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Spinor Currents and Symmetries of Baryons and Mesons*

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By introducing spinor currents in addition to the algebra of vector and axial-vector currents, new symmetries of hadrons are obtained in which both baryons and mesons are grouped together in a supermultiplet. These currents are constructed as bilinear combinations of fundamental fields, both bosonic and fermionic. They form a generalized Jordan algebra, whose representations are constructed. As possible models, two examples are given, $V(6,3)$ and $V(6,21)$. In the latter, all known hadrons are included in the adjoint representation.

1. INTRODUCTION

THE commutation relations of vector and axial-vector currents and other density operators form a Lie algebra, and hadron multiplets are given as its representations. This scheme has proved to be very successful in correlating many phenomena. However, it cannot give any relation between the baryon and meson multiplets; they are two independent representations. The coexistence of baryons and mesons of similar properties, in masses and strong couplings, can be explained and their intimate connection can be obtained if all of them are included in a multiplet. To this end the algebra must contain, as its elements, operators which have matrix elements between baryons and mesons. From the conservation of angular momentum these baryon-number-changing operators have half-odd integer spins and are called spinor currents. Further, for the correct statistics, these operators must obey anticommutation relations. This means that the set of operators under consideration cannot be a Lie algebra and our concept of internal symmetry must be extended to a more generalized algebraic system.

In a quark model in which baryons are regarded as three-quark states while mesons are quark-antiquark states, such baryon-number-changing operators can be constructed by the product of three quark fields.[1] However, anticommutators of two such baryon currents are complicated things, so the commutation relations do not close on a finite number of operators. Thus any algebraic treatment cannot be expected in this scheme. The success of the current-algebra theory lies in the fact that the commutators of two axial-vector currents are given by vector currents, thus forming a finite Lie algebra. This happens, for instance, when currents are written as bilinear combinations of quark and antiquark fields. In this paper we propose an algebraic scheme where anticommutators of two-spinor currents are simply combinations of other currents, so that commutators close on a finite number of operators. Especially we try to extend the algebra $U(6)$ or the nonchiral $U(6)\times U(6)$ which is very successful in classifying hadrons at rest.[2]

To write commutation relations, thereby creating a symmetry scheme of hadrons, it is convenient to inintroduce fundamental fields and to construct operators as their bilinear combinations. These fundamental fields are also useful in obtaining representations of the algebra. We consider only space integrals of current densities, thus avoiding complications of Schwinger terms. An example of this type of symmetry was already discussed in a particular model.[3] Here we shall investigate more generally, obtain representations, and propose an algebra $V(6,21)$ as a realistic but somewhat complicated model.

2. FUNDAMENTAL FIELDS AND COMMUTATION RELATIONS

We suppose that there are fundamental fields, both fermions and bosons, from which current operators are constructed. Fermion fields are written as $\psi_{a\alpha}$, where a represents internal variables, and α is the Dirac index. Similarly, boson fields are written as φ_{ai}, where i denotes the spin direction if it has nonvanishing spin. At some instant t, they satisfy the following canonical commutation relations:

$$\begin{aligned}
&\{\psi_{a\alpha}(x),\psi_{b\beta}{}^\dagger(y)\}=\delta_{a\alpha,b\beta}\delta(x-y)\,,\\
&\{\psi_{a\alpha}(x),\psi_{b\beta}(y)\}=\{\psi_{a\alpha}{}^\dagger(x),\psi_{b\beta}{}^\dagger(y)\}=0\,,\\
&[\varphi_{ai}(x),\pi_{bj}(y)]=[\varphi_{ai}{}^\dagger(x),\pi_{bj}{}^\dagger(y)]=i\delta_{ai,bj}\delta(x-y)\,,\quad(2.1)\\
&[\varphi,\varphi]=[\pi,\varphi^\dagger]=\cdots=0\,.
\end{aligned}$$

π and $\pi^\dagger$ are canonically conjugate fields of φ and $\varphi^\dagger$, respectively. We only consider equal-time commutators, and x or y represents three-dimensional space coordinates. In order to write the commutation relations for bosons in a similar form to those for fermions, we intro-

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[1] R. C. Hwa and J. Nuyts, Phys. Rev. **151**, 1215 (1966).
[2] K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **14**, 48 (1965); **14**, 264 (1965); S. Okubo and R. E. Marshak, *ibid.* **14**, 156 (1965); R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965); **17**, 145 (1965).
[3] H. Miyazawa, Progr. Theoret. Phys. (Kyoto) **36**, 1266 (1966).

