where

$$\Delta=(s^2+4p^2\lambda^2-2sp'^2+p^4+p'^4-2p^2p'^2)$$

and

$$-1\leqslant\beta_0\leqslant 0.$$

The function multiplying $H(\lambda^2,p^2,\beta_0)$ in (A2) is continuous in λ^2, provided $s\geqslant 2(p^2+p'^2)$ and $\lambda^2>p'^2$. We now specialize to the equal-mass case $p^2=p'^2=m^2$ (the generalization for the case $p^2\neq p'^2$ may be carried out in a similar fashion).

Consider the function $g(m^2,s)$ given by

$$g(m^2,s)=\int d\lambda^2\frac{1}{(\lambda^2)^\epsilon}\frac{1}{\sqrt{\Delta}}\times\left\{\ln\frac{(\sqrt{\Delta})+s-4m^2}{(\sqrt{\Delta})-s+4m^2}-\ln\frac{(\sqrt{\Delta})+s-2m^2}{(\sqrt{\Delta})-s+2m^2}\right\}. \quad \text{(A3)}$$

Then, $g(m^2,s)$ is uniformly convergent in s.

Proof: We expand the logarithms in (A3), making use of the fact that

$$\ln\left|\frac{1+x}{1-x}\right|\leq cx,\quad\text{with}\quad 0\leq c<\infty\quad\text{for}\quad |x|<1$$

and noting that $\sqrt{\Delta}>s-2m^2$, for $|x|<1$.

$$|g(m^2,s)|\leq c\left|\int d\lambda^2\left\{\frac{s-4m^2}{\sqrt{\Delta}}-\frac{s-2m^2}{\sqrt{\Delta}}\right\}\frac{1}{(\lambda^2)^\epsilon}\right|$$

$$=2c\int^\infty d\lambda^2\frac{m^2}{4m^2\lambda^2+s^2-2sm^2}\frac{1}{(\lambda^2)^\epsilon}$$

$$<\tfrac{1}{2}c\int^\infty d\lambda^2\frac{1}{(\lambda^2)^{1+\epsilon}}. \quad \text{(A4)}$$

The inequality (A4) ensures the uniform convergence of $g(m^2,s)$ in s.

Assuming that for large enough λ^2, $H(\lambda^2,\beta_0,m^2)$ is bounded by $(\lambda^2)^{-\epsilon}$, and using again the mean-value theorem, one establishes Eq. (8).

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$\rho+\rho'$ Model of Pion-Nucleon Charge-Exchange Scattering and Polarization

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A $\rho+\rho'$ Regge-pole model in which the ρ' is an α-type or nonconspiring trajectory is investigated. It is found to agree with present charge-exchange data and certain superconvergent sum rules. Present polarization measurements are not accurate enough to constrain all the Regge parameters, and a range of solutions emerges consistent with the data.

The ρ' meson with $J^P=1^-$ and $I=1$ has been introduced[1,2] to explain the polarization observed[3] in high-energy pion-nucleon charge-exchange scattering. The recent work of Igi and Matsuda[4] showed that the parameters obtained by Logan, Beaupre, and Sertorio[1] to fit the πp polarization did not satisfy their superconvergence relations.

In this paper, we shall present parameters for the $\rho+\rho'$ model which agree with the πp charge-exchange polarization and scattering data, and the superconvergence relations of Igi, Matsuda,[4] and Olsson.[5,6] In order to satisfy the constraints provided by both the polarization data and the superconvergence relations, Sertorio and Toller[7] introduced a Gribov-Volkov[8] β-type ρ' trajectory (i.e., a conspiring ρ' trajectory). We have found that it is not necessary to introduce a conspiring ρ' trajectory, but that it is possible to satisfy the superconvergence relations and the polarization data with the α-type or nonconspiring ρ'. In fact, the agreement with the superconvergence relations seems to be better with the α-type ρ' trajectory.

Gajdicar and Moffat[9] have recently shown, on the basis of an analysis of the recent Coulomb interference

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measurements[10] of $\alpha_{\pm}=\mathrm{Re}f_{\pm}/\mathrm{Im}f_{\pm}$, that there exists a discrepancy between the calculated forward amplitude $D^{(-)}(\nu)=\frac{1}{2}[D_-(\nu)-D_+(\nu)]$ and $D_\rho^{(-)}(\nu)$ obtained from the single ρ-trajectory model. The latter model is consistent with the charge-exchange data[11–13] and we have found that this discrepancy persists with the introduction of the ρ'.

