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Structure and Extensions of a Theory of Currents*

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We show that Sugawara's theory of currents can be obtained as a formal limit of the massive Yang-Mills theory. In this limit, $g_0 \to 0$ and $m_0 \to 0$ in such a way that $m_0^2/g_0^2 = C$, the constant appearing in Sugawara's theory. The limiting procedure is used to incorporate electromagnetism, the hypothesis of partially conserved axial-vector current, and $SU(3)$ breaking into the theory, retaining canonical structure for electromagnetism.

I. INTRODUCTION

THE possibility of a theory of hadrons using only currents as coordinates has recently been discussed by a number of authors.[1] The motivations for such a theory are quite attractive: (a) Matrix elements of electromagnetic and weak currents are in principle measurable quantities; (b) the success of current algebra[2]; (c) such theories offer the possibility of incorporating all hadrons into a "nuclear democracy," in that no particular particles are singled out by having only their coordinates appear.

More recently, a nontrivial candidate for such a theory was put forth by Sugawara.[3] This theory is non-Lagrangian and noncanonical, consisting of an energy-momentum tensor given as an explicit function of the currents, together with the algebra of fields[4,5] among the currents. The consistency check of the theory involves a remarkable interlocking of internal symmetry [e.g., $SU(3)\otimes SU(3)$], Poincaré invariance, and the Schwinger term of the algebra of fields. Because of the theory's noncanonical structure, and because it allows no obvious breakup into free and interaction Hamiltonians, particle interpretation is not straightforward, apparently involving actual solution of the theory. If the theory is indeed to describe a "nuclear democracy," this is perhaps not surprising. We shall have nothing to say here about the difficult problem of solving the theory, addressing ourselves to the more formal matters of structure and extension to a more physical theory.

In Sec. II, we exhibit Sugawara's theory as a formal limit of the massive Yang-Mills theory.[6] The limit is essentially a scale transformation on the spin-one fields involving the bare coupling and bare mass going to zero with a constant ratio. In this limit, one finds that the momenta canonically conjugate to the spin-one fields vanish. Sugawara's equations of motion turn out to be exactly the statement of this fact. In Sec. III, it is pointed out that the limit procedure allows us to couple large classes of canonical matter fields into the theory, although such "hybrid" theories (being non-Lagrangian but canonical with respect to all but the currents) do not appear useful except in the case of electromagnetism (and perhaps leptons). The incorporation of electromagnetism is given in detail. Towards the end of Sec. III, we extend the notion of the limit procedure to include scalar and pseudoscalar matter fields. Again their bare masses go to zero and their canonical momenta vanish. Their kinetic-energy terms in the energy-momentum tensor vanish also, but, even in the limit, the scalar and pseudoscalar densities have the correct Poincaré transformation properties. Thus, by taking the limit of Yang-Mills theories with the hypothesis of partially conserved axial-vector current (PCAC) and/or $SU(3)$ symmetry breaking, we in-

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[1] R. F. Dashen and D. H. Sharp, Phys. Rev. **165**, 1857 (1968); D. H. Sharp, *ibid.* **165**, 1867 (1968); C. G. Callen, R. F. Dashen, and D. H. Sharp, *ibid.* **165**, 1883 (1968). Properties of such theories have also been discussed by C. Sommerfield (unpublished). We understand that the first suggestion of such theories is due to M. Gell-Mann.

[2] For a recent review, see R. F. Dashen, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 51.

[3] H. Sugawara, Phys. Rev. **170**, 1659 (1968).

[4] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[5] In this context, by "algebra of fields" we mean only the commutators among currents, and not those of currents with their time derivatives. The latter are slightly different in the Sugawara theory. (A certain term in the algebra of fields commutator goes to zero in our limit. See Sec. II for details.)

[6] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954); see also Ref. 4.

corporate these features into a "pure" (noncanonical) theory. The details of this are given in Sec. IV.

II. LIMITING PROCEDURE

We now proceed to show that the Sugawara theory can be obtained as a (formal) limit of the Yang-Mills theory. The limit to be taken is the following: We first express all operators in terms of currents $J_{a\mu}(x)$, related to the original Yang-Mills fields $\phi_{a\mu}(x)$ by

$$\phi_{a\mu}(x)=(g_0/m_0^2)J_{a\mu}(x)\,, \tag{2.1}$$

where a is an internal group index (including also vector or axial-vector labeling); g_0 and m_0 are the bare coupling and the bare mass, respectively. We then let $g_0\to 0$ and $m_0\to 0$, but in such a way that $m_0^2/g_0^2=C$, the constant appearing in Sugawara's theory.

