Asymptotic Behavior in Nucleon-Nucleon Scattering

N. H. Buttimore
Department of Physics, Westfield College, London N.W.3., England
(Received 29 January 1968)

A relation between the real part of an elastic scattering amplitude at arbitrary energy and its asymptotic parameters in the Regge model is derived. Together with finite-energy sum rules, this relation is applied to forward nucleon-nucleon and antinucleon-nucleon scattering where the unphysical region is approximated by a number of pole terms arising from the exchange of meson states. A fit to high-energy total-cross-section data and to forward-angle real parts at all energies indicates that there is little dependence on isospin exchanges in the t channel at high energies. When allowed to vary below unity, the intercept of the leading trajectory prefers the maximum value, 1. In this case the nucleon-nucleon total cross sections each tend to 33 ± 1 mb.

I. INTRODUCTION

Sum rules, of the type introduced by Igi[1] to determine the parameters of the P' trajectory at $t=0$, have since yielded important informaton on the asymptotic behavior of πN, KN, and $\bar{K}N$ forward-elastic-scattering amplitudes.[2,3] Now that forward-dispersion relations have, to a large extent, determined the necessary low-energy parameters of nucleon-nucleon scattering,[4] it is of interest to apply the sum rules to this process.

Formerly the real part at threshold was alone involved, but as real parts have been measured at several energies, in particular at high energy for proton-proton scattering,[5] an extension of the sum rule is made to include a real part at arbitrary energy. Corrections to the real part given by the Regge model[6] are therefore applied before fitting the values obtained from experiment.

II. EXTENSION OF SUM RULE

Consider the spin-averaged forward elastic proton-proton scattering amplitude, $f(E)$, whose imaginary part is related to the total cross section by

$$\operatorname{Im}f(E)=k\sigma_T(E)\,, \tag{1}$$

where k and E are, respectively, the laboratory momentum and energy of the incident proton of mass m. In terms of the helicity amplitudes φ_i $(i=1,2,\cdots,5)$[7] we have $f(E)=2\pi W(\varphi_1+\varphi_3)/m$, W being the total c.m. energy. It follows from crossing symmetry that

$$\operatorname{Im}f(-E)=-k\bar{\sigma}_T(E)\,, \tag{2}$$

where $\bar{\sigma}_T$ is the antiproton-proton total cross section.

At high energy we suppose that $f(E)$ is given by the Regge asymptotic amplitude

$$g(E)=-\sum_n a_n\frac{1+\tau_n\exp(-i\pi\alpha_n)}{\sin\pi\alpha_n}\left(\frac{E}{E_0}\right)^{\alpha_n}. \tag{3}$$

Here τ_n and α_n are, respectively, the signature and trajectory intercept at $t=0$ of the Regge pole n exchanged in the t channel.[8] In particular, for $|E|>\bar{E}$, where $\bar{E}$ is typically 6 GeV, we assume that $\operatorname{Im}f(E)$ is given by the imaginary part of the right side of Eq. (3), whence, from Eqs. (1) and (2),

$$\begin{aligned} k\bar{\sigma}_T(E)&=\sum_n a_n\left(\frac{E}{E_0}\right)^{\alpha_n},\\ k\sigma_T(E)&=\sum_n a_n\tau_n\left(\frac{E}{E_0}\right)^{\alpha_n}. \end{aligned} \tag{4}$$

Including those trajectories P, P', R, ω, and ρ which couple to $f(E)$[9] and whose intercepts are not less than zero, we write an unsubtracted dispersion relation for the amplitude $f(E)-g(E)$,

$$\operatorname{Re}f(E)-\sum_i\frac{R_i}{E_i+E}-\frac{1}{\pi}\int_m^{\bar{E}}dE'\left[\frac{k'\sigma_T(E')}{E'-E}+\frac{k'\bar{\sigma}_T(E')}{E'+E}\right]$$
$$=\operatorname{Re}g(E)-\frac{1}{\pi}\int_0^{\bar{E}}dE'\left[\frac{\operatorname{Im}g(E')}{E'-E}-\frac{\operatorname{Im}g(-E')}{E'+E}\right]. \tag{5}$$

Equation (4) allows the integral from $\bar{E}$ to infinity to be neglected. The quantities R_i and E_i refer, respectively, to the residues and positions of poles corresponding to the exchange of the states π, (3π), ρ, ω, and σ in the u channel.[4] They effectively parametrize the contribution from the unphysical region of antiproton-proton scattering where the position of each pole is given by $E_i=m_i^2/2m-m$, and m_i is the mass of the exchanged meson.[10] In Ref. 4 a negative-parity isovector

