independent amplitudes is reduced from six to four. Moreover, kinematic zeros are introduced into the B's and must be removed.

The second condition just serves to remove B_5 from the list of eligible amplitudes. The first allows us to write

$$A_2=\frac{2B_2}{t-\mu^2}\,,\qquad\text{(A3)}$$

$$A_2=\frac{2B_3-B_2}{s-M_1^2}\,,\qquad\text{(A4)}$$

in two different ways. Equation (A3) does not allow for the singularity in A_2 at $s=M_1^2$ which Eq. (A4) does, and, similarly, Eq. (A3) does not allow for the singularity in A_2 at $t=\mu^2$ which Eq. (A3) does. Therefore, B_2 has a kinematic (gauge invariance) zero at $t=\mu^2$ and $2B_3-B_2$ has one at $s=M_1^2$, and A_2 *is free of kinematic singularities*. The remaining A amplitudes are defined as

$$A_1=B_1-\tfrac{1}{2}(M_1+M_2)B_6\,,\qquad\text{(A5)}$$

$$A_3=-B_8\,,\qquad\text{(A6)}$$

$$A_4=-\tfrac{1}{2}B_6\,.\qquad\text{(A7)}$$

The minus signs and factors are introduced solely on the basis of historical precedent and convenience. The important considerations are not using B_5, and removing the kinematic zeros from B_2 and B_3. The A amplitudes defined in this way are the kinematics-free amplitudes, if gauge invariance is considered a kinematic rather than dynamic phenomenon. They are the amplitudes used in the body of this paper.

We next consider the method involving the crossing matrix between helicity amplitudes. The principle of this method is that some helicity amplitudes would violate angular momentum unless they were to vanish in the forward and/or backward directions. The power of the vanishing is given by the number of units of angular-momentum violation. When these zeros are removed, the set of helicity amplitudes has no kinematic singularities or zeros in the cross channels, but may have some in the direct channel. These zeros are removed from helicity amplitudes in two channels, and the crossing matrix is used to construct a set of amplitudes having the correct properties in both channels. When this procedure is carried out for photoproduction, a set of amplitudes equivalent to the A amplitudes is found.

It follows that the "crossing-matrix method" automatically includes gauge invariance. This is reasonable, as there are only four, rather than six helicity amplitudes. It can be better understood by considering the same process for a massive photon and by taking the limit of its mass approaching zero. The two other amplitudes are the zero-helicity amplitudes (i.e., the photon has helicity zero). If these are expressed in terms of the B invariant amplitudes, they have the form[22]

[20] This method has been studied for processes in general by L. Wang, Phys. Rev. **142**, 1187 (1966).

[21] This is the point of view of Frautschi and Jones (Ref. 7).

[22] See, for example, Högaasen and Salin, Ref. 11, Eq. (5). There is a misprint in their expression for f_{00}^-: the first M should be m.

$$1/m\,[\text{combination of } B\text{'s that vanish if gauge invariance (Eqs. (A1) and (A2)) is imposed}] + m(\cdots),$$

where m is the photon mass. These zero-helicity amplitudes would become infinite as $m \to 0$ unless gauge invariance is imposed. The crossing-matrix method implicitly assumes the finiteness of all amplitudes, and therefore assumes gauge invariance.

The method involving powers of momentum given by the orbital angular momentum gives the kinematic factors of the B amplitudes rather than the A amplitudes. This is because the rule

$$f_{l_{\rm in}l_{\rm out}} \sim k_{\rm in}{}^{l_{\rm in}} k_{\rm out}{}^{l_{\rm out}}$$

is intrinsically nonrelativistic. It requires the velocity to approach zero as the momentum approaches zero. In contrast to this, the crossing-matrix method makes no use of zero-momentum values of s or t. The kinematics come from the forward and backward directions, which make no distinction between zero-mass and nonzero-mass particles.

The rule which replaces the k^l rule occurs in the theory of multipole radiation.[23,24] The intensity of a jth multipole has a factor k^j, for both electric and magnetic multipoles. As a magnetic jth multipole corresponds to $l=j$ [since parity $=(-)^{j-1}$], this is the ordinary rule. For an electric jth multipole, however, $l=j\pm1$ [since parity $=(-)^j$]. For small k, the amplitude is dominated by $l=j-1$; therefore the rule for electric radiation is

$$f_l \sim k^j = k^{l+1}.$$

That this result depends on the masslessness of the photon can be seen from the origin of the extra factor of k. The electric transition involves the divergence of the current, or, equivalently, the time derivative of the charge density. This provides a factor of the energy of the photon, rather than its momentum.

The nature of the kinematics is clouded by the fact that A_2 does have a pole at $t=\mu^2$—the dynamic pion pole. That this pole is dynamic requires, of course, that gauge invariance be considered kinematic. In what follows, we show why this pole should be considered dynamic and therefore why gauge invariance should be considered kinematics.

In perturbation theory the pion pole in A_2 can come from the s-channel proton pole as well as from the t-channel pion pole, depending on the choice of gauge. In particular, in the gauge in which the fourth component of the photon polarization vanishes in the t-channel center-of-mass frame, the pion-exchange diagram is identically zero, yet there is still a pion pole in A_2. Thus, in this gauge it appears that the pole is introduced kinematically. The resolution of this problem is that in gauge-invariant perturbation theory an individual diagram is without meaning. If a gauge-invariant set of diagrams contains the exchange of a particle, the set, rather than just the individual diagram, contributes a pole to the amplitude.

The residue of a dynamic pole is always a one-parameter function of the other variable. The parameter is just the produce of the couplings, while the form is given by the spin of the particle. The residue of a kinematic pole is an arbitrary function of the other variable. The pole in A_2 fits this criterion for a dynamic pole. The pole is

$$A_2 \sim R/(s-M_1{}^2)(t-\mu^2).$$

The quantum numbers of a dynamic pole are given by the particle causing the pole, while a kinematic pole does not respect quantum numbers. The quantum numbers of the pole in A_2 are just those of the pion. For example, there is no pole at $t=\mu^2$ for π^0 photoproduction, and a pion cannot be exchanged in this process.

The division into kinematics and dynamics must be invariant under crossing. If the pole in A_2 is considered kinematic, it should be so in every channel. Then one would have to remove the pole in every channel by multiplying by $(t-\mu^2)(s-M_1{}^2)(u-M_2{}^2)$. The implication is that B_2 and B_3 would contain kinematic poles, contrary to fact.

These considerations, as well as the fact that the crossing-matrix method automatically includes gauge invariance, lead to the conclusion that gauge invariance is a kinematical effect, and that the A amplitudes are the proper kinematics-free amplitudes.

[23] Blatt and Weiskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 595.

[24] I would like to thank Professor Marc Ross for informing me of this result.