Let us start with the massive Yang-Mills[6] theory, the Lagrangian density of which is[7]

$$\mathcal{L}(x)=-\tfrac{1}{4}F_{a\mu\nu}(x)F_a{}^{\mu\nu}(x)+\tfrac{1}{2}m_0^2\phi_{a\mu}(x)\phi_a{}^\mu(x)\,, \tag{2.2}$$

where

$$F_{a\mu\nu}(x)\equiv\partial_\mu\phi_{a\nu}(x)-\partial_\nu\phi_{a\mu}(x) -\tfrac{1}{2}g_0C_{abc}\{\phi_{b\mu}(x)\phi_{c\nu}(x)+\phi_{c\nu}(x)\phi_{b\mu}(x)\}\,. \tag{2.3}$$

C_{abc} are the internal group structure constants. The equations of motion implied by the Lagrangian are

$$\partial^\mu F_{a\mu\nu}(x)+m_0^2\phi_{a\nu}(x) =\tfrac{1}{2}g_0C_{abc}\{F_{b\nu\mu}(x)\phi_c{}^\mu(x)+\phi_c{}^\mu(x)F_{b\nu\mu}(x)\}\,. \tag{2.4}$$

The canonical commutation rules are

$$[\phi_{ak}(\mathbf{x}t),\phi_{br}(\mathbf{y}t)]=0\,, \tag{2.5a}$$

$$[F_{a0k}(\mathbf{x}t),\phi_{br}(\mathbf{y}t)]=i\delta_{ab}g_{kr}\delta^{(3)}(\mathbf{x}-\mathbf{y})\,, \tag{2.5b}$$

$$[F_{a0k}(\mathbf{x}t),F_{b0r}(\mathbf{y}t)]=0\,, \tag{2.5c}$$

where k, r are space component indices ($k,r=1,2,3$).

These commutation rules imply

$$[\phi_{a0}(\mathbf{x}t),\phi_{b0}(\mathbf{y}t)]=i(g_0/m_0^2)C_{abc}\phi_{c0}(\mathbf{x}t)\delta^{(3)}(\mathbf{x}-\mathbf{y})\,, \tag{2.6a}$$

$$[\phi_{a0}(\mathbf{x}t),\phi_{bk}(\mathbf{y}t)]=i(g_0/m_0^2)C_{abc}\phi_{ck}(\mathbf{x}t)\delta^{(3)}(\mathbf{x}-\mathbf{y}) +(i/m_0^2)\delta_{ab}\partial_k{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y})\,, \tag{2.6b}$$

$$[\partial_0\phi_{ak}(\mathbf{x}t)-\partial_k\phi_{a0}(\mathbf{x}t),\,\phi_{br}(\mathbf{y}t)] =i\delta_{ab}g_{kr}\delta^{(3)}(\mathbf{x}-\mathbf{y})+i(g_0/m_0^2)C_{abc}\phi_{ck}(\mathbf{x}t)\partial_r{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y}) -i(g_0^2/m_0^2)C_{ace}C_{bde}\phi_{cr}(\mathbf{x}t)\phi_{dk}(\mathbf{x}t)\delta^{(3)}(\mathbf{x}-\mathbf{y})\,. \tag{2.6c}$$

The (symmetrized) stress-energy tensor $\theta_{\mu\nu}(x)$ is

$$\theta_{\mu\nu}(x)=\tfrac{1}{2}[F_{a\mu\lambda}(x)F^\lambda{}_{a\cdot\nu}(x)+F_{a\nu\lambda}(x)F^\lambda{}_{a\cdot\mu}(x)] +\tfrac{1}{2}m_0^2[\phi_{a\mu}(x)\phi_{a\nu}(x)+\phi_{a\nu}(x)\phi_{a\mu}(x)]-g_{\mu\nu}\mathcal{L}(x)\,. \tag{2.7}$$

We take the Poincaré generators

$$P_\mu=\int\theta_{0\mu}(x)d^3x\,, \tag{2.8a}$$

$$M_{\mu\nu}=\int[x_\mu\theta_{0\nu}(x)-x_\nu\theta_{0\mu}(x)]d^3x\,, \tag{2.8b}$$

and $i[P_\mu,A(x)]=\partial_\mu A(x)$, etc.[8]