[1] K. Igi, Phys. Rev. Letters 9, 76 (1962); Phys. Rev. 130, 820 (1963).
[2] J. J. G. Scanio, Phys. Rev. 152, 1337 (1966).
[3] M. Restignoli, L. Sertorio, and M. Toller, Phys. Rev. 150, 1389 (1966).
[4] D. V. Bugg, Nucl. Phys. B5, 29 (1968).
[5] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters 19, 857 (1967).
[6] W. R. Frazer, Phys. Rev. 131, 491 (1963).
[7] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. J. Wong, Phys. Rev. 120, 2250 (1960).
[8] We approximate the Legendre function by a simple power in E and remark that, at $t=0$, $z_t=E/m$.
[9] A. Ahmadzadeh and E. Leader, Phys. Rev. 134, B1058 (1964).
[10] P. Söding, Phys. Letters 8, 285 (1964); A. A. Carter, Nuovo Cimento 48, 15 (1967).

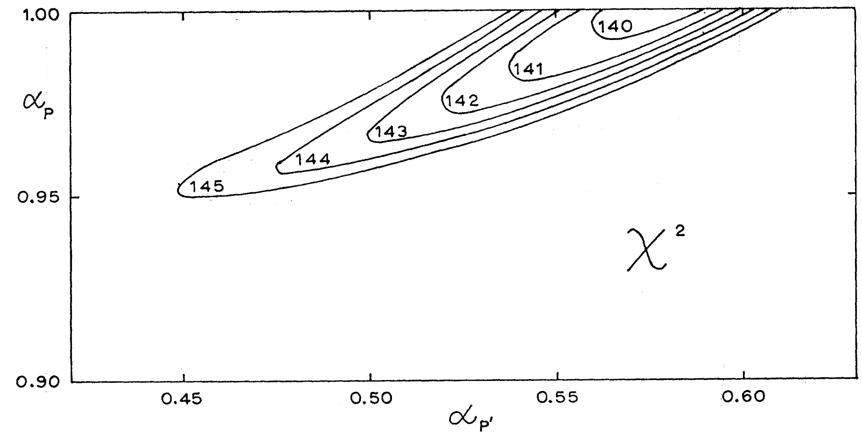

FIG. 1. χ^2 contour as a function of α_P and $\alpha_{P'}$. The intercepts α_R, α_ω, and α_ρ were kept fixed at the values given in Table II.

contribution of even spin, with mass greater than that of the pion, was found necessary to satisfy the subtracted forward-dispersion relations for four of the five nucleon-nucleon amplitudes. This was interpreted as a 3π cut effect, and parametrized as a simple pole. Masses and residues, taken from Ref. 4, are tabulated in Table I.

Other pole-model parameters have been tried in the following analysis, but do not substantially change the conclusions regarding the asymptotic parameters.[11] The effects of higher-mass bosons are generally neglected in the pole models applied to low-energy nucleon-nucleon scattering data and it is not expected that they will make a large contribution to the dispersion relations.

We put $E_0=1$ and using Eq. (3) the right side of Eq. (5) is, for $|E|<\bar{E}$,

$$-\sum_+ a_+\left[\frac{2}{\pi}\frac{\bar{E}^{\alpha_+}}{\alpha_+}F(1,-\tfrac{1}{2}\alpha_+,1-\tfrac{1}{2}\alpha_+,E^2/\bar{E}^2)\right]$$
$$-\sum_- a_-\left[\frac{2}{\pi}\frac{\bar{E}^{\alpha_--1}}{\alpha_--1}EF(1,\tfrac{1}{2}-\tfrac{1}{2}\alpha_-,\tfrac{3}{2}-\tfrac{1}{2}\alpha_-,E^2/\bar{E}^2)\right],$$

where the sum is taken over Regge poles whose signature is even (+) and odd (−). For $|E|>\bar{E}$ we use[12]

$$\sum_+ a_+\left[\frac{2}{\pi}\frac{\bar{E}^{\alpha_++2}}{\alpha_++2}\frac{1}{E^2}F(1,1+\tfrac{1}{2}\alpha_+,2+\tfrac{1}{2}\alpha_+,\bar{E}^2/E^2)-|E|^{\alpha_+}\cot\tfrac{1}{2}\pi\alpha_+\right]$$
$$+\sum_- a_-\left[\frac{2}{\pi}\frac{\bar{E}^{\alpha_-+1}}{\alpha_-+1}\frac{1}{E}F(1,\tfrac{1}{2}+\tfrac{1}{2}\alpha_-,\tfrac{3}{2}+\tfrac{1}{2}\alpha_-,\bar{E}^2/E^2)-E|E|^{\alpha_--1}\tan\tfrac{1}{2}\pi\alpha_-\right].$$

