Fitting of Electromagnetic Nucleon Form Factors in a Vector-Dominance Model*

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By using the Lagrangian theory of vector dominance of Kroll, Lee, and Zumino, a four-parameter fit to the experimental data for the electromagnetic nucleon form factors for spacelike four-momentum transfer squared (q^2) is obtained. The isovector, hypercharge, and baryon-number parts of the nucleon–vector-meson vertices are assumed to have phenomenologically $(1+q^2/\Lambda_I^2)^{-1}$, $(1+q^2/\Lambda_Y^2)^{-1}$, and $(1+q^2/\Lambda_N^2)^{-1}$ momentum dependence. The slope of G_E^n at $q^2=0$ is set equal to the Krohn-Ringo value of 0.457 $(\mathrm{BeV}/c)^{-2}$ $\pm5\%$. The result of the fitting is satisfactory for both large and small q^2.

1. INTRODUCTION

The idea that the electromagnetic nucleon form factors are dominated by known vector mesons has been discussed extensively in the literature.[1] However, there are in existence only three such vector mesons[2] (ρ^0, ϕ^0, ω^0) that are relevant. The actual fitting of the nucleon form factors by using the properties of these mesons has been difficult,[3] especially at large four-momentum transfer squared (q^2). An attempt has been made recently by Massam and Zichichi[4]; they assumed a nonpointlike interaction at the vector-meson–nucleon vertex. While their result on the charge and magnetic form factors for the proton $G_E^p(q^2)$ and $G_M^p(q^2)$ and the magnetic form factor for the neutron, $G_M^n(q^2)$ agrees with the experimental data, the result on the charge form factor for the neutron $G_E^n(q^2)$ has some difficulties. In particular, their best fit gives $dG_E^n/dq^2=0.81$ $(\mathrm{BeV}/c)^{-2}$ at $q^2=0$, in contradiction with the latest experimental value[5] 0.457 $(\mathrm{BeV}/c)^{-2}$ $\pm5\%$. Moreover, in such an analysis one uses a Feynman-diagram approach and assumes a vector-meson–photon vertex which does not vanish at zero q^2, in apparent violation of gauge invariance.[6]

Recently, Kroll, Lee, and Zumino[7] have succeeded in formulating a gauge-invariant Lagrangian field theory of vector dominance. With this, expressions for $G_E^p(q^2)$, $G_M^p(q^2)$, $G_E^n(q^2)$, and $G_M^n(q^2)$ can be derived if one further assumes a $(1+q^2/\Lambda_I^2)^{-1}$ dependence for the isovector vertex $N\to N+\rho^0$ and a $(1+q^2/\Lambda_Y^2)^{-1}$ and $(1+q^2/\Lambda_N^2)^{-1}$ dependence, respectively, for the hypercharge and baryon-number parts of the isoscalar vertices $N\to N+\phi^0$ and $N\to N+\omega^0$. The formulas for the electromagnetic nucleon form factors, which are given by Eqs. (10)–(13) below, then depend on five free parameters only: Λ_I, Λ_Y, Λ_N, $F_2^N(0)$, g_N/g_Y. Among these, $F_2^N(0)$ is the "Pauli" form factor for the baryon-number source current at $q^2=0$; g_N and g_Y are, respectively, the renormalized coupling constants for the baryon-number part and hypercharge part of the nucleon–vector-meson vertices. The number of independent parameters reduces to four when the slope of $G_E^n(q^2)$ at $q^2=0$ is set to be the experimental value. In this paper such a four-parameter fit of the electromagnetic form factors is attempted. The best fit gives a χ^2 of 1.5 per point at 115 degrees of freedom, agreeing very well with the experimental data. The sensitivity of each of the parameters to the fitting as well as the χ^2 is discussed.

In the last section, the form factors for $N\to N+\omega^0$ and $N\to N+\phi^0$ are calculated with the best set of fitted parameters. They come out to be of the same order of magnitude. Using them, the production rates for ω^0 and ϕ^0 are discussed.

2. EXPLICIT FORMULATION AND CURVE FITTING

The essential assumptions of the theory of Kroll *et al.* are (a) that the hadronic electromagnetic current is to be identified with a linear combination of the ρ^0, ϕ^0, ω^0 fields, and (b) that these vector mesons are coupled to conserved currents. These assumptions lead to the following exact expressions for the isovector part and isoscalar part of the electromagnetic form factors[8]:

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[1] Y. Nambu, Phys. Rev. **106**, 1366 (1957); W. R. Frazer and J. R. Fulco, *ibid.* **117**, 1603 (1960); M. Gell-Mann and F. Zachariasen, *ibid.* **124**, 953 (1961); R. Dashen and D. H. Sharp, *ibid.* **133**, B1585 (1964).

[2] We have excluded the new resonance at 1650 MeV which may have the same quantum numbers as $\rho^0(760)$.