Let us now express all equations and commutation rules by $J_{a\mu}(x)$ and $\tilde{F}_{a\mu\nu}(x)$, where $J_{a\mu}(x)$ is defined in Eq. (2.1) and

$$F_{a\mu\nu}(x)=(g_0/m_0^2)\tilde{F}_{a\mu\nu}(x)\,. \tag{2.9}$$

Hence, ($C=m_0^2/g_0^2$)

$$\tilde{F}_{a\mu\nu}(x)=\partial_\mu J_{a\nu}(x)-\partial_\nu J_{a\mu}(x) -(1/2C)C_{abc}\{J_{b\mu}(x)J_{c\nu}(x)+J_{c\nu}(x)J_{b\mu}(x)\}\,. \tag{2.10}$$

The equation of motion (2.4) goes over into

$$\partial^\mu\tilde{F}_{a\mu\nu}(x)+m_0^2J_{a\nu}(x) =(1/2C)C_{abc}\{\tilde{F}_{b\nu\mu}(x)J_c{}^\mu(x)+J_c{}^\mu(x)\tilde{F}_{b\nu\mu}(x)\}\,. \tag{2.11}$$

The canonical commutation rules Eqs. (2.5a)–(2.5c) go into

$$[J_{ak}(\mathbf{x}t),J_{br}(\mathbf{y}t)]=0\,, \tag{2.12a}$$

$$[\tilde{F}_{a0k}(\mathbf{x}t),J_{br}(\mathbf{y}t)]=iCm_0^2\delta_{ab}g_{kr}\delta^{(3)}(\mathbf{x}-\mathbf{y})\,, \tag{2.12b}$$

$$[\tilde{F}_{a0k}(\mathbf{x}t),\tilde{F}_{b0r}(\mathbf{y}t)]=0\,. \tag{2.12c}$$

In the limit described above, $\tilde{F}_{a0k}(\mathbf{x}t)$ commutes with all the canonically independent variables, namely, with $J_{ak}(\mathbf{y}t)$ and $\tilde{F}_{b0r}(\mathbf{y}t)$, and hence has to be a c number. But the vacuum expectation value of $\tilde{F}_{a0k}(\mathbf{x}t)$ is zero. Therefore, $\tilde{F}_{a0k}(\mathbf{x}t)\to 0$ in the limit, and from (2.11b) it follows that $\tilde{F}_{a0k}(\mathbf{x}t)$ vanishes like m_0^2. Thus,

$$\lim_{\substack{m_0\to 0,\, g_0\to 0\\ m_0^2/g_0^2=C}}\tilde{F}_{a0k}(\mathbf{x}t)=0\,. \tag{2.13}$$

This is consistent with Eq. (2.11). Equations (2.10) and (2.13), taken together, lead to

$$\partial_\mu J_{a\nu}(x)-\partial_\nu J_{a\mu}(x) =(1/2C)C_{abc}\{J_{b\mu}(x)J_{c\nu}(x)+J_{c\nu}(x)J_{b\mu}(x)\}\,, \tag{2.14}$$

which is the equation of motion in Sugawara's theory. The commutation rules Eqs. (2.6) go into

$$[J_{a0}(\mathbf{x}t),J_{b0}(\mathbf{y}t)]=iC_{abc}J_{c0}(\mathbf{x}t)\delta^{(3)}(\mathbf{x}-\mathbf{y})\,, \tag{2.15a}$$

$$[J_{a0}(\mathbf{x}t),J_{bk}(\mathbf{y}t)]=iC_{abc}J_{ck}(\mathbf{x}t)\delta^{(3)}(\mathbf{x}-\mathbf{y}) +iC\delta_{ab}\partial_k{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y})\,, \tag{2.15b}$$

$$[\partial_0J_{ak}(\mathbf{x}t)-\partial_kJ_{a0}(\mathbf{x}t),\,J_{br}(\mathbf{y}t)] =iC_{abc}J_{ck}(\mathbf{x}t)\partial_r{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y}) -(i/C)C_{ace}C_{bde}J_{cr}(\mathbf{x}t)J_{dk}(\mathbf{x}t)\delta^{(3)}(\mathbf{x}-\mathbf{y})\,. \tag{2.15c}$$

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[7] Our metric is $g_{00}=1$, $g_{ii}=-1$; i is the spatial index running from 1 to 3.

