of motion are just the usual ones from 1 to 8, plus

$$\partial_\mu \sigma^9 = (1/2C)(\tfrac{2}{3})^{1/2}[A_\mu{}^a, \phi^a]_+ ,$$
$$\partial_\mu \phi^9 = -(1/2C)(\tfrac{2}{3})^{1/2}[A_\mu{}^a, \sigma^a]_+ , \tag{4.18}$$

with summations running from 1 to 8. One cannot drop ϕ^9 (or σ^9) without violating a Jacobi identity.

$SU(3)$ Breakdown

To accomplish this we can simply add a term (to $\theta_{\mu\nu}$) which transforms like the eighth component of the octet[23]

$$\Delta\theta_{\mu\nu} = g_{\mu\nu}\{-\lambda\sigma_8 + \tfrac{1}{4}\lambda^2\} , \tag{4.19}$$

which leaves $\theta_{00} \geqq 0$ ($\sigma_8 = \sigma_8' + \frac{1}{2}\lambda$). Then

$$\partial_\mu V_a{}^\mu = -\lambda f^{a8c}\sigma^c , \quad \partial_\mu A_a{}^\mu = f_\pi \mu^2 \phi_a + \lambda d^{a8c}\phi^c , \tag{4.20}$$

that is, $V_{4,5,6,7}{}^\mu$ are not conserved. The (curl) equations of motion for all the densities are unchanged.

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We understand that Professor C. Sommerfield has independently derived the extension to PCAC. One of the authors (MBH) acknowledges an interesting conversation with Professor M. Vaughn.

[23] Alternately, one could introduce a term in $\theta_{\mu\nu}$ of the form $g_{\mu\nu}d^{8ab}\phi_a\phi_b$. An attempt to add terms of the form $(1/2C)d^{8ab} \times \{[V_\mu{}^a, V_\nu{}^b]_+ - g_{\mu\nu}V_\lambda{}^a V_b{}^\lambda\}$ breaks Poincaré invariance. One could also break $SU(3)$ via an elementary fifth interaction [see Y. Ne'eman, Phys. Rev. **134**, B1355 (1964)] in a hybrid theory (only the 9th meson canonical). On the other hand, a pure theory ($\theta_{\mu\nu}{}^S(\hat{V}^8)$, $\hat{V}^8 = V^8 + \lambda V^9$, $[V_0{}^9(\mathbf{x}), V_i{}^9(\mathbf{y})] = iC\partial_i{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y})$, V^9 commuting with all other coordinates) violates translational invariance.

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Chiral Dynamics of Octet Baryons and Nonleptonic Decays of Hyperons

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Chiral dynamics of the octet baryons is discussed, and a phenomenological chiral-invariant Lagrangian is derived. It is noted that in a chiral-invariant Lagrangian for the octet baryons, the structure of the axial-vector currents and the pattern of baryon-meson couplings are uniquely characterized by two free parameters, corresponding to G_A/G_V and the D/F ratio of the axial-vector currents. Nonleptonic hyperon decays are discussed from the chiral-dynamics point of view. A discussion is given on the transformation properties of the nonleptonic weak interactions under chiral transformations. Conditions under which the usual current-algebra analyses are valid are elaborated.

I. INTRODUCTION

In the present paper, we give a discussion on the construction of a phenomenological chiral-dynamics Lagrangian[1] for the octet baryons, and consider nonleptonic decays of hyperons from the point of view of chiral dynamics. Our views on chiral dynamics have been expressed previously,[2] and the construction of the chiral-dynamics Lagrangian we present here is a direct generalization of the material presented in Chap. IX of Ref. 2.

The construction of the chiral-dynamics Lagrangian for the nonet pseudoscalar mesons has been discussed by Cronin,[3] and will be discussed from a slightly different point of view by us in a separate communication.[4] The Lagrangian for the nonet pseudoscalar mesons is chiral $SU(3)\times SU(3)$ invariant, save for the mass terms for mesons, which make the hypothesis of partially conserved axial-vector current [(PCAC): identity of the divergence of the axial-vector current and the pion field in the sense of perturbative Lagrangian field theory] exactly satisfied for isospin axial-vector currents. To this we superimpose the chiral-invariant Lagrangian for the octet baryons, which then ensures the chiral $SU(3)\times SU(3)$ structure of vector and axial-vector currents, and PCAC for the isotopic-spin axial-vector currents.[5]

Chiral dynamics of the octet baryons is discussed in the next section. We construct a model of the octet

[1] S. Weinberg, Phys. Rev. Letters **18**, 188 (1967); J. Schwinger, Phys. Letters **24B**, 473 (1967); W. A. Bardeen and B. W. Lee (to be published); L. S. Brown, Phys. Rev. **163**, 1802 (1967); J. Wess and B. Zumino, *ibid.* **163**, 1727 (1967); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968), and references therein.

[2] W. A. Bardeen and B. W. Lee, Ref. 1.

[3] J. A. Cronin, University of Chicago, thesis, 1967 (unpublished).

[4] W. A. Bardeen, B. W. Lee, and D. Majumdar (unpublished).

[5] Chiral-dynamics Lagrangian for the nonet pseudoscalar mesons is in some sense unique, once the commutation relation between the axial current and and its divergence is specified. This statement is exactly true for chiral $SU(2)\times SU(2)$. See Ref. 2 and L. S. Brown, Ref. 1. I understand that this point with respect to $SU(3)\times SU(3)$ and the uniqueness of baryon dynamics will be discussed in a forthcoming paper by J. Wess and B. Zumino. I wish to express my thanks to Professor Zumino for this information. See also related discussion by M. Lévy, Nuovo Cimento **52**, 23 (1967).

baryons interacting with the nonet of pseudoscalar mesons. The model contains two free parameters, corresponding to G_A/G_V and the D/F ratio of the axial-vector currents. It is important to note that the principles of chiral dynamics and the number of independent baryon degrees of freedom do not specify either of these parameters. The expected correspondence between the structures of the axial-vector currents and of the baryon–pseudoscalar meson couplings [i.e., $SU(3)$ version of the Goldberger-Treiman relation] is of course maintained.

