P_{11} π-N Scattering and Broken $SU(3)$*

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A dynamical model based on considerations of $SU(3)$ symmetry is presented to explicate the observed behavior of P_{11}, πN scattering. In $SU(3)$-symmetric P_8-B_8 scattering in the $\frac{1}{2}^+$ state, one should expect a $\overline{10}$ resonance as well as an **8** bound state. The possibility that this antidecuplet should be identified with the Roper resonance (1470 MeV) is explored in a broken-symmetry calculation. The model yields values for the nucleon mass, the πN coupling constant, the phase-shift zero, the position and width of the Roper resonance, and the P_{11} scattering length in good agreement with the data. It is argued that this presents a strong case for the view that the Roper resonance is an element of a broken $\overline{10}$ multiplet.

I. INTRODUCTION

THE study of πN scattering data has had a long history. Enough experimental results have accumulated over a wide energy range to permit various groups[1–5] to perform increasingly more accurate phase-shift analyses of this data. The most recent of these efforts has led to rather reliable phase shifts for energies up to, and slightly beyond, 2 BeV.[6–8] One of the most interesting results of this work has been the discovery of a resonance in P_{11} scattering, first obtained by Roper *et al.*[1] and subsequently verified by others. (We shall refer to this as the N'.) Evidence also exists[9–11] for bumps in cross sections corresponding to this resonance.

Although a substantial amount of work has been devoted to dynamical models of the P waves,[12–15] these efforts have not been able to say anything about the N'. In fact, the most popular model for P-wave scattering, the reciprocal bootstrap of Chew,[16] does not yield the zero in δ_{11} at $T=175$ MeV, much less the N'.[17] This is not surprising since the reciprocal bootstrap does not include inelastic effects, and it is known that such effects must play an important role for the N'.[18–20]

Dynamical models always involve a truncation of the unitarity relation and thus a specific selection of inelastic channels. This choice is based on a variety of physical arguments as well as practical considerations. In this paper we will argue, on the basis of $SU(3)$ symmetry, that the appropriate inelastic channels are ηN, $K\Lambda$, and $K\Sigma$. Thus the coupled-channel problem that we treat is one comprising all the P_8-B_8 (pseudoscalar meson octet-baryon octet) states with the quantum numbers of the P_{11} πN system.

At least to some extent the presence of the N' can be viewed as strange. It has all the quantum numbers of the nucleon, but it is certainly a different object. How do N and N' differ, except in mass? It would appear that the most natural answer is based on an $SU(3)$ multiplet structure; i.e., in a world symmetric under $SU(3)$, N and N' belong to different $SU(3)$ multiplets and the observation of the Roper resonance is a remnant of this structure when the symmetry is broken. In what follows we shall take this point of view.

In a symmetric world it is well known[21,22] that the nucleon is a member of an $SU(3)$ octet, a bound state of P_8-B_8 scattering in the spin-parity channel $J^P=\frac{1}{2}^+$. Rather than introducing other channels to understand the N', we will confine ourselves to P_8-B_8 elastic scattering, but consider the possibility that the N' is a member of a different multiplet in $8\otimes 8$, $\frac{1}{2}^+$. It has, in fact, already been established[23] that, in $SU(3)$ symmetry, the same forces which bind the baryon octet also yield a resonant baryon antidecuplet ($\overline{\mathbf{10}}$). To consider only the P_8-B_8 elastic process may at first sight appear to be

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inconsistent with the previous remarks regarding the importance of inelastic contributions for an understanding of the physical N'. However, in the real world where $SU(3)$ is broken, the $I=\frac{1}{2}$, $Y=1$ channel of P_8-B_8 scattering will contain the ηN, $K\Lambda$, and $K\Sigma$ channels, in addition to the πN channel. These additional particle channels provide inelastic contributions to the πN channel. Thus we argue that both the N and N' are contained in P_8-B_8 elastic scattering as members of different multiplets, and when the symmetry is broken, the $I=\frac{1}{2}$ channels additional to πN form the inelastic contributions.

Our task will be to calculate the P_{11} amplitude in terms of broken $SU(3)$ symmetry, producing N, the zero in δ_{11}, and N'. This task is complicated by practical considerations. In particular, a calculation of the mass of the nucleon requires a rather careful treatment of the forces in πN elastic scattering. On the other hand, the solution of a multichannel problem is almost always based on crude approximations. We solve this difficulty by using a method[24] designed for just such problems, especially when a zero is present in the phase shift. In this method the multichannel problem is crudely solved in a pole approximation; this solution leads to a zero in the πN scattering amplitude. The method enables us to introduce this zero as a pole, with known position and residue, in the D function of a calculation of elastic πN scattering. This elastic problem can then be solved with some care.

In Sec. II, we formulate the multichannel problem in terms of $SU(3)$, and in Sec. III we solve this problem in the context of the pole approximation. This solution will already show the main characteristics of P_{11} πN scattering, viz., the nucleon, the zero in δ_{11}, and the Roper resonance. In Sec. IV, we use the results of this solution to recalculate the P_{11} amplitude as indicated above and compare our results with the data. The agreement is quite good. In Sec. V, we return to considerations of $SU(3)$ and amplify the view that the Roper resonance is an element of a broken $SU(3)$ $\overline{\mathbf{10}}$ multiplet. In Sec. VI, we discuss our calculations and results.