[8] Notice the sign difference (as compared with Sugawara) in the relation between $\theta_{\mu\nu}$ and the Poincaré generators. In the limit our $\theta_{\mu\nu}$ will also be the negative of Sugawara's.

Equations (2.15a) and (2.15b) are the commutators which appear in Sugawara's paper. Equation (2.15c) may be derived from Eq. (2.15b) and the equation of motion Eq. (2.14).[9] Notice also that, from Eq. (2.11), it follows that

$$\partial^\nu J_{a\nu}(x)=0. \tag{2.16}$$

This is so because the right-hand side of Eq. (2.11) is the generator of gauge transformations, and hence is conserved. It remains to obtain the limit of the stress-energy tensor. To this end we note that, in our limit, $F_{a\mu\nu}\to 0$ [since $F_{a\mu\nu}(x)=g_0/m_0{}^2\tilde{F}_{a\mu\nu}(x)$ and $\tilde{F}_{a\mu\nu}(x)$ behaves like $m_0{}^2$ in this limit]. Thus, the limit of Eq. (2.7) is

$$\theta_{\mu\nu}(x)=(1/2C)[J_{a\mu}(x)J_{a\nu}(x)+J_{a\nu}(x)J_{a\mu}(x) -g_{\mu\nu}J_{a\lambda}(x)J_a{}^\lambda(x)], \tag{2.17}$$

which is the negative of Sugawara's stress-energy tensor as mentioned in Ref. 8.

Thus, we have shown that Sugawara's theory is a *formal* limit of the massive Yang-Mills theory, where $g_0\to 0$, $m_0\to 0$, while $m_0{}^2/g_0{}^2=C$. We emphasize "formal": Although the Heisenberg equations of motion are quite smooth in the limit, formally the wave-function renormalization of the original fields appears infinite. We see no reason, however, why this should imply ghosts in the limit.

Note finally that, in the limit, it is impossible to have operators canonically conjugate to the spatial currents. Such is easily seen to be inconsistent with the $\mu=i$, $\nu=j$ (purely spatial) part of the equation of motion (2.14). Of course, canonical momenta exist for all finite values of g_0, m_0; in this sense, the scale transform, in the limit, is improper.

III. INTRODUCTION OF ELECTROMAGNETISM; "HYBRID" THEORIES

The advantage of establishing the limit of Sec. II becomes clear when one tries to introduce other interactions into the theory. For example, the introduction of electromagnetic interactions may be achieved by first introducing them in the Yang-Mills theory, as prescribed by Lee and Zumino,[10] and then going to the limit specified above.

As in Ref. 10, we quantize the electromagnetic potentials in the Coulomb gauge. We first state the results in the limit. The commutation relations are

$$\begin{aligned}
&[A_r{}^\perp(\mathbf{x}t),A_k{}^\perp(\mathbf{y}t)]=0,\\
&[\dot{A}_r{}^\perp(\mathbf{x}t),A_k{}^\perp(\mathbf{y}t)]=-i\delta_{rk}{}^\perp\delta^{(3)}(\mathbf{x}-\mathbf{y}),\\
&[A_r{}^\perp(\mathbf{x}t),J_{bk}(\mathbf{y}t)]=0,\\
&[\dot{A}_r{}^\perp(\mathbf{x}t),J_{ak}(\mathbf{y}t)]=-ie_0C\xi_a\delta_{rk}{}^\perp\delta^{(3)}(\mathbf{x}-\mathbf{y}),\\
&[J_{a0}(\mathbf{x}t),J_{bk}(\mathbf{y}t)]\\
&\qquad=iC_{abc}[J_{ck}(\mathbf{x}t)-e_0C\xi_cA_k{}^\perp(\mathbf{x}t)]\delta^{(3)}(\mathbf{x}-\mathbf{y})\\
&\qquad\qquad+iC\delta_{ab}\partial_k{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y}),\\
&[J_{a0}(\mathbf{x}t),A_r{}^\perp(\mathbf{y}t)]=0,\\
&[J_{a0}(\mathbf{x}t),\dot{A}_r{}^\perp(\mathbf{y}t)]=0,
\end{aligned} \tag{3.1}$$