Assuming that $g(E)$ contains all trajectories whose intercepts are not less than -1, we write the following equation[3,13] for the superconvergent amplitude, $f(E)-g(E)$:

$$\frac{1}{\bar{E}}\sum_i R_i+\frac{1}{\pi\bar{E}}\int_m^{\bar{E}} dE'[k'\bar{\sigma}_T(E')-k'\sigma_T(E')]=\sum_- a_-\frac{2}{\pi}\frac{\bar{E}^{\alpha_-}}{\alpha_-+1}. \quad (6)$$

Similar sum rules hold for the pn forward-elastic-scattering amplitude except that, from isospin invariance, the signs of the isovector-modified residue functions, a_R and α_ρ, must be changed.[14] Also, Eqs. (5) and (6) must include the deuteron as an s-channel intermediate state and restrict the u-channel exchanges to the isovector Born terms of π and ρ.[15]

III. DESCRIPTION OF THE FIT TO THE DATA

We have chosen $\bar{E}=6$ GeV, and evaluated the integrals using the currently known total cross-section data,[16] including recent $\bar{p}p$ and $\bar{p}n$ measurements between 1.0 and 3.3 GeV/c.[17] Below $E=1.5$ GeV, each antiproton-nucleon total cross section is assumed to be

[11] A. Scotti, and D. Y. Wong, Phys. Rev. **138**, B145 (1965); R. A. Arndt, R. A. Bryan, and M. H. MacGregor, Phys. Letters **21**, 314 (1966).
[12] L. Sertorio and M. Toller, Phys. Letters **18**, 191 (1965).
[13] A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967).
[14] S. D. Drell, in *Proceedings of the International Conference on High Energy Nuclear Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 897.
[15] A. A. Carter and D. V. Bugg, Phys. Letters **20**, 203 (1966).
[16] W. N. Hess, Rev. Mod. Phys. **30**, 368 (1958); D. V. Bugg, D. C. Salter, G. H. Stafford, R. F. George, K. F. Riley, and R. J. Tapper, Phys. Rev. **146**, 980 (1966).
[17] R. J. Abrams, R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, and D. N. Michael, Phys. Rev. Letters **18**, 1209 (1967).

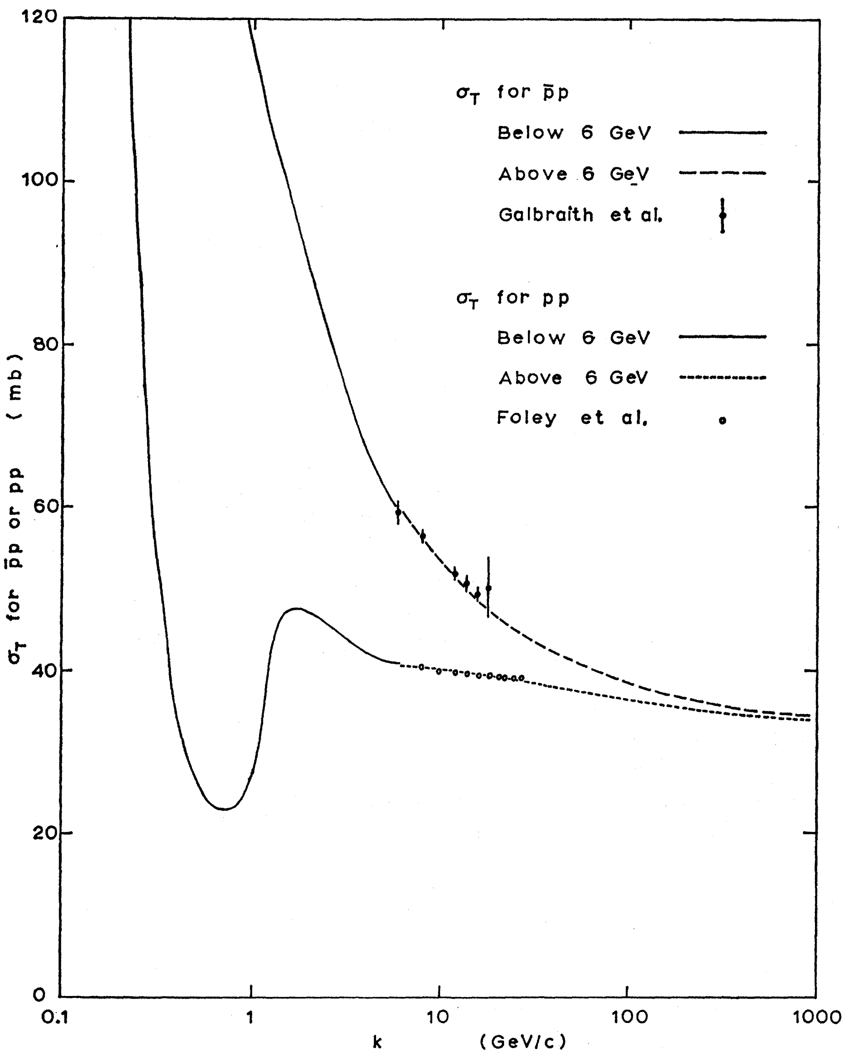

FIG. 2. $\bar{p}p$ and pp total cross sections. Below $E=6$ GeV, values used in the disperion integrals are shown. Above this energy are shown the predicted values, together with the corresponding experimental data used in the fitting program.

inversely proportional to the laboratory velocity of the incoming antiproton; this is consistent with the data from 1.4 to 1.8 GeV which indicate that $k\sigma_T(\bar{p}p)=86.0E$ and $k\sigma_T(\bar{p}n)=83.7E$.