We wish to describe the πN charge-exchange differential cross-section data,[11] the charge-exchange polarization, the πp total cross section $\sigma^{(-)}$,[14,15] and the real forward amplitude $D^{(-)}$,[9,10] using the $\rho+\rho'$ model. The charge-exchange amplitude is defined by

$$A_{\mathrm{ch\ ex}}=f+i\frac{\sigma\cdot\mathbf{q}'\times\mathbf{q}}{q^2}\tilde{f}\,, \tag{1}$$

where f and $\tilde{f}$ are the non-spin-flip and spin-flip amplitudes and $\mathbf{q}$ and $\mathbf{q}'$ are the center-of-mass momenta of the initial and final states. The polarization parameter $\mathbf{P}$ and the differential cross section $d\sigma/dt$ are given by

$$P=-\frac{2\,\mathrm{Im}(f\tilde{f}^*)\,\sin\theta}{|f|^2-(4t/s)|\tilde{f}|^2} \tag{2}$$

and

$$d\sigma/dt=(\pi/q^2)[|f|^2-(4t/s)|\tilde{f}|^2]\,, \tag{3}$$

where θ is the scattering angle in the s channel. We shall assume that f and $\tilde{f}$ are given by $f=f_\rho+f_{\rho'}$ and $\tilde{f}=\tilde{f}_\rho+\tilde{f}_{\rho'}$, where

$$f_i=-[M\mu b_i(t)/4\pi W][(s-M^2-\mu^2)/s_0]^{\alpha_i(t)}\times[i+\tan\tfrac{1}{2}\pi\alpha_i(t)]\,,\quad (i=\rho,\rho') \tag{4}$$

and

$$\tilde{f}_i=(\mu/16\pi)[b_i(t)-\alpha_i(t)\tilde{b}_i(t)][(s-M^2-\mu^2)/s_0]^{\alpha_i(t)}\times[i+\tan\tfrac{1}{2}\pi\alpha_i(t)]\,,\quad (i=\rho,\rho')\,. \tag{5}$$

Here $W=s^{1/2}$ and M and μ are the nucleon and pion masses, respectively; $\alpha_\rho(t)$ and $\alpha_{\rho'}(t)$ denote the ρ and ρ' trajectories, and b_ρ, $b_{\rho'}$ and $\tilde{b}_\rho$, $\tilde{b}_{\rho'}$ are the Regge-pole residues associated with the non-spin-flip and spin-flip amplitudes, respectively. We shall choose $s_0=2M\mu$, whereby $(s-M^2-\mu^2)/s_0=E/\mu$ and the residue functions are only weakly dependent upon t.[16–18] In the present work we have chosen the residues in fact to be constants. In the same spirit we shall adopt simple, linear relationships for the t dependence of $\alpha_\rho(t)$ and $\alpha_{\rho'}(t)$, viz., $\alpha_\rho(t)=\alpha_\rho(0)+\alpha_\rho'(0)t$ and $\alpha_{\rho'}(t)=\alpha_{\rho'}(0)+\alpha_{\rho'}'(0)t$.

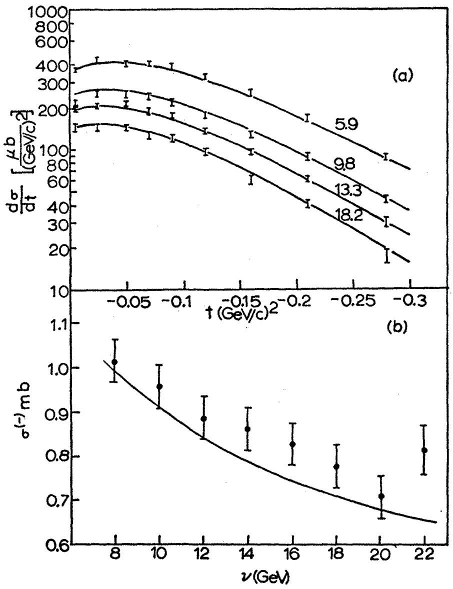

FIG. 1. (a) $\rho+\rho'$ Regge-pole fit to the differential cross sections for $\alpha_{\rho'}(0)=0.2$ at pion lab energies of 5.9, 9.8, 13.3, and 18.2 GeV. (b) $\rho+\rho'$ Regge-pole fit to $\sigma^{(-)}$ for $\alpha_{\rho'}(0)=0.2$.