where $A_r{}^\perp$ is the transverse electromagnetic field and $\xi_a=\delta_{a3}$ for $SU(2)\otimes SU(2)$, $\xi_a=\delta_{a3}+(1/\sqrt{3})\delta_{a8}$ for $SU(3)\otimes SU(3)$. The stress-energy tensor is

$$\begin{aligned}\theta_{\mu\nu}(x)=(1/2C)[&J_{a\mu}(x)J_{a\nu}(x)+J_{a\nu}(x)J_{a\mu}(x)\\ &-g_{\mu\nu}J_{a\lambda}(x)J_a{}^\lambda(x)]+\tfrac{1}{2}[f_{\lambda\mu}(x)f_\nu{}^\lambda(x)\\ &+f_{\lambda\nu}(x)f_\mu{}^\lambda(x)-\tfrac{1}{2}g_{\mu\nu}f_{\lambda\sigma}(x)f^{\sigma\lambda}(x)],\end{aligned} \tag{3.2}$$

where

$$f_{\mu\lambda}(x)=\partial_\mu A_\lambda(x)-\partial_\lambda A_\mu(x), \tag{3.3}$$

with

$$\begin{aligned}&\nabla\cdot\mathbf{A}(x)=0,\\ &A_0(x)=e_0\nabla^{-2}\xi_aJ_{a0}(x).\end{aligned} \tag{3.4}$$

That is, $\theta_{\mu\nu}$ is just Sugawara's $\theta_{\mu\nu}$ plus a "free" photon $\theta_{\mu\nu}$. The interaction is manifest in the commutator structure. The equations of motion are

$$\begin{aligned}&\partial_\mu\tilde{J}_{a\nu}(x)-\partial_\nu\tilde{J}_{a\mu}(x)\\ &\quad=(1/2C)C_{abc}\{\tilde{J}_{b\mu}(x)\tilde{J}_{c\nu}(x)+\tilde{J}_{c\nu}(x)\tilde{J}_{b\mu}(x)\},\end{aligned} \tag{3.5}$$

where

$$\tilde{J}_{a\mu}(x)=J_{a\mu}(x)-e_0C\xi_aA_\mu(x). \tag{3.6}$$

Equation (3.5) is not surprising, in view of our result (2.13) and the way the electromagnetic field is introduced into the theory, namely, one replaces J by $\tilde{J}$ only in the $F_{a\mu\nu}F_a{}^{\mu\nu}$ part of the Yang-Mills Lagrangian, and then adds a free electromagnetic part to it.[10] Thus, the expression for $\theta_{\mu\nu}$ before the limit is different from Eq. (3.2) only by a term

$$\tfrac{1}{2}[F_{a\lambda\nu}(\varphi)F_{a\mu}{}^\lambda(\varphi)+F_{a\lambda\mu}(\varphi)F_{a\nu}{}^\lambda(\varphi)] +\tfrac{1}{4}g_{\mu\nu}F_{a\lambda\sigma}(\varphi)F_a{}^{\lambda\sigma}(\varphi),$$

where $F_{a\mu\nu}(\varphi)$ is defined by Eq. (2.3) with $\phi_{a\mu}$ replaced by $\varphi_{a\mu}=\phi_{a\mu}-(e_0/g_0)A_\mu\xi_a$. As a result of the limiting procedure, in complete analogy with the derivation in Sec. II, $F_{a\mu\nu}(\varphi)\to 0$ (as fast as g_0), which brings us to Eq. (3.2) for $\theta_{\mu\nu}$ and to the equation of motion Eq. (3.5).

Finally, let us mention the divergence equations

$$\partial_\mu J_a{}^\mu(x)=\tfrac{1}{2}e_0\xi_cC_{cab}[A_\mu(x)J_b{}^\mu(x)+J_b{}^\mu(x)A_\mu(x)] \tag{3.7}$$

or, for $SU(2)\otimes SU(2)$,

$$\partial_\mu J_+{}^\mu(x)=-\tfrac{1}{2}ie_0[A_\mu(x)J_+{}^\mu(x)+J_+{}^\mu(x)A_\mu(x)], \tag{3.8}$$

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[9] The two Weinberg sum rules [S. Weinberg, Phys. Rev. Letters **18**, 507 (1967)] follow also in Sugawara's theory as can be seen from Eqs. (2.15b) and (2.15c). The derivation goes through as in Ref. 4 for $SU(2)\otimes SU(2)$. Certain difficulties arise for the case of $SU(3)$. See J. J. Sakurai, Phys. Rev. Letters **19**, 803 (1967).