Using Eqs. (4)–(6), the total cross-section data above 6 GeV[5,18] and the known real parts[19] were simultaneously fitted to obtain the Regge parameters. The parameters α_R and α_ρ were fixed at 0.3 and 0.6, respectively, because χ^2 was insensitive to their variation due to

[18] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).

[19] E. Lohrmann, H. Meyer, and H. Winzeler, Phys. Letters **13**, 78 (1964); L. F. Kirillova, V. A. Nikitin, V. S. Pantuev, V. A. Sviridov, L. N. Strunov, M. N. Khachaturyan, L. G. Khristov, M. G. Shafronova, Z. Korbel, L. Rob, S. Damyanov, A. Zlateva, Z. Zlatanov, V. Iordanov, Kh. Kanazirski, P. Markov, T. Todorov, Kh. Chernev, N. Dalkhazhav, T. Tuvdendorzh, P. Devinski, Z. Zolin, Ngo quant Huy, Nguyen dinh Tu, and Tuong Bien, Yadern. Fiz. **1**, 533 (1965) [English transl.: Soviet J. Nucl. Phys. **1**, 379 (1965)]; Phys. Letters **13**, 93 (1964); *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, Calif., 1967), p. 261; G. Baroni, A. Manfredini, and V. Rossi, Nuovo Cimento **38**, 95 (1965); A. E. Taylor, A. Ashmore, W. S. Chapman, D. F. Falla, W. H. Range, D. B. Scott, A. Astbury, F. Capocci, and T. G. Walker, Phys. Letters **14**, 54 (1965); K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters **14**, 74 (1965); R. A. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966); J. Perring, Rev. Mod. Phys. **39**, 550 (1967); Z. Janout, Yu. M. Kazarinov, and F. Lehar, Dubna Report No. JINR/E-2743, 1966 (unpublished); L. Ashgirey, Yu. Kumekin, M. Mescheryakov, S. Nurushev, V. Solovyanov, and G. Stoletov, Phys. Letters **18**, 203 (1965); J. D. Dowell, R. J. Homer, Q. H. Khan, W. K. McFarlane, J. S. C. McKee, and A. W. O'Dell, *ibid.* **12**, 252 (1964); L. M. C. Dutton, R. J. W. Howells, J. D. Jafor, and H. B. Van der Raay, *ibid.* **25B**, 245 (1967); G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillethun, G. Mattiae, J. Pahl, J. P. Scanlon, J. Walters, A. M. Wetherell, and P. Zanella, *ibid.* **14**, 164 (1965); **19**, 705 (1966); **19**, 341 (1965); A. R. Clyde, B. Cork, D. Keefe, L. T. Kerth, W. M. Layson, and W. A. Wenzel, Bull. Am. Phys. Soc. **10**, 1197 (1965).

An analysis of forward elastic proton-proton scattering using Coulomb and nuclear helicity amplitudes [N. H. Buttimore, Westfield College Report, 1967 (unpublished)] reveals that spin dependence may be included as a single term in a way similar to previous determinations of the real part. Though this term appears to be absent at high energies (L. Kirillova *et al.*, Ref. 19), spin dependence is far from negligible at lower energies [L. M. C. Dutton *et al.*, Phys. Letters **25B**, 245 (1967)].

