The consistency conditions that we have studied are not a bootstrap method in any complete sense, since we have not included unitarity except weakly, in that it is not violated by our model. We expect that full unitarity will provide strong additional conditions that further limit the types of phase contours that may occur. This problem could be studied in the limited case of coupled two-body channels that took into account the resonances on which our model is based. The method of phase contours could also be extended to more general collision amplitudes, although the many variables involved would make their discussion somewhat elaborate.

The most important feature that has been neglected, apart from unitarity, is the local distortion that comes from resonances at low energies. It is not at all obvious how to take account of direct-channel resonances as well as the asymptotic behavior from crossed-channel resonances. However, our study of phase contours provides a new method of approaching this problem, which can certainly be developed much further.

The solutions for phase contours, described in this paper, give some information about fixed-angle behavior. More generally, the study of phase contours and zeros permits a new formulation of the problem of relating asymptotic behavior at fixed momentum transfer and asymptotic behavior at fixed angle. We will consider this in a later paper.

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Current Algebra, Vertex Functions, and Decay Widths of K^*, Q, and ϕ Mesons

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Ward identities are set up for three-point functions obeying $SU(3)\times SU(3)$ chiral algebra. Assuming single-particle dominance and simple momentum dependence of the vertex functions, decay widths of the K^*, Q, and ϕ mesons are obtained with all the mesons on the mass shell. Further, within this scheme, momentum-dependent K_{l3}-decay form factors are also obtained. Our results are in good agreement with experiments.

RECENTLY, Schnitzer and Weinberg[1] have developed a technique to calculate the three-point functions obeying chiral $SU(2)\times SU(2)$ commutation relations with the pions on the mass shell. The central idea is to write down the Ward identities for the vertex functions and supplement the algebra with the assumption of single-particle dominance. In this way, they obtain both $\rho\rightarrow\pi\pi$ and $A_1\rightarrow\rho\pi$ decay widths in agreement with experiment. Similar decay-width calculations have also been performed by Das *et al.*[2]; they work, however, in the limit $p_\pi^2\rightarrow 0$. We attempt, in this note, to calculate the K^*, Q, and ϕ meson decay widths within the framework of $SU(3)\times SU(3)$ algebra, and include "hard meson" effects following Schnitzer and Weinberg.[1] For this purpose, we define the VPP vertex Γ_λ, the AVP vertex $\Gamma_{\nu\lambda}$, and the AAV vertex $\Gamma_{\mu\nu\lambda}$ as in Ref. 1, except that now the indices a, b, and c run from 1 to 8, e.g.,

$$\begin{aligned}M_{\mu\nu\lambda}\equiv\int d^4x d^4y\, e^{-iq\cdot x+ip\cdot y}\langle T(A_a{}^\mu(x),A_b{}^\nu(y),V_c{}^\lambda(0))\rangle_0 &= if_{abc}g_{Vc}{}^{-1}g_{Aa}{}^{-1}g_{Ab}{}^{-1}\Delta_{Aa}{}^{\mu\tau}(q)\Delta_{Ab}{}^{\nu\sigma}(p)\Delta_{Vc}{}^{\lambda\eta}(k)\Gamma_{\tau\sigma\eta}(q,p)\\ &+if_{abc}g_{Vc}{}^{-1}g_{Ab}{}^{-1}(F_aq^\mu/(q^2+m_a{}^2))\Delta_{Ab}{}^{\nu\sigma}(p)\Delta_{Vc}{}^{\lambda\eta}(k)\Gamma_{\sigma\eta}(p,q)+if_{abc}g_{Vc}{}^{-1}g_{Aa}{}^{-1}(F_bp^\nu/(p^2+m_b{}^2))\\ &\times\Delta_{Aa}{}^{\mu\tau}(q)\Delta_{Vc}{}^{\lambda\eta}(k)\Gamma_{\tau\eta}(q,p)+if_{abc}g_{Vc}{}^{-1}(F_aF_bq^\mu p^\nu/(q^2+m_a{}^2)(p^2+m_b{}^2))\Delta_{Vc}{}^{\lambda\eta}(k)\Gamma_\eta(q,p)\,. \qquad (1)\end{aligned}$$

[1] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967). Throughout, we follow the notation of this paper.
[2] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 1067 (1967); see also S. G. Brown and G. B. West, Phys. Rev. Letters **19**, 812 (1967).

