Short- and Long-Range Charge Independence*

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The difference between the proton-proton (p-p) and neutron-proton (n-p) $^1S_0$ interactions attributable to effects of other than electromagnetic origin is discussed in connection with evidence concerning long-range charge independence. The latter evidence is based on the adjustment of the amplitude of the one-pion-exchange (OPE) group of phase parameters to secure best agreement with p-p and n-p scattering data. It is concerned mainly with distant collisions and long-range effects. The $^1S_0$ phase shift is appreciably influenced by short-range interactions. Although the pion-mass difference is only $\approx 3.3\%$, an appreciably larger fractional difference of effective p-p and n-p ranges, $(^1r_0)_{pp}-(^1r_0)_{np}$, is shown to be conceivable. In particular, the approximately 10% effect on $^1r_0$ obtained by Noyes in his effective-range-type analysis, employing a literal acceptance of the published low-energy n-p data, is not out of the question. The argument used for these conclusions is only semiquantitative. It differs from other earlier treatments in that the reconciliation of short-range charge dependence with long-range charge independence is not made to depend on the adjustment of the shape of the energy curve of the N-N potential versus distance at a distance comparable to the core radius, such as results from an adjustable cutoff radius for the OPE interaction. The adjustments in the shape of the potential-energy curve are those needed to reproduce the phenomenological phase-parameter fits to p-p scattering data, with some direct guidance from the data. The semiquantitative procedure applied to a hard-core potential of the Hamada-Johnston type then yields a fair, though not an exact, reproduction of the phenomenological requirements on the $^1S_0$-(n-p) phase shift in the 0–350 MeV incident laboratory energy range. Although the hard-core-potential work makes a 10% difference in the effective ranges conceivable, it favors a smaller effect, such as $(^1r_0)_{np}=2.7$ F. The same procedure, when applied to a soft-core potential somewhat similar to that of Reid, does not give nearly as satisfactory a reproduction of n-p data. It appears possible, though far from certain, that the soft-core potential does not give as satisfactory results because it does not include sufficiently the effect of the smeared out δ function of the OPE potential. The procedure used hybridizes the viewpoints of the S matrix and of the equivalent static nonrelativistic potential, with p-p data fitting playing the role of an analog-computer determination of the parameters of the potential. The speculative character of the frequently made assumption of exact equality of the nonelectromagnetic part of the N-N interaction in $T=1$ states with $L>0$ is emphasized. Evidence regarding the failure of such a view for $L=0$ in the 0–350 MeV energy range is reviewed. The desirability of improving existing experimental information on n-p scattering to the point of making it possible to determine both the $T=1$ and $T=0$ phase parameters for the lower L and J is discussed.

I. INTRODUCTION

The words "long-range charge independence" are used below for a hypothesized lack of dependence of the pion-nucleon coupling constant g^2 derived from the one-pion-exchange (OPE) interaction on the total charge of the two-nucleon system. It implies an essential lack of dependence of specific nucleon-nucleon interactions on the nucleon charge at large internucleon distances apart from direct electromagnetic effects and those attributable to the differences in the masses of charged as compared with neutral pions. The words "short-range charge independence" as applied to nucleon-nucleon interactions have often been used to mean that, after direct electromagnetic nucleon-nucleon interaction effects are taken into account, the p-p and n-p interactions are equal for states with relatively low values of the orbital angular momentum Lh. Although the early comparisons of p-p with n-p scattering were concerned entirely with $^1S_0$ states, the approximate equality of the p-p and n-p interactions in that state has been often interpreted in the sense of applying to other states as well. Long-range charge independence is more closely related to "particle physics," short-range charge independence to nuclear-structure and nuclear-reaction applications. The early evidence was that the low-energy specifically nuclear n-p interaction is slightly the stronger in the $^1S_0$ state. Except for a temporary opinion[1,2] regarding exact equality of p-p and n-p forces which was based on calculations with attractive Yukawa potentials, the applicability of which is improbable in the light of present experimental evidence, the early indications have changed only regarding the exact value of the difference between the n-p and p-p interactions. At zero incident energy the existence of such a difference may be inferred from a comparison of $(^1a)_{pp}$, the $^1S_0$-state scattering length in p-p scattering, with $(^1a)_{np}$, the corresponding length in n-p scattering.

An indication of an additional difference appeared in

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[1] L. E. Hoisington, S. S. Share, and G. Breit, Phys. Rev. **56**, 884 (1939).
[2] J. Schwinger, Phys. Rev. **78**, 135 (1950).

the work of Noyes,[3] according to which $(^1r_0)_{np}$, the singlet effective range in n-p scattering, is at least 10% smaller than $(^1r_0)_{pp}$, the corresponding quantity in p-p scattering. His data-analysis calculations were confirmed by Breit, Friedman, and Seamon.[4] An analysis of the theory of the experiments[4] performed by the latter authors showed, however, that it was difficult to exclude the possibility of $(^1r_0)_{np}$ being as high as 2.7 F in place of 2.5 F, the latter value being essentially the same as obtained by Noyes. The larger value appeared more plausible because the difference between charged and neutral pion masses is $\approx 3\%$, as compared with the approximately 10% effect on the value of $^1r_0$ if it is to be taken as 2.5 F, and also because a number of analyses of p-p and n-p scattering data in the 10–350 MeV energy region have been performed without running into obvious contradictions and without assuming differences between p-p and n-p interactions other than those caused by the Coulomb potential supplemented in some cases by corrections for the charge dependence of ^1a and effects of nucleon magnetic moments. The value 2.7 F for $^1r_0$ was used therefore in recent analyses[5,6] of n-p data. It was mentioned in the latter connection that the data analysis in the 10–350 MeV energy region did not exclude the possibility $(^1r_0)_{np}=2.5$ F, an attempt having been made to see whether data at energies essentially higher than those in Noyes' analysis showed a preference for one over the other value of $(^1r_0)_{np}$. One of the objects of the present note is to present a semiquantitative argument showing that, although a 10% effect on the effective range appears to be rather large in comparison with the relatively small pion-mass difference, it is conceivable from a certain viewpoint that this small fractional difference may be responsible for much larger fractional differences of scattering lengths and effective ranges in the p-p and n-p cases, and that these differences are of the order of those mentioned above.

The semiquantitative argument is described in Sec. II. It is far from rigorous. The viewpoint taken is intermediate between those of ordinary static-potential descriptions and that of the S matrix. A rigorous transition from the latter to the former is difficult. In particular, any inference concerning the magnitude of the local nonrelativistic potential from the structure of the S matrix is in the end a purely formal one, implying little more than that the employment of the potential in the wave equation reproduces the S matrix. In practice, even this not too significant connection is usually not established. Instead a "potential" is frequently introduced by requiring it to reproduce the S matrix in the first Born approximation. Since the S matrix is not known for nucleon-nucleon (N-N) scattering, the first Born approximation is made to reproduce only some of the terms. For example, in the one-boson exchange-potential models the terms selected for reproduction are the contributions to the S matrix caused by one-boson exchanges. Since the wave equation employing the potential so derived has no further justification than that it reproduces the S matrix in a certain questionable approximation, the wave function obtained may not be assumed to be the one that should be used for the usual calculation of mean densities and other statistical predictions concerning the relative motion of the two nucleons. Such predictions are needed, however, in the calculation of electromagnetic effects, especially at the lower energies. For example, the calculation of electrostatic effects depends on the availability of the density of representative points in the space of relative coordinates of the two nucleons. Quantitative reasoning with theoretically derived potentials which depends on localization of the nucleons in space, such as is common in cutoff procedures, is thus clearly unjustified, as will be discussed more fully in Sec. IV. The need for some other approach—even if it should be only supplementary—is apparent, however, without the discussion just mentioned.