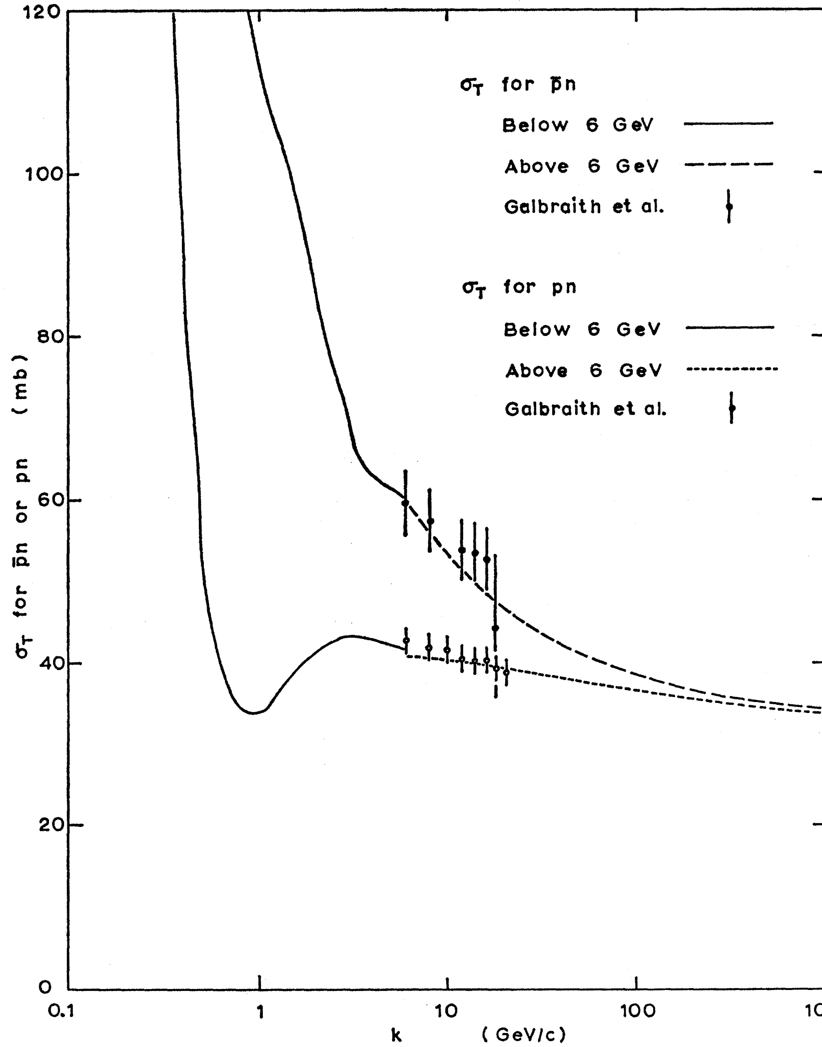

FIG. 3. $\bar{p}n$ and pn total cross sections. The situation is similar to that of Fig. 2.

these poles being almost decoupled from forward nucleon-nucleon scattering. This result is in agreement with recent analyses of pn and $\bar{p}p$ charge-exchange reactions for near-forward angles.[20] With α_P fixed at 1.00 and 87 data points, a minimum in χ^2 at 139, for 80 deg of freedom, was found at the values of the parameters shown in Table II.

IV. DISCUSSION OF RESULTS

We notice that $\alpha_{P'}$ is somewhat lower than the value 0.69 obtained by Scanio[2] from an analysis of a πN sum rule and the value 0.66 which Restignoli, Sertorio, and Toller[3] derived with the inclusion of KN and $\bar{K}N$ sum rules. This slight discrepancy may indicate the presence of either a cut in the angular-momentum plane or another trajectory, P'', with the quantum numbers of the vacuum.[21] We would expect the 2^+ meson f' to lie on this trajectory.[22] A realistic estimate of the error

TABLE I. Masses and residues of exchanged mesons taken from D. V. Bugg[a] where the effect of three-pion exchange in an S state is considered important. These refer to pp scattering.

Meson	π	(3π)	ρ	ω	σ
m_i (GeV/c^2)	0.135	0.750	0.750	0.750	0.613
R_i (mb GeV^2/c)	0.17	3.17	25.97	43.57	−29.50

[a] Reference 4.

TABLE II. Regge parameters for forward elastic $\bar{p}p$ scattering. Parameters for pp, pn, and $\bar{p}n$ amplitudes may be obtained using isospin invariance. The quantity $\epsilon_i=[2(A^{-1})_{ii}]^{1/2}$ is shown in the third row, where $A_{jk}=\partial^2\chi^2/\partial a_j\partial a_k$. The error in a_i $(i=P, P', \cdots, \rho)$ is expected to be a number of times this.

Parameter	Regge subscript of parameter P	P'	R	ω	ρ
α_i	1.00	0.58	0.30	0.22	0.60
a_i (mb GeV/c)	32.85	35.75	0.06	40.38	0.04
ϵ_i (mb GeV/c)	0.06	0.20	0.04	1.22	0.24

[20] R. J. N. Phillips, Nucl. Phys. **B2**, 394 (1967); F. Arbab and J. W. Dash, Phys. Rev. **163**, 1603 (1967).

[21] V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).

[22] B. R. Desai and P. G. O. Freund, Phys. Rev. Letters **16**, 622 (1966).

