Let us now consider the medium-strong mass splittings. Equations (10a), (11a), and (12) predict

$$2\mu_0[2\mu_K+\mu_\eta-3\mu_\pi]=2m_\phi^2+m_{K^*}^2-2m_\omega^2. \quad (14)$$

Experimentally, the right-hand side is 1.03 BeV^2 and the left-hand side is 0.975 BeV^2, in good agreement.

For the medium-strong **27** we again get the sign-inversion characteristic of **27**-type splittings. Equations (10b), (11b), and (13) may be combined to give

$$\tfrac{3}{4}\mu_0[4\mu_K-3\mu_\eta-\mu_\pi]=-[2m_{K^*}^2-m_\omega^2-m_\phi^2]. \quad (15)$$

Experimentally, the right-hand side is 0.059±0.004 BeV^2 and the left-hand side is 0.065 BeV^2, again in good agreement. This last result is rather surprising since the inclusion of scalar mesons in the spectral functions or other possible second-order[12] effects may give a sizable correction to Eq. (15). Another approximation, perhaps related to our exclusion of scalar mesons from the spectral functions, is our neglect of σ terms (commutators of the axial charge with pseudoscalar meson fields) in the reduction process.

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Asymptotic Symmetry and ϕ Decay*

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Isospin violations in ϕ decay are investigated under the assumptions of asymptotic $SU(2)$ symmetry.

THE assumption that $SU(3)$ becomes an exact symmetry at high energy[1] has led to several interesting results, including a calculation[2] of the $K\bar{K}$ decay mode of the ϕ meson in good agreement with experiment. In the present work, the assumption that $SU(2)$ violations also vanish at infinite momentum transfer is used to explain isospin violation in the $\phi\to K\bar{K}$ mode and to predict the rates of the electromagnetic modes $\phi\to 2\pi$ and $\phi\to\omega\pi$.

We begin by considering the Fourier transforms of the $SU(2)$ forbidden currents

$$\int d^4x\, e^{iq\cdot x}\langle\pi\pi|V_\mu{}^8(x)|0\rangle=(q_1-q_2)_\mu F_1(q^2), \quad (1)$$

$$\int d^4x\, e^{-iq\cdot x}\langle\pi\pi|V_\mu{}^0(x)|0\rangle=(q_1-q_2)_\mu F_2(q^2). \quad (2)$$

Since these form factors are assumed to vanish at infinity we may write unsubtracted dispersion relations for them. Then, since the two-π state has zero hypercharge and baryon number, the $F_i(q^2)$ vanish at $q^2=0$ also, so that we may write

$$F_i(0)=\frac{1}{\pi}\int dq'^2\,\frac{\mathrm{Im}F_i(q'^2)}{q'^2-i\epsilon}=0. \quad (3)$$

We now assume that the imaginary part of F is saturated by the vector mesons ρ, ω, and ϕ so that Eq. (3) becomes

$$g_{\rho\pi\pi}(G_\rho{}^8/m_\rho{}^2)+g_{\omega\pi\pi}(G_\omega{}^8/m_\omega{}^2)+g_{\phi\pi\pi}(G_\phi{}^8/m_\phi{}^2)=0, \quad (4)$$

$$g_{\rho\pi\pi}(G_\rho{}^0/m_\rho{}^2)+g_{\omega\pi\pi}(G_\omega{}^0/m_\omega{}^2)+g_{\phi\pi\pi}(G_\phi{}^0/m_\phi{}^2)=0, \quad (5)$$

where we have used the definitions

$$\langle\pi\pi|\phi^i\rangle=\epsilon\cdot(q_1-q_2)g_{\phi^i\pi\pi},\quad \langle\phi^i|V_\mu{}^j|0\rangle=\epsilon_\mu G_i{}^j, \quad (6)$$

where $|\phi^i\rangle$ is the vector-meson nonet. The problem now is to solve Eqs. (4) and (5) for the coupling constant $g_{\phi\pi\pi}$ from which we directly obtain the decay rate as given by

$$\Gamma(\phi\to\pi\pi)=\tfrac{2}{3}(g_{\phi\pi\pi}{}^2/4\pi)(p_{\pi\pi}{}^3/m_\phi{}^2). \quad (7)$$

The coupling constants as defined by Eq. (6) contain all the effects of $\phi\rho$ mixing and isospin violation in the decay vertex, since $|\phi^i\rangle$ is the physical state.

