Asymptotic Behavior of Form Factors and the Ratio of the Renormalization Constants Z_1/Z_3

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The asymptotic behavior of the form factor of three scalar particles has been investigated using the Deser-Gilbert-Sudarshan-Ida-Nakanishi representation. For finite Z_3, we show that the form factor tends at most to a constant for large s. The compositeness condition $Z_3=0$ implies that $Z_1=0$ provided the renormalization function $Z_3(s)$ does not tend to zero faster than $1/s$ for $s\to\infty$. The relevance of our results to the Lee model and to the Zachariasen model is briefly discussed.

WITH the extension of the experimental information on form factors $F(s)$ to a very large momentum transfers s, increasing interest has been paid[1] to the behavior of $F(s)$ as $|s|\to\infty$. To investigate the problem from a general point of view, we would like to advocate the use of the Deser-Gilbert-Sudarshan-Ida-Nakanishi (DGSIN)[2] representation for the chronological product

$$(2\omega)^{1/2}\langle p|T(J(x)J(y))|0\rangle=\int\frac{d\lambda^2d\beta d^4q}{i(2\pi)^4}\times\frac{\exp\{-iq\cdot(x-y)-ip\cdot y+i\beta p\cdot(x-y)\}}{q^2-\lambda^2+i\epsilon}\times H(\lambda^2,\beta,p^2). \quad (1)$$

In Eq. (1), p is the momentum of a single-particle state (we restrict ourselves to scalar particles). Support conditions on the spectral function H are $-1\leqslant\beta\leqslant 0$ and $\lambda^2>0$.

The form factor is defined by

$$\langle p|J(0)|p'\rangle=\frac{1}{(4\omega\omega')^{1/2}}F[(p-p')^2], \quad (2)$$

where p and p' are the momenta of the respective single-particle states and $s=(p-p')^2$ is the momentum transfer. The form factor [Eq. (2)] can be related to the DGSIN representation (1) by the LSZ formalism[3]:

$$F(s)=\int d\lambda^2d\beta\frac{H(\lambda^2,\beta,p^2)}{\alpha^2-\lambda^2+i\epsilon}+\sum_{n=0}^{N}a_ns^n =F_0(s)+\sum a_ns^n, \quad (3)$$

where $\alpha=p'+p(1+\beta)$, $\sum a_ns^n$ is a polynomial, and $F_0(s)$ is the nonpolynomial part of $F(s)$.

A few remarks on the validity of Eq. (3) should be made. Shortly after the proposal of the DGSIN representation, Minguzzi and Streater[4] demonstrated that it does not follow solely from microcausality and the spectral conditions. It is, however, well known that the DGSIN representation has subsequently been proved to exist to all orders of perturbation theory by Araki and Nakanishi.[5] This proof suggests that the representation is generally valid, at least when two particles are on the mass shell.[6] (One might say that the use of the DGSIN representation is justified in the same way as the use of the Mandelstam representation.)

In the following we shall assume the DGSIN representation to hold for $p^2=m^2$, $p'^2=m'^2$ on the mass shell. The polynomial which occurs in Eq. (3) has to be considered carefully since it has a crucial effect on the asymptotic behavior of $F(s)$.

If the weight function $H(\lambda^2,\beta)$ satisfies a Hölder condition for $\lambda^2>R$ (R is a positive, β-independent number), is a well behaved distribution in λ^2 in the rest of the interval, and contains only integrable singularities in β, then it follows from Eq. (3) by means of a straightforward generalization of a theorem by Nakanishi[7] that, for large enough s,

$$F_0(s)<C/|s|^\delta, \quad (4)$$

where C is a positive number and δ a small positive constant. For a more rigorous formulation the reader is referred to Nakanishi's paper[7] (see also the Appendix).