duce new quantities

$$\varphi_{a\ i+1}=\left(\sqrt{\frac{m}{2}}\right)\varphi_{ai}+i\left(\sqrt{\frac{1}{2m}}\right)\pi_{ai}{}^\dagger\,,$$
$$\bar\varphi_{a\ i+1}=\varphi_{a\ i+1}{}^\dagger=\left(\sqrt{\frac{m}{2}}\right)\varphi_{ai}{}^\dagger-i\left(\sqrt{\frac{1}{2m}}\right)\pi_{ai}\,,$$
$$\varphi_{a\ i-1}=\left(\sqrt{\frac{m}{2}}\right)\varphi_{ai}-i\left(\sqrt{\frac{1}{2m}}\right)\pi_{ai}{}^\dagger\,, \quad (2.2)$$
$$\bar\varphi_{a\ i-1}=\varphi_{a\ i-1}{}^\dagger=\left(\sqrt{\frac{m}{2}}\right)\varphi_{ai}{}^\dagger+i\left(\sqrt{\frac{1}{2m}}\right)\pi_{ai}\,,$$

where m has the dimension of mass. The satisfy

$$[\varphi_{ai\xi}(x),\bar\varphi_{bj\eta}(y)]=\delta_{ai,bj}\beta_{\xi\eta}\,,$$
$$[\varphi_{ai\xi}(x),\varphi_{bj\eta}(y)]=[\bar\varphi_{ai\xi}(x),\bar\varphi_{bj\eta}(y)]=0\,, \quad (2.3)$$

where ξ or η stands for ±1 and

$$\beta_{\xi\eta}=\delta_{\xi\eta}\epsilon_\xi\,,$$
$$\epsilon_\xi=1,\quad \xi=+1\,,$$
$$\epsilon_\xi=-1,\quad \xi=-1\,.$$

Instead of the Dirac index α, we use $i\xi$, where i is the eigenvalue of σ_z, and ξ is the eigenvalue of $\gamma_4=\beta$. Then $(\bar\psi=\psi^\dagger\beta)$,

$$\{\psi_{ai\xi}(x),\bar\psi_{bj\eta}(y)\}=\delta_{ai,bj}\beta_{\xi\eta}. \quad (2.4)$$

We also define

$$\hat\varphi_{ai\xi}=\bar\varphi_{ai\eta}\beta_{\eta\xi},\quad \hat\psi_{ai\xi}=\bar\psi_{ai\eta}\beta_{\eta\xi}=\psi_{ai\xi}{}^\dagger\,, \quad (2.5)$$

which obey commutation relations

$$[\varphi_A(x),\hat\varphi_B(y)]=\{\psi_A(x),\hat\psi_B(y)\}=\delta_{AB}\delta(x-y)\,,$$

where A stands for $ai\xi$, etc. Summarizing, when φ_A or ψ_A is written as Q_A,

$$[Q_A(x),\hat Q_B(y)]_\pm=\delta_{AB}\delta(x-y)\,,$$
$$[Q_A(x),Q_B(y)]_\pm=[\hat Q_A(x),\hat Q_B(y)]=0\,, \quad (2.6)$$

where $[\ ]_\pm$ denotes the anticommutator when the two operators inside are both fermions, and the commutator in other cases. In this way a charged boson of spin direction i can be described by four quantities: either by φ_i, $\varphi_i{}^\dagger$, π_i, and $\pi_i{}^\dagger$, or by $\varphi_{i\pm1}$ and $\hat\varphi_{i\pm1}$.

From n-component fermions ψ_A, $A=1,\cdots,n$, and m-component bosons φ_A, $A=n+1,\cdots,n+m$, we can make $(n+m)^2$ operators,

$$G_B{}^A=\int\hat Q_A(x)Q_B(x)dx. \quad (2.7)$$

When either A or B is a fermion and the other is a boson, $G_B{}^A$ is a Fermi-like operator (written as $F_B{}^A$) and in other cases it is Bose-like ($B_B{}^A$). They satisfy the commutation relations

$$[B_B{}^A,B_D{}^C]=B_D{}^A\delta_{BC}-B_B{}^C\delta_{AD}\,,$$
$$[F_B{}^A,B_D{}^C]=F_D{}^A\delta_{BC}-F_B{}^C\delta_{AD}\,, \quad (2.8)$$
$$\{F_B{}^A,F_D{}^C\}=B_D{}^A\delta_{BC}+B_B{}^C\delta_{AD}\,,$$

or, in short,

$$[G_B{}^A,G_D{}^C]_\pm=G_D{}^A\delta_{BC}\pm G_B{}^C\delta_{AD}. \quad (2.9)$$

In addition there are reality conditions of the form

$$G_B{}^{A\dagger}=\pm G_A{}^B\,, \quad (2.10)$$

where the minus sign holds when only one of A and B is a boson, and $\xi=-1$.