In Sec. III we discuss nonleptonic decays of hyperons from the chiral-dynamics point of view. First a thorough discussion is given of the transformation properties of nonleptonic weak interactions under chiral algebra. It is emphasized that the almost chiral-invariant strong interactions play an important role in enhancing the strength of nonleptonic weak interactions, without simultaneously altering its transformation properties under chiral transformations. The standard results of current-algebra analyses are reproduced under certain assumptions. It is pointed out that the absence of direct parity-conserving, derivative $\bar{B}'B\pi$ couplings[6] with $\Delta S=\pm 1$ is essential to justify the usual current-algebra analyses. The role of the parity-violating $B'-B$ ($\Delta S=\pm 1$) process,[7] which is forbidden in the $SU(3)$ limit, is discussed.

II. CHIRAL DYNAMICS OF OCTET BARYONS

We first discuss a chiral-dynamics Lagrangian for the baryon octet. The pseudoscalar-meson Lagrangian will be described elsewhere[3,4]; the addition of the pseudoscalar-meson Lagrangian to our scheme does not alter our discussions below. We shall assume the existence of the nonet of pseudoscalar mesons belonging to the $(3,\bar{3})+(\bar{3},3)$ representation of the chiral algebra, so that[8]

$$M_{\dot{\beta}}{}^{\alpha}=(\Sigma+i\Pi)_{\alpha\beta}\,,\qquad M_{\beta}{}^{\dot{\alpha}}=(\Sigma-i\Pi)_{\alpha\beta}\,. \tag{2.1}$$

Here Π is the 3×3 matrix representation for the pseudoscalar nonet, and Σ is to be determined from the constraint[9]

$$M_{\dot{\beta}}{}^{\alpha}M_{\kappa}{}^{\dot{\beta}}=f_\pi{}^2\delta_\kappa{}^\alpha\,,\qquad M_{\beta}{}^{\dot{\alpha}}M_{\dot{\kappa}}{}^{\beta}=f_\pi{}^2\delta_{\dot{\kappa}}{}^{\dot{\alpha}}\,. \tag{2.2}$$

We shall designate by $N_\beta{}^\alpha$ a set of nucleon fields which transform like (8,1):

$$N_\beta{}^\alpha=[(1+\gamma_5)N]_{\alpha\beta},\quad (\bar{N})_\alpha{}^\beta=[\bar{N}(1-\gamma_5)]_{\beta\alpha}\,,$$
$$N_{\dot{\beta}}{}^{\dot{\alpha}}=[(1-\gamma_5)N]_{\alpha\beta},\quad (\bar{N})_{\dot{\alpha}}{}^{\dot{\beta}}=[\bar{N}(1+\gamma_5)]_{\beta\alpha}\,. \tag{2.3}$$

It is possible to contruct other nucleon fields which transform differently from (8,1) or (1,8). There are two ways of constructing $(3,\bar{3})$ and $(\bar{3},3)$:

$$\begin{aligned}(N_1)_{\dot{\beta}}{}^{\alpha}&=(1/f_\pi)N_\kappa{}^\alpha M_\beta{}^\kappa\\&=(1/f_\pi)[(1+\gamma_5)N(\Sigma+i\Pi)]_{\alpha\beta}\,,\\(N_1)_\beta{}^{\dot{\alpha}}&=(1/f_\pi)N_{\dot{\kappa}}{}^{\dot{\alpha}}M_\beta{}^{\dot{\kappa}}\\&=(1/f_\pi)[(1-\gamma_5)N(\Sigma-i\Pi)]_{\alpha\beta}\,,\end{aligned} \tag{2.4a}$$

$$\begin{aligned}(N_2)_{\dot{\beta}}{}^{\alpha}&\equiv(1/f_\pi)M_{\dot{\kappa}}{}^\alpha N_{\dot{\beta}}{}^{\dot{\kappa}}\\&=(1/f_\pi)[(\Sigma+i\Pi)(1-\gamma_5)N]_{\alpha\beta}\,,\\(N_2)_\beta{}^{\dot{\alpha}}&=(1/f_\pi)M_\kappa{}^{\dot{\alpha}}N_\beta{}^\kappa\\&=(1/f_\pi)[(\Sigma-i\Pi)(1+\gamma_5)N]_{\alpha\beta}\,.\end{aligned} \tag{2.4b}$$

We are now in a position to write down a Lagrangian for massive octet baryons which is chiral $SU(3)\times SU(3)$ invariant. The baryon part of the Lagrangian is

$$\begin{aligned}\mathfrak{L}_B=&\tfrac{1}{2}(1-\alpha_1-\alpha_2)[\bar{N}_\beta{}^\alpha i\gamma\cdot\partial N_\alpha{}^\beta+\bar{N}_{\dot{\beta}}{}^{\dot{\alpha}}i\gamma\cdot\partial N_{\dot{\alpha}}{}^{\dot{\beta}}]\\&+\tfrac{1}{2}\alpha_1[(\bar{N}_1)_\beta{}^{\dot{\alpha}}i\gamma\cdot\partial(N_1)_{\dot{\alpha}}{}^\beta+(\bar{N}_1)_{\dot{\beta}}{}^\alpha i\gamma\cdot\partial(N_1)_\alpha{}^{\dot{\beta}}]\\&+\tfrac{1}{2}\alpha_2[(\bar{N}_2)_\beta{}^{\dot{\alpha}}i\gamma\cdot\partial(N_2)_{\dot{\alpha}}{}^\beta+(\bar{N}_2)_{\dot{\beta}}{}^\alpha i\gamma\cdot\partial(N_2)_\alpha{}^{\dot{\beta}}]\\&-\tfrac{1}{4}m[(\bar{N}_1)_{\dot{\beta}}{}^\alpha(N_2)_\alpha{}^{\dot{\beta}}+(\bar{N}_1)_\beta{}^{\dot{\alpha}}(N_2)_{\dot{\alpha}}{}^\beta\\&+(\bar{N}_2)_{\dot{\beta}}{}^\alpha(N_1)_\alpha{}^{\dot{\beta}}+(\bar{N}_2)_\beta{}^{\dot{\alpha}}(N_1)_{\dot{\alpha}}{}^\beta]\,.\end{aligned} \tag{2.5}$$