[10] T. D. Lee and B. Zumino, Phys. Rev. **163**, 1667 (1967).

etc., and similar expressions for $SU(3)\otimes SU(3)$. These can be easily derived from the energy-momentum tensor Eq. (3.2) and the commutation rules Eqs. (3.1). Equation (3.7) is essentially a consequence of gauge invariance. It can be used, together with PCAC (as introduced in Sec. IV), to derive low-energy theorems to first order in e_0.[11] Also, Cottingham's formula follows from Eq. (3.7) plus current algebra, so the usual approach to electromagnetic mass differences[12] is correct in Sugawara's theory. It should finally be mentioned that the limit of Schwinger's theory[13] of Yang-Mills plus electromagnetism is just the theory presented above with

$$J_{a\mu}(x)\to J_{a\mu}(x)+e_0C\xi_aA_\mu(x).$$

Other Hybrid Theories

The limit process used above to incorporate electromagnetism into the theory of currents can be used to incorporate any matter field (fields other than spin-one mesons) into the theory, but, as defined, the limit will leave invariant the canonical structure of the matter fields. Hence we call these "hybrid" theories: In general, to couple a matter field ψ to the massive Yang-Mills theory, one replaces (in the matter-field Lagrangian)

$$\partial_\mu\psi\to D_\mu\psi=\partial_\mu\psi-g_0T_a\phi_\mu{}^a\psi\,, \tag{3.9}$$

where T_a is the relevant internal symmetry generator. Then our scale transformation on the vector-meson structure yields in the limit a large class of theories

$$\theta_{\mu\nu}=\theta_{\mu\nu}{}^M(\psi,D_\mu\psi)+\theta_{\mu\nu}{}^S\,, \tag{3.10}$$

where $\theta_{\mu\nu}{}^M$ is the stress-energy tensor of the matter field, $\theta_{\mu\nu}{}^S$ is Sugawara's $\theta_{\mu\nu}$, and

$$D_\mu\psi=\partial_\mu\psi-(1/C)T_aJ_{a\mu}\psi\,. \tag{3.11}$$

Note that although no obvious Lagrangian exists for the system, the canonical structure of the matter fields persists (hence hybrid)—i.e., variables canonically conjugate to the matter fields persist, just as discussed above for electromagnetism.

As a very simple example, consider a system of π mesons and ρ mesons

$$\mathcal{L}=\tfrac{1}{2}(D_\mu\phi^aD^\mu\phi^a)-\tfrac{1}{2}\mu_0{}^2\phi^a\phi^a+\mathcal{L}_{\text{YM}}\,, \tag{3.12}$$

where μ_0 is the bare pion mass, $\mathcal{L}_{\text{YM}}$ is the massive Yang-Mills Lagrangian, and

$$D_\mu\phi^a=\partial_\mu\phi^a-\tfrac{1}{2}g_0\epsilon^{abc}[\rho_\mu{}^b,\phi^c]_+\,. \tag{3.13}$$

Going over to $\theta_{\mu\nu}$,

$$\theta_{\mu\nu}=\theta_{\mu\nu}{}^{\text{YM}}+\tfrac{1}{2}[D_\mu\phi^a,D_\nu\phi^a]_+ -\tfrac{1}{2}g_{\mu\nu}\{D_\lambda\phi^aD^\lambda\phi^a-\mu_0{}^2\phi^a\phi^a\}\,, \tag{3.14}$$

we obtain in the limit

$$\theta_{\mu\nu}=\theta_{\mu\nu}{}^S+\tfrac{1}{2}[D_\mu\phi^a,D_\nu\phi^a]_+ -\tfrac{1}{2}g_{\mu\nu}\{D_\lambda\phi^aD^\lambda\phi^a-\mu_0{}^2\phi^a\phi^a\} \tag{3.15}$$

(here $\theta_{\mu\nu}{}^S$ is just the vector part of $\theta_{\mu\nu}$; also see Ref. 8), with

$$D_\mu\phi^a=\partial_\mu\phi^a-(1/2C)\epsilon^{abc}[V_\mu{}^b,\phi^c]_+\,,\quad \rho_\mu{}^a=(g_0/m_0{}^2)V_\mu{}^a\,, \tag{3.16}$$

still canonically conjugate to the pion. Although such theories are completely consistent, we shall not present any details here because we consider them unacceptable within the rules of the game, in that the matter fields remain canonical and, in this sense, elementary. We feel such hybrid theories are useful[14] only for incorporating electromagnetism and perhaps leptons, for which canonical structure seems better founded.

