Calculation of γ-3π Coupling Constant Using Current Algebra

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In the framework of the algebra of currents and hypothesis of partially conserved axial-vector current, the γ-3π coupling constant has been calculated without using the soft-pion limit. The weak-amplitude term is then calculated in a pole-dominant model, and a sum rule involving the γ-3π coupling constant, the $\rho\pi\gamma$ coupling strength, and the neutral-pion decay lifetime is obtained, allowing the determination of the γ-3π coupling. Our result has been compared with previous determinations and is in good agreement.

RECENTLY, a large number of phenomena in elementary-particle interactions have been successfully understood on the basis of the algebra-of-currents approach. The purpose of the present paper is to discuss the γ-3π coupling on the basis of (1) equal-time commutation rules[1] and (2) the hypothesis of partially conserved axial-vector current (PCAC).[2] This particular problem was first considered by Kawarabayashi and Suzuki[3] (KS). In addition to the hypotheses enumerated in (1) and (2), they also assumed soft-pion emission. Since in the process $\gamma+\pi\rightarrow\pi+\pi$ the pion pair must be predominantly in p wave, one does not expect the soft-pion method to be justifiable. This was originally pointed out by Rubinstein and Veneziano,[4] who raise this kind of question for a class of meson decay phenomena, and also by Schnitzer,[5] who discussed the application of current algebra to the p-wave pion-nucleon scattering length. In the original approach, KS not only used the soft-pion limit but also dispersed the pair of pions successively. Here we abandon the soft-pion limit; however, we continue to work in the off-shell limit and disperse both the pions and photon simultaneously, following Weinberg.[6] The full T product is decomposed and analyzed term by term. The nonvanishing T-product amplitudes are all discussed and obtained in a model-dependent calculation. In calculating such terms we encounter the vertex function for the ρ-A_1-A_1 interaction, an estimate of which has been obtained from the recent calculation of Schnitzer and Weinberg.[7]

In terms of this coupling and from the known value of the $\rho\pi\gamma$ coupling as well as the $\pi^0\rightarrow 2\gamma$ decay lifetime, we determine the value of the γ-3π coupling constant, which is defined to be the value of the form factor for the γ-3π interaction at the symmetric point $s=t=u=m_\pi^2$, where s, t, and u are the usual Mandelstam variables for the process $\gamma+\pi\rightarrow\pi+\pi$. It should be remarked that our present approach allows us to evaluate the form factor only at the point where the pair of final pions are of vanishing masses. However, a simple Born-approximation calculation shows that the extrapolation of this value of the γ-3π form factor up to the physical pion mass is quite justifiable. We now discuss our sum rule involving various coupling parameters and neutral pion lifetime, and compare the value thus obtained for coupling strength with other results.

The general form for the matrix element for the process $\gamma+\pi\rightarrow\pi+\pi$ can be written as

$$\langle\pi_\alpha\pi_\beta|V_\lambda(0)|\pi_\gamma\rangle=i(2\pi)^{-9/2}(8p_{10}p_{20}p_{30})^{-1/2}\times\epsilon_{\alpha\beta\gamma}\epsilon_{\lambda\mu\nu\rho}p_1{}^\mu p_2{}^\nu p_3{}^\rho F(s,t,u)\,,\quad(1)$$

where V_λ denotes the isoscalar photon. In writing down the above matrix element we have used G invariance, Lorentz invariance, and gauge invariance as usual. The variables s, t, and u are defined as $s=-(k+p_1)^2$, $t=-(k-p_2)^2$, and $u=-(k-p_3)^2$; $s+t+u=3m_\pi^2$. Further, the γ-3π coupling constant Λ, defined through[8]

$$(e\Lambda/\mu^3)\tfrac{1}{2}\epsilon_{\lambda\mu\nu\rho}A_\lambda(\partial_\mu\boldsymbol{\phi}_\pi\cdot\partial_\nu\boldsymbol{\phi}_\pi\times\partial_\rho\boldsymbol{\phi}_\pi)\,,$$

is related to $F(s,t,u)$ by

$$e\Lambda/\mu^3=F(s=t=u=\mu^2)\,.\quad(2)$$

We now follow Weinberg and disperse symmetrically both the pions and γ. Using PCAC, we have