Equation (2.5) is *not* the most general chiral-invariant structure, but it is sufficiently general for our discussions. The parameters α_1 and α_2 in Eq. (2.5) are arbitrary. Eq. (2.5) may be cast in a simpler form:

$$\begin{aligned}\mathfrak{L}_B=\mathrm{Tr}[&(1-\alpha_1-\alpha_2)\bar{N}i\gamma\cdot\partial N+\alpha_1\bar{N}_1i\gamma\cdot\partial N_1\\&+\alpha_2\bar{N}_2i\gamma\cdot\partial N_2-\tfrac{1}{2}m(\bar{N}_2N_1+\bar{N}_1N_2)]\,,\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}N_1&=(1/f_\pi)N(\Sigma+i\Pi\gamma_5)\equiv(1/f_\pi)(N\Sigma+i\gamma_5N\Pi)\,,\\N_2&=(1/f_\pi)(\Sigma-i\Pi\gamma_5)N\,.\end{aligned} \tag{2.7}$$

Actually it is more convenient to define *the* baryon field B in such a way that the "mass" term in Eq. (2.6) becomes diagonal[10]:

$$\tfrac{1}{2}\,\mathrm{Tr}(\bar{N}_2N_1+\bar{N}_1N_2)=\mathrm{Tr}\bar{B}B\,. \tag{2.8}$$

To this end we consider a unitary transformation (which depends on the pseudoscalar fields nonlinearly) on N:

$$N=U(i\Pi\gamma_5)BU^\dagger(i\Pi\gamma_5)\,, \tag{2.9}$$

such that

$$U^\dagger(i\Pi\gamma_5)=U^{-1}(i\Pi\gamma_5)=U(-i\Pi\gamma_5)\,. \tag{2.10}$$

Under this class of unitary transformations, we have

$$\mathrm{Tr}N^\dagger N=\mathrm{Tr}N_1{}^\dagger N_1=\mathrm{Tr}N_2{}^\dagger N_2=\mathrm{Tr}B^\dagger B$$

and

$$\begin{aligned}\tfrac{1}{2}\,\mathrm{Tr}(\bar{N}_2N_1+\bar{N}_1N_2)=&(1/f_\pi{}^2)\\&\times\mathrm{Tr}\bar{N}(\Sigma-i\Pi\gamma_5)N(\Sigma+i\Pi\gamma_5)=(1/f_\pi{}^2)\\&\times\mathrm{Tr}\bar{B}[U(\Sigma-i\Pi\gamma_5)U]B[U^\dagger(\Sigma+i\Pi\gamma_5)U^\dagger]\,.\end{aligned}$$

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[6] Here and in the following, B and B' stand for baryons, π for pions.

[7] A. Kumar and J. C. Pati, Phys. Rev. Letters 18, 1230 (1967).

[8] Undotted superscripts (subscripts) are $SU(3)$ cogredient (contragredient) spinor indices under $Q^\alpha+Q_5{}^\alpha$; dotted ones under $Q^\alpha-Q_5{}^\alpha$, where $Q^\alpha\pm Q_5{}^\alpha=\int d^3x[V_0{}^\alpha(x)\pm A_0{}^\alpha(x)]$, $V_\mu{}^\alpha$ and $A_\mu{}^\alpha$ being octet vector and axial-vector currents. See Ref. 2.

[9] f_π is the pion-decay constant. See Refs. 2 and 4.

[10] We wish to emphasize, as in Refs. 1 and 2, the fact that it is largely irrelevant which one of the canonically equivalent fields we consider to be the baryon field, insofar as the perturbative construction of the S matrix on the mass shell is concerned.

Equation (2.8) is, therefore, obtained if we choose U so that

$$U(i\Pi\gamma_5)(\Sigma-i\Pi\gamma_5)U(i\Pi\gamma_5)=1$$

or

$$U(i\Pi\gamma_5)=\left(\frac{f_\pi+\Sigma}{2f_\pi}\right)^{1/2}\left(1+i\Pi\gamma_5\frac{1}{f_\pi+\Sigma}\right). \qquad (2.11)$$

Equation (2.11) implies that

$$\begin{aligned} U^2(i\Pi\gamma_5)&=(1/f_\pi)(\Sigma+i\Pi\gamma_5),\\ U^{+2}(i\Pi\gamma_5)&=(1/f_\pi)(\Sigma-i\Pi\gamma_5). \end{aligned} \qquad (2.12)$$

Equation (2.5) may be written in terms of the baryon field B:

$$\begin{aligned} \mathcal{L}_B=\mathrm{Tr}(\bar{B}i\gamma\cdot\partial B-m\bar{B}B+\bar{B}i\gamma_\mu[s^\mu,B]&\\ +(1-\alpha_1-\alpha_2)\bar{B}\gamma_\mu[p^\mu,B]&\\ +(\alpha_1-\alpha_2)\bar{B}\gamma_\mu\{p^\mu,B\}),& \qquad (2.13) \end{aligned}$$

where

$$\begin{aligned} s^\mu&=\tfrac{1}{2}[U^\dagger\partial^\mu U+U\partial^\mu U^\dagger]\\ &=[1/2(2f_\pi)^2][\Pi,\partial^\mu\Pi]+O(\Pi^4), \end{aligned} \qquad (2.14)$$

$$\begin{aligned} p^\mu&=(1/2i)[U^\dagger\partial^\mu U-U\partial^\mu U^\dagger]\\ &=(1/2f_\pi)\gamma\partial^\mu\Pi+O(\Pi^3). \end{aligned} \qquad (2.15)$$