II. INITIAL CONSIDERATIONS OF $SU(3)$

As indicated in the Introduction, we shall confine ourselves to a discussion of P_8-B_8 scattering in the $J^P=\frac{1}{2}^+$ channel. For low energies, it is well known[21,22] that the scattering in this channel is dominated by the u-channel exchange of the baryon octet (B_8, $\frac{1}{2}^+$) and decuplet (B_{10}, $\frac{3}{2}^+$). The exchange of the decuplet contributes the strongest force and leads to an octet bound state and an antidecuplet resonance.[23] If the Born term for **10** and **8** exchange contributions in the $SU(3)$ basis (tilded) is written as

$$\tilde{B}_{\gamma\alpha}(s)=\tilde{\beta}_{\gamma\alpha}{}^{(10)}b^{(10)}(s)+\tilde{\beta}_{\gamma\alpha}{}^{(8)}b^{(8)}(s)\,, \qquad (1)$$

[24] J. J. Brehm and L. F. Cook, preceding paper, Phys. Rev. **170**, 1381 (1968).

one finds for the nonzero elements

$$\begin{aligned}
&\tilde{\beta}_1{}^{(10)}=-\tfrac{5}{2}\,, &&\tilde{\beta}_1{}^{(8)}=-12/25\,,\\
&\tilde{\beta}_{10}{}^{(10)}=\tfrac{1}{2}\,, &&\tilde{\beta}_{10}{}^{(8)}=-72/25\,,\\
&\tilde{\beta}_{\overline{10}}{}^{(10)}=\tfrac{1}{2}\,, &&\tilde{\beta}_{\overline{10}}{}^{(8)}=24/25\,,\\
&\tilde{\beta}_{27}{}^{(10)}=\tfrac{1}{6}\,, &&\tilde{\beta}_{27}{}^{(8)}=-28/25\,, \qquad (2)\\
&\tilde{\beta}_{8,8}{}^{(10)}=1\,, &&\tilde{\beta}_{8,8}{}^{(8)}=42/25\,,\\
&\tilde{\beta}_{8',8'}{}^{(10)}=0\,, &&\tilde{\beta}_{8',8'}{}^{(8)}=6/25\,,\\
&\tilde{\beta}_{8,8'}{}^{(10)}=-\tfrac{1}{2}\sqrt{5}\,, &&\tilde{\beta}_{8,8'}{}^{(8)}=0\,,
\end{aligned}$$

where a single index represents an element on the diagonal, and where, for **8** exchange, we have taken $f/d=\frac{2}{3}$. We use

$$\begin{aligned}b^{(10)}(s)=&-(2\pi g'^2/p^4)(p_0+m)^2\{[U+V(m-W)]Q_1(\lambda)\\&-[p/(p_0+m)]^2[U+V(m+W)]Q_0(\lambda)\}\,, \qquad (3)\end{aligned}$$

with

$$U=(M+m)\left[\mu^2-\tfrac{1}{2}(s+M^2)+\frac{(M^2+m^2-\mu^2)^2}{6M^2}+\tfrac{1}{3}m^2\right]+(M^2-m^2-\mu^2)[(M+m)^2-\mu^2]/6M\,,$$

$$V=\mu^2-\tfrac{1}{2}(s+M^2)+(M^2+m^2-\mu^2)\frac{(M-m)^2-\mu^2}{6M^2}-\tfrac{1}{3}m^2\,,$$

$$\lambda=1+\frac{2m^2+2\mu^2-M^2-s}{2p^2}\,,\quad \frac{g'^2}{4\pi}=0.4m_\pi{}^{-2}\,;$$

and

$$\begin{aligned}b^{(8)}(s)=&-(2\pi g^2/p^4)(p_0+m)^2[(W-m)Q_1(\lambda)\\&+(p/(p_0+m))^2(W+m)Q_0(\lambda)]\,, \qquad (4)\end{aligned}$$

with

$$\lambda=1+(m^2+2\mu^2-s)/2p^2\,,\quad g^2/4\pi=15\,.$$

In the above, μ, m, and M are the symmetric masses of P_8, B_8, and B_{10}; p and p_0 are the c.m. momentum and energy of the baryon, $W=\sqrt{s}$ is the total c.m. energy; and Q_0 and Q_1 are Legendre functions of the second kind. To see the content of this quantitatively, let us evaluate $b^{(10)}$ and $b^{(8)}$ at threshold. There $b^{(10)}/b^{(8)}\sim 10$, $b^{(10)}>0$, and from Eqs. (1) and (2) one then obtains

$$\begin{aligned}
\tilde{B}_1(s_T)&\sim b_0{}^{(10)}(-5/2)\,,\\
\tilde{B}_{10}(s_T)&\sim b_0{}^{(10)}(+1/5)\,,\\
\tilde{B}_{\overline{10}}(s_T)&\sim b_0{}^{(10)}(+3/5)\,,\\
\tilde{B}_{27}(s_T)&\sim b_0{}^{(10)}(+1/15)\,, \qquad (5)\\
\tilde{B}_{8,8}(s_T)&\sim b_0{}^{(10)}(+6/5)\,,\\
\tilde{B}_{8',8'}(s_T)&\sim b_0{}^{(10)}(+1/40)\,,\\
\tilde{B}_{8,8'}(s_T)&\sim b_0{}^{(10)}(-9/8)\,.
\end{aligned}$$

Thus, in addition to the strong attraction in the **8** channel, the exchange of **8** and **10** leads to an attractive force in the $\overline{\mathbf{10}}$ channel with approximately half the strength of that in the **8** channel. All other channels are weak or repulsive. A $\overline{\mathbf{10}}$, $\frac{1}{2}^+$ resonance in P_8-B_8 scattering is therefore to be expected. Since a $\overline{\mathbf{10}}$ multiplet contains a $I=\frac{1}{2}$, $Y=1$ member, the N' is a natural candidate for this multiplet. As we have indicated, our purpose is to investigate dynamically how reasonable such an identification is. The initial suggestion that the N' belongs to a $\overline{\mathbf{10}}$ was made by Lovelace.[25]

In order to explore this possibility, we consider the breaking of the symmetry, but confine ourselves to P_{11} meson-baryon scattering and treat the coupled channels πN, ηN, $K\Lambda$, $K\Sigma$ for $J^P=\frac{1}{2}^+$. In this way we have a finite problem and one for which the results can be directly compared with the data. We recognize at the outset that the model is incapable of giving a complex phase shift in the neighborhood of the N'. We are therefore presuming that those states which participate in the inelastic decay of the Roper resonance do not participate significantly in the dynamics of the N, the zero, and the N'.