The form factor is related to the vertex part $\Gamma(s)$ by

$$F(s)=\frac{\Delta'(s)}{\Delta(s)}\Gamma(s), \quad (5)$$

where Δ and Δ' are the free and the renormalized propagators, respectively. In the limit $|s|\to\infty$ Eq. (5) becomes

$$F(s\to\infty)=Z_3^{-1}\Gamma(s\to\infty), \quad (6)$$

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where Z_3 is the wave-function renormalization constant. From the established[8] bound on $\Gamma(s)$

$$\lim_{s\to\infty}\left[\frac{\Gamma(s)}{\sqrt{s}}\right]=0, \tag{7}$$

one concludes using Eqs. (3), (4), and (6) that for finite Z_3,[9] or if $Z_3(s)\equiv\Delta(s)/\Delta'(s)$ goes to zero slower than $1/\sqrt{s}$ as $s\to\infty$, one has

$$\lim_{s\to\infty}F(s)=\lim_{s\to\infty}F_0(s)+a_0 \tag{8}$$

and

$$F(s=\infty)=a_0. \tag{9}$$

Experiments seem to suggest that the form factors actually do decrease as the second power of invariant variable s. If this behavior should persist in the very high-energy region not yet accessible to experiment, then one is necessarily led to conclude that the constant a_0 in Eq. (9) is indeed zero.[9a]

If one accepts the so-called Symanzik conjecture[10]

$$\lim_{s\to\infty}\Gamma(s)=Z_1, \tag{10}$$

which is obviously a stronger assumption than Eq. (7), another conclusion may be drawn from Eqs. (6), (9), and (10), viz.,

$$\frac{Z_1}{Z_3}=\lim_{s\to\infty}F(s)=a_0. \tag{11}$$

Equation (11) is valid for finite Z_3 or for $Z_3(s)$ approaching zero slower than $1/s$ as $s\to\infty$. If $Z_3\neq 0$, Eq. (11) yields a "representation" for Z_1:

$$Z_1=a_0\int d\kappa^2\sigma(\kappa^2),$$

where $\sigma(\kappa^2)$ is the spectral function for the renormalized propagator, $a_0\neq 0$. Clearly Z_1 does vanish for finite Z_3 if $a_0=0$. This corresponds to the situation in the Zachariasen model[11] when the A particle is "elementary." If $Z_3=0$, then Eq. (11) tells us that, under the restrictions stated immediately after Eq. (11), Z_1 *must be zero*. This result is of special interest in connection with compositeness criteria suggested by Salam, Weinberg, and others[12] and with the question of renormalizability of field theories for higher-spin particles.[13] It is however in apparent contradiction with the Lee model when the V particle is "composite"[14] which is resolved as follows: (a) $Z_3(s)\to 0$ slower than $1/s$. We may assume for example $s^{\epsilon}<Z_3^{-1}(s)<s^{\delta}$ with $0<\epsilon<\delta<1$. This behavior cannot be generated by the representation Eq. (3).[9] (b) $Z_3(s)\to 0$ as $1/s$ or faster. Then $Z_1=1$ is consistent with our results.

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APPENDIX

For $s\to\infty$, a simple proof of our results can be given. Without loss of generality, we may assume that $H(\lambda_1^2\beta)$ is continuous in λ^2 and β. Distributions which may be expressed as nth order derivatives of continuous functions in λ^2 can be treated analogously by first carrying out n partial integrations.

Separating the first term on the right-hand side of (3) into real and imaginary parts, we obtain

$$F_0(s)=-\int d\lambda^2 d\beta\, H(\lambda^2,\beta)\times\left\{P\frac{1}{\lambda^2-\alpha^2}-i\pi\delta(\lambda^2-\alpha^2)\right\}. \tag{A1}$$

Choosing $s=(p-p')^2>3p^2+p'^2$ and $\lambda^2>p'^2$ [this is permissible since we are interested in the limit of $F(s)$ for large s, the lower limit of the λ^2 integration[1] is clearly $>p'^2$] one can easily establish that the term $\delta(\lambda^2-\alpha^2)$ makes no contribution since the argument of the δ function never vanishes.

The principal-value integral must be evaluated carefully using a generalized mean-value theorem:

$$F_0(s)=\int d\lambda^2\frac{-1}{\sqrt{\Delta}}\left\{\ln\frac{(\sqrt{\Delta})+s-3p^2-p'^2}{(\sqrt{\Delta})-s+3p^2+p'^2}-\ln\frac{(\sqrt{\Delta})+s-p^2-p'^2}{(\sqrt{\Delta})-s+p^2+p'^2}\right\}H(\lambda^2,p^2,\beta_0), \tag{A2}$$

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[9] An increasing fractional-power behavior of $F(s)$ as $s\to\infty$ cannot be generated by the representation Eq. (3), since the first term on the right-hand side of Eq. (3) $\to 0$ for large s, and the second term is a polynomial.