Relation (2.9) contains both commutators and anticommutators, so the set of operators $G_B{}^A$ does not form a Lie algebra. An algebra with anticommutation as multiplication is called a Jordan algebra.[4] Our system is a mixture: Multiplication is defined by an anticommutator between Fermi-like elements and by a commutator between any other combinations. Similar to the Jacobi identity, our multiplication obeys the following rule:

$$[F,\{F',F''\}]+[F',\{F'',F\}]+[F'',\{F,F'\}]=0\,,$$
$$[F,[B,B']]+[B,[B',F]]+[B,'[F,B]]=0\,, \quad (2.11)$$
$$[B,\{F,F'\}]+\{F,[F',B]\}-\{F',[B,F]\}=0.$$

Unlike a Lie algebra, a Jordan algebra or our "algebra"[5] does not generate a continuous group, so we do not have a symmetry group, but only a symmetry algebra. The operators $G_B{}^A$ and the commutation relations (2.9) are similar but, of course, not equal to the generators of the unitary group of $n+m$ dimensions. Tentatively we let $V(n,m)$ denote the algebra defined by (2.9), where n is the number of Fermi-like indices, and m is the number of Bose-like indices.

The commutator of two Fermi-like current densities $F_B{}^A(x)=\hat Q_A(x)Q_B(x)$ and $F_D{}^C(y)=\hat Q_C(y)Q_D(y)$ is a complicated quantity and does not vanish even when x and y are separated by a finite distance. However, this will not conflict with causality, which requires that two observables at two points with spacelike separation should commute. We think a Fermi-like quantity cannot be measured completely. Its phase will be unmeasurable because of the strict conservation of the fermion number. Only its absolute value will be observable. $|F(x)|^2=F(x)^\dagger F(x)$ and $|F'(y)|^2$ commute when $x\neq y$.

3. ALGEBRA $V(n,m)$ AND ITS REPRESENTATION

A set of operators X_α, $\alpha=1,\cdots,\nu$, satisfying

$$[G_B{}^A,X_\alpha]_\pm=(C_B{}^A)_{\alpha\beta}X_\beta\,, \quad (3.1)$$

[4] For Jordan algebra, see H. Braun and M. Koecher, *Jordanalgebren* (Springer-Verlag, Berlin, 1961).

[5] Our system is not an algebra since addition of a Fermi-like element and a Bose-like element is not defined. Nevertheless we call it an algebra for lack of a suitable word.

defines a linear transfromation in ν-dimensional space for each $G_B{}^A$. Because of the generalized Jacobi identity (2.11), we see that the correspondence

$$G_B{}^A \to (G_B{}^A)_{\alpha\beta} = (C_B{}^A)_{\beta\alpha} \quad (3.2)$$

is a representation of the algebra. That is, the matrices $(G_B{}^A)_{\alpha\beta}$ satisfy the same commutation relation as (2.9):

$$(C_B{}^A)_{\beta\alpha}(C_D{}^C)_{\gamma\beta} \pm (C_D{}^C)_{\beta\alpha}(C_B{}^A)_{\gamma\beta} = (C_D{}^A)_{\gamma\alpha}\delta_{BC} \pm (C_B{}^C)_{\gamma\alpha}\delta_{AD}. \quad (3.3)$$

The fundamental representation of $V(n,m)$ is obtained by $n+m$ vectors $\hat{Q}_A$ used as the X_α of Eq. (3.1). This representation is given by

$$G_B{}^A \to E_{AB}, \quad (3.4)$$

where E_{AB} is the matrix whose (A,B) element only is unity and other elements are zero:

$$(E_{AB})_{ab} = \delta_{Aa}\delta_{Bb}.$$

Another $(n+m)$-dimensional representation is obtained from Q_A. Some complication arises for baryon-number-changing operators:

$$\begin{aligned} &B_B{}^A \to -E_{BA}, \\ &F_B{}^A \to -E_{BA}; \quad A\text{: Bose, } B\text{: Fermi} \\ &F_B{}^A \to +E_{BA}, \quad A\text{: Fermi, } B\text{: Bose.} \end{aligned} \quad (3.5)$$

For this antiparticle representation, $F_B{}^A$ and $F_A{}^a$ are anti-Hermitian conjugate to each other.