In order to characterize the breaking of the $SU(3)$ symmetry we must adjoin a mass-breaking term which transforms like λ_8 under $SU(3)$ to the Lagrangian (2.13). We must pause here, however, to discuss how the mass-breaking term transforms under chiral $SU(3)\times SU(3)$. Simple possibilities are that it transforms like the chiral $SU(2)\times SU(2)$ invariant part of a $(3,\bar{3})\oplus(\bar{3},3)$ representation or a $(8,1)\oplus(1,8)$. Consequences of either assignment are very similar, and we choose to discuss the first possibility in some detail. The matrix

$$(\lambda_8)_\beta{}^\alpha=\frac{1}{\sqrt{3}}\begin{bmatrix}1&&\\&1&\\&&-2\end{bmatrix}_{\alpha\beta}$$

is a mixture of $(\frac{1}{2},\frac{1}{2})$ and $(0,0)$ of chiral $SU(2)\times SU(2)$, so that if we were to use $(\lambda_8)_\beta{}^\alpha$ and $(\lambda_8)_\beta{}^\alpha$ to construct the mass-breaking term, the term would also destroy the chiral $SU(2)\times SU(2)$ symmetry. Since we assume the isospin chiral symmetry is broken only by the pion mass term (PCAC), we shall instead use the matrix

$$(\Delta)_\beta{}^\alpha=\begin{bmatrix}0&&\\&0&\\&&1\end{bmatrix}_{\alpha\beta}=-\tfrac{1}{3}[\sqrt{3}(\lambda_8)_\beta{}^\alpha-\delta_\beta{}^\alpha], \qquad (2.16)$$

which transforms like $8\oplus1$ under $SU(3)$, and which is invariant under isospin chiral transformations.

Thus the baryon mass-splitting term in the Lagrangian may be written as

$$\begin{aligned} \mathcal{L}_\Delta{}^{(1)}&=-\tfrac{1}{2}\,\mathrm{Tr}[\gamma(\bar{N}\Delta N_1+\bar{N}_1\Delta N)\\ &\qquad+\delta(\bar{N}N_2\Delta+\bar{N}_2N\Delta)]\\ &=-\tfrac{1}{2}\,\mathrm{Tr}[\gamma\bar{B}(U\Delta U+U^\dagger\Delta U^\dagger)B\\ &\qquad+\delta\bar{B}B(U\Delta U+U^\dagger\Delta U^\dagger)] \qquad (2.17)\\ &=-\mathrm{Tr}(\gamma\bar{B}\Delta B+\delta\bar{B}B\Delta)+O(\Pi^2). \end{aligned}$$

The parameters γ and δ are related to the baryon mass differences:

$$\begin{aligned} \gamma&=M_\Xi-M_\Sigma,\\ \delta&=M_N-M_\Sigma. \end{aligned} \qquad (2.18)$$

It is noteworthy that the mass-breaking term (2.17) does *not* induce changes in pseudovector-$\bar{B}B\Pi$ vertices. [This, incidentally, is true also for the other assignment, $(8,1)\oplus(1,8)$.]

The interaction of the pseudoscalar mesons with the baryons is described, to second order in Π, by the phenomenological Lagrangian

$$\begin{aligned} \mathcal{L}_{\mathrm{int}}\simeq\frac{1-2\alpha_2}{2f_\pi}&\mathrm{Tr}\bar{B}\gamma_\mu\gamma_5(\partial^\mu\Pi)\\ &-\frac{1-2\alpha_1}{2f_\pi}\mathrm{Tr}\bar{B}\gamma_\mu\gamma_5B\partial^\mu\Pi \qquad (2.19)\\ &+\frac{i}{2}\frac{1}{(2f_\pi)^2}\mathrm{Tr}\bar{B}\gamma_\mu[[\Pi,\partial^\mu\Pi],B]. \end{aligned}$$

The nucleon-pion coupling constant (pseudoscalar coupling) g is given by

$$g/2m_N=(1-2\alpha_2)/2f_\pi. \qquad (2.20)$$

The part of the axial-vector current arising from the baryon Lagrangian $\mathcal{L}_B$, namely, $(A_\mu{}^\alpha)_B$, is most readily computed from Eq. (2.6). Under infinitesimal axial-isocharge transformations the fields N, N_1, and N_2 transform as

$$\begin{aligned} \delta_\omega N&=-\gamma_5[\tfrac{1}{2}\lambda^\alpha,N]\omega_\alpha,\\ \delta_\omega N_1&=-\gamma_5\{\tfrac{1}{2}\lambda^\alpha,N_1\}\omega_\alpha,\\ \delta_\omega N_2&=+\gamma_5\{\tfrac{1}{2}\lambda^\alpha,N_2\}\omega_\alpha, \end{aligned} \qquad (2.21)$$

so that

$$\begin{aligned} (A_\mu{}^\alpha)_B=(1-\alpha_1-\alpha_2)\,&\mathrm{Tr}\bar{N}\gamma_5\gamma_\mu[\tfrac{1}{2}\lambda^\alpha,N]\\ &+\alpha_1\,\mathrm{Tr}\bar{N}_1\gamma_5\gamma_\mu\{\tfrac{1}{2}\lambda^\alpha,N_1\}\\ &-\alpha_2\,\mathrm{Tr}\bar{N}_2\gamma_5\gamma_\mu\{\tfrac{1}{2}\lambda^\alpha,N_2\}, \qquad (2.22a)\\ =(1-2\alpha)\,&\mathrm{Tr}\bar{B}\gamma_5\gamma_\mu(\tfrac{1}{2}\lambda^\alpha)B\\ &-(1-2\alpha_1)\,\mathrm{Tr}\bar{B}\gamma_5\gamma_\mu B(\tfrac{1}{2}\lambda^\alpha)+O(\Pi). \qquad (2.22b) \end{aligned}$$