We begin by maintaining the symmetry and transforming the force matrix given by Eqs. (1) and (2) from the $SU(3)$ basis (tilded) to the particle basis (untilded). Thus,

$$B_{ij}=U_{\alpha i}\tilde{B}_{\alpha\beta}U_{\beta j}=\beta_{ij}{}^{(10)}b^{(10)}+\beta_{ij}{}^{(8)}b^{(8)}\,, \tag{6}$$

where the transformation matrix is given, e.g., in deSwart.[26] Ordering the channels as πN, ηN, $K\Lambda$, $K\Sigma$, one obtains for $\beta^{(10)}$ and $\beta^{(8)}$

$$\beta^{(10)}=\begin{bmatrix} \frac{4}{3} & 0 & \frac{1}{2} & -\frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{3} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{6} \end{bmatrix}, \tag{7}$$

$$\beta^{(8)}=\begin{bmatrix} 1 & \frac{3}{5} & 6/25 & -\frac{2}{5} \\ \frac{3}{5} & -3/25 & -18/25 & -6/25 \\ 6/25 & -18/25 & -18/25 & -\frac{3}{5} \\ -\frac{2}{5} & -6/25 & -\frac{3}{5} & 1 \end{bmatrix}.$$

To analyze the four-channel physical situation with broken symmetry, we maintain the force structure given by Eqs. (6) and (7) but break the threshold degeneracy of the channels. In so doing, the calculation is enormously simplified by not breaking the threshold degeneracy entirely, but by simply splitting the πN channel from the remaining three channels.

The threshold for the πN channel is given by the sum of the physical masses, $M_\pi+M_N$. We choose the three-channel, degenerate threshold as $M_K+\bar{M}$, where M_K is the physical mass of the K meson and $\bar{M}$ is the average of the physical masses M_Λ and M_Σ. This choice, which ignores the physical ηN threshold, is based on the observation that the ηN channel is weakly coupled to the πN channel [as one sees in Eq. (7)]. In any event, the physical thresholds for all three channels lie above the energy range of interest, viz., $s<(1450\text{ MeV})^2$, and the general features of our results will not depend markedly on the exact position of the three-channel, degenerate threshold. This broken-symmetry scheme then yields a four-channel problem with a force structure given by $SU(3)$ and two-channel kinematics.

To facilitate the solution of this problem, in particular, to take advantage of the two-channel kinematics, it would be convenient to choose a basis in which the 3×3 degenerate sector of the force matrix is diagonal. Of course, an energy-independent diagonalization of the degenerate sector of the entire force matrix is not possible, but one can diagonalize the 3×3 part of either the contribution from **10** exchange or from **8** exchange, independently of s. Since **8** exchange contributes only corrections to the force due to **10** exchange (because of the relative magnitudes of $b^{(10)}$ and $b^{(8)}$), we choose a basis which diagonalizes the degenerate sector of the **10**-exchange term. The eigenvalues and normalized eigenvectors which form the basis are

$$\begin{aligned} &\lambda_1=-0.721:\\ &|\psi_1\rangle=0.541|\eta N\rangle-0.319|K\Lambda\rangle-0.780|K\Sigma\rangle,\\ &\lambda_2=0.854:\\ &|\psi_2\rangle=0.324|\eta N\rangle-0.771|K\Lambda\rangle+0.553|K\Sigma\rangle,\\ &\lambda_3=0.209:\\ &|\psi_3\rangle=0.770|\eta N\rangle+0.553|K\Lambda\rangle+0.322|K\Sigma\rangle. \end{aligned} \tag{8}$$

Transforming to the basis $|\pi N\rangle$, $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$, denoted by circumflex, we obtain

$$\hat{B}=\hat{\beta}^{(10)}b^{(10)}+\hat{\beta}^{(8)}b^{(8)}\,, \tag{9}$$

in which

$$\hat{\beta}^{(10)}=\begin{bmatrix} 1.333 & 0.101 & -0.570 & 0.170 \\ 0.101 & -0.721 & 0 & 0 \\ -0.570 & 0 & 0.854 & 0 \\ 0.170 & 0 & 0 & 0.209 \end{bmatrix} \tag{10}$$

and

$$\hat{\beta}^{(8)}=\begin{bmatrix} 1 & 0.560 & -0.212 & 0.465 \\ 0.560 & 0.653 & -0.521 & 0.202 \\ -0.212 & -0.521 & 0.650 & 0.598 \\ 0.465 & 0.202 & 0.598 & -1.130 \end{bmatrix}. \tag{11}$$

Pursuing the observation that $b^{(10)}$ exchange is the dominant contribution to the force, we approximate $\hat{\beta}^{(8)}$ in Eq. (11) by including only the diagonal 3×3 elements as corrections to $\hat{B}^{(10)}$ and ignoring the off-diagonal terms. Thus we take

$$\hat{\beta}^{(8)}\rightarrow\begin{bmatrix} 1 & 0.560 & -0.212 & 0.465 \\ 0.560 & 0.653 & 0 & 0 \\ -0.212 & 0 & 0.650 & 0 \\ 0.465 & 0 & 0 & -1.130 \end{bmatrix} \tag{12}$$

for $T=\frac{1}{2}$, $Y=1$, $\frac{1}{2}^+$ scattering. Equations (10) and (12) together with (9), (3), and (4) provide the driving terms for our calculations.

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III. MULTICHANNEL MODEL

Even with the (principally) kinematical simplifications discussed in Sec. II, the analysis of the resulting four-channel problem is quite complicated. The solutions to multichannel problems are nearly always approximate ones and are characterized by treating each channel on about the same footing. Although this is a reasonable procedure in some cases, it is not appropriate here. In particular, we wish to perform a calculation in which the πN channel is treated much more carefully than the other, inelastic, channels. Specifically, we wish to calculate the position of the bound-state nucleon as well as the low-energy, πN phase shift, especially the zero at $T=175$ MeV. The nucleon mass depends strongly on the treatment of the forces in the elastic (πN) channel, while the phase-shift zero is sensitive to the inelastic contributions.