[3] J. R. Dunning, Jr., K. W. Chen, A. A. Cone, G. Hartwig, N. F. Ramsay, J. K. Walker, and R. Wilson, Phys. Rev. Letters **13**, 631 (1964); L. H. Chan, K. W. Chen, J. R. Dunning, Jr., N. F. Ramsay, J. K. Walker, and R. Wilson, Phys. Rev. **141**, 1298 (1966); E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, *ibid.* **139**, B458 (1965).

[4] T. Massam and A. Zichichi, Nuovo Cimento **43**, 1137 (1966).

[5] V. E. Krohn and G. R. Ringo, Phys. Letters **18**, 297 (1965).

[6] G. Feldman and P. Mathews, Phys. Rev. **132**, 823 (1963).

[7] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

[8] $\langle N|J_\mu{}^b(x)|N\rangle = u_N{}^\dagger\gamma_4(\gamma_\mu F_1{}^b(q^2)+(q_\nu/2M)\sigma_{\nu\mu}F_2{}^b(q^2))u_N e^{iq_\lambda x_\lambda}$, where $J_\mu{}^b$ with $b=\gamma$, ρ, ϕ, or ω denotes the corresponding renormalized source currents for γ, ρ^0, ϕ^0, or ω^0. M is the physical mass of the nucleon. $J_\mu{}^\rho$ is normalized to $-i\int J_4{}^\rho(x)d^3x=I_z$ (z component of isospin operator) while $J_\mu{}^\phi$, $J_\mu{}^\omega$ are normalized by setting their coupling constants equal to unity. θ_Y and another mixing angle θ_N are defined as

$$\begin{pmatrix} g_Y J_\mu{}^Y \\ g_N J_\mu{}^N \end{pmatrix} = \begin{pmatrix} \cos\theta_Y & -\sin\theta_Y \\ \sin\theta_N & \cos\theta_N \end{pmatrix} \begin{pmatrix} J_\mu{}^\phi \\ J_\mu{}^\omega \end{pmatrix},$$

with $-i\int J_4{}^Y(x)d^3x=Y$ (hypercharge operator) and

$$-i\int J_4{}^N(x)d^3x=N$$

(baryon-number operator). $F_a{}^I(q^2)$, $F_a{}^Y(q^2)$, and $F_a{}^N(q^2)$ are defined by setting $b=I$, Y, or N in the first equation of this note. Here we want to emphasize that $J_\mu{}^\rho$, $J_\mu{}^Y$, and $J_\mu{}^N$ are not the isospin, hypercharge, and baryon-number currents.

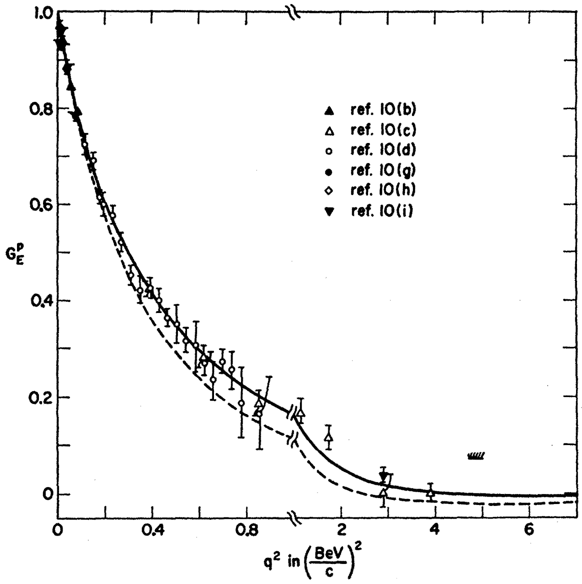

FIG. 1. The solid curve is our best fit to $G_E{}^p$ for spacelike momentum transfer (see Table I). The dashed curve is for the case with $\Lambda_Y=\Lambda_N$. Note the change of scale of q^2 at 1 $(\text{BeV}/c)^2$. The shaded symbol is an upper bound determined by Ref. 10(c).

$$F_a{}^\gamma(q^2)_{I=1}=\frac{1}{1+q^2/m_\rho{}^2}F_a{}^\rho(q^2),$$

$$F_a{}^\gamma(q^2)_{I=1}=\frac{1}{2g_Y}\left[\frac{1}{1+q^2/m_\phi{}^2}F_a{}^\phi(q^2)\cos\theta_Y-\frac{1}{1+q^2/m_\omega{}^2}F_a{}^\omega(q^2)\sin\theta_Y\right], \quad (1)$$

where $a=1$ or 2, denoting "Dirac" or "Pauli" form factors. m_ρ, m_ϕ, m_ω are the physical masses for ρ^0, ϕ^0, ω^0, respectively; and $F_a{}^\rho(q^2)$, $F_a{}^\phi(q^2)$, $F_a{}^\omega(q^2)$ are the similarly defined nucleon form factors for meson emission. Equations (1) give expressions similar to those obtained by assuming a direct vector-meson–photon vertex which is not zero at $q^2=0$, but the theory is now gauge invariant.