A direct and practical S-matrix treatment, free of questionable approximations, is still out of reach. The semiquantitative considerations of Sec. II attempt to use, therefore, some of the properties of the scattering matrix concerned with distant collisions but rely otherwise on a static-potential representation. The latter is arranged to give the long-range behavior of the potential in such a way as to represent correctly the long-distance-collision S-matrix properties. The hard-core and soft-core nonrelativistic static potentials used here are results of work to be submitted for publication in the near future. The soft-core potential is similar to that devised by Reid, but has some additional terms of the same general type and the coefficients of the terms used by Reid are also different. Both have been made to fit the phenomenological phase parameters of the Yale Y-IV series. Starting with the potential representing

[3] H. P. Noyes, Phys. Rev. **130**, 2025 (1963); *Comptes Rendus du Congrès International de Physique Nucléaire*, (Centre National de la Recherche Scientifique, Paris 1964), Vol. II, Paper 1/C 104, p. 6, 172; Phys. Rev. Letters **12**, 171 (1964); Nucl. Phys. **74**, 508 (1965). In the last reference, making use of the hypothesis of charge independence, of dispersion-theoretical considerations and of potential models, Noyes arrives on p. 514 of his paper at (2.73±0.03) F for the value of $(^1r_0)_{np}$. On p. 513 he obtains for the same quantity 2.758 F for the value 14.4 of his G^2 and 2.739 F for his $G_{op}^2=14.002$. Linear extrapolation from the last two values to $G_{op}^2=16.0$, which appears now to be the probable value, gives $(^1r_0)_{np}=2.83$ F. This extrapolation is admittedly of doubtful significance and is mentioned only as an illustration of the possible sensitivity of the derived value of the effective range to the assumptions made. Since the probable value of G_{op}^2 at the time of Noyes' paper was appreciably lower than now and since his Table I, which is used in the discussion on p. 514 of his paper, employs a one-pion potential tail with $g_0^2=14.4$, and since the accuracy of approximations made is hard to be sure of, it is not althogether clear that (2.73±0.03) F should be considered as a firm prediction but it is of interest that this value fits in nicely with other values quoted in the present paper.

[4] G. Breit, K. A. Friedman, and R. E. Seamon, Progr. Theoret. Phys. (Kyoto) Suppl. (Yukawa Issue), 449 (1965).

[5] G. Breit, Rev. Mod. Phys. **19**, 560 (1967). This report was based on collaboration with K. A. Friedman, R. D. Haracz, J. M. Holt, A. Prakash, and R. E. Seamon.

[6] R. E. Seamon, K. A. Friedman, G. Breit, R. D. Haracz, J. M. Holt, and A. Prakash, Phys. Rev. **165**, 1579 (1968).

p-p scattering and making use of a somewhat speculative consideration regarding the effect of the mass difference of charged and neutral pions on the mathematical form of the potential, the n-p potential is calculated for different values of the pion-nucleon coupling constant g^2. The value of this coupling constant needed in order to account for the experimental value of the n-p scattering length ${}^1a_{np}$ is compared with that needed for the n-p effective range $({}^1r_0)_{np}$ being alternatively 2.5 F and 2.7 F. In the hard-core case the agreement is rather close, with some preference for the larger of the two values of $({}^1r_0)_{np}$. For the soft-core potential the agreement is not as good but even in this case, it is conceivable that some effects omitted in the present crude considerations could remove the discrepancy. The values of the ${}^1S_0$ state phase shift K_0 corresponding to the different g^2 have been calculated up to laboratory energies of 350 MeV. In the case of the hard-core potential this procedure gives a reasonably consistent approximation to the n-p potential with phenomenologically adjusted parameters. In the case of the soft-core potential, however, the agreement is poor. It may be noted that this potential has no direct equivalent of the repulsive $\delta(\mathbf{r})$-type term arising from the OPE interaction. The repulsive Yukawa-potential-like parts may perhaps be collectively picturing the repulsion caused by vector-meson exchanges without accounting for the δ-function term. It will be remembered that, according to the views of Lévy and Bethe, the phenomenologically desirable hard core may be the result of spreading the effect of the δ function through the hard-core radius. The hard core becomes a softish one in the process, but its shape need not be given by a superposition of the Yukawa-potential-type terms used in the soft-core potential. Perhaps the relatively larger consistency of the hard-core calculation is caused by a closer simulation of the δ-function term.

In Sec. III the possible effect of magnetic interactions on the comparison of p-p with n-p interactions in the ${}^1S_0$ state is reviewed and the result of a rough approximate estimate in the case of the soft-core potential is briefly reported on. The estimates indicate that the conclusions of Sec. II are not appreciably affected by the magnetic interactions.

In Sec. IV the relationship of the comparisons made here with other work is reviewed and the way in which localization of cutoffs enters such work is discussed. The remarks made concerning such localization may perhaps be regarded as unduly critical, especially because the approach used in Sec. II of the present paper is rather crude, lacks mathematical rigor, and is speculative. But since these weaknesses have been pointed out more than once in the present paper and, since no claim is being made to have proven charge independence of the π-N interaction on the basis of ${}^1S_0$-state evidence, the discussion is believed to present the situation fairly.

The indications of an appreciable charge dependence of the effective range and of the scattering length in comparisons of p-p with n-p scattering, combined with the possibility of explaining this dependence without contradiction with charge independence of the nonelectromagnetic part of the pion-nucleon interaction, raise the question of the safety of the frequently made assumption of equality of specifically nonelectromagnetic N-N interactions in inferring n-p phase shifts from p-p scattering data. In particular, it is pointed out that there is little direct evidence from N-N scattering that such equalities apply to states with orbital-angular-momentum quantum number $L>0$, and that for $L=0$ the present experimental evidence indicates a lack of equality which is outside the error of the phenomenological phase-shift determination. The unavailability of accurate n-p scattering data is emphasized in connection with the lack of certainty of the applicability of charge independence to interactions with $L>0$, and the uncertainties regarding the applicability of the usual n-p phase shift analysis to problems in low-energy nuclear physics.

It is assumed throughout most of the paper that the pion-nucleon coupling is pseudoscalar and rotationally symmetric in isospin space. Most of the considerations may become inapplicable if it should turn out that the coupling is predominantly of the pseudovector type as discussed more fully in Sec. IV.

II. CALCULATIONS

Use is being made of fits to p-p scattering by means of static potentials which were obtained by fitting the new Yale phenomenological phase-shift sets of the Y-IV series. For the ${}^1S_0$ phase shift K_0 one of the potentials is a modification of the Reid soft-core potential[7] and has the general form

$$V^{\mathrm{SC}}=V_{\mathrm{OPE}}+(A_3e^{-3x}+A_4e^{-4x}+A_6e^{-6x}+A_7e^{-7x})/x\,,\qquad x=m_\pi cr/\hbar=\mu r\,, \tag{1}$$

with r representing the internucleon distance. The terms in e^{-3x} and e^{-6x} were added to those used by Reid because an improvement in representing the new phenomenological fit was thus obtained. The values of the A_n with $n=4$ and 7 are also different from Reid's and the one-pion-exchange (OPE) potential (OPEP) was used with $g_0^2=15$ corresponding to the "$pp+np$ combined" fit of Ref. 6. The quantity g_0^2 is g^2 expressed in rational units which make its value ≈ 15. This potential is referred to below as $\mathrm{Y_{RM}}$, with Y, R, and M standing for Yale, Reid, and modified, respectively. A modification of the Hamada-Johnston[8] potential adjusted to give an over-all fit to the same phenomeno-

[7] The writers are indebted to Professor H. A. Bethe and R. V. Reid for furnishing them with the soft-core potential fitted by Reid to an older set of phenomenological phase shifts.