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where

$$\Delta=(s^2+4p^2\lambda^2-2sp'^2+p^4+p'^4-2p^2p'^2)$$

and

$$-1\leqslant\beta_0\leqslant 0.$$

The function multiplying $H(\lambda^2,p^2,\beta_0)$ in (A2) is continuous in λ^2, provided $s\geqslant 2(p^2+p'^2)$ and $\lambda^2>p'^2$. We now specialize to the equal-mass case $p^2=p'^2=m^2$ (the generalization for the case $p^2\neq p'^2$ may be carried out in a similar fashion).

Consider the function $g(m^2,s)$ given by

$$g(m^2,s)=\int d\lambda^2\frac{1}{(\lambda^2)^\epsilon}\frac{1}{\sqrt{\Delta}}\times\left\{\ln\frac{(\sqrt{\Delta})+s-4m^2}{(\sqrt{\Delta})-s+4m^2}-\ln\frac{(\sqrt{\Delta})+s-2m^2}{(\sqrt{\Delta})-s+2m^2}\right\}. \quad (A3)$$

Then, $g(m^2,s)$ is uniformly convergent in s.

Proof: We expand the logarithms in (A3), making use of the fact that

$$\ln\left|\frac{1+x}{1-x}\right|\leq cx, \quad \text{with} \quad 0\leq c<\infty \quad \text{for} \quad |x|<1$$

and noting that $\sqrt{\Delta}>s-2m^2$, for $|x|<1$.

$$|g(m^2,s)|\leq c\left|\int d\lambda^2\left\{\frac{s-4m^2}{\sqrt{\Delta}}-\frac{s-2m^2}{\sqrt{\Delta}}\right\}\frac{1}{(\lambda^2)^\epsilon}\right|$$
$$=2c\int^\infty d\lambda^2\frac{m^2}{4m^2\lambda^2+s^2-2sm^2}\frac{1}{(\lambda^2)^\epsilon}$$
$$<\tfrac{1}{2}c\int^\infty d\lambda^2\frac{1}{(\lambda^2)^{1+\epsilon}}. \quad (A4)$$

The inequality (A4) ensures the uniform convergence of $g(m^2,s)$ in s.

Assuming that for large enough λ^2, $H(\lambda^2,\beta_0,m^2)$ is bounded by $(\lambda^2)^{-\epsilon}$, and using again the mean-value theorem, one establishes Eq. (8).

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$\rho+\rho'$ Model of Pion-Nucleon Charge-Exchange Scattering and Polarization

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A $\rho+\rho'$ Regge-pole model in which the ρ' is an α-type or nonconspiring trajectory is investigated. It is found to agree with present charge-exchange data and certain superconvergent sum rules. Present polarization measurements are not accurate enough to constrain all the Regge parameters, and a range of solutions emerges consistent with the data.

THE ρ' meson with $J^P=1^-$ and $I=1$ has been introduced[1,2] to explain the polarization observed[3] in high-energy pion-nucleon charge-exchange scattering. The recent work of Igi and Matsuda[4] showed that the parameters obtained by Logan, Beaupre, and Sertorio[1] to fit the πp polarization did not satisfy their superconvergence relations.

In this paper, we shall present parameters for the $\rho+\rho'$ model which agree with the πp charge-exchange polarization and scattering data, and the superconvergence relations of Igi, Matsuda,[4] and Olsson.[5,6] In order to satisfy the constraints provided by both the polarization data and the superconvergence relations, Sertorio and Toller[7] introduced a Gribov-Volkov[8] β-type ρ' trajectory (i.e., a conspiring ρ' trajectory). We have found that it is not necessary to introduce a conspiring ρ' trajectory, but that it is possible to satisfy the superconvergence relations and the polarization data with the α-type or nonconspiring ρ'. In fact, the agreement with the superconvergence relations seems to be better with the α-type ρ' trajectory.

Gajdicar and Moffat[9] have recently shown, on the basis of an analysis of the recent Coulomb interference

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