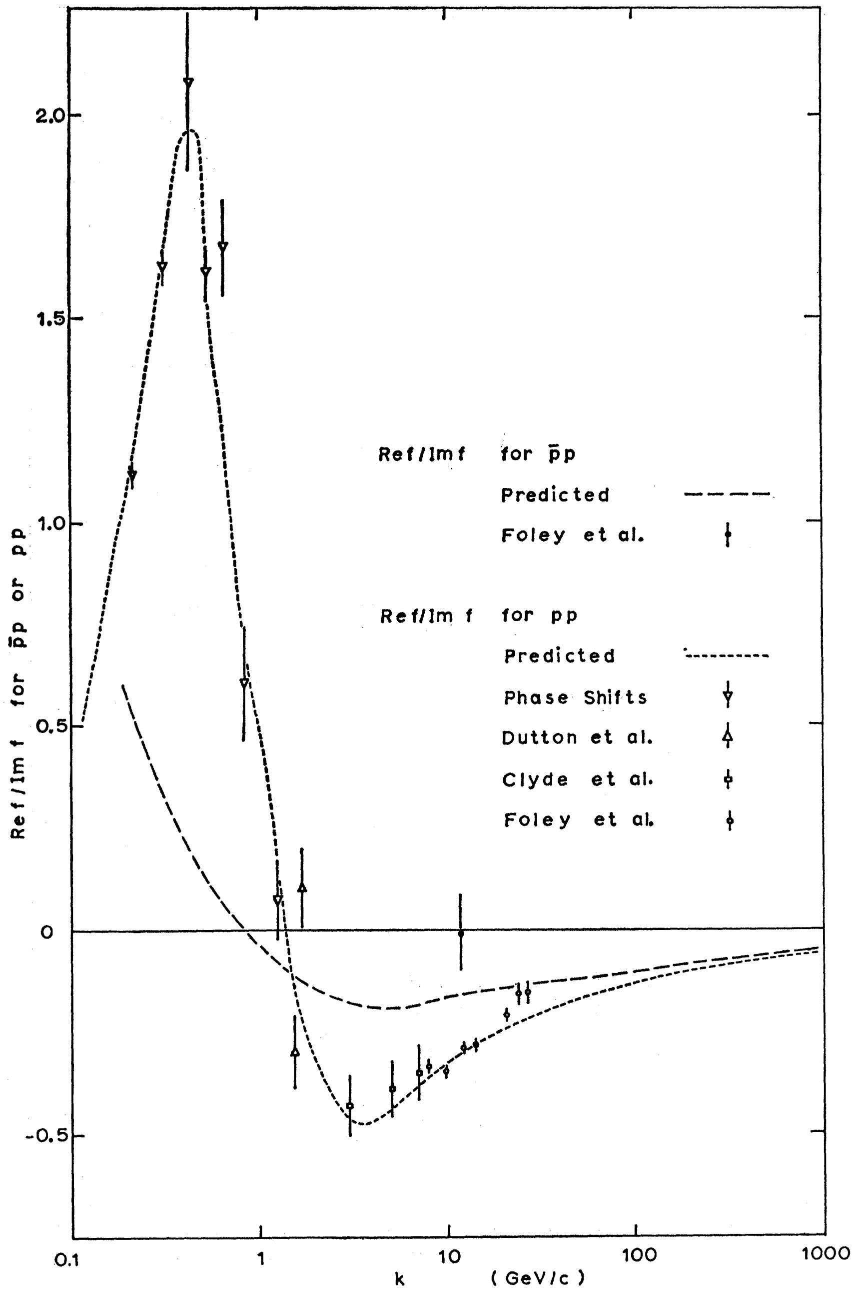

FIG. 4. Values of Ref/Imf for $\bar{p}p$ and pp forward scattering. The more-recent experimental data are shown, part of a selection used in the fitting program. Notice that, in the $\bar{p}p$ case, the low α_ω intercept depresses Ref/Imf below that value given, for example, by the calculation of Söding in Ref. 10.

of $\alpha_{P'}$ would be 0.05; a variation either way by this amount raises χ^2 by about 10.

To test the possibility of vanishing total cross sections at high energy[23] α_P was allowed to vary. In Fig. 1 different constant values of χ^2 are shown as a function of α_P and $\alpha_{P'}$, keeping α_ω fixed at 0.22. We remark that $\alpha_P = 1.00$ is favored both because χ^2 is a minimum there and because of previous determinations of $\alpha_{P'}$.

The intercept of the ω trajectory is somewhat lower than expected and may represent some average of ω and φ exchange. That ω is far more copiously produced than φ in backward $K^- p \rightarrow \Lambda\omega(\varphi)$ scattering,[24] where a proton is exchanged, suggests that φ is only relatively weakly coupled to the $\bar{N}N$ vertex and therefore $\alpha_\omega = 0.22$ is probably a reasonable value. Again a

[23] N. Cabibbo, L. Horwitz, J. Kokkedee, and Y. Ne'eman, Nuovo Cimento **45**, 275 (1966).