[8] T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962).

logical phases as Y_{RM} was also tried. It has the form

$$V^{HC}=Kg_0^2(e^{-x}/x)[1+A(e^{-x}/x)+B(e^{-x}/x)^2], \quad (r>r_c)$$
$$V^{HC}=+\infty, \quad (r<r_c). \tag{2}$$

The superscripts SC and HC on V refer to soft and hard cores, respectively. The factor K is independent of r. The readjusted Hamada-Johnston potential, Eq. (2), is referred to as Y_{H-J} below. The first term in square brackets, together with the factor in front, reproduce the first term in Eq. (1), and represent the effect of OPE at $r>r_c$, where r_c is the hard-core radius. Both V^{SC} and V^{HC} are added to e^2/r to give the total potential for use in the nonrelativistic Schrödinger equation.

The first term in square brackets in Eq. (2) and the V_{OPE} in Eq. (1), as used in the calculations, do not include the well-known δ-function term, whose insertion amounts to the replacement

$$e^{-x}/x \to e^{-x}/x - 4\pi\delta(\mu\mathbf{r}).$$

In the case of V^{HC} the inclusion of $-4\pi\delta(\mathbf{r})$ has no effect, since it amounts to making the infinite potential inside the core more infinite. In the case of V^{SC} the large short-range repulsion caused by a_4e^{-4x}/x similarly makes the presence of the $\delta(\mu\mathbf{r})$ term only of secondary importance.

In dealing with long-range charge independence, it is customary[9] to modify V_{OPE} for $T=1$ in applications to n-p data analysis to take account of the difference in charged and neutral pion masses $m_{\pi^+}-m_{\pi^0}$, calculating phase shifts as

$$2\delta(m_{\pi^+})-\delta(m_{\pi^0}), \tag{3}$$

where $\delta(m_\pi)$ is the phase shift obtained from the pseudoscalar coupling theory for the case of equal pion masses $m_{\pi^+}=m_{\pi^0}=m_\pi$. If, excluding electromagnetic effects, V_{OPE} is the longest-range part of the N-N potential, then the long-range effects of the potential will not be correctly reproduced unless the substitution

$$V_{OPE}(m_{\pi^0}) \to 2V_{OPE}(m_{\pi^+})-V_{OPE}(m_{\pi^0}) \tag{4}$$

in Eq. (1) and the first term in Eq. (2) is similarly modified. This reasoning is far from rigorous because of the difficulty[10] of making a clear distinction between long- and short-range effects in the case of s terms. Even apart from the diffuseness in the definition of the effective impact parameter for low L obvious from the last-mentioned reference, the presence of non-OPE terms in V^{SC} and V^{HC} affects the wave function appreciably at any energy, there being no centrifugal potential barrier for $L=0$. The first-order effect of V_{OPE} on the phase shift as calculated by means of field-free wave functions and the phase shift corresponding to the first-order effect of OPE on the S matrix are not equal even at zero energy, depending in this case on the ratio of the range of $V-V_{OPE}$ to $\hbar/m_\pi c$, the range of V_{OPE}. Arguments for the employment of V_{OPE} in the Schrödinger equation are not compelling, therefore, many possible forms of V_{OPE} being equally logical. Equation (4) is, nevertheless, assumed to be applicable partly as a matter of uniformity with the usual treatment of higher L states.

The parts of the potential containing e^{-nx}/x with $n>1$ are presumably partly caused by multiple pion exchange and by the exchange of other mesons, such as ω and ρ. They are presumably also partly caused by OPE because of the lack of unique correspondence of the effect of OPE on the S matrix and on the potential, the primary meaning of OPE being that for the S matrix. A complete calculation of the effect of $m_{\pi^+}-m_{\pi^0}$ is clearly very difficult and is beyond the scope of this paper. A general idea of what may be expected can be obtained, however, by considering the effect of successive one-pion exchanges as in ladder diagrams.

If the ladder rungs were connecting nucleon lines corresponding to nucleons in real rather than virtual states, then each exchange would give rise to the factor

$$Q\equiv g_{0+}^2\mathcal{V}(m_{\pi^+},r)[(\boldsymbol{\tau}^a\cdot\boldsymbol{\tau}^b)-\tau_3^a\tau_3^b] + g_{00}^2\mathcal{V}(m_{\pi^0},r)\tau_3^a\tau_3^b, \tag{5}$$

where, neglecting relativistic effects,

$$\mathcal{V}(m_\pi,r)=\tfrac{1}{3}\hbar c\left(\frac{m_\pi}{2M}\right)^2\left\{\frac{-4\pi\delta(\mathbf{r})}{\mu^2} + \left[(\boldsymbol{\sigma}^a\cdot\boldsymbol{\sigma}^b)+\left(1+\frac{3}{x}+\frac{3}{x^2}\right)S_{ab}\right]\frac{e^{-x}}{r}\right\}. \tag{6}$$

The two nucleons are designated by a and b, the corresponding isospin operators by τ^a and τ^b, and the relations between x, μ, and r are as in Eq. (1). If the second subscript on g_0 is $+$ it indicates that the value for the charged pion is meant; if it is 0, the value for the neutral pion is understood similarly to m_{π^+} and m_{π^0} denoting the masses of charged and neutral pions, respectively. For $T=1$ in p-p scattering, $(\boldsymbol{\tau}^a\cdot\boldsymbol{\tau}^b)=1$, $\tau_3^a\tau_3^b=1$ so that the first term in Eq. (5) disappears. The whole OPE interaction is due to π^0 in this case. In n-p scattering $\tau_3^a\tau_3^b=-1$ and π^+, π^- exchanges then enter also. For n one-pion exchanges there arise the factors occurring in the equations

$$Q^n\chi^\tau=[2g_{0+}^2\mathcal{V}(m_{\pi^+},r)-g_{00}^2\mathcal{V}(m_{\pi^0},r)]^n\chi^\tau, \quad (n\text{-}p) \tag{7a}$$
$$Q^n\chi^\tau=[g_{00}^2\mathcal{V}(m_{\pi^0},r)]^n\chi^\tau \quad (p\text{-}p). \tag{7b}$$

If the part of V^{HC} having the form const$\times(e^{-x}/x)^n$ in Eq. (2) is associated with n-pion exchange, then the relationship between (7a) and (7b) suggests employment for n-p scattering of the result of the replacement

$$(Q^n)_{pp}\to(Q^n)_{np}, \tag{8}$$

the eigenvalues of Q^n for the two cases being the factors multiplying the isospin function χ^τ. Assuming, following

[9] G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960); R. E. Schneider and R. M. Thaler, *ibid.* **137**, B874 (1965).
[10] G. Breit and M. H. Hull, Jr., Nucl. Phys. **15**, 216 (1960).

Kemmer,[11] that

$$g_{0+}^2 = g_{00}^2, \tag{9}$$

the difference between Q_{np} and Q_{pp} in the ${}^1S_0$ case consists in the substitution for the attractive Yukawa potential a repulsive one and the introduction of an additional attractive one with approximately twice the depth parameter. Because of the factor m_π^2 in Eq. (6), the change to m_{π^+} in the first term of (7a) makes the depth parameter still somewhat deeper and the range somewhat shorter. These changes are in the direction of giving a smaller effective range in the n-p case.