$F_a{}^\rho(q^2)$, $F_a{}^\phi(q^2)$, and $F_a{}^\omega(q^2)$ are related to the "isospin," "hypercharge," and "baryon-number nucleon form factors" $F_a{}^I(q^2)$, $F_a{}^Y(q^2)$, and $F_a{}^N(q^2)$ by [8]

$$F_a{}^\rho(q^2)=F_a{}^I(q^2),$$

$$F_a{}^\phi(q^2)=\frac{1}{\cos(\theta_Y-\theta_N)}\times[g_YF_a{}^Y(q^2)\cos\theta_N+g_NF_a{}^N(q^2)\sin\theta_Y], \quad (2)$$

$$F_a{}^\omega(q^2)=\frac{1}{\cos(\theta_Y-\theta_N)}\times[-g_YF_a{}^Y(q^2)\sin\theta_N+g_NF_a{}^N(q^2)\cos\theta_Y].$$

The functional dependence of $F_a{}^I(q^2)$, $F_a{}^Y(q^2)$, and $F_a{}^N(q^2)$ on q^2 is not known. We approximate them, in the spacelike region of q^2, by

$$\begin{aligned}F_a{}^I(q^2)/F_a{}^I(0)&=1/(1+q^2/\Lambda_I{}^2),\\ F_a{}^Y(q^2)/F_a{}^Y(0)&=1/(1+q^2/\Lambda_Y{}^2),\\ F_a{}^N(q^2)/F_a{}^N(0)&=1/(1+q^2/\Lambda_N{}^2).\end{aligned} \quad (3)$$

This assumption would predict a $(q^2)^{-2}$ dependence for the electromagnetic nucleon form factors at large q^2, in agreement with the present experimental results.[9]

For the Dirac form factors we make use of the isospin, hypercharge, and baryon number of the nucleons to get

$$F_1{}^I(0)=\tfrac{1}{2},\quad F_1{}^Y(0)=1,\quad F_1{}^N(0)=1.$$

Thus

$$F_1{}^\gamma(q^2)_{I=1}=\frac{1}{2}\frac{1}{1+q^2/\Lambda_I{}^2}\frac{1}{1+q^2/m_\rho{}^2}, \quad (4)$$

$$F_1{}^\gamma(q^2)_{I=0}=\frac{1}{2}\frac{1}{\cos(\theta_Y-\theta_N)}\left[\frac{1}{1+q^2/\Lambda_Y{}^2}\left(\frac{\cos\theta_Y\cos\theta_N}{1+q^2/m_\phi{}^2}+\frac{\sin\theta_Y\sin\theta_N}{1+q^2/m_\rho{}^2}\right)+\frac{g_N/g_Y}{1+q^2/\Lambda_N{}^2}\cos\theta_Y\sin\theta_Y\left(\frac{1}{1+q^2/m_\phi{}^2}-\frac{1}{1+q^2/m_\omega{}^2}\right)\right]. \quad (5)$$

For the Pauli form factors, we have

$$F_2{}^\gamma(q^2)_{I=1}=\frac{1}{2}\frac{1}{1+q^2/\Lambda_I{}^2}\frac{F_2{}^I(0)}{1+q^2/m_\rho{}^2}, \quad (6)$$

[9] L. Y. Mo and his group at SLAC have measured the electromagnetic form factors for protons from 2.5 to 25.0 $(\text{BeV}/c)^2$. Their results still obey the empirical formula

$$G_E{}^p=G_M{}^p/\mu_p=1/(1+q^2/0.71)^2$$

(private communication). However, this formula cannot be right because it predicts a double pole in the timelike region of q^2.

$$F_2{}^{\gamma}(q^2)_{I=0}=\frac{1}{2}\frac{1}{\cos(\theta_Y-\theta_N)}\left[\frac{F_2{}^Y(0)}{1+q^2/\Lambda_Y{}^2}\left(\frac{\cos\theta_Y\cos\theta_N}{1+q^2/m_\phi{}^2}+\frac{\sin\theta_Y\sin\theta_N}{1+q^2/m_\omega{}^2}\right)+\frac{(g_N/g_Y)F_2{}^N(0)}{1+q^2/\Lambda_N{}^2}\cos\theta_Y\sin\theta_Y\left(\frac{1}{1+q^2/m_\phi{}^2}-\frac{1}{1+q^2/m_\omega{}^2}\right)\right], \quad (7)$$

where, on account of the known nucleon magnetic moments,

$$F_2{}^I(0)=3.70 \quad \text{and} \quad F_2{}^Y(0)=-0.12.$$

The proton and neutron form factors are given by

$$F_a{}^p(q^2)=F_a{}^{\gamma}(q^2)_{I=0}+F_a{}^{\gamma}(q^2)_{I=1}, \quad F_a{}^n(q^2)=F_a{}^{\gamma}(q^2)_{I=0}+F_a{}^{\gamma}(q^2)_{I=1}. \quad (8)$$