[24] J. Badier, M. Demoulin, J. Goldberg, B. P. Gregory, C. Pelletier, A. Rouge, R. Barloutaud, A. Derem, A. Leveque, J. Meyer, P. Schlein, A. Verglas, D. J. Holthuizen, W. Hoogland, J. C. Kluyver, and A. G. Tenner (Paris-Saclay-Amsterdam Collaboration), Report No. CEA R-3037 (unpublished); J. D. Jackson, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, Calif., 1967), p. 170; see also the note on p. 1084 of Ref. 21.

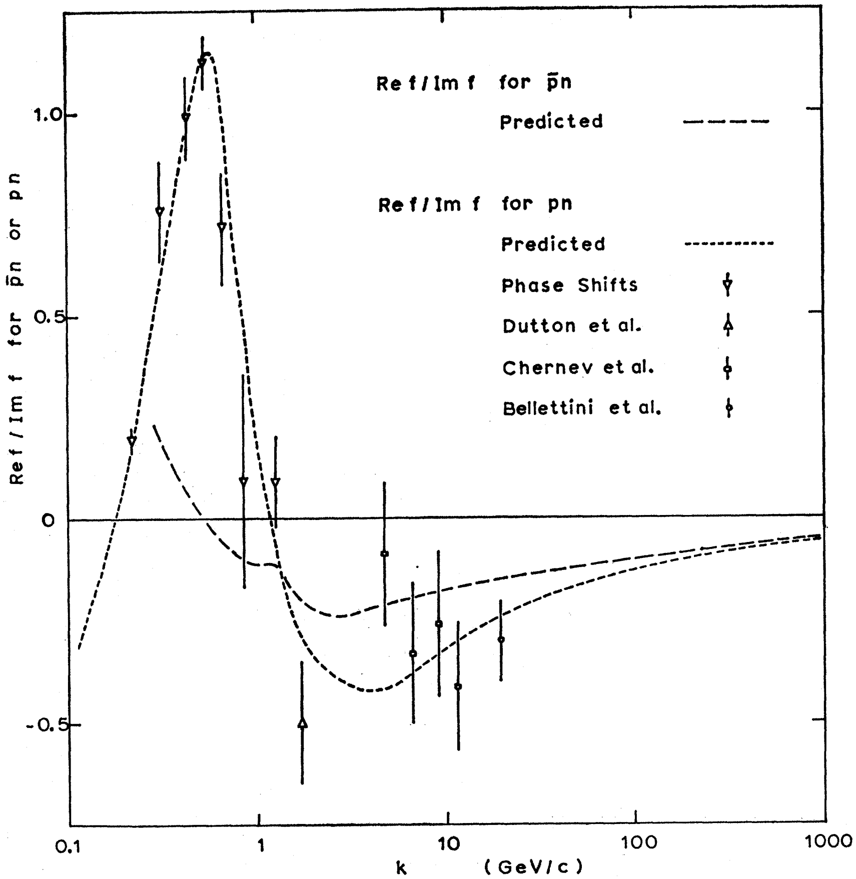

FIG. 5. Values of Ref/Imf for $\bar{p}n$ and pn forward scattering. Higher-energy data in this case are not very accurate.

variation of 0.08 in this quantity produces an increase of 10 in χ^2.

The values of the Regge parameters obtained using sum rules are in general agreement with a recent analysis of πp, pp, and $\bar{p}p$ data carried out by Rarita, Riddell, Chiu, and Phillips.[25] They include the πN sum rule with enlarged error and in solutions 2 and 3 find that $\alpha_{P'}=0.75$, $\alpha_\omega=0.21$, and $\alpha_{P'}=0.57$, $\alpha_\omega=0.36$, respectively.

In Figs. 2 and 3 we show the values of the total cross sections used in the integrals. Above 6 GeV the same quantities, obtained from Eq. (4), are plotted using the set of parameters given in Table II. The real part of $f(E)$ may be obtained, at each energy, from Eq. (5) and Ref/Imf is displayed in Figs. 4 and 5, again using the parameters of Table II. It is stressed, however, that the rate at which Ref/Imf tends to zero with increase in energy depends upon the values of $\alpha_{P'}$ and α_ω. A similar remark applies to the rate at which each total cross section approaches its asymptotic value of about 33 mb.

[25] W. Rarita, R. J. Riddell, C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968).