It does not appear justifiable to apply the substitution of Eq. (8) to every term of V^{HC} or V^{SC}, because: (a) the OPE and the multiple pion exchanges have clearly distinguishable effects on the S matrix but not on the effective potential to be used in the Schrödinger wave equation; (b) the nucleon lines of ladder diagrams refer to virtual nucleon states rather than on-energy-shell conditions; (c) diagrams other than those of the ladder type contribute also. Because of (c) the order of neutron and proton portions of nucleon lines becomes reshuffled. Because of (a) some OPE effects should be apparent in $(e^{-x}/x)^2$ and $(e^{-x}/x)^3$ parts of V. Because of (b) the exact exponential dependence on integral multiples of x becomes questionable. A literal association of Q^n of Eqs. (7) and (8) with terms in e^{-nx} of V^{HS} and V^{SC} therefore appeared unjustifiable. In a somewhat experimental and speculative spirit, several variations were tried in modifying the literal assignment of Eq. (8). Among them the one that came closest to reproducing the experimental values of ${}^1a_{np}$ and $({}^1r_0)_{np}$ consists in associating the nth power of g_0^2 with a term in e^{-nx} but employing the combination with coefficients 2, -1 as in Eq. (7a) and the $m_{\pi^+}^2$, $m_{\pi^0}^2$ factors only once for the whole potential. The m_{π^+}-m_{π^0} mass difference is thus affecting every term but not quite as strongly as Eqs. (7) and (8) would require. Explicitly,

$$(V^\alpha)_{np} = 2V^\alpha(m_{\pi^+}, g_0^2, r) - V^\alpha(m_{\pi^0}, g_0^2, r), \tag{10a}$$

where

$$\alpha = \mathrm{HC}, \mathrm{SC}, \tag{10b}$$

and

$$V^{\mathrm{HC}}(m_\pi, g_0^2, r) = g_0^2 \mathcal{V}(m_\pi, r) \times \left[1 + a\frac{g_0^2 e^{-x}}{15x} + b\left(\frac{g_0^2 e^{-x}}{15x}\right)^2\right]_{x=m_\pi c r/\hbar} \tag{11a}$$

for

$$r > r_c, \tag{11b}$$

where r_c is the core radius for the p-p potential.

The choice of the core radius is usually made phenomenologically. In the present case it has been determined by a fit to p-p data. It was assumed in making the fit that $V^{\mathrm{HC}} = \infty$ when $r < r_c$. Out of all possible points of view regarding correlating the values of r_c in going from p-p to n-p cases, two extremes appear to deserve mention. In the first the value of x corresponding to r_c is taken to be the same for charged and neutral pions because r enters the calculations primarily through x. If the hard core had its origin primarily in many pion exchanges and if the effects of charged and neutral pions could be separated from each other, this would be a valid viewpoint. Actually such a separation is not possible. If, nevertheless, it is assumed that

$$(r_c)_{\pi^+} = \hbar x_c / m_{\pi^+} c, \quad (r_c)_{\pi^0} = \hbar x_c / m_{\pi^0} c \tag{12}$$

with the same x_c, then

$$(r_c)_{\pi^0} > (r_c)_{\pi^+}. \tag{13}$$

According to Eq. (4) the "hard core" arising from π^0 exchange acts as an attractive potential. It follows from Eq. (13) that in the short length $(r_c)_{\pi^0} - (r_c)_{\pi^+}$ there is a residual infinite attractive potential, the effect of which on the wave function has to be evaluated by first replacing the infinite value of the potential within the core radius by a finite one, then calculating the wave function, and finally taking its limit as the potential becomes infinite. Since the result depends on the relationship of the core depth to $(r_c)_{\pi^0} - (r_c)_{\pi^+}$, the answer is indeterminate. Since an infinitely hard core is only an idealization of a soft core, the employment of a hard core in the x_c=const viewpoint is unphysical even apart from the unconvincing basis for the choice of the quantity to keep constant in the limiting process just described.

Another simple procedure is to use the same r_c throughout. Part of the reason for doing so is the expectation[12] of a hard core arising from the $\delta(\mathbf{r})$ term of Eq. (6). The term in V_{OPE} containing $\delta(\mathbf{r})$ is m_π-independent. If the δ function is smeared out, its spatial extension may be supposed to be determined mainly by the possibility of localizing a nucleon. The important length for this is $\hbar/Mc$, the Compton wavelength of the nucleon divided by 2π. This length does not depend on m_π. Another possible reason for the existence of the core is ω-meson exchange. This also gives the same core independently of which term in Eq. (4) is under consideration. For these reasons the value of r_c obtained by adjustment to p-p data was used for both parts of $(V^{\mathrm{HC}})_{np}$ in Eq. (10a), i.e., for $(V^{\mathrm{HC}})_{np}$ as a whole.

For V^{SC} a form similar to that of Eq. (11a) was used as follows:

$$V^{\mathrm{SC}}(m_\pi, g_0^2, r) = g_0^2 \mathcal{V}(m_\pi, r) + \frac{\hbar}{m_\pi c r}\left[a_3\left(\frac{g_0^2}{15}e^{-x}\right)^3 + a_4\left(\frac{g_0^2}{15}e^{-x}\right)^4 + a_6\left(\frac{g_0^2}{15}e^{-x}\right)^6 + a_7\left(\frac{g_0^2}{15}e^{-x}\right)^7\right]_{x=m_\pi c r/\hbar}. \tag{14}$$

[11] N. Kemmer, Proc. Cambridge Phil. Soc. 34, 354 (1938); H. Fröhlich, W. Heitler, and N. Kemmer, Proc. Roy. Soc. (London) A166, 154 (1938).

[12] M. Lévy, Phys. Rev. 88, 725 (1952); H. A. Bethe and P. Morrison, *Elementary Nuclear Theory* (John Wiley & Sons, Inc., New York, 1956), especially p. 155.

In this case all terms apply through practically the whole range of values of r, with the exception of a very small region for $x<0.0001$ inside of which a hard core was used in some of the calculations, this being more convenient in starting the numerical integration of the Schrödinger equation. Tests indicate that the effects of the radius of the narrow hard core on the scattering length, the effective range, and the phase shifts is negligible, for present purposes.

Figure 1 shows the dependence of the reciprocal of the singlet scattering length $1/{}^1a$ and of the singlet effective range on g_0^2 for the hard-core case. The convention regarding sign is such that ${}^1a$ is of the same sign as the small phase shift at small positive energies. In the present physical case, therefore, ${}^1a>0$. The horizontal lines intersecting the curves correspond to the experimental values of the quantities. For ${}^1r_0$ the full horizontal short line corresponds to the nominal experimental value 2.50 F, which is essentially the value following[3,4] from a literal use of data as in print before the fall of 1967. The short dashed horizontal line corresponds to ${}^1r_0=2.70$ F, a value which is hard to exclude[4] as a reasonable upper limit in view of uncertainties in the theory of the experiments. Houk and Wilson[13] have recently completed measurements of the neutron-proton total cross section in hydrogen gas and of the neutron-carbon total cross section in pyrolytic graphite between 0.3 and 400-eV laboratory energy. Employing these measurements and those of Koester[14] and on revising some of Koester's derivations of values from experimental quantities, they arrive at ${}^1r_0=(2.59\pm0.08)$ F, a value which may be considered as agreeing with either 2.50 F or 2.70 F. The values of g_0^2 for which the experimental ${}^1a$ and ${}^1r_0$ are reproduced are seen to fall close together. This is especially true if the value from ${}^1r_0=2.70$ F is compared with that from ${}^1a$ which was used as (23.679 ± 0.028) F. The graphical comparison made in Fig. 1 is unaffected by the small statistical uncertainty in the value of ${}^1a$. All the values of g_0^2 are within the statistical uncertainty of the recent determinations[5,6] of $(g_0^2)_{pp}$ though lower than the most probable value and higher by about 1 than the most probable value of $[(g_0^2)+\Delta(g_0^2)]_{np}$, where (Δg_0^2) is the statistical uncertainty of g_0^2. Both $(g_0^2)_{pp}$ and $(g_0^2)_{np}$ are from work on long-range charge independence, which makes no use of either S- or P-wave interactions except for a secondary effect of ${}^3P_2$ through its coupling to ${}^3F_2$. The analysis of pp data being the more reliable, the comparison with the long-range value of $(g_0^2)_{pp}$ is presumably the more significant.