The experimental data are usually reported by the more easily analyzed charge and magnetic form factors

$$G_E{}^{p,n}(q^2)=F_1{}^{p,n}(q^2)-(q^2/4M^2)F_2{}^{p,n}(q^2), \quad G_M{}^{p,n}(q^2)=F_1{}^{p,n}(q^2)+F_2{}^{p,n}(q^2), \quad (9)$$

where M stands for nucleon mass. Combining Eqs. (4)–(9), we arrive at the final expressions:

$$G_E{}^p=\frac{1}{2}\left[\frac{1}{1+q^2/\Lambda_I{}^2}\frac{1}{1+q^2/m_\rho{}^2}\left(1-\frac{3.70q^2}{4M^2}\right)+\frac{1}{\cos(\theta_Y-\theta_N)}\frac{1}{1+q^2/\Lambda_Y{}^2}\left(\frac{\cos\theta_Y\cos\theta_N}{1+q^2/m_\phi{}^2}+\frac{\sin\theta_Y\sin\theta_N}{1+q^2/m_\omega{}^2}\right)\left(1+\frac{0.12q^2}{4M^2}\right)+\frac{1}{1+q^2/\Lambda_N{}^2}\frac{\cos\theta_Y\sin\theta_Y}{\cos(\theta_Y-\theta_N)}\left(\frac{1}{1+q^2/m_\phi{}^2}-\frac{1}{1+q^2/m_\omega{}^2}\right)\left(1-\frac{q^2}{4M^2}F_2{}^N(0)\right)\frac{g_N}{g_Y}\right], \quad (10)$$

$$G_M{}^p=\frac{1}{2}\left[\frac{1}{1+q^2/\Lambda_I{}^2}\frac{4.70}{1+q^2/m_\rho{}^2}+\frac{1}{1+q^2/\Lambda_Y{}^2}\frac{0.88}{\cos(\theta_Y-\theta_N)}\left(\frac{\cos\theta_Y\cos\theta_N}{1+q^2/m_\phi{}^2}+\frac{\sin\theta_Y\sin\theta_N}{1+q^2/m_\omega{}^2}\right)+\frac{g_N}{g_Y}\frac{1+F_2{}^N(0)}{1+q^2/\Lambda_N{}^2}\frac{\cos\theta_Y\cos\theta_N}{\cos(\theta_Y-\theta_N)}\left(\frac{1}{1+q^2/m_\phi{}^2}-\frac{1}{1+q^2/m_\omega{}^2}\right)\right], \quad (11)$$

$$G_E{}^n=-\frac{1}{2}\left[\frac{1}{1+q^2/\Lambda_I{}^2}\frac{1}{1+q^2/m_\rho{}^2}\left(1-\frac{3.70q^2}{4M^2}\right)-\frac{1}{1+q^2/\Lambda_Y{}^2}\frac{1}{\cos(\theta_Y-\theta_N)}\left(\frac{\cos\theta_Y\cos\theta_N}{1+q^2/m_\phi{}^2}+\frac{\sin\theta_Y\sin\theta_N}{1+q^2/m_\omega{}^2}\right)\left(1+\frac{0.12q^2}{4M^2}\right)-\frac{1}{1+q^2/\Lambda_N{}^2}\frac{\cos\theta_Y\sin\theta_Y}{\cos(\theta_Y-\theta_N)}\left(\frac{1}{1+q^2/m_\phi{}^2}-\frac{1}{1+q^2/m_\omega{}^2}\right)\left(1-\frac{q^2}{4M^2}F_2{}^N(0)\right)\frac{g_N}{g_Y}\right], \quad (12)$$

$$G_M{}^n=-\frac{1}{2}\left[\frac{1}{1+q^2/\Lambda_I{}^2}\frac{4.70}{1+q^2/m_\rho{}^2}-\frac{1}{1+q^2/\Lambda_Y{}^2}\frac{0.88}{\cos(\theta_Y-\theta_N)}\left(\frac{\cos\theta_Y\cos\theta_N}{1+q^2/m_\phi{}^2}+\frac{\sin\theta_Y\sin\theta_N}{1+q^2/m_\omega{}^2}\right)-\frac{g_N}{g_Y}\frac{1+F_2{}^N(0)}{1+q^2/\Lambda_N{}^2}\frac{\cos\theta_Y\cos\theta_N}{\cos(\theta_Y-\theta_N)}\left(\frac{1}{1+q^2/m_\phi{}^2}-\frac{1}{1+q^2/m_\omega{}^2}\right)\right]. \quad (13)$$