In order to treat the elastic channel carefully and also to include the inelastic effects, we use a technique developed earlier[24] which is particularly useful if a zero exists in the πN amplitude in its elastic region. This technique solves the multichannel problem in an approximate way, treating all channels on an equal footing. If a zero is present in this approximation to the elastic amplitude, this is introduced into a one-channel, N/D treatment of the low-mass channel as a pole in the D function. The one-channel analysis treats the elastic forces much more carefully than does the approximate multichannel solution. In this section, we shall set up and solve, approximately, the multichannel model.

As indicated in Sec. II, we maintain a symmetric force matrix, i.e., not only are the contributions of the exchanged multiplets to various channels determined by $SU(3)$ crossing, but the singularity structure of the left-hand cuts is taken to be degenerate and determined by the symmetric masses. The symmetry is broken on the right-hand cuts by breaking the degeneracy of the thresholds. To solve the multichannel problem we use a pole approximation and, since the left-hand singularity structure is assumed symmetric and degenerate, we use the same single pole in all channels; thus we write

$$\mathrm{Im}\hat{B}_{\mathrm{pole}}=-\pi\delta(s-z_0)\begin{bmatrix} a & b_1 & b_2 & b_3\\ b_1 & c_1 & 0 & 0\\ b_2 & 0 & c_2 & 0\\ b_3 & 0 & 0 & c_3\end{bmatrix} \tag{13}$$

as the input to a multichannel N/D problem.

If we write the elastic πN amplitude as

$$M=F/\mathfrak{D}, \tag{14}$$

we find

$$F=[a/(s-z_0)]D_1^{(2)}D_2^{(2)}D_3^{(2)}+Q_{00}^{(2)}(b_1^2D_2^{(2)}D_3^{(2)}+b_2^2D_1^{(2)}D_3^{(2)}+b_3^2D_1^{(2)}D_2^{(2)}), \tag{15}$$

and

$$\mathfrak{D}=D^{(1)}D_1^{(2)}D_2^{(2)}D_3^{(2)}-(s-z_0)^2Q_{00}^{(1)}Q_{00}^{(2)}\times(b_1^2D_2^{(2)}D_3^{(2)}+b_2^2D_1^{(2)}D_3^{(2)}+b_3^2D_1^{(2)}D_2^{(2)}), \tag{16}$$

where

$$\begin{aligned} D^{(1)}&=1-a(s-z_0)Q_{00}^{(1)},\\ D_i^{(2)}&=1-c_i(s-z_0)Q_{00}^{(2)},\\ Q_{00}^{(k)}&=\int\frac{dx\,\rho_k(x)}{(x-s)(x-z_0)^2},\quad \rho_k=\frac{2}{(4\pi)^3}\frac{p_k}{W}(p_{0k}-M_k). \end{aligned} \tag{17}$$

The index $k=1$ or 2; 1 refers to the πN kinematics and 2 refers to the kinematics of the ψ_1, ψ_2, and ψ_3 channels. The normalization is such that

$$e^{2i\delta_{11}}=1+2\pi i\rho_1 M. \tag{18}$$

The solution is thus given completely in terms of the parameters a, b_i, c_i, and z_0.

To calculate $\hat{B}=\hat{B}^{(10)}+\hat{B}^{(8)}$ as determined in Sec. II, we use the degenerate P_8, B_8, and B_{10} masses: $\mu=2.93$, $m=8.22$, and $M=9.90$; the scale is the physical pion mass. Evaluating $\hat{B}$ at the symmetric threshold $m+\mu$, we determine the parameters a, b_i, and c_i by fixing z_0 and matching $\hat{B}_{\mathrm{pole}}$ to $\hat{B}$ at this single threshold. One could argue that the matching should be done at the physical πN and degenerate ψ_1, ψ_2, and ψ_3 thresholds depending on the channel, but $\hat{B}_{\mathrm{pole}}$ should represent an approximation to the fully symmetric $SU(3)$ force matrix. In order that the weighting of the channels in the force matrix maintain this symmetry, the matching should be done at a single value of the energy. Because it lies between the physical πN and degenerate ψ_1, ψ_2, ψ_3 thresholds, the symmetric threshold is a convenient and representative point at which to match; further, the determination of the parameters is not sensitive to the choice of the single matching point in this energy range.

Only the parameter z_0 of $\hat{B}_{\mathrm{pole}}$ is not determined by the above procedure. By varying z_0 over a wide range of values, the general features of this multichannel solution emerge. For a range $-100<z_0<0$, i.e., for any reasonable z_0, one finds the following: F, and therefore M, has a zero between the physical πN and degenerate ψ_1, ψ_2, and ψ_3 thresholds; $\mathfrak{D}$ has two zeros, one below the πN threshold and one slightly above the zero of F. Thus this solution already exhibits the general behavior of P_{11} πN scattering: There is a nucleon, the first zero of $\mathfrak{D}$; δ_{11} has a zero, the zero of F; a resonance exists above the zero of δ_{11}, the second zero of $\mathfrak{D}$.

The transformation (8) to the circumflexed basis is not only convenient for obtaining the pole-model solution, but it also helps us to isolate the principal inelastic contribution. We find that the zero of F occurs to the right of a zero appearing in $D_2^{(2)}$. In fact, the zero of $D_2^{(2)}$ is necessary for the zero of F to occur. Thus the elastic amplitude zero has an origin closely associated with an uncoupled bound state in the channel described by the wave function ψ_2.

Even though this solution exhibits the general features of P_{11} πN scattering, it is not satisfactory. In particular, for $z_0<-60$, the bound state becomes a ghost. This behavior simply reflects the very approximate nature of the multichannel solution, particularly in those regions where a careful treatment of the forces in the elastic πN channel is necessary. This situation will be corrected in Sec. IV, where a more careful treatment of these forces will be given. On the other hand, one should expect the general behavior of the elastic amplitude between the two thresholds to be comparatively better in this approximation. In this energy region, the behavior of the elastic amplitude is governed not so much by the details of the elastic forces as by the coupling between the channels through unitarity. This the N/D pole approximation accomplishes. We shall see in Sec. IV that this is verified, i.e., the behavior of the amplitude between the thresholds is not substantially altered by a more careful treatment of the elastic forces.