In order to proceed we must define the spectral functions of the vector currents $\rho_V{}^{ij}(m^2)$:

$$-i\int d^4x\, e^{-iq\cdot x}\langle 0|T(V_\mu{}^i(x)V_\nu{}^j(0))|0\rangle=\int_0^\infty dm^2\rho_V{}^{ij}(m^2)\left(\frac{\delta_{\mu\nu}+q_\mu q_\nu/m^2}{q^2+m^2-i\epsilon}-\frac{\delta_{\mu4}\delta_{\nu4}}{m^2}\right). \quad (8)$$

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[1] S. Weinberg, Phys. Rev. Letters 18, 506 (1967); T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967).

[2] T. Das, V. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967); Divakaran and Pandit, *ibid.* 19, 535 (1967).

The spectral functions are assumed to satisfy the asymptotic symmetry requirements[3]

$$\int dm^2 \frac{\rho_V^{ij}(m^2)}{m^2} = 2c^2\delta_{ij}, \tag{9}$$

$$\int dm^2 \rho_V^{ij}(m^2) = 2c^2 m_{ij}^2, \tag{10}$$

where m_{ij}^2 is the vector nonet mass matrix:

$$m_{ij}^2 = m_0^2\delta_{ij} + \delta m^2 d_{8ij} + \text{electromagnetic corrections}, \tag{11}$$

$$m_0^2 = \tfrac{1}{3}(2m_\omega^2 + m_\phi^2) \qquad \delta m^2 = -(m_\phi^2 - m_\omega^2)/\sqrt{3}. \tag{12}$$

Equations (9) and (10) applied to $ij = 33, 44, 88, 00$ give

$$(G_\rho^3)^2 = 2c^2 m_\rho^2, \tag{13}$$

$$(G_{K^*}^4)^2 = 2c^2 m_{K^*}^2, \tag{14}$$

$$(G_\phi^8)^2 = 2c^2 m_\phi^2 (m_{88}^2 - m_\omega^2/m_\phi^2 - m_\omega^2) = \tfrac{4}{3}c^2 m_\phi^2, \tag{15}$$

$$(G_\omega^8)^2 = 2c^2 m_\omega^2 (m_\phi^2 - m_{88}^2/m_\phi^2 - m_\omega^2) = \tfrac{2}{3}c^2 m_\omega^2, \tag{16}$$

$$(G_\phi^0)^2 = 2c^2 m_\phi^2 (m_0^2 - m_\omega^2/m_\phi^2 - m_\omega^2) = \tfrac{2}{3}c^2 m_\phi^2, \tag{17}$$

$$(G_\omega^0)^2 = 2c^2 m_\omega^2 (m_\phi^2 - m_0^2/m_\phi^2 - m_\omega^2) = \tfrac{4}{3}c^2 m_\omega^2. \tag{18}$$

Equations (12) and (13) are the generalized KS–RF relations[4] for ρ and K^*. Equations (15) and (16) are equivalent to the solutions of Das, Mathur, and Okubo[2] if the vector-meson masses satisfy the Gell-Mann–Okubo formula. Equations (9) and (10) applied to $ij = 08$ yield

$$G_\omega^0 G_\omega^8 + G_\phi^0 G_\phi^8 = 2c^2 \delta m^2 d_{808}, \tag{19}$$

$$(G_\omega^0 G_\omega^8/m_\omega^2) + (G_\phi^0 G_\phi^8/m_\phi^2) = 0. \tag{20}$$

These provide a consistency check on the extension of the spectral-function sum rules to off-diagonal elements and determine that $G_\omega^0 G_\omega^8$ is positive while $G_\phi^0 G_\phi^8$ is negative.