Here, $k=(p-q)$ and the hypothesis of partially conserved axial-vector current (PCAC) is used in the form

$$\partial_\mu A_a{}^\mu(x)=F_a m_a{}^2\phi_a(x). \tag{2}$$

Similarly, one can write expressions for M_λ and $M_{\nu\lambda}$. Next, we assume the $SU(3)\times SU(3)$ algebra of currents and write down all the independent Ward identities for M_λ, $M_{\nu\lambda}$, and $M_{\mu\nu\lambda}$. We use the Källén-Lehmann representation for the axial-vector and vector-current propagators and substitute these in the definitions of M_λ, etc. A little algebra gives the following Ward identities for the proper vertices:

$$q^\tau\Gamma_{\tau\sigma\eta}(q,p)=C_{Aa}{}^{-1}g_{Aa}\{2g_{Ab}g_{Vc}[(\Delta_{Vc}{}^{-1}(k))_{\sigma\eta}-(\Delta_{Ab}{}^{-1}(p))_{\sigma\eta}]-F_a\Gamma_{\sigma\eta}(p,q)\}, \tag{3}$$

$$p^\sigma\Gamma_{\sigma\eta}(p,q)=C_{Ab}{}^{-1}g_{Ab}[2F_a g_{Vc}q^\lambda(\Delta_{Vc}{}^{-1}(k))_{\lambda\eta}-F_b\Gamma_\eta(q,p)], \tag{4}$$

$$k^\lambda\Gamma_\lambda(q,p)=2C_{Vc}{}^{-1}g_{Vc}\left[\frac{F_a m_a{}^2}{F_b m_b{}^2}(p^2+m_b{}^2)-\frac{F_b m_b{}^2}{F_a m_a{}^2}(q^2+m_a{}^2)\right], \tag{5}$$

$$k^\eta\Gamma_{\sigma\eta}(p,q)=2C_{Vc}{}^{-1}g_{Vc}g_{Ab}F_a(\Delta_{Ab}{}^{-1}(p))_{\nu\sigma}(q^\nu-(m_a{}^2/m_b{}^2)p^\nu), \tag{6}$$

and

$$k^\eta\Gamma_{\tau\sigma\eta}(q,p)=2g_{Aa}g_{Ab}g_{Vc}C_{Vc}{}^{-1}[(\Delta_{Ab}{}^{-1}(p))_{\tau\sigma}-(\Delta_{Aa}{}^{-1}(q))_{\tau\sigma}]. \tag{7}$$

At the same time, the noncovariant terms in the Ward identities lead to the relations

$$C_{Vc}=C_{Ab}+F_b{}^2=C_{Aa}+F_a{}^2. \tag{8}$$

It is important to point out at this stage that in the derivation of identities (3) and (4) we have dropped the contribution of the so-called scalar term. Further, in the derivation of Eqs. (5)–(7), we have explicitly assumed the conserved-vector-current (CVC) hypothesis.[3]

Now, as in Ref. 1, we make the assumption of single-particle dominance and maintain that the proper vertices are smooth functions of the four-momenta. Thus, we take the AAV vertex $\Gamma_{\mu\nu\lambda}$ to be linear in the four-momenta p and q, and the vector and axial-vector spectral functions to be given by[4]

$$\Delta_{Aa}{}^{\mu\nu}(p)\simeq g_{Aa}{}^2(p^2+M_{Aa}{}^2)^{-1}(g^{\mu\nu}+p^\mu p^\nu/M_{Aa}{}^2), \tag{9}$$

and a similar expression for $\Delta_{Vc}{}^{\mu\nu}(k)$; also

$$C_{Aa}\simeq g_{Aa}{}^2/M_{Aa}{}^2,\quad C_{Vc}\simeq g_{Vc}{}^2/M_{Vc}{}^2. \tag{10}$$

Now, $\Gamma_{\mu\nu\lambda}$ can be written as a sum of six terms with six parameters. However, it is not possible to solve for these six unknowns from identities (3) and (4) only. Hence, we have to use the identity (7) as well, in which the explicit assumption of CVC has been included. Now, once we assume the CVC hypothesis, three of the six terms in $\Gamma_{\mu\nu\lambda}$ drop out and we are left with the form

$$\Gamma_{\mu\nu\lambda}(q,p)\simeq\Gamma_1(p+q)_\lambda g_{\mu\nu}+\Gamma_2(g_{\mu\lambda}k_\nu-g_{\nu\lambda}k_\mu)+\Gamma_3(g_{\mu\lambda}p_\nu+g_{\nu\lambda}q_\mu). \tag{11}$$