Before embarking on a more detailed analysis of the elastic forces, however, let us fix the remaining parameter, z_0. Since the critical inputs from the multichannel calculation into an effective, one-channel calculation[24] are the position and residue of the pole in F^{-1}, it is desirable that these input values be independent of the parameters of the multichannel model, in this case z_0. Unfortunately, this is not entirely the case. If we restrict ourselves to a reasonable range of z_0, i.e., $-100<z_0<0$, to represent the singularity structure, then as seen in Table I, the residue ξ, where

$$\xi=\left(\frac{\mathfrak{D}}{dF/ds}\right)_{s=s_c}, \tag{19}$$

does become quite insensitive to variations of z_0. On the other hand, s_c is approximately linear as a function of z_0. Since we cannot eliminate entirely a dependence on z_0, we minimize it by choosing the parameter in the middle of the acceptable region where $d\xi/dz_0\sim 0$; thus we fix $z_0=-90$, where $s_c=76$ and $\xi=5.52\times10^{-3}$. For this choice the other parameters are (all multiplied by 10^5)

$$\begin{aligned} a&=\ 9.94, & c_1&=-4.45,\\ b_1&=\ 1.13, & c_2&=\ 6.37,\\ b_2&=-4.08, & c_3&=\ 0.56.\\ b_3&=\ 1.53, \end{aligned}$$

One may certainly argue at this point that the z_0 dependence cannot be removed from our multichannel model, and a different criterion should be imposed. The natural view here would be to fix s_c by experiment and, by Table I, determine z_0 and ξ. Proceeding in this direction, we observe that the zero in δ_{11} occurs at $T_\pi=175$ MeV or $s_c=76$. But this is just the value where the z_0 dependence is minimized and is identical with the above choice.

IV. EFFECTIVE ONE-CHANNEL MODEL

In order to perform a more careful calculation of the πN amplitude itself, we employ the method of Ref. 24.

TABLE I. The residue and position of the pole in F^{-1} for various values of z_0. The πN threshold is at $s=60$, that for the degenerate ψ_1, ψ_2, and ψ_3 channel is at $s=138$.

z_0	$10^3\xi$	s_c
0	3.76	138
−10	4.34	133
−20	4.77	127
−30	5.05	120
−40	5.22	113
−50	5.34	106
−60	5.42	99
−70	5.47	91
−80	5.50	83
−90	5.52	76
−100	5.52	68

The inelastic effects are simulated in an effective one-channel calculation by introducing a D-function pole at $s=s_c$. This one-channel problem is defined by the following set of equations:

$$\begin{aligned} M(s)&=N(s)/[D(s)+C(s)],\\ N(s)&=\frac{1}{\pi}\int_L\frac{ds'}{s'-s}[D(s')+C(s')]\,\mathrm{Im}B(s'),\\ D(s)&=1-(s-s_0)\int_R\frac{ds'\rho_1(s')N(s')}{(s'-s)(s'-s_0)},\\ C(s)&=\xi N(s_c)/(s-s_c). \end{aligned} \tag{20}$$

The parameters ξ and s_c are the residue and position, respectively, of the pole in F^{-1} obtained in the multichannel calculation given in Sec. III; s_0 is a subtraction point which ultimately drops out of the problem.

We solve this set of equations by using a method due to Pagels[27] in which the integral

$$J(s)=\int_R\frac{dx\,\rho_1(x)}{x^2(x-s)} \tag{21}$$

is approximated (when needed in the analysis on the left) by a pole, i.e.,

$$J(s)\to\gamma/(s-\alpha),\quad s\in L \tag{22}$$

valid on L where $B(s)$ satisfies

$$B(s)=\frac{1}{\pi}\int_L\frac{ds'}{s'-s}\,\mathrm{Im}B(s'). \tag{23}$$

In principle, α and γ are independent of the details of the scattering problem depending only on the phase-space integral J.

Since this approximation preserves the independence of the N/D solution on the subtraction point s_0, we can pick s_0 on the basis of convenience.[27] Choosing $s_0=0$,

[27] H. Pagels, Phys. Rev. 140, B1599 (1965).

one obtains

$$N(s)=B(s)-\frac{sB(s)-\alpha B(\alpha)}{s-\alpha}\gamma\alpha N(\alpha)+\frac{B(s)-B(s_c)}{s-s_c}\xi N(s_c), \quad (24)$$

$$D(s)=1-s^2[J(s)-\gamma/(s-\alpha)]N(s)-[s\alpha/(s-\alpha)]\gamma N(\alpha),$$

in which

$$N(\alpha)=\delta^{-1}\left\{B(\alpha)[1-\xi B'(s_c)]+B(s_c)\xi\frac{B(\alpha)-B(s_c)}{\alpha-s_c}\right\},$$

$$N(s_c)=\delta^{-1}\left\{B(s_c)[1+\gamma\alpha[B(\alpha)+\alpha B'(\alpha)]]-B(\alpha)\gamma\alpha\frac{\alpha B(\alpha)-s_cB(s_c)}{\alpha-s_c}\right\},$$

where

$$\delta=[1-\xi B'(s_c)][1+\gamma\alpha[B(\alpha)+\alpha B'(\alpha)]]+\xi\alpha\gamma\frac{B(\alpha)-B(s_c)}{\alpha-s_c}\frac{\alpha B(\alpha)-s_cB(s_c)}{\alpha-s_c}.$$