We therefore choose the negative root of Eq. (17) for G_ϕ^0 and the positive root for all others.

We now apply Eqs. (9) and (10) to $ij = 38$ and 30, obtaining

$$(G_\rho^3 G_\rho^8/m_\rho^2) + (G_\omega^3 G_\omega^8/m_\omega^2) + (G_\phi^3 G_\phi^8/m_\phi^2) = 0, \tag{21}$$

$$(G_\rho^3 G_\rho^0/m_\rho^2) + (G_\omega^3 G_\omega^0/m_\omega^2) + (G_\phi^3 G_\phi^0/m_\phi^2) = 0, \tag{22}$$

$$G_\rho^3 G_\rho^8 + G_\omega^3 G_\omega^8 + G_\phi^3 G_\phi^8 = 2c^2 m_{38}^2, \tag{23}$$

$$G_\rho^3 G_\rho^0 + G_\omega^3 G_\omega^0 + G_\phi^3 G_\phi^0 = 2c^2 m_{30}^2, \tag{24}$$

where m_{38}^2 and m_{30}^2 are the ρ-ω_8 and ρ-ω_0 mixing masses. Combining these with our previous results we find the consistency condition

$$m_{30}^2 = -m_{38}^2/\sqrt{2}, \tag{25}$$

and[5]

$$G_\phi^3 = c m_\phi\,(\sqrt{3} m_{38}^2/m_\phi^2 - m_\omega^2), \tag{26}$$

$$\sqrt{2} G_\rho^8 - G_\rho^0 = -c m_\rho\,(3 m_{38}^2/m_\phi^2 - m_\omega^2). \tag{27}$$

We may now return to Eqs. (4) and (5) solving for $g_{\phi\pi\pi}$:

$$\frac{g_{\phi\pi\pi}}{g_{\rho\pi\pi}} = \frac{m_\phi^2}{m_\rho^2}\left(\frac{\sqrt{2}G_\rho^8 - G_\rho^0}{\sqrt{2}G_\phi^8 - G_\phi^0}\right) = \frac{m_\phi}{m_\rho}(\sqrt{\tfrac{3}{2}})\frac{m_{38}^2}{m_\omega^2 - m_\phi^2}. \tag{28}$$

In the approximation of octet dominance, we have

$$m_{38}^2 = (m_{K^{*+}}^2 - m_{K^{*0}}^2)/\sqrt{3},$$

so that

$$\frac{g_{\phi\pi\pi}}{g_{\rho\pi\pi}} = \frac{m_\phi}{\sqrt{2} m_\rho}\frac{m_{K^{*0}}^2 - m_{K^{*+}}^2}{(m_\phi^2 - m_\omega^2)}. \tag{29}$$

Using the most recent data on the K^* splitting,[6] we find

$$\Gamma(\phi\to\pi\pi)/\Gamma(\phi\to\text{all}) = 0.024 \pm 0.018. \tag{30}$$

The prediction [Eq. (30)] is significantly higher than previous theoretical estimates[7] but is still well below current experimental[6] upper limits:

$$[\Gamma(\phi\to\pi\pi)/\Gamma(\phi\to\text{all})]_{\text{exp}} \le 0.20.$$

It is interesting to note that we could have obtained Eq. (30) without assuming the unsubtracted dispersion relations [Eqs. (4) and (5)] by applying the soft-pion procedure to the $\phi\pi\pi$-overlap integral. Thus,

$$g_{\phi\pi\pi}\epsilon\cdot(q_1 - q_2) = \langle\pi^+\pi^-|\phi\rangle = \frac{(q_1 - q_2)_\mu}{2c^2} \times \left\langle 0\left|\left[\int A_0^+(\mathbf{x},0)d^3x, A_\mu^-(0)\right]\right|\phi\right\rangle = \frac{(q_1 - q_2)_\mu}{2c^2}\epsilon_\mu G_\phi^3, \tag{31}$$