By construction, the axial-vector currents satisfy the commutation relations of $SU(3)\times SU(3)$. We recognize the relation[11]

$$\left(\frac{G_A}{G_V}\right)=1-2\alpha_2, \qquad (2.23)$$

[11] One recognizes the combination of Eqs. (2.20) and (2.23) as the Goldberger-Treiman relation.

and the fact that the D/F ratios of the axial-vector currents and the baryon-meson (pseudovector) couplings are the same:

$$\left(\frac{D}{F}\right)_{\text{axial}}=\left(\frac{D}{F}\right)_{\text{pseudovector}}=\frac{\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2}. \quad (2.24)$$

In concluding this section, we emphasize that once we identify the number of baryon degrees of freedom, which transforms irreducibly under $SU(3)$, the pattern of the baryon-meson couplings, Eq. (2.19) and the structure of the axial-vector current, Eq. (2.22b), are uniquely characterized by two parameters α_1 and α_2, or equivalently, by G_A/G_V and the (D/F) ratio, Eq. (2.24). [This statement is to be contrasted with the naive expectation that, if the baryons belong to $(8,1)+(1,8)$, the (D/F) ratio is zero, while if they belong to $(3,\bar{3})+(\bar{3},3)$ the (F/D) ratio is zero; or, to assign the baryons to a mixture of $(8,1)+(1,8)$ and $(3,\bar{3})+(\bar{3},3)$ called for the existence of another set of baryons.] The transformation properties of the baryons B under chiral transformations are characterized by a representation in terms of a local, nonlinear function of the pseudoscalar-meson fields,[12] since

$$\begin{aligned}[][Q_5{}^\alpha,B(x)]&=[Q_5{}^\alpha,U^\dagger N(x)U]\\ &=[F^\alpha(i\gamma_5\Pi(x)),B(x)], \end{aligned}\quad (2.25)$$

where $Q_5{}^\alpha\equiv\int d^3x\,A_0{}^\alpha(x)$ is the generator of the axial-charge transformation, and [13]

$$\begin{aligned}F^\alpha(i\gamma_5\Pi(x))=&[Q_5{}^\alpha,U^\dagger(i\gamma_5\Pi(x))]U(i\gamma_5\Pi(x))\\ &-U^\dagger(i\gamma_5\Pi(x))\gamma_5(\tfrac{1}{2}\lambda^\alpha)U(i\gamma_5\Pi(x)).\end{aligned}\quad (2.26)$$

We note also that $\partial_\mu B$ does not transform like B. The quantity that does is

$$\begin{aligned}U^\dagger(i\gamma_5\Pi)(\partial_\mu N)U(i\gamma_5\Pi)&=\partial_\mu B+[U^\dagger\partial_\mu U,B]\\ &=\partial_\mu B+[s_\mu+ip_\mu,\,B]\equiv D_\mu B,\end{aligned}$$

which may be called the "covariant derivative" of chiral dynamics.[12]

III. NONLEPTONIC DECAYS OF HYPERONS

As an application of the preceding considerations, we shall discuss nonleptonic decays of the hyperons.[14] We shall assume the current×current interaction as being responsible for nonleptonic decays. As discussed elsewhere,[14] the current×current Lagrangian transforms like a mixture of **8** and **27** under $SU(3)$, if the currents are of the Cabibbo type.[15] The Cabibbo currents are in turn a representation (8,1) of the chiral algebra. Once one accepts the premise of the current ×current theory, then experimental indications are that the octet part of the interaction is selectively enhanced,[16] with, perhaps, an accompanying suppression of the **27** part.[17] Since the enhancement is presumed to be due to the intervention of strong interactions[18] which are almost chiral invariant (except for the finite mass of the pseudoscalar mesons), it is reasonable to assume that the dominant part of the nonleptonic weak interactions transforms like (8,1) under chiral $SU(3)\times SU(3)$. CP invariance of the weak interactions then dictates that it transform like $(\lambda_6)_\beta{}^\alpha$.[16]

In constructing the phenomenological Lagrangian for nonleptonic decays of hyperons, we will be guided by the following considerations. We shall first consider nonderivative form of the Lagrangian, which gives the "smoothest" amplitudes in pion momentum, and show that this gives the "classical" results of Hara and Nambu,[19] and Brown and Sommerfield.[20] We shall then relax the condition that phenomenological vertices be momentum-independent, and allow couplings involving first derivatives of fields. In this case, we will reproduce essentially the results of Kumar and Pati[7] under a rather restrictive additional condition.