The adjoint representation is the one with $G_D{}^C$ themselves as X_α. It is $(n+m)^2$-dimensional. The trace $G_C{}^C$ commutes with all elements and forms an invariant subspace, so the representation is reducible, but not fully reducible. $G_B{}^A$ is represented by a matrix of the form

$$G \to \begin{bmatrix} 0 & \\ 0 & \\ \vdots & G' \\ 0 & \end{bmatrix}, \quad (3.6)$$

when $G_C{}^C$ is chosen as the first basis vector.

The general tensor representations can be obtained from products of a number of Q and $\hat{Q}$'s.

$$T_{CD\cdots}{}^{AB\cdots} = \hat{Q}_A(x)\hat{Q}_B(y)\cdots Q_C(z)Q_D(u)\cdots. \quad (3.7)$$

They are not irreducible and must be decomposed according to symmetry properties. From the generalized Jacobi identity, we have, for instance,

$$[F_A{}^C, \hat{Q}_A(x)\hat{Q}_B(y) \pm \hat{Q}_B(z)\hat{Q}_A(y)] = \hat{Q}_C(x)\hat{Q}_B(y) \mp \hat{Q}_B(x)\hat{Q}_C(y), \quad \text{(not summed over } A) \quad (3.8)$$

when A and B are Fermi-like and C is Bose-like. This means that a set of T^{AB} which are symmetric in A, B when both A and B are Fermi-like and antisymmetric in other cases forms an invariant subspace. We define the symmetrization symbol ( ) and antisymmetrization symbol [ ] by

$$\begin{aligned} T^{(AB)} &= T^{AB} - T^{BA}, \quad A, B\text{: Fermi-like} \\ &= T^{AB} + T^{BA}, \quad \text{otherwise} \end{aligned} \quad (3.9)$$

and

$$\begin{aligned} T^{[AB]} &= T^{AB} + T^{BA}, \quad A, B\text{: Fermi-like} \\ &= T^{AB} - T^{BA}, \quad \text{otherwise.} \end{aligned} \quad (3.10)$$

Then T^{AB} is decomposed into two invariant subspaces,

$$T^{AB} = T^{(AB)} + T^{[AB]}.$$

General irreducible representations are given by $T_{CD\cdots}{}^{AB\cdots}$ where it is symmetrized or antisymmetrized, in the sense of (3.9) and (3.10), in the upper indices $AB\cdots$ and also in the lower indices $CD\cdots$. The traceless parts alone do not furnish a representation.

4. EXTENSION OF $U(6)$ SYMMETRY

For the extension of the $U(6)$ or $U(6)\times U(6)$, we consider subalgebras of the complete algebra (2.9). To avoid complications, take a simple case of one Dirac field and one complex scalar field as fundamental fields. Writing indices explicitly, the following nine operators

$$B_j{}^i = \int \hat{\psi}_{i\xi}\psi_{j\xi}dx, \quad B_3{}^3 = \int \hat{\varphi}_\xi\varphi_\xi dx,$$

$$F_3{}^i = \int \hat{\psi}_{i\xi}\varphi_\xi dx, \quad F_i{}^3 = \int \hat{\varphi}_\xi\psi_{i\xi}dx, \quad (i, j=1, 2)$$

or, in short,

$$G_b{}^a = \int \hat{Q}_{a\xi}(x)Q_{b\xi}(x)dx, \quad (a, b=1, 2, 3) \quad (4.1)$$

satisfy the commutation relations (2.9) and form $V(2,1)$. The reality conditions are imposed only on Bose-like operators:

$$B_b{}^{a\dagger} = B_a{}^b, \quad \text{but} \quad F_b{}^{a\dagger} \neq F_a{}^b. \quad (4.2)$$

Fundamental representations are **3** and $\bar{\mathbf{3}}$, and the adjoint representation is nine-dimensional.