Substituting Eq. (11) in Eq. (7), we obtain

$$\Gamma_1=-\Gamma_3=2g_{Vc}{}^{-1}M_{Vc}{}^2,\quad g_{Aa}{}^2=g_{Ab}{}^2. \tag{12}$$

For Γ_2, we introduce an unknown free parameter δ (as in Ref. 1) and write it as

$$\Gamma_2=\Gamma_1(2+\delta). \tag{13}$$

It is easy to verify that the above form of $\Gamma_{\mu\nu\lambda}$ coupled with the CVC results[3] satisfies all the Ward identities [Eqs. (3)–(7)]. Though the $\Gamma_{\mu\nu\lambda}$ vertex has been obtained in the CVC limit, we shall include the symmetry-breaking effects by taking the experimental masses for the particles.[5] Henceforth, we shall use only the identities (3) and (4). Substituting for $\Gamma_{\mu\nu\lambda}$ in Eqs. (3) and (4), we obtain the proper AVP and VPP vertices

$$\Gamma_{\nu\lambda}(p,q)\simeq-\frac{2g_{Vc}{}^{-1}g_{Aa}M_{Vc}{}^2}{M_{Aa}{}^2F_a}[q_\nu(p+q)_\lambda-(q_\lambda p_\nu+g_{\nu\lambda}q^2)+(2+\delta)(q_\lambda k_\nu-k\cdot q g_{\nu\lambda})]$$
$$+2g_{Ab}g_{Vc}F_a{}^{-1}\{g_{Vc}{}^{-2}[(M_{Vc}{}^2+k^2)g_{\nu\lambda}-k_\nu k_\lambda]-g_{Ab}{}^{-2}[(M_{Ab}{}^2+p^2)g_{\nu\lambda}-p_\nu p_\lambda]\} \tag{14}$$

and

$$\Gamma_\lambda(q,p)\simeq(p+q)_\lambda\left[\frac{g_{Vc}{}^{-1}g_{Aa}g_{Ab}M_{Vc}{}^2}{M_{Aa}{}^2M_{Ab}{}^2F_aF_b}k^2(1+\delta)+\frac{(k^2+M_{Vc}{}^2)}{g_{Vc}}\left(\frac{F_a}{F_b}+\frac{F_b}{F_a}\right)-k^2g_{Vc}M_{Vc}{}^{-2}F_a{}^{-1}F_b{}^{-1}\right]$$
$$+(p-q)_\lambda\left[(q^2-p^2)\left(\frac{g_{Vc}{}^{-1}g_{Aa}g_{Ab}M_{Vc}{}^2}{M_{Aa}{}^2M_{Ab}{}^2F_aF_b}(1+\delta)+g_{Vc}{}^{-1}\left(\frac{F_a}{F_b}+\frac{F_b}{F_a}\right)-\frac{g_{Vc}}{M_{Vc}{}^2}F_a{}^{-1}F_b{}^{-1}\right)+M_{Vc}{}^2g_{Vc}{}^{-1}\left(\frac{F_b}{F_a}-\frac{F_a}{F_b}\right)\right]. \tag{15}$$

[3] This gives the relations $m_a{}^2=m_b{}^2$, $F_a=F_b$. See also Ref. 12.

[4] Note that it is only the spin-one part. The spin-zero part is given by $\sim F_a{}^2(p^2+m_a{}^2)^{-1}p^\mu p^\nu$.

[5] All the experimental data have been taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 39, 1 (1967).

It is easy to see that for the case when a, b refer to pions, we produce all the results of Ref. 1. In particular, Eq. (8) reads

$$g_\rho^2 M_\rho^{-2} = g_{A_1}^2 M_{A_1}^{-2} + F_\pi^2 . \tag{16}$$

Now, we consider the case when a refers to the pion and b to the K meson. Then, Eq. (8) gives

$$g_{K^*}^2 M_{K^*}^{-2} = g_{A_1}^2 M_{A_1}^{-2} + F_\pi^2 = g_Q^2 M_Q^{-2} + F_K^2 . \tag{17a}$$