It is characteristic of Pagels' method that $D(s)$ has a left-hand cut arising from the presence of $N(s)$ in the expression for $D(s)$ given in Eq. (24). In the usual application of this method, one is interested in $D(s)$ only for $s>$ threshold, in which case this presents no problem. In our case, however, since we wish to obtain the nucleon as a bound state, this term should be reduced to a negligible amount. To accomplish this we determine γ such that

$$\gamma=(\hat{s}-\alpha)J(\hat{s}) \quad (25)$$

for a given choice of α; $\hat{s}$ is the position of the most positive branch point arising from the forces driving the amplitude. Since the pole $\gamma/(s-\alpha)$ is in fact a very good approximation to $J(s)$ near the matching point, we replace the expression for $D(s)$ in Eq. (24) by

$$\begin{aligned}D(s)&=1-s^2[J(s)-\gamma/(s-\alpha)]N(s)-[s\alpha/(s-\alpha)]\gamma N(\alpha), \quad s\geqslant\hat{s} \quad (26)\\&=1-[s\alpha/(s-\alpha)]\gamma N(\alpha), \quad s\leqslant\hat{s}.\end{aligned}$$

In practice it is known that the Pagels solution is sensitive to the choice of α. In particular, it is possible to pick α such that $N(s)$ and $B(s)$ differ in sign over some interval of s; such solutions are physically unacceptable. On the other hand, solutions with acceptable interpretation are generated when $N(s)$ and $B(s)$ are approximately equal in the energy region of interest. Therefore, to stabilize our solutions, we pick α such that

$$N(\alpha)=B(\alpha). \quad (27)$$

This will lead to a value of α which is somewhat above the πN threshold. With this choice the solutions are not sensitive to variations of α.

To complete the determination of the input necessary to evaluate the amplitude, we need only specify $B(s)$. Referring back to (6) and (7), we see that

$$B(s)=\tfrac{4}{3}b^{(10)}(s)+1b^{(8)}(s), \quad (28)$$

where $b^{(10)}(s)$ and $b^{(8)}(s)$ should now be evaluated using the physical masses for π, N, and N^*. Strictly speaking, this choice is not compatible with the Pagels method. In particular, (28) does not satisfy Eq. (23) since it diverges for large s. However, since $B(s)$ given by (28) is monotonically decreasing over the energy range of interest, $60<s<140$, we will use it in (24) and argue that the Pagels method yields a reasonable amplitude for $s<140$. We will return to this point in Sec. V.

The conditions given in Eqs. (25) and (27) yield $\alpha=67$, $\gamma=-9.68\times10^{-7}$. Using these values and $B(s)$ as given Eq. (28), we may now evaluate the amplitude. In Fig. 1, the denominator function $D(s)+C(s)$ is plotted. It exhibits the following rather remarkable results (in MeV):

nucleon position = 915 (940),

phase-shift zero = 1220 (1220),

resonance position = 1330 (1470) (Ref. 7);

the experimental values are given in parentheses. This confirms the remarks made in Sec. III; in particular, the behavior of the amplitude between the thresholds is not markedly altered by a more careful treatment of the elastic forces, whereas the nucleon has been substantially changed, its position being altered from that of a ghost to one which is acceptable.

The phase shift δ_{11} is plotted in Fig. 2. This yields the full width

$\Gamma=80$ MeV (cf. expt. πN partial width: 140 MeV),

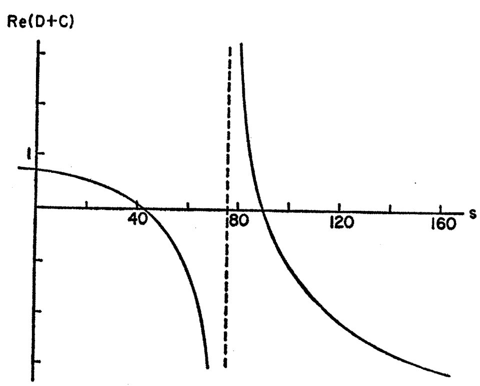

FIG. 1. The real part of the function $D+C$, plotted against s (in m_π^2).

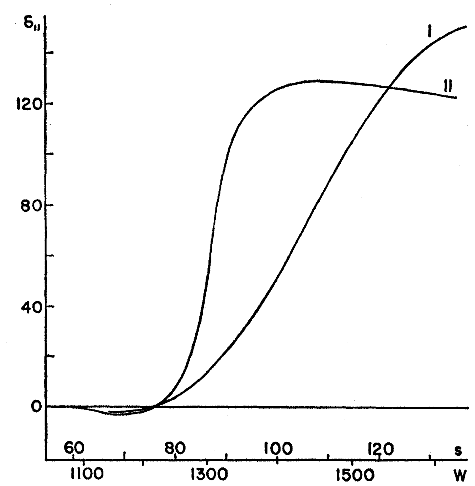

FIG. 2. The P_{11}, πN phase shift. Curve I is given by the phase-shift analysis of Ref. 7; curve II is the calculated phase shift. The units are s in m_π^2, and W in MeV.

as well as the P-wave scattering length[28]

$$a_{11}=-0.105\ (-0.101\pm0.007).$$

Finally, we calculate the pion-nucleon coupling constant from N and $d[D+C]/ds$ and obtain

$$g^2/4\pi=15.7\ (14.7).$$

Thus we see that the model presented here leads to a rather good fit to the data.

V. FURTHER CONSIDERATIONS OF $SU(3)$

In Secs. III and IV, we have attempted to show that a dynamical model based on considerations of $SU(3)$ can reproduce the characteristics of P_{11} πN scattering. We believe that the model presented there accomplishes this. The role of $SU(3)$ has been crucial in defining the model, i.e., in the selection of states and in the force matrix. If the model is to be a realistic one for P_8-B_8 scattering, it is essential that a substantial remnant of the symmetry be present in P_{11} scattering.

Since the model is predicated on the existence, in the $SU(3)$ symmetry limit, of a bound octet and a resonant antidecuplet, one should see elements of this in P_{11} πN scattering. In order to explicate this we must know the contributions of octet and antidecuplet components, respectively, in the residues of the poles when the symmetry is broken. In general, since we ultimately make a one-channel calculation, this is not possible. However, as we have indicated, the behavior given by the multichannel solution between thresholds is not substantially changed in the one-channel calculation; in particular, the one-channel denominator function $D+C$ vanishes near the position where $\mathfrak{D}$ vanishes. Thus we will assume that the pole-model, multichannel solution is good enough near the resonance position to estimate the composition of the resonant wave function. This is not possible for the bound-state nucleon because the characteristics of the multichannel model are altered so drastically in this region.