Besides the two angles θ_Y and θ_N, which can be predicted by assuming a particular ϕ-ω mixing model, the above expressions contain five independent parameters Λ_I, Λ_Y, Λ_N, $F_2{}^N(0)$, and g_N/g_Y. By imposing the condition[5]

$$dG_E{}^n/dq^2=0.457\ (\text{BeV}/c)^{-2}\pm5\% \quad \text{at} \quad q^2=0 \quad (14)$$

as required by slow-neutron–electron interactions, the number of free parameters is reduced to four. With the existing data,[10] the best set of parameters is found for each of the current-mixing, mass-mixing, and mass-mixing (a variation) models.[7] The results are listed in Table I. It is found that within one standard deviation, the best sets of parameters are the same for all the three models; thus we have plotted the detailed the-

[10] (a) C. Akerloff, K. Berkelman, G. Rouse, and M. Tigner, Phys. Rev. **135**, B810 (1964); (b) D. J. Drickey and L. N. Hand, Phys. Rev. Letters **9**, 521 (1962); (c) J. R. Dunning, Jr., K. W. Chen, A. A. Cone, G. Hartwig, N. F. Ramsay, J. K. Walker, and R. Wilson, Phys. Rev. **141**, 1286 (1966); J. R. Dunning *et al.* (Ref. 3); (d) T. Janssens, E. B. Hughes, M. R. Yearian, and R. Hofstadter, Phys. Rev. **142**, 922 (1966); (e) P. Stein, R. W. McAllister, B. D. McDaniel, and W. M. Woodward, Phys. Rev. Letters **9**, 403 (1962); (f) E. B. Hughes *et al.* (Ref. 3); (g) B. Dundelgab, G. Sauvage, and P. Lehmann, Nuovo Cimento **28**, 18 (1963); (h) P. Lehmann, R. Taylor, and R. Wilson, Phys. Rev. **126**, 1183 (1962); (i) W. Bartel, B. Dudelzak, H. Krehbiel, J. M. Elroy, U. Meyer-Berkhout, R. J. Morrison, H. Nguyen-Ngoc, W. Schmidt, and G. Weber, DESY Report 67/18 (unpublished). We have discarded the neutron data of Ref. 10(b). They are obtained from the electron-deuteron elastic cross section by assuming the nonrelativistic deuteron wave function derived from nucleon-nucleon scattering experiments. These data suggest that $G_E{}^n=0$ out to $q^2=0.25$ (BeV/c)^2 in contrast to the nonzero initial slope of $G_E{}^n$ (Ref. 4). Neutron data of other references are obtained from quasi-elastic electron-deuteron scattering.

TABLE I. Best sets of fitted parameters $[\Lambda_I, \Lambda_Y, \Lambda_N, F_2^N(0)]$ and χ^2 per point. There are 40 experimental points for G_E^p, 37 for G_M^p, 22 for G_E^n, and 21 for G_M^n. The errors shown are one standard deviation. They are statistical errors.

	Current mixing	Mass mixing	Mass mixing (a variation)
θ_Y	33°	32°	39°
θ_N	21°	32°	39°
Λ_I	0.933±0.007	0.993±0.007	0.993±0.007
Λ_Y	0.440±0.006	0.442±0.008	0.440±0.006
Λ_N	0.702±0.020	0.705±0.020	0.690±0.026
$F_2^N(0)$	−0.752±0.026	−0.749±0.026	−0.741±0.026
g_N/g_Y	10.8 ±0.6	11.2 ±0.6	10.7 ±0.6
$\chi^2(G_E^p)$ per point	1.07	1.07	1.07
$\chi^2(G_M^p)$ per point	1.98	1.97	1.97
$\chi^2(G_E^n)$ per point	1.64	1.64	1.63
$\chi^2(G_M^n)$ per point	1.18	1.18	1.18
Total χ^2 per point	1.47	1.47	1.47

oretical results for only one model (mass-mixing) in Figs. 1–4.

The fitting is extremely sensitive to Λ_I, the reason being that Λ_I is the only parameter contained in the isovector magnetic form factor $(G_M^p - G_M^n)$ and that the experimental data for magnetic form factors are much more accurate than those for the electric form factors.[11] Furthermore, $G_M^p(q^2)$ and $G_M^n(q^2)$ are dominated by their corresponding isovector parts as is seen in Eqs. (11) and (13). All these enable us to determine Λ_I to less than 1% accuracy. The next dominant terms of Eqs. (11) and (13) are the second terms which contain the sum of information for ϕ^0 and ω^0 while the third terms contain the difference. A similar term appears in Eq. (10) too, which is the main contribution to $G_E^p(q^2)$ near $q^2 = 1$ (BeV/c)^2. The determining parameter Λ_Y of these terms is therefore sensitive to the fitting too (2% accuracy). The rest of the parameters Λ_N, $F_2^N(0)$, and g_N/g_Y determine the last terms of Eqs. (10)–(12). They are less sensitive (~4 or 5% accuracy).