On the other hand, as discussed in the next paragraph, at least scalar and pseudoscalar matter fields may be incorporated in the "pure" sense by extending our scale transformation to include them. In fact, it seems necessary to do this to incorporate PCAC and $SU(3)$ breaking into the pure theory.

Extended Limit Procedure

For concreteness, consider first the simple case of the π-ρ system discussed above. Suppose, at the level of the Yang-Mills Lagrangian, we introduce, in addition to our scale transformation on the vector fields, the analogous limit or scale transformation for the pion field

$$\phi_a=\phi_a'/g_0\,,\quad \mu_0\to 0\,,\quad g_0\to 0\,,\quad \mu_0/g_0=C'\,, \tag{3.17}$$

where ϕ_a' remains finite and C' is a constant. Then by reasoning entirely analogous to that for the vector fields, we learn that the variable canonically conjugate to the pion field goes to zero (like g_0), yielding the constraint equation on ϕ_a'

$$\partial_\mu\phi_a'=(1/2C)\epsilon^{abc}[V_\mu{}^b,\phi_c']_+\,. \tag{3.18}$$

With this extended-limit procedure, the stress-energy-momentum tensor goes over to[15]

$$\theta_{\mu\nu}=\theta_{\mu\nu}{}^S+\tfrac{1}{2}g_{\mu\nu}(C')^2\phi_a'\phi_a' \tag{3.19}$$

[11] See, e.g., S. M. Berman and Y. Frishman, Phys. Rev. **165**, 1555 (1968).

[12] T. Das, G. Guralnik, V. Mathur, F. Low, and J. Young, Phys. Rev. Letters **18**, 759 (1967). In particular, the divergence of the pion electromagnetic mass difference persists for physical pion mass, at least formally, using Bjorken's method. The calculation is essentially identical to that of M. B. Halpern and G. Segrè, Phys. Rev. Letters **19**, 611 (1967); **19**, 1000 (1967).

[13] See, e.g., J. Schwinger, Phys. Rev. Letters **19**, 1154 (1967).

[14] The hybrid theories may also be useful as an approximation scheme. For example, in the ρ bootstrap of S-matrix theory, one assumes the pion (approximately) elementary and looks for a composite ρ. The analog here might be the hybrid π-ρ system discussed above.

[15] Notice that C' may be taken to be zero; thus we have a choice whether or not ϕ_a' need appear in $\theta_{\mu\nu}$. The Poincaré invariance and equations of motion, etc., are independent of C' because $\phi_a'\phi_a'$ is a c number in the theory. Note also that in this toy theory ϕ_a' is not necessarily an observable. This will not be the case in the more realistic models of Sec. IV where ϕ_a' is proportional to the divergence of the axial-vector current.

(where $\theta_{\mu\nu}{}^S$ here is just the vector part), with the commutation relations

$$[V_0{}^a(\mathbf{x}),\phi_b'(\mathbf{y})]=i\epsilon^{abc}\phi_c'(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}),$$
$$[V_i{}^a(\mathbf{x}),\phi_b'(\mathbf{y})]=[\phi_a'(\mathbf{x}),\phi_b'(\mathbf{y})]=0,$$

plus those of the algebra of fields (among the currents). The equation of motion of ϕ_a' in the limit is easily calculated from Eq. (3.19) and turns out to be precisely Eq. (3.18). Moreover, the scaled pion field ϕ_a' (or pseudoscalar density) transforms as it should under the Poincaré group. Finally, note that the Hamiltonian stays explicitly positive semidefinite in the limit, etc.

In summary, considering theories of spin one plus scalar and pseudoscalar particles, we have learned that there is a hierarchy of consistent theories: If the limiting procedure is applied only to the spin-one fields, we obtain generally unacceptable hybrid theories with no Lagrangian but canonical matter fields; on the other hand, if the extended limit procedure is applied to all fields including matter fields, we obtain non-Lagrangian, noncanonical pure theories. In any case, the limiting procedure is characterized by the following mnemonic: Roughly speaking, one writes a Yang-Mills Lagrangian including matter fields, goes over to $\theta_{\mu\nu}$, and sets the desired canonical momenta to zero. All equations of motion are essentially this constraint (now in terms of the scaled densities). In the case of fermionic matter fields the limit procedure, if it works at all, is more complicated and is under consideration.