In Fig. 2 a similar comparison is made for the soft-core potential with conventions similar to those for Fig. 1. The agreement among the three values of g_0^2 is seen to be not as good as in Fig. 1. In this case consistency between the value from ${}^1a$ and that from ${}^1r_0$ favors ${}^1r_0=2.50$ F over the larger value. The values of g_0^2 now fall within the statistical error limits of the $(g_0^2)_{np}$ values corresponding to long-range interaction. In this respect there is perhaps better self consistency in the soft-core case, but the usual uncertainty regarding the np data analysis should be borne in mind in drawing conclusions from this fact. An exact agreement with the long-range interaction value of g_0^2 may be too much to

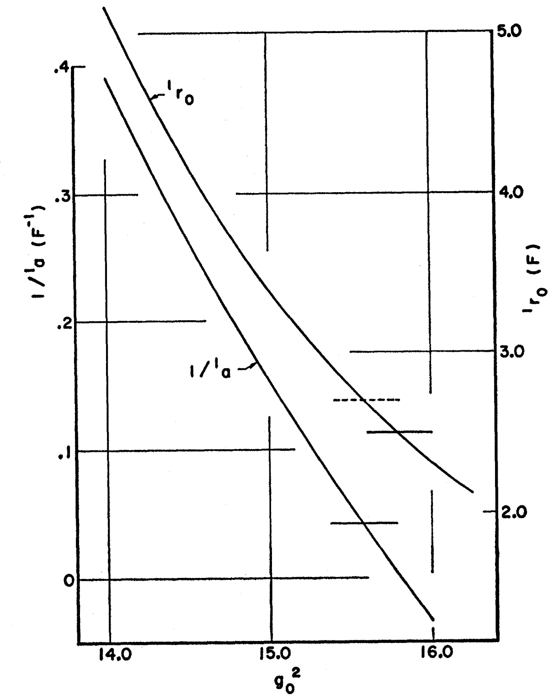

FIG. 1. Plots of n-p values of $1/{}^1a$ and ${}^1r_0$ against g_0^2 as calculated from Eq. (10) for the hard-core potential. Full and dashed horizontal lines for ${}^1r_0=2.5$ F and ${}^1r_0=2.7$ F, respectively.

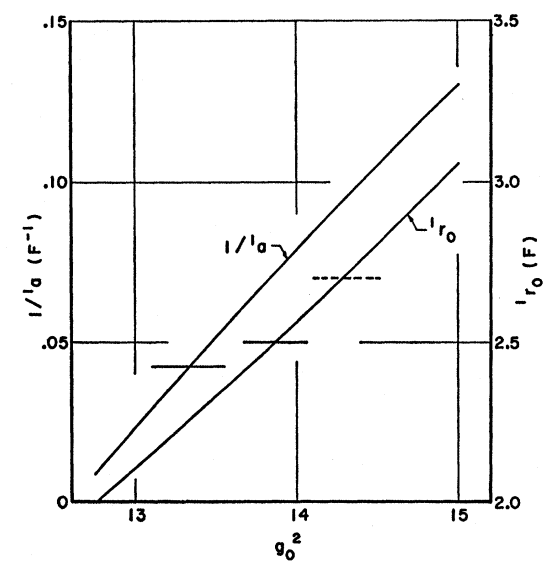

FIG. 2. Plots of n-p values of $1/{}^1a$ and ${}^1r_0$ against g_0^2 as calculated from Eq. (10) for the soft-core potential. Full and dashed horizontal lines for ${}^1r_0=2.5$ F and ${}^1r_0=2.7$ F, respectively.

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[13] Theodore L. Houk and Richard Wilson, Rev. Mod. Phys. **39**, 546 (1967). According to a private communication from Professor Wilson, a re-evaluation of their results gave $({}^1r_0)_{np}=2.69$ F.

[14] L. Koester, Z. Physik **198**, 187 (1967).

require in view of the omission of several effects, mentioned previously.

For V^{HC} the calculated dependence of K_0 on the incident laboratory energy E is similar to that obtained from the phenomenological $(\mathrm{Y\text{-}IV})_{pp}$ fit on applying corrections for the effect of the Coulomb potential on the phase shift and to the result of refitting this potential to n-p data with adjustments to simulate ${}^1r_0 = 2.7$ F, ${}^1a = 23.679$ F. Through the energy region 2–360 MeV the values of K_0 for these two phenomenological fits are bracketed between $g_0^2 = 15.25$ and $g_0^2 = 15.75$ as used in Eqs. (10) and (11) for the computation of potential and phase shift. In these respects the hard-core potential used here comes reasonable close to general consistency as a static potential, provided numerical accuracy requirements are not made too high.

The situation is quite different for the soft-core potential used for Fig. 2. The phase shift calculated from Eqs. (10) and (11) is too large for $g_0^2 = 13$ even at 10 MeV and is consistently much larger than the phenomenological fit at higher energies. The K_0 node is moved up to $E > 300$ MeV even for $g_0^2 = 14.5$. There is no consistency of description of K_0 as a whole by means of Eqs. (10) and (11) if the potential is used as a static one. The possibility of employing energy-dependent parameters remains, but so far soft-core potentials of this type have been used in nuclear-matter work[15] with energy-independent parameters.

The values of g_0^2 obtained from Eq. (10a) by means of graphs on coordinate paper, but otherwise such as those in Figs. 1 and 2 are shown in Table I. In the first column is given the potential type. Entries in parentheses explain the origin of the potential. Thus $\mathrm{Y_{H\text{-}J}}$ refers to the Yale modification of the Hamada-Johnston potential and $\mathrm{Y_{RM}}$ to the Yale modification of the Reid soft-core potential. In Fig. 3 the values of $(K_0)_{np}$ obtained by means of Eqs. (10) and (11) are compared with a phenomenological data fit for the g_0^2 occurring in Table I in the case of the hard-core potential. The difference in the curves corresponding to $g_0^2 = 15.57$ and 15.59 would be barely perceptible in a printed reproduction of the main part of the figure. Accordingly the plot for $g_0^2 = 15.57$ is not shown. The curve marked "Refit $(K_0)_{np}$" was calculated using a potential giving ${}^1a = 23.735421$ F, ${}^1r_0 = 2.70$ F, and $[(K_0)_{pp}]_{e^2/r=0} = -0.22416$ at 350 MeV. The adjustment of the potential to the data was made employing such linear combinations of phase parameters that for them the statistical parallel-shift uncertainties are uncorrelated. This procedure will be described more fully in a forthcoming publication, for comparison with the manuscript of which it may be identified as the δ_{unc} phenomenological potential search. The inset at the upper right of Fig. 3 illustrates the relative values at low energies. The preference of the "Refit $(K_0)_{np}$" curve for the two

TABLE I. Values of g_0^2 corresponding to assigned values of ${}^1a$ and ${}^1r_0$.

Potential	Assigned quantity and its value (F)	g_0^2
HC ($\mathrm{Y_{H\text{-}J}}$)	${}^1a = 23.679$	15.57
	${}^1r_0 = 2.5$	15.79
	${}^1r_0 = 2.7$	15.59
SC ($\mathrm{Y_{RM}}$)	${}^1a = 23.679$	13.33
	${}^1r_0 = 2.5$	13.86
	${}^1r_0 = 2.7$	14.28

smaller g_0^2 in comparison with $g_0^2 = 15.79$ corresponds to ${}^1r_0 = 2.7$ F having been used in obtaining the "Refit $(K_0)_{np}$" values at low energies in contrast with ${}^1r_0 = 2.5$ F corresponding to $g_0^2 = 15.79$. The over-all agreement of the "Refit $(K_0)_{np}$" values with the $g_0^2 = 15.59$ plots may be considered satisfactory in view of the crudeness of the justification of Eqs. (10) and (11). The relatively poor agreement of the soft-core potential with phenomenology mentioned in the text shortly before Table I makes a graphical comparison for this case unnecessary.