Also, from Eq. (12), we have

$$g_{A_1}^2 = g_Q^2, \quad g_{A_1} = \pm g_Q . \tag{17b}$$

We make the choice that both g_{A_1} and g_Q have the same sign. Hereafter, we shall use the current-algebra result[6]

$$g_\rho^2 = 2M_\rho^2 F_\pi^2 , \tag{18a}$$

and the Weinberg[7] sum rule

$$g_{A_1}^2 = g_\rho^2 . \tag{18b}$$

These immediately lead to[8]

$$M_{A_1}^2 = 2M_\rho^2, \quad F_K/F_\pi \simeq 1.17 . \tag{19}$$

From Eqs. (14)–(19) and the definitions of Γ_λ, $\Gamma_{\nu\lambda}$ it is straightforward to obtain the on-mass-shell $K^*K\pi$, $Q\rho K$, and $QK^*\pi$ vertices and hence the decay widths.[9] The results obtained are

$$\begin{aligned} \Gamma_{K^*K\pi} &= 50.77(0.596 - 0.352\delta + 0.052\delta^2) , \\ \Gamma_{Q\rho K} &\simeq 11.63(3.977 + 3.0\delta + 0.59\delta^2) , \\ \Gamma_{QK^*\pi} &\simeq 94.5(2.818 + 2.473\delta + 0.56\delta^2) . \end{aligned} \tag{20}$$

As a further application, we calculate the decay width of ϕ meson. From Eqs. (8) and (16), we find

$$g_{\phi_8}^2 M_{\phi_8}^{-2} = g_\rho^2 M_\rho^{-2} . \tag{21}$$

Using the ω-ϕ mixing angle,[10] we obtain

$$\Gamma_{\phi K^+K^-} \simeq 1.19(0.496 - 0.417\delta + 0.087\delta^2) . \tag{22}$$

Thus, with $\delta = -1$, $\Gamma_{K^*} \simeq 50.77$ MeV, $\Gamma_Q \simeq 18.2 + 85.5 = 103.7$ MeV, and $\Gamma_{\phi K^+K^-} \simeq 1.2$ MeV. These results are to be compared with the experimental values[5] $\Gamma_{K^*} = 49.8 \pm 1.7$ MeV, $\Gamma_Q \simeq 80 \pm 20$ MeV, and $\Gamma_{\phi K^+K^-} = 1.9 \pm 0.5$ MeV.

With a single parameter δ, which can be identified with the anomalous magnetic moment of the Q meson, we are able to obtain consistently the K^*, Q, and ϕ meson decay widths in agreement with experiments. Lastly, from Γ_λ vertex, we obtain the energy dependence of K_{l3} decay form factors $F_\pm(k^2)$ given by

$$F_\pm(k^2) = F_\pm(0)(1 - \lambda_\pm k^2/m_\pi^2) . \tag{23}$$

Our analysis predicts

$$\begin{aligned} &F_+(0) \simeq 1.012, && \lambda_+ \simeq (0.015 - 0.0046\delta) , \\ &F_-(0) \simeq (-0.031 + 0.0556\delta), && \lambda_- \simeq (0.0044 - 0.0013\delta)/ \\ & && \quad (0.031 - 0.0556\delta) . \end{aligned} \tag{24}$$

With $\delta = -1$, we obtain $\lambda_+ \simeq 0.02$, $\lambda_- \simeq 0.066$, $F_-(0) \simeq -0.087$. It is interesting to note that whereas $F_+(0)$ is independent of δ, $F_-(0)$ is much sensitive to the value of δ. Similarly, λ_- is much more sensitive to δ than λ_+. Our results for $F_+(0)$ and λ_+ are in very good agreement with a recent analysis of K_{l3} form factors carried out by Auerbach *et al.*[11]; however, our result for F_- is not in good agreement with experiment. Perhaps it can be improved by including the scalar mesons in the theory.[12]

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[6] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966); see also D. A. Geffen, Phys. Rev. Letters **19**, 770 (1967).

[7] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[8] H. T. Nieh, Phys. Rev. Letters **19**, 43 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, *ibid.* **19**, 139 (1967).

[9] In our calculations, we take F_π from the Goldberger-Treiman relation; $0.22\pi F_\pi^2 = m_\pi^2$.

[10] The mixing angle is taken from Ref. 5. In Eq. (23), ϕ_8 is the unphysical eighth component of the vector meson octet.

[11] L. B. Auerbach *et al.*, Phys. Rev. Letters **19**, 464 (1967).

[12] One way to do this is to use the hypothesis of partially conserved vector current [see Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **11**, 42 (1963)]. Then, the vector-current propagator will have a spin-zero part too. In this case, we can redefine from Eq. (1) onwards and set up the Ward identities. However, solutions for the proper vertices become much complicated. We will present these in a separate paper.