V. SECOND FINITE-ENERGY SUM RULE

Finally, in addition to Eq. (6), we mention another finite-energy sum rule,[26]

$$\frac{1}{\bar{E}^2}\sum_i E_i R_i + \frac{1}{\pi \bar{E}^2}\int_m^{\bar{E}} dE'\, E'[k'\bar{\sigma}_T(E') + k'\sigma_T(E')] = \sum_+ a_+ \frac{2}{\pi}\frac{\bar{E}^{\alpha+}}{\alpha_+ + 2}. \quad (7)$$

The right sides of Eqs. (6) and (7) are compared with the left sides in Table III. In the first case, neither sum rule was included in the fit, and inclusion of Eq. (6) in the second case tended to depress the value of the ω intercept. The sum rule Eq. (7) is reasonably well satisfied for proton-proton scattering but not for proton-neutron scattering. However, we remember that the extraction of pn and $\bar{p}n$ total cross sections from deuteron measurements, using the Glauber correction, is a process not without error[27]; this should be borne in mind when interpreting the values of a_R and α_ρ.

[26] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. Letters **19**, 402 (1967); Phys. Rev. **166**, 1768 (1968).

[27] M. N. Kriesler, L. W. Jones, M. J. Longo, and J. R. O'Fallon, Phys. Rev. Letters **20**, 468 (1968).

TABLE III. Comparison of the left-hand side (l.h.s.) and right-hand side (r.h.s.) of the finite-energy sum rules Eq. (6) involving the negative-signature poles (−) and Eq. (7) involving the positive-signature poles (+). In the first row the sum rule Eq. (6) was not included in the fit, whereas in the second row this condition was imposed. The sum rule Eq. (7) was not imposed in either case. The error in the left-hand side arises both from the integrals and from the uncertainty in the parameters of the unphysical region.

	$pp-$	$pn-$	$pp+$	$pn+$
R.h.s. without Eq. (6) imposed (mb GeV/c)	30.7	30.4	66.8	66.7
R.h.s. with Eq. (6) imposed (mb GeV/c)	31.3	31.2	66.8	66.7
L.h.s. (mb GeV/c)	32.8	31.2	66.3	64.8
Error in l.h.s. (mb GeV/c)	1.1	1.9	0.9	1.5

The finite-energy sum rules, Eqs. (6) and (7), have been applied to KN and $\bar{K}N$ scattering data alone,[28] where it is found that χ^2 is a minimum for the following values of two of the parameters: $\alpha_{P'}=0.30\pm0.13$ and $\alpha_\omega=0.47\pm0.06$. It is evident that a wider selection of experimental data should be analyzed in order to clarify the picture of the complex angular-momentum plane near $t=0$.

VI. CONCLUSIONS

We have found that the asymptotic behavior of forward elastic nucleon-nucleon scattering amplitudes may be related to these amplitudes at lower energies, through dispersion relations and superconvergence relations. These relations assume that the high-energy behavior is that given by the Regge model. When applied to the available data in the forward direction, the following assumptions were made:

(i) The effects of higher-mass boson exchanges, when approximating the antinucleon-nucleon unphysical region by a series of poles, are negligible.

(ii) From $E=m$ at threshold to $E=1.5$ GeV the antiproton-nucleon total cross sections are inversely proportional to the laboratory velocity of the antiproton.

(iii) Total cross sections involving a target neutron have been correctly extracted from the corresponding deuteron measurements using the Glauber correction.

An over-all fit to the data provided the following information on nucleon-nucleon asymptotic behavior:

(i) There is little dependence on the isovector exchanges of ρ and $R(A_2)$ in the t channel.

(ii) The preferred value of the leading trajectory's intercept is unity.

(iii) The intercepts $\alpha_{P'}=0.58$ and $\alpha_\omega=0.22$ are lower than values found from previous work. Though possibly due to an error in one of the assumptions made, particulary assumption (i), this may indicate that the analyticity of the angular-momentum plane is more complicated than implied by a model with only two important vacuum poles.

In addition to measurements which would clarify assumptions (i)–(iii), more measurements of the real part of a forward scattering amplitude would be very useful in obtaining asymptotic parameters. Only proton-proton values of this quantity are known with reasonable accuracy over most of the available energy range. Predicted values of Ref/Imf for both $\bar{p}p$ and $\bar{p}n$ scattering at lower energies are particularly sensitive to assumptions (i) and (ii) and in Figs. 4 and 5 should not be taken seriously when they become positive. Measurements of antiproton-nucleon real parts at lower energies therefore would be very welcome. At higher energies they are equally useful, since parameters of the P' and ω trajectories are sensitive to these real parts.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Elliot Leader for his advice and encouragement. The author also wishes to thank both Dr. D. V. Bugg and Dr. A. A. Carter for helpful conversations. This work was all but completed at the Cavendish Laboratory, and the author expresses his gratitude to Churchill College, Cambridge, for the award of a Gulbenkian Studentship. The cooperation of the computing staff at the Mathematical Laboratory, Cambridge is also greatly appreciated.

[28] M. Lusignoli, M. Restignoli, G. Violini, A. Borgese, and M. Colocci, Nuovo Cimento 51, 1136 (1967).