The trace $B_i{}^i$ and $B_3{}^3$ are the fermion-number and the boson-number operators, respectively. The traceless parts of $B_j{}^i$ can be identified with the angular momentum. Then the adjoint representation is decompoesd as

$$9 = \text{spin}1 + \text{spin}\tfrac{1}{2} + (\text{spin}\tfrac{1}{2})^* + \text{spin}0 + \text{spin}0,$$

where (spin $\frac{1}{2}$)* means the antiparticle representation of spin $\frac{1}{2}$.

Similar to the nonchiral $U(6)\times U(6)$, we can add

$$G'_b{}^a = \int \hat{Q}_{a\xi}(x)\beta_{\xi\eta}Q_{b\eta}(x)dx$$

to $G_b{}^a$. Taking combinations $1\pm\beta$, the operators

$$(G^+)_b{}^a = \int \hat{Q}_{a+1}Q_{b+1}dx, \quad (G^-)_b{}^a = \int \hat{Q}_{a-1}Q_{b-1}dx$$

form $V^+(2,1)\times V^-(2,1)$. The reality condition requires

$$(B^\pm)_b{}^{a\dagger}=(B^\pm)_a{}^b,\quad (F^\pm)_b{}^{a\dagger}=\pm(F^\pm)_a{}^b.$$

Comparing this with Eqs. (3.4) and (3.5), we see that $V^+(2,1)$ has only particle representations while $V^-(2,1)$ has only antiparticle representations. A general representation of $V^+(2,1)\times V^-(2,1)$ is given again by $T^{ab\cdots}\times T_{cd\cdots}$ which is identical to that of $V(2,1)$. The space parity can be defined by the element of the algebra:

$$P=(-1)^{\bar{N}},\quad \bar{N}=(B^-)_1{}^1+(B^-)_2{}^2.$$

The adjointlike representation $(3,\bar{3})$ is given as $(3,\bar{3})$ = vector+spinor+spinor*+scalar+pseudoscalar. Here 3 and $\bar{3}$ mean the number of upper and lower indices, respectively.

For practical application the simplest example will be provided by $V(6,3)$, a system consisting of the quark triplet and a triplet of scalar mesons arranged in the form (4.1):

$$G_B{}^A=\int \hat{Q}_{\alpha a\xi}(x)Q_{\beta b\xi}(x)dx\,,\quad (\alpha,\beta=1,2,3;\ a,b=1,2,3). \tag{4.3}$$

α and β represent the unitary index and $A=(\alpha,a)$ and $B=(\beta,b)$. We can also consider $V(6,3)\times V(6,3)$, which is a natural extension of the $U(6)\times U(6)$. This is the model suggested in Ref. 3. Its representations are the same as those of $V(6,3)$.

The adjoint representation is 81-dimensional and is composed of vector mesons, spin-$\frac{1}{2}$ baryons, spin-$\frac{1}{2}$ antibaryons, pseudoscalar mesons, and scalar mesons, each occurring as a unitary singlet and an octet:

$$81=(9,\bar{9})=V_1+V_8+B_1+B_8+B_1{}^*+B_8{}^*+P_1+P_8+S_1+S_8.$$

Except for scalar mesons all the corresponding particles are actually found. Mass formulas similar to that of Gell-Mann and Okubo have already been discussed.[3] They are essentially equal to the $U(6)$ results plus one relation between baryons and mesons:

$$m(\Xi)-m(\Sigma)=m(K^*)-m(\rho)=m(K)-m(\pi). \tag{4.4}$$

Two-particle states, 81×81, are decomposed into four irreducible components, by symmetrizing and antisymmetrizing in the upper and lower indices:

$$(9,\bar{9})\times(9,\bar{9})=(s,s)+(s,a)+(a,s)+(a,a)=(39,\overline{39})+(39,\overline{42})+(42,\overline{39})+(42,\overline{42}). \tag{4.5}$$

s and a stand for symmetric and antisymmetric, respectively. The approximate symmetry of the S matrix implies that the transition occurs between the same representations. Thus the hadron-hardron scattering amplitudes can be expressed in terms of four invariant amplitudes, denoted by (s,s), (s,a), (a,s), and (a,a). Invariance under charge conjugation requires $(a,s)=(s,a)$ and there are only three independent amplitudes. The hadron-hadron elastic scattering amplitudes are given as follows:

$$(\pi^+\pi^+)=\tfrac{1}{4}[(s,s)+3(a,a)],$$
$$(pp)=\tfrac{1}{4}[(s,s)+3(a,s)],\quad (pn)=\tfrac{1}{2}[(s,s)+(a,s)],$$
$$(p\pi^+)=(pK^+)=(\pi^+K^+)=\tfrac{1}{8}[(s,s)+4(a,s)+3(a,a)], \tag{4.6}$$
$$(p\pi^-)=(pK^-)=(nK^+)=(nK^-)=(p\bar{p})=(p\bar{n})=(\pi^+\pi^-)=(\pi^+K^-)=\tfrac{1}{4}[(s,s)+2(a,s)+(a,a)].$$

For baryon-baryon scattering, the amplitudes are averaged over spins. These can be regarded as relations among total cross sections which are proportional to the imaginary parts of the forward-scattering amplitudes. Relations such as $\sigma(p\pi^-)=\sigma(pK^-)=\sigma(p\bar{p})$ are not in accord with experiments. As in the case of masses, the symmetry is considerably violated.

5. $V(6,21)$ SYMMETRY

In the example $V(6,3)$ given in the previous section, the decimet of baryon resonances is not included in the multiplet. This is a serious defect of the model, since the decimet is known to play important roles in hadron physics. This also means that the nice features of the $U(6)$ theory are lost. The reason is traced back to the fact that the fundamental scalar triplet of $V(6,3)$ is not a representation of the $U(6)$. In view of the success of the $U(6)$ theory, we try to extend our algebra so that the theory is invariant under the $U(6)$.[6]

To this end, the fundamental bosons must be a representation of the $U(6)$. The simplest choice is a singlet scalar meson, but this is insufficient. The next possibilities are **15** and **21**, and we see that $\overline{\mathbf{21}}$ is most convenient. The $\overline{\mathbf{21}}$ consists of a unitary triplet of scalar mesons and a unitary sextet of axial-vector mesons.

The adjoint representation of $V(6,21)$ or $V(6,21)\times V(6,21)$ consists of, in terms of $U(6)$ multiplets,

$$(27,\overline{27})=(6+\overline{21},\bar{6}+21)=\mathbf{1}+\mathbf{35}+\mathbf{56}+\overline{\mathbf{56}}+\mathbf{70}+\overline{\mathbf{70}}+\mathbf{1}+\mathbf{35}+\mathbf{405}. \tag{5.1}$$

The parity of the first two terms, **1** and **35**, is negative. They are the well-established negative-parity mesons [V_8, V_1, P_8, and $P_1=X_0(960)$]. The **56** is the baryon octet and decimet. The $U(6)$ 70-plet of baryons are not yet established. $\overline{\mathbf{56}}$ and $\overline{\mathbf{70}}$ are their antiparticles. The remaining **1**+**35**+**405** are positive parity mesons. **1**+**35** contain A_8, A_1, S_8, and S_1, where A stands for the 1^+ meson and S for the 0^+ meson. **405** consists, in terms of $SU(3)\times SU(2)$, of

$$\mathbf{405}=T_{27}+T_8+T_1+A_{27}+A_{10}+A_{10}{}^*+A_8+A_8+A_8+A_1+S_8+S_1, \tag{5.2}$$

[6] In order to include the decimet, disregarding the $U(6)$ theory, one can also consider higher representations (such as $9\times9\times9$ or $9\times9\times\bar{9}\times\bar{9}$) of $V(6,3)$.

where T denotes the 2^+ meson. T_8+T_1 could be identified with the tensor nonet [$f(1250)$, $A_2(1300)$, $K_V(1420)$, and $f'(1500)$]. Thus the adjoint representation includes all of the well-established hadrons.[7] Results of the $U(6)$ theory also hold here.

For the mass formulas, symmetry breaking is assumed to belong not only to the adjoint representation of $V(6,21)$ but also to the adjoint or the singlet representation of the subalgebra $U(6)$. The singlet term splits the average baryon masses from the meson masses in such a way that

$$m(M_-)+m(M_+)=2m(B)\,, \tag{5.3}$$

where $m(M_\pm)$ is the average mass of positive- and negative-parity mesons, respectively, and $m(B)$ is the average of the baryon masses. Although little is known about positive-parity mesons, the relationship seems to be roughly satisfied experimentally.