The superconvergence relations obtained from the non-spin-flip amplitude in the limit $\nu\to\infty$ take the form

$$\{b_\rho(\mu\bar{E}/\sqrt{2})[1/(1+\alpha_\rho)](\bar{E}/\mu)^{\alpha_\rho}+b_{\rho'}(\mu\bar{E}/\sqrt{2})[1/(1+\alpha_{\rho'})](\bar{E}/\mu)^{\alpha_{\rho'}}\}=-4\pi^2f^2+\int_\mu^{\bar{E}}dE(E^2-\mu^2)^{1/2}\sigma^{(-)}(E)\,, \tag{6}$$

$$(b_\rho/4\pi\alpha_\rho)\tan(\tfrac{1}{2}\pi\alpha_\rho)(\bar{E}/\mu)^{\alpha_\rho}+(b_{\rho'}/4\pi\alpha_{\rho'})\tan(\tfrac{1}{2}\pi\alpha_{\rho'})(\bar{E}/\mu)^{\alpha_{\rho'}}=\sqrt{2}\pi f^2/\mu^2+\frac{1}{\sqrt{2}\mu}\int_\mu^{\bar{E}}\frac{dE}{(E^2-\mu^2)^{1/2}}\mathrm{Re}f^{(-)}(E)\,. \tag{7}$$

If we choose the asymptotic lab energy $\bar{E}=5$ BeV, we get from Eqs. (6) and (7) the relations

$$[b_\rho/(\alpha_\rho+1)](\bar{E}/\mu)^{\alpha_\rho}+[b_{\rho'}/(\alpha_{\rho'}+1)](\bar{E}/\mu)^{\alpha_{\rho'}}=60.9\pm6.6\,, \tag{8}$$

$$(b_\rho/\pi\alpha_\rho)\tan(\tfrac{1}{2}\pi\alpha_\rho)(\bar{E}/\mu)^{\alpha_\rho}+(b_{\rho'}/\pi\alpha_{\rho'})\tan(\tfrac{1}{2}\pi\alpha_{\rho'})\times(\bar{E}/\mu)^{\alpha_{\rho'}}=61.8\pm12.7\,. \tag{9}$$

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Initially, we have eight free parameters. We use Eq. (8) to obtain $b_{\rho'}$ in terms of $\alpha_{\rho'}$, α_ρ, and b_ρ, reducing the number of free parameters to seven. In view of the error involved in the determination of the integral, in Eq. (7), we do not use (9) to reduce the number of parameters further, but only attempt to fit this relation.

In fitting the data two types of solutions emerge—those with $\alpha_{\rho'}(0)>0$ and those with $\alpha_{\rho'}(0)<0$. Both solutions give good χ^2 values in fitting the present data. The fit to $d\sigma/dt$ for $\alpha_{\rho'}(0)=0.2$ is shown in Fig. 1(a). The fit to the total cross sections $\sigma^{(-)}$ is shown in Fig. 1(b) for $\alpha_{\rho'}(0)=0.2$. In fitting the polarization P, we find that the behavior is quite different in the two cases when $\alpha_{\rho'}(0)$ is positive or negative. The results for $\alpha_{\rho'}(0)=0.2$ and $\alpha_{\rho'}(0)=-0.5$ are displayed in Fig. 2(a). For $\alpha_{\rho'}(0)=0.2$ the polarization P decreases with increasing $-t$, whereas for $\alpha_{\rho'}(0)=-0.5$ it shows a definite increasing trend. This is an interesting prediction as future, more accurate experiments on the charge-exchange polarization will be able to choose between the two possible solutions $\alpha_{\rho'}(0)>0$ and $\alpha_{\rho'}(0)<0$.