$$i(2\pi)^{-3/2}(2p_{30})^{-1/2}\epsilon_{\alpha\beta\gamma}\epsilon_{\lambda\mu\nu\rho}p_1{}^\mu p_2{}^\nu p_3{}^\rho F(s,t,u)$$
$$=F_\pi{}^{-2}\mu^{-4}(p_1{}^2+\mu^2)(p_2{}^2+\mu^2)\int\int d^4x d^4y\, e^{-ip_1\cdot x}e^{-ip_2\cdot y}$$
$$\times\langle 0|T\{\partial_\mu A_\alpha{}^\mu(x),\partial_\nu A_\beta{}^\nu(y),V_\lambda(0)\}|\pi_\gamma\rangle.\quad(3)$$

Expansion of the right-hand-side T product in Eq. (3), as in Ref. 6 [Eq. (4)], leads to eight terms. The four terms like

$$\delta(x_0)\delta(y_0)\langle 0|T\{A_\alpha{}^0(x),[A_\beta{}^0(y),V^\lambda(0)]\}|\pi_\gamma\rangle\,,$$

which entail the commutator of an isoscalar vector current with an axial-vector current, are zero; and two of the remaining terms are dropped for the same reason as in Refs. 6 and 13. We are thus left with the following

[1] M. Gell-Mann, Physics 1, 63 (1964).
[2] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Letters 4, 380 (1960).
[3] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966).
[4] H. R. Rubinstein and G. Veneziano, Phys. Rev. Letters 18, 411 (1967).
[5] H. Schnitzer, Phys. Rev. 158, 1471 (1967).
[6] S. Weinberg, Phys. Rev. Letters 17, 336 (1966).
[7] H. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).
[8] K. Kawarabayashi and A. Sato, Progr. Theoret. Phys. (Kyoto) 28, 173 (1962); 28, 667 (1962); see, also, S. Okubo and B. Sakita, Phys. Rev. Letters 11, 50 (1963).

two terms:

$$p_1{}^{\mu}p_2{}^{\nu}\langle 0|T\{A_{\alpha}{}^{\mu}(x),A_{\beta}{}^{\nu}(y),V^{\lambda}(0)\}|\pi_{\gamma}\rangle \quad (4)$$

and

$$-i\epsilon_{\alpha\beta\gamma}\left(\frac{\partial}{\partial y^{\nu}}-\frac{\partial}{\partial x^{\nu}}\right)\langle 0|T\{V_{\gamma}{}^{\nu}(x),V^{\lambda}(0)\}|\pi_{\gamma}\rangle. \quad (5)$$

It should be remarked that KS dropped the term designated in (4). We will calculate this term in a vector-meson dominance model by saturating the T product with suitable allowed intermediate-vector-meson states (if any).

The matrix element given in (5) can be easily identified with the physical $\pi^0\to 2\gamma$ decay amplitude. In (5) we encounter the decay of the π^0 into an isoscalar photon and an isovector photon. This can be easily related, using $SU(3)$ invariance, to the real photons.

The matrix element for the $\pi^0\to 2\gamma$ decay can be written as

$$\langle\gamma_q\gamma_p|\pi_k{}^0\rangle = i(2\pi)^{-9/2}(8p_0q_0k_0)^{-1/2}i(2\pi)^4\delta^4(k-p-q) \times\epsilon_{\lambda\nu\rho\sigma}e_{1\lambda}{}^*e_{2\nu}{}^*q_\rho p_\sigma f(-q^2,-p^2,-k^2), \quad (6)$$

where the neutral-pion lifetime $\tau(\pi^0)$ is given in terms of f as

$$\tau = 64\pi f^{-2}\mu^{-3}. \quad (7)$$

Dominating the intermediate one-particle state by a vector meson (ρ) only, we can evaluate the matrix element for the term in (4) as

$$\frac{4m_\rho{}^2}{g_\rho}\frac{(2+\delta)}{2p_1\cdot p_2-m_\rho{}^2}\frac{g_{\rho\pi\gamma}}{\mu}(p_1\cdot p_2). \quad (8)$$

For the ρ-A_1-A_1 vertex we have used the result of Weinberg and Schnitzer,[7] namely,

$$\Gamma_{\mu'\nu'}{}^{\rho-A_1-A_1}(p_1,p_2) = (2m_\rho{}^2/g_\rho)\{g_{\mu'\nu'}(p_1+p_2)_{\lambda'} +(2+\delta)[g_{\mu'\lambda'}(p_1-p_2)_{\nu'}-g_{\nu'\lambda'}(p_1-p_2)_{\mu'}] -g_{\mu'\lambda'}p_{1\nu'}-g_{\nu'\lambda'}p_{2\mu'}\}, \quad (9)$$

where δ is a free parameter and $g_{\rho\pi\gamma}$ denotes the $\rho\to\pi\gamma$ decay coupling parameter. The limit $p_1{}^2=p_2{}^2=0$ is taken in evaluating (8).