An attempt has been made to let $\Lambda_N = \Lambda_Y$ so that only one parameter will be used for the isoscalar vertices $N \to N + \omega^0$ (or ϕ^0). The best set of parameters is

$$\Lambda_I = 1.12 \text{ BeV},$$
$$\Lambda_Y = \Lambda_N = 0.70 \text{ BeV},$$
$$g_N/g_Y = 1.7,$$
$$F_2^N(0) = -4.6.$$

The last two values differ totally from those with Λ_Y differing from Λ_N. The calculated curve for $G_E^p(q^2)$ lies consistently below the experimental data (Fig. 1), while that for $G_M^n(q^2)$ lies too high (Fig. 3). The χ^2 per point is as high as 3.0 making the fitting unacceptable.

[11] In extracting G_M and G_E from the measured cross sections at particular q^2, the differential cross sections are plotted against $\tan^2(\frac{1}{2}\theta)$, where θ is the scattering angle. G_M is given by the slope of the best-fitted straight line. The uncertainty is usually small. $(G_E)^2$ is given by the intercept at a small negative value of $\tan^2(\frac{1}{2}\theta)$ which in turn depends on G_M. The uncertainty is usually large.

With $\Lambda_I = \Lambda_N$, the fitting is quite different, the best set of parameters

$$\Lambda_N = \Lambda_I = 1.00 \pm 0.01 \text{ BeV},$$
$$\Lambda_Y = 0.50 \pm 0.02 \text{ BeV},$$
$$g_N/g_Y = 7.5 \pm 0.6,$$
$$F_2^N = -0.84 \pm 0.03,$$

which although differing from those in Table I by an appreciable amount, gives a χ^2 per point of 1.584, just slightly bigger than our best value. The fitted curves in fact do not differ from our best ones significantly. The curves for G_E^p, G_M^p/μ_p, and G_M^n/μ_n actually coincide with our corresponding best curves at the scale used in Figs. 1–3. The only discrepancy is that this fitting predicts values for $|G_E^n(q^2)|$ that are too high at large q^2 (Fig. 4).

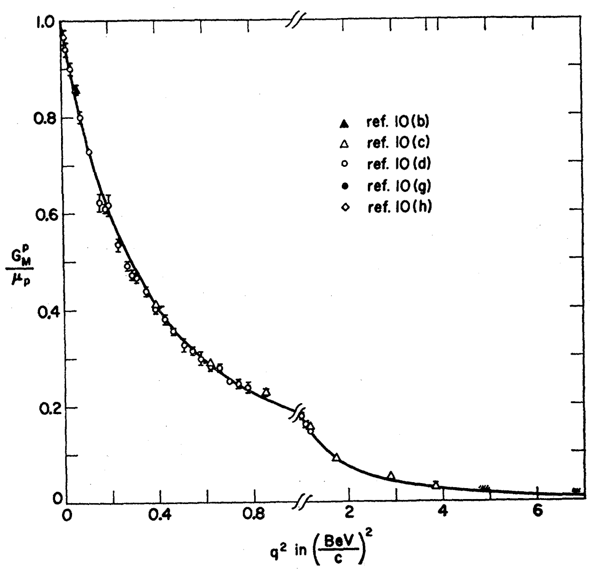

FIG. 2. Our best fit to G_M^p/μ_p for spacelike momentum transfer (see Table I). The curve for $\Lambda_Y = \Lambda_N$ does not differ from this appreciably. Note the change of scale of q^2 at 1 (BeV/c)^2. The shaded symbols are upper bounds determined by Ref. 10(c).

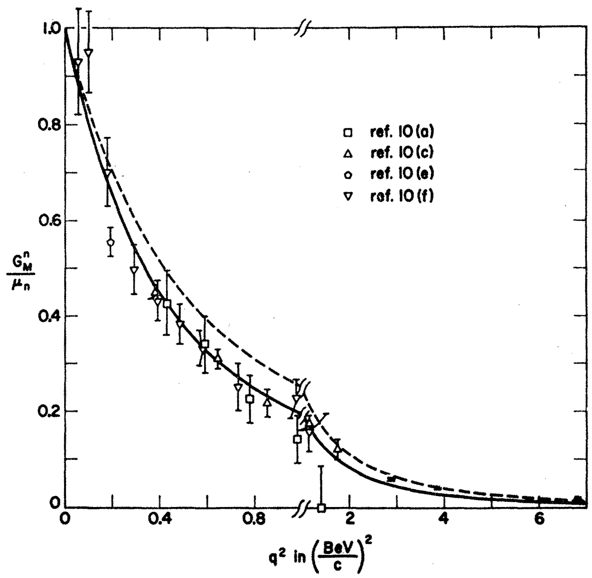

FIG. 3. The solid curve is our best fit to $G_M{}^n/\mu_n$ for spacelike momentum transfer (see Table I). The dashed curve is for the case with $\Lambda_Y=\Lambda_N$. Note the change of scale of q^2 at 1 $(\mathrm{BeV}/c)^2$. The shaded symbols are upper bounds determined by Ref. 10(c).