There are two nonderivative, CP-conserving couplings which transform like the member $(\lambda_6)_\beta{}^\alpha$ of (8,1) under chiral algebra, which are capable of mediating processes: $B'\to B$ and $B'\to B+\pi$. These are

$$\begin{aligned}\mathcal{L}_{\rm NL}{}^{(1)}=&\tfrac{1}{2}(d-f)\,\mathrm{Tr}[\bar{N}(1-\gamma_5)(\Sigma+i\Pi)N(\Sigma-i\Pi)\lambda_6\\ &\qquad+\bar{N}(1+\gamma_5)(\Sigma-i\Pi)N\lambda_6(\Sigma+i\Pi)]\\ &+\tfrac{1}{2}(d+f)\,\mathrm{Tr}[\bar{N}(1-\gamma_5)\lambda_6(\Sigma+i\Pi)N(\Sigma+i\Pi)\\ &\qquad+\bar{N}(1+\gamma_5)(\Sigma-i\Pi)\lambda_6N(\Sigma+i\Pi)]\\ =&\tfrac{1}{2}(d-f)\,\mathrm{Tr}\{\bar{B}B\lambda_6+(i/2f_\pi)\bar{B}B[\lambda_6,\Pi]\}\\ &+\tfrac{1}{2}(d+f)\,\mathrm{Tr}\{\bar{B}\lambda_6B+(i/2f_\pi)\bar{B}[\lambda_6,\Pi]B\}+O(\pi^2)\end{aligned}\quad (3.1)$$

and, where d and f are free parameters, Eq. (3.1) states the essential results of Refs. 19 and 20 that the parity-violating (s-wave) decay amplitudes are related to the parity-conserving $B'\to B$ ($\Delta S=\pm1$) transition by the substitution rule

$$(i/2f_\pi)[\lambda_6,\Pi]\to\lambda_6. \quad (3.2)$$

To compute the s- and p-wave amplitudes we must incorporate Eqs. (2.19) and (3.1), and compute them to order f^{-1}. The results are essentially those of Lee and Swift[21] based on a dynamical model. We record

[12] This observation is due to W. A. Bardeen and we borrow his argument. [See also, S. Weinberg, Phys. Rev. **166**, 1568 (1968).] I thank Dr. Bardeen for informing me of his results in advance of publication.

[13] Of course $[Q_5{}^\alpha,U^\dagger(i\gamma_5\Pi)]$ is a function of the Π field.

[14] See, for previous reviews on the subject, for instance, B. W. Lee, in Proceedings of the Argonne International Conference on Weak Interactions, Argonne National Laboratory Report No. ANL-7130, 1965, p. 421 (unpublished).

[15] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963); B. W. Lee, *ibid.* **12**, 83 (1964).

[16] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).

[17] H. T. Nieh (to be published).

[18] M. Gell-Mann, Ref. 16; R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1964).

[19] Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters **16**, 380 (1966).

[20] L. S. Brown and C. Sommerfield, Phys. Rev. Letters **16**, 751 (1966); S. Badier and C. Bonchiat, Phys. Letters **20**, 529 (1966).

[21] B. W. Lee and A. R. Swift, Phys. Rev. **136**, B228 (1964).

the results[22]:

$$
\begin{aligned}
B(\Lambda_-^0) &= \left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Lambda+m_N}{2\sqrt{3}}\left[-\frac{(d-f)}{m_\Sigma-m_N}(1+x) -\frac{(3f+d)}{m_\Lambda-m_N}-\frac{C(2-x)}{\mu_\pi^2\mu_k^2}\right],\\
B(\Sigma_+^+) &= i\left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Sigma+m_N}{6\sqrt{2}}(1+x)\left[\frac{3(d-f)}{m_\Sigma-m_N} +\frac{(3f+d)}{m_\Lambda-m_N}\right],\\
B(\Sigma_-^-) &= \left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Sigma+m_N}{6\sqrt{2}}\left[\frac{(3f+d)}{m_\Lambda-m_N}(1+x) +3\frac{(d-f)}{m_\Sigma-m_N}(1-x)+\frac{6XC}{\mu_\pi^2-\mu_k}\right],\\
B(\Xi_-^-) &= i\left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Xi+m_\Lambda}{2\sqrt{3}}\left[\frac{(3f-d)}{m_\Xi-m_\Lambda}x -\frac{(d+f)}{m_\Xi-m_\Sigma}(1+x)-\frac{C(1-2X)}{\mu_\pi^2-\mu_k^2}\right];
\end{aligned}
\tag{3.3}
$$

and

$$
\begin{aligned}
A(\Lambda_-^0) &= (i/4\sqrt{3}f)(-d-3f),\\
A(\Sigma_+^+) &= 0,\\
A(\Sigma_-^-) &= (i/2\sqrt{2}f)(d-f),\\
A(\Xi_-^-) &= (i/4\sqrt{3}f)(d-3f),
\end{aligned}
\tag{3.4}
$$

where A and B are the usual invariant s-wave and p-wave amplitudes[14]; the term proportional to C is the K-pole contribution with C denoting the strength of the K-π transition; and x is defined so that

$$\frac{1-x}{x}=\left(\frac{D}{F}\right)_{\text{axial}}=\left(\frac{D}{F}\right)_{\text{pseudovector}}$$

in Eq. (2.24).

When we allow terms linear in the first derivatives of fields, there are 12 additional terms which have the desired transformation properties under CP and chiral $SU(3)\times SU(3)$ transformations. These terms are recorded in the Appendix. When we expand these terms in the particle fields B and π, we obtain 12 independent terms which mediate the processes $B'\to B$ and $B'\to B+\pi$ ($\Delta S=\pm1$). Collecting these terms, we write the addition to $\mathfrak{L}_{\mathrm{NL}}^{(1)}$ in Eq. (3.1).