Keeping the terms (4) and (5) for the right-hand side of (3), we obtain, after comparison with (6), (7), and (8), the following sum rule:

$$F_\pi{}^2F(s,t,u) = F_W(s,t,u)+\sqrt{3}(64\pi/\tau\mu^3)^{1/2}, \quad (10)$$

where Eq. (9) is evaluated in the two-pion off-shell limit, $p_1{}^2=p_2{}^2=0$, and F_W is the weak-amplitude term as given in (4) above.

In the particular limit we are considering, we have $s=-2k\cdot p_1$, $u=-2p_1\cdot p_2$, and $t=-2k\cdot p_2$; and also $s+t+u=\mu^2$. The γ-3π coupling is defined at the symmetric point $s=t=u$ of F. In the off-shell limit then, we choose the symmetric point to be $s=t=u=\frac{1}{3}\mu^2$, so $p_1\cdot p_2=-\frac{1}{6}\mu^2$.

Thus our sum rule becomes

$$F_\pi{}^2F(\tfrac{1}{3}\mu^2,\tfrac{1}{3}\mu^2,\tfrac{1}{3}\mu^2) = F_W(\tfrac{1}{3}\mu^2,\tfrac{1}{3}\mu^2,\tfrac{1}{3}\mu^2)+(64\pi/\mu^3\tau)^{1/2}. \quad (11)$$

Substituting the relevant expressions for F_W from (8), and using the definitions for Λ in terms of F and the observed pion lifetime[9] τ, we determine

$$\Lambda = 0.03. \quad (12)$$

This value of Λ obtained here should be compared with that given by Donnachie and Shaw,[10]

$$\Lambda = 0.04\pm 0.15,$$

from the analysis of photoproduction data. In our analysis to obtain (12), we have used $g_{\rho\pi\gamma}{}^2/4\pi\simeq 0.007$, which gives the observed $\rho\to\pi\gamma$ decay width of 0.6 eV.[11]

In our analysis above we have admittedly assumed that off-shell continuation of $F(s,t,u)$ from the point $s=t=u=\mu^2$ to $\frac{1}{3}\mu^2$ is smooth. This may perhaps be justifiable if we look at the Born-approximation calculation of the $\gamma+\pi\to\pi+\pi$ amplitude, i.e., $F(s,t,u)$, by calculating the three ρ-dominant Feynman diagrams for the process. It may be remarked that there also exist some dispersion-theoretic calculations[12] leading to a value of the γ-3π coupling constant which does not differ very much from the present calculation. However, for such a comparison one needs to maintain crossing symmetry in the current algebra (which obviously means dispersing all the pions together); this point needs further investigation. Finally, we may mention that a similar sum rule to that given in (10) for γ-3π coupling is also found for the $\eta\to 2\pi\gamma$ decay amplitude,[13] relating its decay width to $g_{\rho\eta\gamma}$ coupling through the weak amplitude and the $\eta_0\to 2\gamma$ lifetime.

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[9] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[10] A. Donnachie and G. Shaw, Ann. Phys. (N. Y.) **37**, 333 (1966).

[11] G. Fidecaro, M. Fidecaro, J. A. Poirier, and P. Schiaron, Phys. Letters **23**, 163 (1966).

[12] H. Wong, Phys. Rev. Letters **5**, 70 (1960). The value of $|\Lambda|$ obtained by him is quite large: $|\Lambda|\sim 1.3e$. For extrapolation of current-algebra calculation by means of a dispersion calculation for the γ-3π amplitude, an attempt was made by S. N. Biswas, J. Smith, and J. A. Campbell [Adelaide University Report (unpublished)].

[13] For the soft-pion calculation see J. Pashupati and R. Marshak, Phys. Rev. Letters **17**, 888 (1966).