so that we have

$$G_\phi^3 = 2c^2 g_{\phi\pi\pi} \tag{32}$$

analogous to the usual KS–RF relation $G_\rho^3 = 2c^2 g_{\rho\pi\pi}$. It is easy to see that Eq. (32) is equivalent to our previous results [Eqs. (13), (26), and (28)]. Although the procedure used in Eq. (31) has been criticized because

[3] We have assumed the KS-RF relation $2c^2 = m_\rho^2/f_\rho^2$ (see Ref. 4) in Eqs. (9) and (10). The extension of Eqs. (9) and (10) to the $SU(3)$ singlet channels ($i, j = 0$) without additional terms has no sound basis in group theory. We regard our success here, however, as no more (nor less) mysterious than the analogous nonet symmetry of Eq. (11) which is well-satisfied empirically.

[4] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[5] Unfortunately we cannot isolate G_ω^3 which would permit a calculation of the $\omega\to 2\pi$ rate. It is, however, possible to obtain a relation between G_ω^3 and G_ρ^8, G_ρ^0 which, using the techniques of Eqs. (38) and (39), leads to a relation between the $\rho\to 3\pi$ and $\omega\to 2\pi$ rates. The prediction is $\Gamma(\rho\to 3\pi) = 75$ keV, in which case this mode may compete favorably with the strong decay $\rho\to 4\pi$.

[6] A. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[7] P. Singer, Phys. Rev. Letters **12**, 524 (1964); J. Yellin, Phys. Rev. **147**, 1080 (1966).

of the large extrapolations involved, it leads to the well-satisfied KS–RF relations when applied to ρ and K^* decay and reproduces results derived in another way for $\phi\to 2\pi$ decay. We therefore propose to trust this technique to give us the electromagnetic corrections to $\phi\to K\bar{K}$ decay. We write

$$\langle K^+K^-|\phi\rangle=\frac{(q_1-q_2)_\mu}{2c^2}\langle 0|[Q_5{}^{(4+i5)/\sqrt{2}},A_\mu{}^{(4-i5)/\sqrt{2}}]|\phi\rangle$$

with a similar expression for $\langle K^0\bar{K}^0|\phi\rangle$ from which we obtain

$$4c^2g_{\phi K^+K^-}=\sqrt{3}G_\phi{}^8+G_\phi{}^3\,, \quad (33)$$

$$4c^2g_{\phi K_1K_2}=\sqrt{3}G_\phi{}^8-G_\phi{}^3\,. \quad (34)$$

In the approximation of vanishing $G_\phi{}^3$ these relations have been derived and compared with experiment by Wada.[8] Because of the possibility of interference, the effect of electromagnetism is expected to be greater here than in the case of $\rho\to 2\pi$. We predict

$$A(\phi\to K^+K^-)/A(\phi\to K_1K_2)\cong 1+2G_\phi{}^3/\sqrt{3}G_\phi{}^8\,. \quad (35)$$

Putting in our previously derived values for $G_\phi{}^3$ and $G_\phi{}^8$, and taking phase space into account we predict

$$\Gamma(\phi\to K^+K^-)/\Gamma(\phi\to K_1K_2)=1.44\pm0.08\,. \quad (36)$$

Phase space alone would give

$$\Gamma(\phi\to K^+K^-)/\Gamma(\phi\to K_1K_2)=1.52\pm0.04\,. \quad (37)$$

The experimental branching ratio[6] is 1.22 ± 0.20. The effect we predict is of the right sign and is sufficient in magnitude to obtain agreement within one standard deviation.