There are two terms among the adjoint representation of $V(6,21)$ which behave as a 35-plet with respect to $U(6)$. These two are responsible for the additional mass splitting. Experimentally, the coefficients of these two terms are nearly equal. If they are equal, we have mass formulas identical to those of the $U(6)$ theory and the relation (4.4).

In the exact symmetry, the elastic-scattering amplitudes are again expressed by three invariant amplitudes (s,s) (a,s) and (a,a).

$$\begin{aligned}(27,\overline{27})\times(27,\overline{27})&=(s,s)+(s,a)+(a,s)+(a,a)\\&=(372,\overline{372})+(372,\overline{357})+(357,\overline{372})+(352,\overline{352})\,.\end{aligned} \tag{5.4}$$

Relations among elastic scattering amplitudes or total cross sections are

$$\begin{aligned}
(\pi^+\pi^+)&=\tfrac{1}{4}[(s,s)+3(a,a)]\,,\\
(\pi^+K^+)&=\tfrac{1}{8}[(s,s)+4(a,s)+3(a,a)]\,,\\
(p\pi^+)=(pK^+)&=\tfrac{1}{6}[(s,s)+3(a,s)+2(a,a)]\,,\\
(p\pi^-)=(nK^+)&=(1/24)[5(s,s)+12(a,s)+7(a,a)]\,,\\
(pp)&=(1/216)[31(s,s)+162(a,s)+23(a,a)]\,,\\
(pn)&=(1/162)[44(s,s)+81(a,s)+37(a,a)]\,,\\
(pK^-)=(p\bar{p})=(p\bar{n})=(nK^-)=(\pi^+\pi^-)=(\pi^+K^-)&\\
&=\tfrac{1}{4}[(s,s)+2(a,s)+(a,a)]\,.
\end{aligned}$$

Again we have $\sigma(pK^-)=\sigma(p\bar{p})$, which is violated by experiments.

In the extremely high-energy region where only the Pomeranchuk-Regge trajectory is exchanged, all hadron-hadron total cross sections become equal if the Pomeranchukon is $V(6,21)$-invariant. However, the Pomeranchukon-hadron-hadron coupling will contain symmetry-breaking terms similar to hadron-mass terms.

6. CONCLUSIONS AND DISCUSSION

By extending the notion of the algebra of currents to a generalized Jordan algebra, it is possible to construct a symmetry theory of baryons and mesons. Although in this theory no continuous transformation group is generated, one can construct representations, determine multiplets, and calculate matrix elements.

In the $V(6,21)$ symmetry, all known hadrons are included in one super-supermultiplet, the adjoint representation, so that many of their properties and their matrix elements are related by Clebsch-Gordan coefficients. Although the symmetry seems to be badly broken, it is often convenient to describe things as (symmetry)+(symmetry breaking).

In this paper only adjoint representations are considered, which corresponds to QQ^* configurations. It is of course possible to try other representations, such as QQQ. In this case, however, the fundamental fields must have fractional charges which may cause some trouble. So far as QQ^* states are considered, all fundamental fields Q can have integral charges and the usual statistics.

A trilinear vertex of three hadrons can be constructed in a $V(6,21)$-invariant way but it does not conserve parity. In the nonchiral $V(6,21)\times V(6,21)$, an invariant trilinear vertex cannot be made; only an even number of hadrons can make an invariant. To construct a vertex, we include the differential operator as a spurion, regarding it as behaving like a member of the adjoint representations. This means considering a collinear algebra, which is not attempted in this paper.

In order to develop a similar theory of generalizing the chiral algebra consisting of V_0 and A_0, it is necessary to include fundamental bosons of a parity doublet, that is, bosons of both positive and negative parity. Then the axial vector A_0 can be constructed also from bosons and a relation of the form

$$[A_0{}^i,A_0{}^j]=iC_{ijk}V_o{}^k \tag{6.1}$$

is satisfied. The Adler-Weisberger relation follows from (6.1). However, commutation relations for the space part of vector and axial-vector currents are different from those of the quark model.

The algebra $V(6,21)$ considered here will be a minimum algebra which reproduces results of the $U(6)$ theory and unifies mesons and baryons. It will be useful at least as an approximate symmetry, which is a consistent scheme to correlate all hadrons. To test the validity of this algebra, we have to check the relation of the form $\{F,F\}=B$. Probably spinor currents are connected with baryon fields through the partially conserved spinor-current hypothesis,[1,3] but the actual application has not been attempted.

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[7] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).