$$
\begin{aligned}
\mathfrak{L}_{\mathrm{NL}}^{(2)} =& \tfrac{1}{2}(d'-f')\,\mathrm{Tr}\{\bar{B}i\gamma\cdot\partial B\lambda_6+(i/2f)\bar{B}i\gamma\cdot\partial B[\lambda_6,\pi]+\mathrm{H.c.}\}+\tfrac{1}{2}(d'+f')\,\mathrm{Tr}\{\bar{B}\lambda_6 i\gamma\cdot\partial B\\
&+(i/2f)\bar{B}[\lambda_6,\pi]i\gamma\cdot\partial B+\mathrm{H.c.}\}+\tfrac{1}{2}(d''-f'')\,\mathrm{Tr}\{\bar{B}\gamma_5 i\gamma\cdot\partial B\lambda_6+(i/2f)\bar{B}\gamma_5 i\gamma\cdot\partial B[\lambda_6,\pi]+\mathrm{H.c.}\}\\
&+\tfrac{1}{2}(d''+f'')\,\mathrm{Tr}\{\bar{B}\lambda_6\gamma_5 i\gamma\cdot\partial B+(i/2f)\bar{B}[\lambda_6,\pi]\gamma_5 i\gamma\cdot\partial B+\mathrm{H.c.}\}+a_1\bar{B}[\lambda_6,\partial_\mu\pi]_+\gamma^\mu B+a_2\bar{B}\gamma_\mu B[\lambda_6,\partial^\mu\pi]_+\\
&+a_3\bar{B}\gamma_\mu\partial^\mu\pi B\lambda_6+a_4\bar{B}\gamma_\mu\lambda_6 B\partial^\mu\pi+b_1\bar{B}[\lambda_6,\partial_\mu\pi]_+\gamma_5\gamma^\mu B+b_2\bar{B}\gamma_5\gamma_\mu B[\lambda_6,\partial^\mu\pi]_++b_3B\gamma_5\gamma_\mu\partial^\mu\pi B\lambda_6\\
&+b_4\bar{B}\gamma_5\gamma_\mu\lambda_6 B\partial^\mu\pi.
\end{aligned}
\tag{3.5}
$$

We shall make several remarks on the Lagrangian (3.5):

(1) The first two terms are equivalent on the mass shell to Eq. (3.1) in $SU(3)$ limit. In broken $SU(3)$, these terms give a first-order deviation from Eq. (3.1) proportional to baryon mass differences.

(2) The third and fourth terms vanish identically on the mass shell in the $SU(3)$ limit. This is in accordance with the observation of Gell-Mann[16] and Lee and Swift[21] that in this limit the parity-violating process $B'\to B$ ($\Delta S=\pm1$) is forbidden if the interaction responsible transforms like λ_6. In broken $SU(3)$, this process is allowed and is of the first order in the symmetry breaking. These two terms could represent phenomenologically the mechanism proposed by Kumar and Pati,[7] in which the parity-violating $\Delta S=\pm1$ transition $B'\to B$ is mediated by $B'\to B+K_1^0$ (strong interactions) followed by $K_1^0\to$ vacuum [the latter process is also of the first order in the $SU(3)$ symmetry breaking]. In the phenomenological approach one need not commit oneself to any particular underlying mechanism for this transition.

(3) The first four terms clearly satisfy the soft-pion limit applied to nonleptonic decays of hyperons, i.e., the processes $B'\to B+\Pi$ and $B'\to B$ (both $\Delta S=\pm1$) are related by substitution rule (3.2).

(4) The parity-violating derivative couplings (terms multiplied by a_i's) vanish in the $SU(3)$ limit. They are first order in symmetry breaking on the mass shell. These couplings vanish in the soft-pion limit.

(5) The parity-conserving derivative couplings (terms multiplied by b_i's) vanish in the soft-pion limit. However they are of the zeroth order in the $SU(3)$ breaking. Therefore the presence of these terms is catastrophic and invalidates the usual current-algebra analyses. *The extant current-algebra analyses make the tacit assumption that these terms are absent.*

Thus the "success" of the current algebra with regard to the nonleptonic hyperon decays must be

[22] These results differ from those of Refs. 19 and 20 by the K-meson pole term. Since K mesons are coupled to the baryons derivatively [see Eq. (2.19)], the K-meson pole contributions vanish in the soft-pion limit.

understood in the light of the comment (5) above. The salient feature of including derivative couplings (3.5) is that it allows the parity-violating $\Delta S=\pm 1$ process $B'\to B$ [see comments (2) and (3)]. Contributions of the third and fourth terms of Eq. (3.5) to hyperon decays have already been considered by Kumar and Pati. Denoting these contributions to the s- and p-wave amplitudes by A' and B', we have

$$
\begin{aligned}
A'(\Lambda_-^0)&=-i\left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Lambda-m_N}{2\sqrt{3}}\left[(d''-f'')(1+x)\right.\\
&\qquad\left.\times\frac{m_\Sigma-m_N}{m_\Sigma+m_N}+(d''+3f'')\frac{m_\Lambda-m_N}{m_\Lambda+m_N}\right],\\
A'(\Sigma_+^+)&=-i\left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Sigma-m_N}{6\sqrt{2}}(1+x)\left[3(f''-d'')\right.\\
&\qquad\left.\times\frac{m_\Sigma-m_N}{m_\Lambda+m_N}-(d''+3f'')\frac{m_\Sigma-m_N}{m_\Lambda+m_N}\right],\\
A'(\Sigma_-^-)&=-i\left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Sigma-m_N}{6\sqrt{2}}\\
&\qquad\times\left[-(3f''+d'')(1+x)\frac{m_\Lambda-m_N}{m_\Lambda+m_N}\right.\\
&\qquad\qquad\left.+3(f''-d'')(1-x)\frac{m_\Sigma-m_N}{m_\Sigma+m_N}\right],\\
A'(\Xi_-^-)&=-i\left(\frac{1}{f_\pi}\frac{G_A}{G_V}\right)\frac{m_\Xi-m_\Lambda}{2\sqrt{3}}\left[(d''3f'')\frac{m_\Xi-m_\Lambda}{m_\Xi+m_\Lambda}\right.\\
&\qquad\left.+(d''+f'')(1+x)\frac{m_\Xi-m_\Sigma}{m_\Xi+m_\Sigma}\right];
\end{aligned}
\tag{3.6}
$$

and

$$
\begin{aligned}
B'(\Lambda_-^0)&=(i/4\sqrt{3}f_\pi)(m_\Lambda-m_N)(-d''-3f''),\\
B'(\Sigma_+^+)&=0,\\
B'(\Sigma_-^-)&=(i/2\sqrt{2}f_\pi)(m_\Sigma-m_N)(d''-f''),\\
B'(\Xi_-^-)&=(i/4\sqrt{3}f_\pi)(m_\Xi-m_\Lambda)(d''-3f'').
\end{aligned}
\tag{3.7}
$$