[28] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963).

In addition to the numerator function for the πN amplitude given in Eq. (15), the appropriate quantities for the three channels $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$ are

$$\begin{aligned} F_1&=[b_1/(s-z_0)]D_2^{(2)}D_3^{(2)}, \\ F_2&=[b_2/(s-z_0)]D_1^{(2)}D_3^{(2)}, \\ F_3&=[b_3/(s-z_0)]D_1^{(2)}D_2^{(2)}. \end{aligned} \tag{29}$$

These are the numerator functions for the three production amplitudes in the 4-channel pole model. At the resonant zero of $\mathfrak{D}$ we calculate F and F_i to obtain the four residues necessary to construct the wave function of the Roper resonance. One finds

$$|\psi_R\rangle=0.156|\pi N\rangle+0.055|\psi_1\rangle +0.973|\psi_2\rangle+0.169|\psi_3\rangle. \tag{30}$$

Transforming back successively to the particle basis and then to the $SU(3)$ basis, one obtains

$$|\psi_R\rangle=0.156|\pi N\rangle+0.475|\eta N\rangle -0.674|K\Lambda\rangle+0.549|K\Sigma\rangle$$

and

$$|\psi_R\rangle=-0.045|27\rangle-0.927|\overline{10}\rangle +0.306|8\rangle-0.222|8'\rangle. \tag{31}$$

This leads to the probabilities

$$\begin{aligned} 27&:<1\%, \\ \overline{10}&:86\%, \\ 8&:9\%, \\ 8'&:5\%. \end{aligned} \tag{32}$$

Thus we see that the physical amplitude with broken symmetry retains a large fraction of its $SU(3)$ behavior, and this demonstrates the over-all consistency of our approach.

We believe that this result, together with those of Sec. IV, presents a strong case for the view proposed in the Introduction, viz., that the Roper resonance is a member of a broken $SU(3)$ $\overline{10}$ multiplet with $J^P=\frac{1}{2}^+$. If this is the case, then the remaining members of the multiplet should exist. There are three such objects: $I=0$, $Y=2$; $I=1$, $Y=0$; $I=\frac{3}{2}$, $Y=-1$, in addition to the $I=\frac{1}{2}$, $Y=1$ member. We will call these particles Z_0', Y_1', $\Xi_{3/2}'$, respectively, in addition to $N_{1/2}'$.

To check the consistency of our model further, we should now calculate the masses of these three hypothetical particles within our broken-symmetry scheme. Unfortunately, this is complicated by several considerations. Since the Z_0' can exist in but a single channel of P_8-B_8 scattering, viz., KN, this would appear to be a simple one-channel calculation driven by Y_1^*, Σ, Λ,

u-channel exchange. However, Y_1^* exchange will again lead to a Born term which diverges for large s, and in contrast to the situation for the $N_{1/2}'$ calculation, it will be increasing in the energy range of interest. Thus, although the Born term is sufficiently attractive, we have no reliable way of calculating the Z_0' mass.

In the case of the Y_1' and $\Xi_{3/2}'$ one suffers from an almost total lack of information. Specifically, our method[24] selects a particular particle channel in which to make ultimately a one-channel calculation. Both the Y_1' and the $\Xi_{3/2}'$ are multichannel calculations; for the Y_1': $\pi\Lambda$, $\pi\Sigma$, $\eta\Sigma$, $K\Xi$, $\bar{K}N$; for the $\Xi_{3/2}'$: $\pi\Xi$, $\bar{K}\Sigma$. We have no knowledge to guide us in the selection of a single channel.

Turning to the experimental data, we see that there is some evidence for the Z_0'. Recently, data have been presented for an $I=0$ object with $Y=+2$ (Z_0^*) in the vicinity of 1850 MeV.[29,30] Moreover, analysis[31] based on early K^+ experiments in deuterium[32] favor the assignment[25] $J^P=\frac{1}{2}^+$. If real, the $I=0$ object would be a natural candidate for Z_0'. Such an identification, however, carries with it some difficulties which cannot be resolved within the context of the present data. The most obvious difficulty involves the masses of Y_1' and $\Xi_{3/2}'$. Since $\overline{\mathbf{10}}$ is a triangular representation, the broken multiplet would satisfy an equal-spacing mass rule if the symmetry breaking were proportional to $M^{(8)}$ (i.e., transforms like **8**). The identification given above would then lead to

$$\begin{aligned} Z_0'&: 1850 \text{ MeV},\\ N_{1/2}'&: 1470 \text{ MeV},\\ Y_1'&: 1090 \text{ MeV},\\ \Xi_{3/2}'&: 710 \text{ MeV}, \end{aligned}$$

which is very difficult to believe, to say the least.

Aside from the obvious possibility that the Z_0' is not to be identified with the experimental Z_0^*, two alternatives suggest themselves in this context. First, one may eliminate pure octet-type splitting and introduce terms like $M^{(27)}$ or $M^{(64)}$ in the symmetry-breaking perturbation. Independent of a lack of desire to add such terms, one finds that anomalously large reduced matrix elements are necessary to yield reasonable masses for Y_1' and $\Xi_{3/2}'$. For example, if we neglect $M^{(64)}$, we obtain

$$\Xi_{3/2}'-Z_0'=3(Y_1'-N_{1/2}').$$

If we choose Y_1' to be some reasonable value, say, 1700 MeV, we find $\Xi_{3/2}'\sim 2600$ MeV. More significant, however, are the values of the reduced matrix elements: $M_1\sim 2000$ MeV, $M_8\sim 1000$ MeV, and $M_{27}\sim 1600$ MeV. This seems untenable; it may be that $M^{(27)}$ plays a role, but not with this strength. The second possibility is more reasonable, viz., that in the energy region in question, 1500–2000 MeV, mixing between baryon multiplets may very well be a serious problem in disentangling the remnants of an $SU(3)$ structure. If such mixing is in fact significant in this energy range, then it is inappropriate, within the context of the present data, to make speculations about assignments based on mass formulas.