3. χ^2 DISCUSSION

From Figs. 1–4, our fittings appear to be reasonably satisfactory. The χ^2 per point (120 points) is around 1.5, which is not too large. Further reduction of the χ^2 seems to be impossible, probably because of the inconsistency among some of the experimental data. For example, in Fig. 3 the $G_M{}^n$ data of Akerlof *et al.*[10] are much lower than those of Dunning *et al.*[10] From Figs. 1 and 2 we see that the results of Dunning *et al.*[10] are consistently higher than those of Janssens *et al.*[10] Also the point obtained by Stein *et al.*[10] for $G_M{}^n$ is lower than the average (Fig. 3).

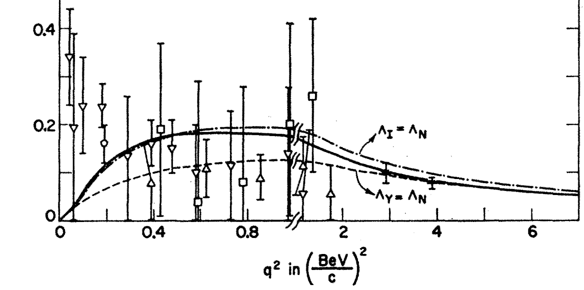

FIG. 4. The solid curve is our best fit to $|G_E{}^n|$ for spacelike momentum transfer (see Table I). The dashed curve is for $\Lambda_Y=\Lambda_N$ and the dot-dashed curve for $\Lambda_I=\Lambda_N$. The slope at $q^2=0$ is 0.457 $(\mathrm{BeV}/c)^{-2}$ for all the three curves. Note the change of scale of q^2 at 1 $(\mathrm{BeV}/c)^2$. The shaded symbols are upper bounds determined by Ref. 10(c).

TABLE II. Calculated form factors $F_a{}^\omega(q^2)$, $F_a{}^\phi(q^2)$ for $N\to N+\omega^0$ (or ϕ^0) [see Eq. (15)].

	Current mixing	Mass mixing	Mass mixing (a variation)
$Y_1{}^\phi$	0.96	0.85	0.63
$N_1{}^\phi$	6.0 ±10%	6.0 ±10%	6.8 ±10%
$Y_2{}^\phi$	−0.12	−0.10	−0.08
$N_2{}^\phi$	−4.5 ±10%	−4.5 ±10%	−5.0 ±10%
$Y_1{}^\omega$	−0.36	−0.53	−0.78
$N_1{}^\omega$	9.2 ±10%	9.5 ±10%	8.3 ±10%
$Y_2{}^\omega$	0.04	0.06	0.09
$N_2{}^\omega$	−6.9 ±10%	−7.1 ±10%	−6.2 ±10%
$F_1{}^\phi(0)/g_Y$	6.9 ±10%	6.8 ±10%	7.4 ±10%
$F_2{}^\phi(0)/g_Y$	−4.6 ±10%	−4.6 ±10%	−5.1 ±10%
$F_1{}^\omega(0)/g_Y$	7.9 ±10%	9.0 ±10%	7.6 ±10%
$F_2{}^\omega(0)/g_Y$	6.8 ±10%	−7.1 ±10%	−6.1 ±10%

4. REMARKS

With the help of Eqs. (2) the nucleon form factors for ϕ^0 and ω^0 emission can be calculated using the results in Table I. They are of the form

$$F_a{}^\phi(q^2)=g_Y\left[\frac{Y_a{}^\phi}{1+q^2/\Lambda_Y{}^2}+\frac{N_a{}^\phi}{1+q^2/\Lambda_N{}^2}\right],$$
$$F_a{}^\omega(q^2)=g_Y\left[\frac{Y_a{}^\omega}{1+q^2/\Lambda_Y{}^2}+\frac{N_a{}^\omega}{1+q^2/\Lambda_N{}^2}\right]. \tag{15}$$

The constants $Y_a{}^{\phi,\omega}$, $N_a{}^{\phi,\omega}$ are listed in Table II. We see that the constants for ϕ^0 are of the same order of magnitude as those for ω^0. The near equality of $F_a{}^\phi(q^2)$ and $F_a{}^\omega(q^2)$ does not necessarily contradict the fact that the ϕ^0 production is greatly suppressed when compared with the ω^0 production.[12] Let us consider the production reaction

$$\pi^-+p\to\pi^-+p+\omega^0\quad(\text{or }\phi^0),$$

which is assumed to proceed via a combination of one-

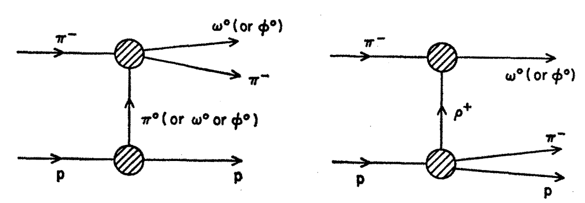

FIG. 5. One-particle-exchange graphs for the reaction $\pi^-+p\to\pi^-+p+\omega^0$ (or ϕ^0).