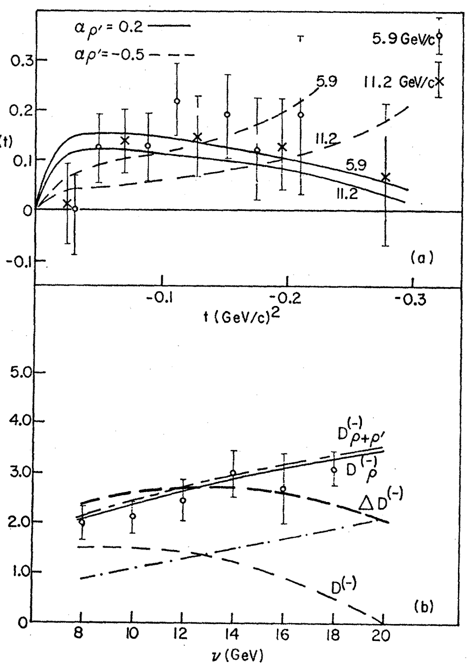

FIG. 2. (a) Fit to polarization data for $\alpha_{\rho'}(0)=0.2$ and $\alpha_{\rho'}(0)=-0.5$. (b) Fit of $D_{\rho+\rho'}^{(-)}$ for $\alpha_{\rho'}(0)=0.2$ to the forward real antisymmetric amplitude calculated from charge-exchange data. Also shown, for comparison, is $D_\rho^{(-)}$ from the single ρ model, and $D^{(-)}$ calculated from Coulomb interference data. $\Delta D^{(-)}$ is the result of adding the systematic error (straight dashed line) to $D^{(-)}$.

TABLE I. $\rho+\rho'$ parameters, which give good χ^2 fits to present charge-exchange data, are shown for $\alpha_{\rho'}(0)=0.2$ (solution A) and $\alpha_{\rho'}(0)=-0.5$ (solution B). The residues are given in mb$^{1/2}$ BeV^{-1}.

	Sol. A	Sol. B
$\alpha_\rho(0)$	0.576	0.575
$\alpha_{\rho'}(0)$	0.200	-0.500
$\alpha_\rho'(0)$	0.78	0.87
$\alpha_{\rho'}'(0)$	0.71	0.47
b_ρ	13.0	12.5
$\bar{b}_\rho$	172	225
$b_{\rho'}$	-2.29	-4.28
$\bar{b}_{\rho'}$	775	-763

With $\alpha_{\rho'}(0)=0.2$ and $\alpha_{\rho'}(0)=-0.5$ the superconvergence relation [Eq. (9)] is satisfied giving values 69.5 and 68.6, respectively. The effect of the superconvergence relations is to limit the magnitude of the ρ' contribution to the non-spin-flip amplitude to be small, but they have no effect on the contribution to the spin-flip amplitude. In fact, we had to take a rather large ρ' contribution to the spin flip in order to obtain the experimentally observed polarization. (See Table I.)

Figure 2(b) shows that $D_{\rho+\rho'}^{(-)}$ with $\alpha_{\rho'}(0)=0.2$ is almost the same as D_ρ.[9] This remains true for all $\alpha_{\rho'}(0)$ which gave us good fits to the charge-exchange data.

In view of the large number of parameters in the present calculation it was not possible to limit the range of acceptable values for both $\alpha_{\rho'}(0)$ and $\alpha_{\rho'}'(0)$. These values of $\alpha_{\rho'}(0)$ range between -0.5 and 0.25, with a transition region in-between in which the increasing polarization behavior becomes a decreasing one.

In Table I, the various parameters obtained from two solutions at $\alpha_{\rho'}(0)=0.2$ and $\alpha_{\rho'}(0)=-0.5$ are displayed.

In conclusion, we can make the following comments:

(1) The $\rho+\rho'$ model fits the present charge-exchange data with a range of values for the ρ' trajectory.

(2) The two solutions $\alpha_{\rho'}(0)>0$ and $\alpha_{\rho'}(0)<0$ which emerge predict an increasing polarization and a decreasing polarization, respectively. More accurate polarization experiments to determine the behavior of P for increasing $-t$ should discriminate between the two ranges of solutions, and provide some information about a possible branch cut simulated by the ρ' trajectory.

(3) The two superconvergence sum rules for the forward non-spin-flip amplitude are well satisfied by the $\rho+\rho'$ model.