We now wish to comment on the decays $\phi\to\rho\pi$ and $\phi\to\omega\pi$. We therefore consider the currents

$$\int d^4x\, e^{-iq\cdot x}\langle\pi\rho|\sqrt{2}V_\mu{}^8(x)-V_\mu{}^0(x)|0\rangle =\epsilon_{\mu\nu\lambda\sigma}\epsilon_\nu q_{1\lambda}q_{2\sigma}F_3(q^2)\,, \quad (38)$$

$$\int d^4x\, e^{-iq\cdot x}\langle\pi\omega|\sqrt{2}V_\mu{}^8(x)-V_\mu{}^0(x)|0\rangle =\epsilon_{\mu\nu\lambda\sigma}\epsilon_\nu q_{1\lambda}q_{2\sigma}F_4(q^2)\,. \quad (39)$$

[8] W. W. Wada, Phys. Rev. Letters **16**, 956 (1966).

Since the coupling of the vector current to a vector and a pseudoscalar meson must be of d type, these currents must vanish at infinity[9] so that $F_{3,4}(q^2)$ are superconvergent. We may therefore disperse in $q^2F(q^2)$ which vanishes at zero. Then we obtain the following sum rules analogous to Eqs. (4) and (5):

$$g_{\rho\pi\rho}(\sqrt{2}G_\rho{}^8-G_\rho{}^0)+g_{\rho\pi\omega}(\sqrt{2}G_\omega{}^8-G_\omega{}^0) +g_{\rho\pi\phi}(\sqrt{2}G_\phi{}^8-G_\phi{}^0)=0\,, \quad (40)$$

$$g_{\omega\pi\rho}(\sqrt{2}G_\rho{}^8-G_\rho{}^0)+g_{\omega\pi\pi}(\sqrt{2}G_\omega{}^8-G_\omega{}^0) +g_{\omega\pi\phi}(\sqrt{2}G_\phi{}^8-G_\phi{}^0)=0\,. \quad (41)$$

Since the first term in Eq. (40) is of second order in α, we have

$$g_{\phi\rho\pi}/g_{\omega\rho\pi}=-(\sqrt{2}G_\omega{}^8-G_\omega{}^0)/(\sqrt{2}G_\phi{}^8-G_\phi{}^0) \quad (42)$$

which vanishes upon substitution of Eqs. (15)–(18). Experimentally,[10]

$$|g_{\phi\rho\pi}/g_{\omega\rho\pi}|=0.03\pm0.02\,. \quad (43)$$

From Eq. (41) we have

$$\frac{g_{\phi\omega\pi}}{g_{\omega\rho\pi}}=-\frac{\sqrt{2}G_\rho{}^8-G_\rho{}^0}{\sqrt{2}G_\phi{}^8-G_\phi{}^0}=\frac{m_\rho{}^2}{m_\phi{}^2}\frac{G_\phi{}^3}{G_\rho{}^3}=-0.014\pm0.009\,, \quad (44)$$

where we have used our previous results for the G's. From Eqs. (44) and (43) we note the interesting result that the isospin violating rate $\Gamma(\phi\to\omega\pi)$ might be as large as the rate for $\phi\to\rho\pi$. To date no attempt to observe the $\omega\pi$-decay mode of ϕ has been reported.

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[9] The vanishing of Eq. (38) depends on the interference of the two currents, while in Eq. (39) $SU(3)$ forbids each current separately. Also, in contrast to our previous results, $\pi\eta$ mixing could affect the convergence properties of $F_4(q^2)$ so that perhaps the sum rule [Eq. (41)] is somewhat less certain than the others.

[10] We have assumed the $\phi\to 3\pi$ rate is entirely $\phi\to\rho\pi$ and have used the phase-space calculation of Yellin (Ref. 7). Our prediction [Eq. (42)] merely reproduces the nonet symmetry result. It is clear that if in Eqs. (15)–(18) we had taken into account deviations from the canonical mixing angle in the ϕ-ω complex, a nonzero result for Eq. (42) would be obtained without much effect on our other results.