Less speculative and more interesting tests of a possible antidecuplet assignment for a given particle would be those involving $SU(3)$ selection rules. Donnachie[33] has proposed such a test in terms of photoproduction from deuterons: A selection rule forbids $\gamma+p\to N'^+$ but allows $\gamma+n\to N'^0$. There also exists preliminary evidence for a $Y_1^*(1700)$ which does not decay appreciably to $Y_1^*(1385)+\pi$.[34] [$SU(3)$ symmetry implies that $B_{\overline{10}}\not\to B_{10}+P_8$.]

VI. DISCUSSION

We have attempted to show that there is a remnant of a $\overline{\mathbf{10}}$ $SU(3)$ symmetric resonant multiplet which plays an important role in P_{11}, πN scattering. We believe that this attempt has been successful, and that the Roper resonance should be viewed as a member of a broken $\overline{\mathbf{10}}$ multiplet. As is always the case in dynamical calculations of this sort, one may criticize the results by arguing that the physical content cannot be separated from the approximations. We believe that the physics is clear on the basis of $SU(3)$; considerable care has been taken to ensure that the approximations are under control at all points in the calculation. The role of the approximation parameters is the best test of this control. At every stage the parameters were chosen to represent the input functions in the dispersion relations in such a way that the results were as insensitive to parameter variation as possible while still behaving in a physically sensible manner when the physical parameters (e.g., the coupling constants) were changed. The single exception to this is the variation of the results of the multichannel calculation as z_0 is varied, although even there a case can be made for the choice of z_0.

We have not explored the role of other forces in any great detail, but it is appropriate that we comment on them. The most natural candidate for an additional force would be some t-channel process. It has been shown[17] that the addition of such an ingredient can improve the results of the static reciprocal bootstrap. Typically, N^* and N exchange in a single-channel model do not bind the nucleon if physical couplings are em-

[29] R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters **17**, 102 (1966).
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[31] R. L. Warnock and G. Frye, Phys. Rev. **138**, B947 (1965).
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[34] D. C. Colley, F. MacDonald, B. Musgrave, W. M. R. Blair, I. S. Hughes, R. M. Turnbull, S. J. Goldsack, K. Paler, L. K. Sisteron, W. Blum, W. W. M. Allison, D. H. Locke, L. Lyons, P. J. Finney, C. M. Fisher, and A. M. Segar, Phys. Letters **24B**, 489 (1967); also, M. Derrick, T. Fields, J. Loken, R. Ammar, R. E. P. Davis, W. Kropac, J. Mott, and F. Schweingruber, Phys. Rev. Letters **18**, 266 (1967).

ployed.[19] However, such calculations have not included the additional channels arising in P_8-B_8 scattering. By ignoring the inelastic contributions in the calculation given here, we find the same result. In particular, if we take ξ, the residue of the pole of the denominator function $D+C$, as zero, then the "nucleon" becomes a resonance; i.e., the elastic channel forces alone are not sufficiently strong to yield a bound nucleon. This occurs for our choice of the N^* width, the experimental value, and of course if we anomalously increase the coupling constant g', we obtain a bound nucleon. Thus we argue that the principal contributions which will yield the nucleon are given by u-channel exchange and the inelastic effects; t-channel exchange could only be a correction to these contributions.

The role of other inelastic contributions is a serious problem. The competing channels of interest are πN^* and σN. Certainly within the context of our model it is consistent to ignore the πN^* channel; it is a part of P_8-B_{10} scattering. Since $\mathbf{8}\otimes\mathbf{10}$ does not contain $\overline{\mathbf{10}}$, P_8-B_{10} scattering would play no role for a $\overline{\mathbf{10}}$ resonance. This argument, of course, begs the question somewhat, but other calculations[19] have already indicated that the role of the πN^* channel is not a large one.

The σN channel, on the other hand, surely plays a role.[20] By "σ" we mean an S-wave, $\pi\pi$ enhancement of some type so that effective, two-body kinematics prevails. This inelastic state is of particular interest because of the effect it must have on η, the elasticity factor. It is clear that our model includes no open inelastic channels to participate in the decay of the Roper resonance. Thus in this model $\eta\equiv 1$, in contrast to the data,[7] which indicate $\eta<0.4$ in the vicinity of the resonance. This is certainly a shortcoming of the model presented here, but we do not believe that it militates against the validity of the fundamental dynamics which we have assumed, viz., that dictated by $SU(3)$. It is possible to contemplate including the σN channel in an effort to reproduce η, although the methods[24] employed in Secs. III and IV would have to be extended. In particular, we would have to learn how to simulate, by means of D-function poles, the three closed channels already being used but, in this case, in the presence of two open channels (πN and σN), both satisfying unitarity. The work of Ball, Shaw, and Wong[20] indicates that some effort should be made to incorporate the σN channel using realistic estimates of the interchannel couplings. It is to be expected that the presence of the σN channel would change the position and width of the resonance; the interesting aspect here concerns the direction of the change. We recognize that the nucleon must be more strongly bound; however, we see no reason why the N' should not be shifted to higher energies. The pertinent observation here is that the N' corresponds to the second zero of a D function, and the usual arguments do not apply.

Finally, let us remark on the implications of this calculation with respect to the reciprocal bootstrap. We emphasize that the inelastic effects in our model are crucial in the determination of the nucleon at approximately the correct position with approximately the correct coupling constant; it does so with a physical input value for the $N^*N\pi$ coupling constant g'. A one-channel πN calculation fails in this respect, requiring a substantially larger value for g'. Thus we would conclude that it is not a reasonable approximation to consider the nucleon as a bound state of the πN system alone—in contrast to the basic, dynamical assumption of the πN reciprocal bootstrap—and that those strange-particle channels dictated by $SU(3)$ must be included as well.

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