Equation (3.6) and (3.7) are to be added to Eqs. (3.4) and (3.3), respectively. The numerical comparison of this model with experiment was performed by Kumar and Pati,[7] and we shall not repeat that here. There is a minor difference between our expressions (3.3), (3.4), (3.6), and (3.7), and the corresponding ones in Kumar and Pati. This difference stems from the fact that Kumar and Pati assume the pseudoscalar baryon-meson couplings obey $SU(3)$, while our consideration in the previous section dictates that it be the pseudovector couplings that obey the $SU(3)$ pattern. This difference is insignificant numerically, however.

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APPENDIX

The 12 independent derivative couplings for nonleptonic decays of hyperons, which are CP-invariant and transform like $(\lambda_6)_\beta{}^\alpha$, may be written as

$$(\bar{N})_\beta{}^\alpha i\gamma\cdot\partial N_\kappa{}^\beta(\lambda_6)_\alpha{}^\kappa+\text{H.c.},\tag{A1}$$

$$(\bar{N})_\beta{}^\alpha(\lambda_6)_\kappa{}^\beta i\gamma\cdot\partial N_\alpha{}^\kappa+\text{H.c.},\tag{A2}$$

$$(\bar{N})_\beta{}^{\dot\alpha}(\lambda_6)_\kappa{}^\beta i\gamma\cdot\partial(N_1)_{\dot\alpha}{}^\kappa+\text{H.c.},\tag{A3}$$

$$(\bar{N}_1)_{\dot\beta}{}^\alpha i\gamma\cdot\partial(N_1)_\kappa{}^{\dot\beta}(\lambda_6)_\alpha{}^\kappa+\text{H.c.},\tag{A4}$$

$$(\bar{N}_2)_\beta{}^{\dot\alpha}(\lambda_6)_\kappa{}^\beta i\gamma\cdot\partial(N_2)_{\dot\alpha}{}^\kappa+\text{H.c.},\tag{A5}$$

$$(\bar{N})_{\dot\beta}{}^\alpha i\gamma\cdot\partial(N_2)_\kappa{}^{\dot\beta}(\lambda_6)_\alpha{}^\kappa+\text{H.c.},\tag{A6}$$

$$\bar{N}_\beta{}^\alpha\gamma_\mu(N_1)_{\dot\kappa}{}^\beta i(\partial^\mu M)_\lambda{}^{\dot\kappa}(\lambda_6)_\alpha{}^\lambda+\text{H.c.},\tag{A7}$$

$$\bar{N}_{\dot\beta}{}^{\dot\alpha}\gamma_\mu(N_2)_\kappa{}^{\dot\beta}(\lambda_6)_\lambda{}^\kappa i(\delta^\mu M)_{\dot\alpha}{}^\lambda+\text{H.c.},\tag{A8}$$

$$\bar{N}_\beta{}^\alpha\gamma_\mu i(\partial^\mu M)_{\dot\kappa}{}^\beta(N_2)_\lambda{}^{\dot\kappa}(\lambda_6)_\alpha{}^\lambda+\text{H.c.},\tag{A9}$$

$$\bar{N}_\beta{}^\alpha\gamma_\mu(\lambda_6)_\kappa{}^\beta(N_1)_\lambda{}^\kappa(i\partial^\mu M)_\alpha{}^{\dot\lambda}+\text{H.c.},\tag{A10}$$

$$\bar{N}_\beta{}^\alpha\gamma_\mu(\lambda_6)_\kappa{}^\beta(i\partial^\mu M)_{\dot\lambda}{}^\kappa(N_2)_\alpha{}^{\dot\lambda}+\text{H.c.},\tag{A11}$$

$$\bar{N}_{\dot\beta}{}^{\dot\alpha}\gamma_\mu(i\partial^\mu M)_\kappa{}^{\dot\beta}(\lambda_6)_\lambda{}^\kappa(N_2)_{\dot\alpha}{}^\lambda+\text{H.c.}\tag{A12}$$

CP invariance of the above expressions follows from

$$PC\,N_\beta{}^\alpha(\mathbf{x},t)C^{-1}P^{-1}=C(\bar{N}_\alpha{}^\beta)^t(-\mathbf{x},t),$$

where C is a 4×4 matrix satisfying $C\gamma_\mu C^{-1}=-\gamma_\mu{}^t$, and the superscript t refers to the transposition in the Dirac spinor space, and

$$PC\,M_{\dot\beta}{}^\alpha(\mathbf{x},t)C^{-1}P^{-1}=M_\alpha{}^{\dot\beta}(-\mathbf{x},t).$$

The definitions of N_1 and N_2 appear in Eqs. (2.4a) and (2.4b). To show the completeness of the above 12 expressions, it is necessary to keep in mind the identities

$$M_{\dot\beta}{}^\alpha M_\gamma{}^{\dot\beta}=f_\pi{}^2\delta_\gamma{}^\alpha,$$

$$M_\beta{}^{\dot\alpha}M_{\dot\kappa}{}^\beta=f_\pi{}^2\delta_{\dot\kappa}{}^{\dot\alpha},$$

from which follow

$$(\partial_\mu M_{\dot\beta}{}^\alpha)M_\kappa{}^{\dot\beta}=M_{\dot\beta}{}^\alpha(\partial_\mu M_\kappa{}^{\dot\beta}),\quad\text{etc}.$$