III. MAGNETIC INTERACTION EFFECTS

In the considerations presented above the effect of magnetic interactions between the nuclei has been omitted. This has been introduced into the comparison of p-p with n-p interactions by Schwinger[2] in connection with purely attractive ${}^1S_0$ potentials. It has since been pointed out by Salpeter[16] that these effects are much

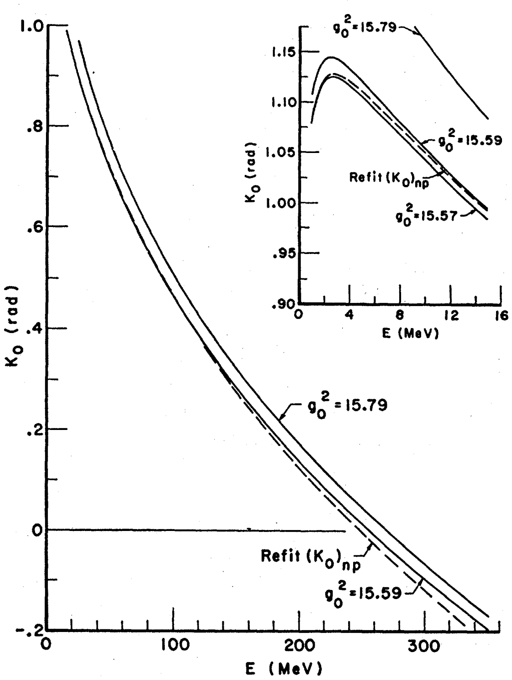

FIG. 3. Comparison of values of K_0 in n-p scattering as a function of incident neutron energy (lab) with phenomenological potential fit "Refit $(K_0)_{np}$."

[15] R. Rajaraman and H. A. Bethe, Rev. Mod. Phys. 34, 745 (1967).

[16] E. E. Salpeter, Phys. Rev. 91, 994 (1953).

reduced if the potentials have repulsive cores. Some of the more important terms become in fact exactly zero if the picture of point sources of magnetic fields used by Schwinger[2] is taken literally. Salpeter distributed the repulsion of the hard-core model through the core radius r_c but with a softish core of approximately the magnitude $Mc^2/2$. He also considered the effect of spreading out the nucleon magnetic moment over a finite distance, replacing the δ functions in Schwinger's formulas for the magnetic interaction energy by constants with values $\frac{3}{4}\pi r_c^3$ when $r<r_c$ and zero when $r>r_c$. His results are expressed in terms of ${}^1a_{np}$ and ${}^1a_{pp}$. The latter is the scattering length calculated by means of a p-p potential first adjusted to reproduce the phenomenological p-p scattering length and effective range, then removing the Coulomb part of the potential energy assumed to be given by e^2/r and calculating the scattering length for the fictitious chargeless protons. If charge independence of nucleon-nucleon interactions were exact and if e^2/r were the true representation of electromagnetic interactions, then it would be expected that ${}^1a_{np}={}^1a_{pp}$. The values tabulated by Salpeter correspond to ${}^1a_{np}/{}^1a_{pp}=1.43$, 1.50, and 1.56 for $r_c=0.3$ F, 0.6 F, and 1.2 F, respectively. The corrections for magnetic effects gave him changes $\delta(1/{}^1a_{pp})$ and $\delta(1/{}^1a_{np})$ in the reciprocals of the two scattering lengths which correspond to the double ratio

$$\frac{(1/{}^1a_{pp})}{(1/{}^1a_{np})}\Bigg/\frac{(1/{}^1a_{pp})+\delta(1/{}^1a_{pp})}{(1/{}^1a_{np})+\delta(1/{}^1a_{np})} \tag{15}$$

having the values 1.12, 1.05, 1.02(5) for $r_c=0.3$ F, 0.6 F, and 1.2 F, respectively. The change caused by the magnetic corrections is therefore relatively insignificant in comparison with the discrepancy of the values of ${}^1a_{pp}$ and ${}^1a_{np}$, as stated, in another form, by Salpeter himself. Of the three values, $r_c=0.6$ F comes closest to the phenomenological core radius. A 5% allowance for the effect of magnetic corrections appears therefore as a fair estimate for the hard-core case. An inspection of Fig. 1 shows that such a change in $1/{}^1a$ is of no consequence for the kind of comparison of the values of g_0^2 as has been made with the aid of that figure.

A later calculation of magnetic effects in comparisons of ${}^1a_{np}$ and ${}^1a_{pp}$ was carried out by Schneider and Thaler,[17] who made use of the nucleon electromagnetic form factors of the Stanford group.[18] The parameters for the potential wells were adjusted to fit a phenomenological p-p scattering length of 7.68 F and effective range of 2.65 F. These phenomenological quantities are meant for use in the effective-range representation of p-p scattering data. In terms of a potential-well model "explaining" these data, the Coulomb potential e^2/r has to be included in distinction from the models giving the ${}^1a_{pp}$ of Schwinger and Salpeter as well as of the present paper. The authors give two kinds of results regarding the comparison of ${}^1a_{pp}$ and ${}^1a_{np}$. The first kind is summarized in their Table I. It gives values of ${}^1a_{np}$ and $({}^1r_0)_{np}$ expected from strict charge independence for the four combined possibilities of point versus distributed charges and point versus distributed magnetic moment distributions. The values of ${}^1a_{np}$ obtained are listed by them for comparison with the experimental and, in the convention regarding signs used by Schwinger and here, the expected ${}^1a_{np}$ is consistently much the smaller, being generally very close to ${}^1a_{nn}$. These calculations were made with a core radius of 0.388 F which corresponds to the "local Yukawa" fit of Giltinan and Thaler.[19] This fit was evolved in connection with work on nonlocal potentials for ${}^1S_0$ and ${}^1D_2$ states. This "local Yukawa" potential has a smaller core radius than the one used here and an exponential decay length of 0.74 F which is about $\frac{1}{2}$ of the value used by Schneider and Thaler in a later part of their paper concerned with OPE calculations. The potential used in Ref. 19 differs from V^{HC} of Eq. (2) by the large-distance cutoff at 4.63 F which is removed in Ref. 17 with a simultaneous change of spatial decay length from 0.70 to 0.696 F. This potential is not comparable with that in Eq. (2), much of the difference arising from having a pure Yukawa shape for $r>r_c$, no allowance being made for two-pion and multiple-pion exchange. However, according to Schneider and Thaler's Table I, the ratio of ${}^1a_{np}$ to the value of this quantity calculated from p-p scattering and the assumption of charge independence lies between 1.37 and 1.50 which does not differ much from corresponding numbers in Salpeter's work already discussed in connection with Eq. (15). The relative importance of magnetic interaction corrections, as crudely inferred from the effects of changing point to extended distributions, is also similar. There appears to be no qualitative difference between Salpeter's results and those of Schneider and Thaler. The inclusion of the Stanford electromagnetic form factors, while more conscientious, does not necessarily bring one closer to the true answer because the experimental form factors are for free rather than interacting nucleons. In view of the greater similarity of Salpeter's core radii to the usual, including those employed for Fig. 1, his estimates will be considered good enough for the purposes of this paper.

For V^{SC} the magnetic effects have been estimated only crudely, there being difficulty in representing n-p data by means of Eq. (10a) in more than a narrow energy range, as mentioned in connection with Fig. 2. The wave function is very small at $r=0$ in this case. The terms[2] containing $\delta(r)$ have been omitted therefore. Schwinger's convection-current term affecting p-p scattering was estimated employing the results of numerically integrating the wave function. The estimate

[17] R. E. Schneider and R. M. Thaler, Phys. Rev. **137**, B874 (1965).