[12] Y. Y. Lee, W. D. C. Moebs, Jr., B. P. Roe, D. Sinclair, and J. C. Vander Velde, Phys. Rev. Letters **11**, 508 (1963).

particle-exchange processes[13] (Fig. 5). From experiments[14] we learn that the coupling constants for $\phi \to \rho\pi$ and $\phi \to 3\pi$ are very much less than the corresponding constants for $\omega \leftrightarrow \rho\pi$ and $\omega \to 3\pi$ (the ratio is roughly 0.18). Therefore the Feynman graphs in Fig. 5 will produce a cross section of ϕ^0 production equal to about 3% of that for ω^0. The same graphs in the crossed channel also describe the reaction

$$p+\bar{p} \to \pi^+ + \pi^- + \omega^0 \quad (\text{or } \phi^0).$$

As a result, the ϕ^0 production by this reaction is also suppressed.

[13] Y. Y. Lee *et al.*, Ref. 12; D. Berley and N. Gelfand, Phys. Rev. **139**, B1097 (1966); G. W. London, R. R. Rau, N. P. Samios, S. S. Yamamoto, M. Goldberg, S. Lichtman, M. Prime, and J. Leitner, *ibid.* **143**, 1034 (1966).

[14] P. L. Connolly, E. L. Hart, K. W. Lai, G. London, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamoto, M. Goldberg, M. Gundzik, J. Leitner, and S. Lichtman, Phys. Rev. Letters **10**, 371 (1963); J. J. Sakurai, *ibid.* **9**, 472 (1962); D. C. Miller, Nevis Cyclotron Report No. 131, 1965 (unpublished).

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Experimental Restrictions in the Determination of Invariant Amplitudes*

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Necessary conditions are proven concerning the type of polarization experiments that must be carried out as part of a set of experiments aimed at uniquely determining the invariant amplitudes of a reaction involving particles of arbitrary spins.

In this paper a somewhat abstractly formulated result about the experimental requirements for the determination of invariant amplitudes in reactions involving particles of arbitrary spins will be proved, and then a number of quite practical consequences of this result will be given. These consequences are listed at the end of the paper under Lemmas 1–5.

With the advent of sophisticated polarization techniques and high-current accelerators, increased attention is directed toward a detailed experimental determination of the reaction *amplitudes*, since it is felt that this is a much more searching and reliable test of dynamical theories, particularly when they contain a large number of adjustable parameters, as they usually do. At low energies such a determination has been carried out primarily in terms of phase shifts, but with the attention turning to high-energy phenomena where the inelastic channels and the large number of partial waves annihilate the advantages of phase shifts, the amplitudes themselves come to the foreground.

A recent paper[1] made a significant contribution to this problem by showing that the amplitudes cannot be determined by experiments which fail to yield at least one spin correlation between any two sets formed out of all the particles participating in the reaction. The proof of this statement consists of an elegant application of rather formal theorems pertaining to finite-dimensional vector spaces.

The results of the present paper are in some respects more general, in other respects complementary to that of Ref. 1. They are presented here partly because of their relevance to the planning of future experimental equipment and techniques, and partly to demonstrate the power of very simple arguments based on a recent formalism[2,3] dealing with nondynamical properties of reactions involving particles with arbitrary spins. It is hoped that an application of this technique in wider circles would soon result in a complete solution of the problem of how to determine the invariant amplitudes of a given reaction from the simplest set of experiments possible through the set of available experimental techniques.

The considerations here will deal with a parity-conserving four-particle reaction, since the proof is somewhat simpler here than for general rotation-invariant processes, and because many practical applications will be to parity-conserving reactions. Similar results, however, can be derived for the rotation-

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[1] M. Simonius, Phys. Rev. Letters **19**, 279 (1967).

[2] P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Ann. Phys. (N. Y.) **41**, 1 (1967).

[3] M. J. Moravcsik, Lectures on Non-Dynamical Tests of Conservation Laws in Particle Reactions, College of William and Mary, Williamsburg, 1966 (unpublished). These lectures also give a fairly up-to-date list of other papers of ours relating to the nondynamical structure of particle reactions.