[18] C. de Vries, R. Hofstadter, and R. Herman, Phys. Rev. Letters **8**, 381 (1962).

[19] D. Giltinan and R. M. Thaler, Phys. Rev. **131**, 8 05 (1963)

gives a change of roughly 2% in $1/{}^1a_{pp}$ and indicates that for the crude considerations of the present paper the magnetic effects may be neglected in the soft-core case. Although the assumed core is "soft," the repulsion at small r is high, so that, qualitatively speaking, Salpeter's considerations apply.

IV. DISCUSSION

The paper of Schneider and Thaler includes estimates of the effect of $m_{\pi^+}-m_{\pi^0}$ on ${}^1a_{np}$. They cut off the OPE interaction at values of r smaller than an adjustable value r_{cutoff}. Varying the cutoff and the core radii with an adjustable value of the characteristic length denoted by them as b, they find it possible to obtain reasonably good agreement between the calculated and observed ${}^1a_{np}$ with effective-range values close to 2.7 F. They say regarding this apparent success: "We believe that this should not be taken too seriously, one way or the other, until more reliable estimates of the two-pion-exchange effects are made." The modification of the OPEP needed to take $m_{\pi^+}-m_{\pi^0}$ into account as stated by Schneider and Thaler[17] differs in form only from that available in a paper,[9] the results of which have been previously used by Giltinan and Thaler.[19] The equations in question are (5.1), (5.3) in Ref. 9 and their slight generalization, Eq. (5) of the present paper. The skeptical attitude of Schneider and Thaler regarding the reality of their version of the explanation of the difference between ${}^1a_{np}$ and ${}^1a_{pp}$ appears appropriate in view of the arbitrariness of an adjustable low-r cutoff of V^{OPE} combined with an adjustable core radius and an adjustable range constant for the non-OPE part of the potential. There appears to be some confusion in Ref.17 regarding the applicability of V^{OPE} at small distances. In that reference attention is drawn to the fact that there are other than OPE effects at small distances, some of which are expected to cancel, and the view is taken that a serious error may be therefore committed by the inclusion of OPE effects at small r. But since the insertion of a hard core of adjustable radius modifies the interaction at small distances and provides some flexibility for taking care of such effects and the adjustment of the range constant provides even more, the additional suppression of V^{OPE} supplies mainly an additional fitting parameter. Schneider and Thaler's adjustments are concerned only with values of the scattering length and effective range. Experience in the work reported here shows that many potentials have to be discarded even though they give correctly the first two terms of the effective-range expansion. It should be stated, nevertheless, that, to the writers' knowledge, Schneider and Thaler have been the first to point out in semiquantitative fashion the possibility of explaining the difference in scattering lengths in terms of the effect of the difference in pion masses as in Ref. 9, which was already used there and later elsewhere in tests of long-range charge independence.

Another consideration having a bearing on both long-range and short-range charge independence has been made by Lassila and Peltola,[20] who estimate a contribution of 0.03 F to the observed n-n and n-p scattering-length difference in the ${}^1S_0$ state caused by the instability of π^0 and find no "measurable" effects of π^0 instability on other N-N scattering quantities. The effect of π^0 instability is clearly too small to be taken into account on the rather coarse scale of the present paper.

The combined effect of the exchange of a photon and pion has been considered by Leung and Nogami.[21] The equivalent γ-pion exchange potential ($\gamma\pi$EP) associated with this exchange was found by them to have a range comparable with that of the OPEP but the charge-dependent part of the $\gamma\pi$EP in the ${}^1S_0$ state is only a few percent of the OPEP at $r\approx\hbar/m_\pi c$ and decreases more rapidly than the OPEP with increasing r. They conclude, therefore, that the $\gamma\pi$EP cannot produce a long-range correction of the strength indicated by Noyes' phenomenological analysis. The $\gamma\pi$EP is relatively very large at short distances and is considered by the authors to be unreliable in that region. Even though they consider it "futile to attempt to explain" the difference in scattering lengths "with present theoretical techniques," they adjust the short-range part of the potential difference $\Delta V=V_{pn}-V_{pp}$ to obtain the value of ${}^1a_{pn}-{}^1a_{pp}$. In this adjustment they make use of an assumed 2% fractional difference $(\Delta g^2)/g^2$ as contributing to ΔV_{cc}, the charge-dependent part of the OPEP. The possibility of a difference in the three coupling constants for π^+, π^-, and π^0 caused by electromagnetic corrections has been realized by many but seems to have been first pointed out in print by Sugie.[22] The magnitude of this fractional difference is quite uncertain[21] but, judging by the numbers given in connection with Eq. (4.4) of Ref. 21, the effect of $\Delta(g^2)/g^2$ is not major. Employing an updated version of the Hamada-Johnston potential and the short-range adjustment of ΔV, the difference $({}^1r_0)_{np}-({}^1r_0)_{pp}=-0.3$ F following from Ref. 3 is cut down in Ref. 21 to -0.13 F, i.e., to about one half of the original value. The $\gamma\pi$ potential is introduced in Ref. 21 by requiring that it reproduce the $\gamma\pi$ exchange effect on the S matrix in first Born approximation when used in the Schrödinger equation. Such a definition does not make it possible to localize the effect of the "potential" with any certainty, especially at small distances, nor does this definition allow for the effects of wave function distortion caused by other than the $\gamma\pi$E interaction. However, this distortion enters the calculations with the final potential and the $\gamma\pi$ potential is used in a

[20] K. E. Lassila and E. I. Peltola, Rev. Mod. Phys. 39, 591 (1967).

[21] J. S. Leung and Y. Nogami, Ann. Phys. (N. Y.) (to be published). The writers are very grateful to the authors of this paper for communicating their results before publication and for discussions concerning them.

[22] A. Sugie, Progr. Theoret. Phys. (Kyoto) 11, 333 (1954).

localized manner. The same criticism can be applied, of course, to the usual employment of the OPEP. However, the main application of the latter is to long-range charge-independence considerations for which the OPE effect may be introduced without the OPEP, the considerations being then confined to high L. In the present paper the OPEP is used literally but only as a part of the whole potential for which the employment of Eq. (10) is tested empirically over a wide energy range. Furthermore, the OPEP does not increase as rapidly at small r as the $\gamma\pi$EP and no adjustments for short-range modifications are usually made for the OPEP. The illegitimacy of localizing to $\gamma\pi$EP has been mentioned to the authors of Ref. 21 by one of the writers (GB) in an earlier stage of the $\gamma\pi$EP work when it appeared that the $\gamma\pi$ effect even at large r might be appreciable. Literal local significance could be attached to the $\gamma\pi$EP if the whole N-N interaction could be treated in an adiabatic, Born-Oppenheimer–type approximation. But there is no reason to believe this to be the case. It may also be pointed out that a complete consideration of the $\gamma\pi$E effect should contain not only a consideration of electromagnetic form-factor effects for free nucleons, but also of the change in the charge and current densities around each nucleus caused by the N-N interaction. Both of these effects may be expected to be relatively serious at small internucleon distances at which the $\gamma\pi$EP of Ref. 21 is large. For these reasons, as well as the presence of complications caused by the mass difference between $\rho^{\pm}$ and ρ^0, the differences in ρ-nucleon coupling constants, the ρ^0-φ-ω mixing as well as other effects pointed out in Ref. 21, the short-range N-N interaction can hardly be considered to be understood in detail. The application of local potentials to it appears especially doubtful, but if the combination of the $\delta(\mathbf{r})$-effect argument and of vector-meson effects subdues its importance, the error committed by literal localization may not be too serious, provided the details of space dependence of such effects as the $\gamma\pi$E and the OPE do not become essential for the problem in hand.

According to the estimates presented above, the small difference in the masses of charged and neutral pions can conceivably cause relatively large effects on the differences between phase shifts of n-p as compared with corresponding p-p states at the same energy. The phenomenologically derived phase parameters are usually obtained, however, on the assumption that the $T=1$ phases are the same in n-p and p-p scattering except for the electromagnetic interaction effects. An exception is often made for the $^1S_0$ state, some definite information regarding the charge dependence of the interaction at low energies being available in this case. The accuracy and the number of n-p scattering data are too small to make a meaningful determination of both $T=0$ and $T=1$ phase parameters for the n-p case possible at this time. The poorly founded assumption of exact charge independence for most $T=1$ states present in all analyses affects applications to the theories of finite nuclei, of nuclear matter, of the photodisintegration of the deuteron, of p-p bremsstrahlung, and of other phenomena. It is obviously unsatisfactory to have so many important developments subject to the present uncertainty in the values of n-p phase parameters. The desirability of obtaining more high-accuracy n-p scattering data, preferably in experiments on the scattering of neutrons by hydrogen, so as to eliminate uncertainties caused by corrections for the effect of the spectator particle in p-d scattering experiments, is obvious.

In the newer Yale phenomenological fits the $^1S_0$ phase shift K_0 was adjusted to n-p data simultaneously with the $T=0$ parameters. The other $T=1$ parameters were adjusted to p-p data. In Table II there are shown some values of $(K_0)_{pp}$ and of $(K_0)_{np}-(K_0')_{pp}$ obtained in this manner. The quantity $(K_0')_{pp}$ is the value of K_0 for p-p scattering corrected for the supposed presence of the Coulomb potential energy e^2/r in the Yale potential, approximately readjusted to the Y-IV phenomenological fit. The V^{SC} values were obtained for K_0 computed by means of the V^{SC} potential, Eq. (14), adjusted separately to the p-p and n-p phenomenological-fit values. The Coulomb correction for this case was calculated by means of the V^{SC} potential. The first of the two V^{SC} rows corresponds to $(^1r_0)_{np}=2.7$ F, $(^1a)_{np}=23.735$ F, the second, marked $(V^{SC})'$, to $(^1r_0)_{np}=2.5$ F, $(^1a)_{np}=23.675$ F. The row marked $(\Delta K_0)_{n-p}$ contains parallel shift uncertainties[9] for fit $(\text{Y-IV})_{pp+np}$ with factor $D^{1/2}$ included, employing n-p data alone and varying $T=1$ phases only. The row marked $(\Delta K_0)_{p-p}$ shows parallel-shift uncertainties with factor $D^{1/2}$ included for the same fit, employing pp and np data together in the phase variations of $T=0$

TABLE II. Intercomparisons of n-p and Coulomb-corrected p-p values of $^1S_0$ phase shift K_0 in radians.

E (MeV)	14.16	30.0	70.0	137.0	310.0	Source
$(K_0)_{pp}$	0.938	0.807	0.570	0.317	−0.128	$(\text{Y–IV})_{pp+np}$
$(K_0)_{np}-(K_0')_{pp}$	0.045	0.031	0.0238	0.0184	0.0143	$(\text{Y–IV})_{pp+np}$
$(K_0)_{np}-(K_0')_{pp}$	0.046	0.033	0.0162	0.0016	−0.0169	V^{SC}
$(K_0)_{np}-(K_0')_{pp}$	0.074	0.064	0.043	0.0183	−0.0147	$(V^{SC})'$
$(\Delta K_0)_{n-p}$	0.010	0.010	0.034	0.034	0.082	$(\text{Y–IV})_{pp+np}$
$(\Delta K_0)_{pp}$	0.0002	0.0002	0.0059	0.0059	0.0070	$(\text{Y–IV})_{pp+np}$
$(K_0')_{pp}-(K_0)_{pp}$	0.0136	−0.0054	−0.0127	−0.0137	−0.0127	$(\text{Y–IV})_{pp+np}$

and $T=1$ phases. Since p-p data have the dominant influence in this case, the ΔK_0 is taken to be representative of p-p data. The parallel shifts for both $(\Delta K_0)_{np}$ and $(\Delta K_0)_{pp}$ correspond to energy intervals 0–69, 69–155, 155–350, with incident laboratory energies in MeV. Within an interval the values of ΔK_0 close to the upper interval bound are probably too small, those close to the lower bound are probably too large as compared with using continuous phase variations. At the lower energies the values of $(K_0)_{np}-(K_0')_{pp}$ shown in the table tend to be larger than either $(\Delta K_0)_{np}$ or $(\Delta K_0)_{pp}$. At the higher energies the $|(K_0)_{np}-(K_0')_{pp}|$ are smaller than $(\Delta K_0)_{np}$ but larger than $(\Delta K_0)_{pp}$. Were the parallel-shift uncertainties obtained by using phase variations in smaller energy intervals, they would have been larger, as is known to be the case for $(\Delta K_0)_{pp}$. It is difficult, however, to obtain meaningful parallel-shift values of $(\Delta K_0)_{np}$ for small energy intervals. The large energy intervals were used in both cases therefore. In spite of some indefiniteness in the uncertainties of the phenomenological-fit values of K_0, the purely phenomenological charge dependence of this quantity is seen not to be negligible in comparison with the uncertainties of the phenomenological-fit values. It may be remarked that the low-E anchor of the phenomenological fit corresponds to the same $({}^1r_0)_{np}$ and $({}^1a)_{np}$ as the V^{SC} rather than the $(V^{\text{SC}})'$ fit. This accounts for some of the largest $(K_0)_{np}-(K_0')_{pp}$ differences occurring for $(V^{\text{SC}})'$. But even apart from this, the differences $(K_0)_{np}-(K_0')_{pp}$ are seen to be appreciable. Since in the searches giving $(K_0)_{np}$ the $T=1$ phases with $L>0$ were not searched, even larger shifts of $(K_0)_{np}$ from $(K_0')_{pp}$ are conceivable. No direct information regarding corresponding differences for phases with $L>0$ is available. Furthermore, changes in $T=1$ phases may produce changes in $T=0$ phases as well. The desirability of improvement in the phenomenology of n-p scattering is apparent.

The cautions regarding the possibility of a nonnegligible charge dependence which were made shortly before the discussion of Table II are seen to be substantiated and so is the desirability of obtaining additional n-p scattering information emphasized in that connection.

The calculations reported on in Sec. II depend on Eq. (4), which is a consequence of the supposition that the pion-nucleon coupling is pseudoscalar and rotationally symmetric in isospin space. If these assumptions should be shown to be inapplicable, the arguments regarding reconciliation of long-range charge independence with short-range charge dependence would be vitiated. It appears relevant to recall that the well-known Fermi-Yang model of the pion leads to a linear combination of pseudovector and pseudoscalar coupling. Although the original form of the model relies in its formulation on an improbable weak interaction, it is clear from qualitative considerations[23] that the main features of the model are preserved if vector-meson coupling is substituted for the weak interaction and that the admixture of a pseudovector pion-nucleon coupling is expected also for the modified Fermi-Yang model. There is little doubt, therefore, that Eq. (4) has no literal significance. Consequently, the reconciliation of long-range with short-range properties of the N-N interaction may not be considered as more than qualitative even apart from the approximations mentioned in Sec. II.

If it should turn out that there is no pseudoscalar π-N coupling or that such coupling is very weak, the considerations of Sec. II will become inapplicable. In a recent proposal, Pradhan, Sudarshan, and Saxena[24] formulate a theory of strong interactions which leads to a pseudovector pion-nucleon coupling. The theory is in a preliminary form and its possibilities are not obvious. But if it will not provide for a pseudoscalar π-N coupling admixture or some other near-equivalent to Eq. (4), then either an explanation of the short-range phenomena along quite different lines from those under discussion will have to be found or else the new theory of strong interactions will have to be discarded.

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