IV. APPLICATIONS

In this section, we want to show how to incorporate PCAC and $SU(3)$ symmetry breaking in a pure theory. We have in mind writing down a generalized Yang-Mills model with these features[16] and taking the limit, but this is straightforward, as discussed above, leaving only the mass terms of the spin-zero mesons. Thus in this section, we will present only the limit or pure theories. The reader will observe that all equations of motion are essentially the statement that canonical momenta in the original theory are set to zero.

Our first extension is to include PCAC in an $SU(2)\otimes SU(2)$ model. For this purpose, we may write down a Yang-Mills σ model[17] with bare parameters μ_0, $f_\pi{}^0$, g_0, m_0, where μ_0 and $f_\pi{}^0$ are the pion bare mass and bare decay amplitude, respectively, and perform the scale transformation

$$\phi_a=(1/g_0)\phi_a',\quad \sigma=(1/g_0)\sigma',\quad \phi_{a\mu}=(g_0/m_0{}^2)J_{a\mu},$$
$$g_0\to 0,\quad m_0\to 0,\quad \mu_0\to 0,\quad f_\pi{}^0\to\infty,\qquad (4.1)$$
$$(m_0{}^2/g_0{}^2)=C,\quad \tfrac{1}{2}(\mu_0{}^2/g_0{}^2)=C',\quad \mu_0{}^2f_\pi{}^0/g_0=\mu^2 f_\pi,$$

where μ and f_π are the physical pion mass and decay amplitude, respectively. In the limit, the commutation relations involving the pseudoscalar and scalar densities[18] are (dropping all primes)

$$[V_0{}^a(\mathbf{x}),\phi^b(\mathbf{y})]=i\epsilon^{abc}\phi^c(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}),$$
$$[V_0{}^a(\mathbf{x}),\sigma(\mathbf{y})]=[V_i{}^a(\mathbf{x}),\sigma(\mathbf{y})]=[V_i{}^a(\mathbf{x}),\phi^b(\mathbf{y})]=0,$$
$$[A_0{}^a(\mathbf{x}),\phi^b(\mathbf{y})]=i\sigma(\mathbf{x})\delta^{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y}),\qquad (4.2)$$
$$[A_0{}^a(\mathbf{x}),\sigma(\mathbf{y})]=-i\phi^a(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}),$$
$$[A_i{}^a(\mathbf{x}),\phi^b(\mathbf{y})]=[A_i{}^a(\mathbf{x}),\sigma(\mathbf{y})]=0,$$
$$[\phi^a(\mathbf{x}),\phi^b(\mathbf{y})]=[\phi^a(\mathbf{x}),\sigma(\mathbf{y})]=0,$$

plus the usual algebra of fields among the currents. These are to be taken together with the stress-energy-momentum tensor (taking $C'=1$)

$$\theta_{\mu\nu}=\theta_{\mu\nu}{}^S+g_{\mu\nu}(\sigma^2+\phi^a\phi^a-f_\pi\mu^2\sigma+\tfrac{1}{4}f_\pi{}^2\mu^2),\qquad (4.3)$$

where f_π is the physical pion decay amplitude and μ is the physical pion mass. This $\theta_{\mu\nu}$ is symmetric, satisfies Schwinger's condition[19]

$$[\theta_{00}(\mathbf{x}),\theta_{00}(\mathbf{y})]=i\{\theta_{0i}(\mathbf{x})+\theta_{0i}(\mathbf{y})\}\partial_i{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y}),\qquad (4.4)$$

and is conserved.

Before exhibiting the dynamical content of the theory, one comment is important. The combination $\sigma^2+\phi^2$ is left in $\theta_{\mu\nu}(C'\neq 0)$ only to guarantee positive semidefiniteness of the Hamiltonian in an obvious way: The transformation

$$\sigma=\sigma'+\tfrac{1}{2}f_\pi\mu^2\qquad (4.5)$$

allows $\theta_{\mu\nu}$ to be written in an explicitly positive-definite fashion. Because $\phi^2+\sigma^2$ commutes with all the operators in the theory, it can be taken to be a constant c number —and can be omitted ($C'=0$) if one had reason to believe $\theta_{\mu\nu}{}^S$ was large enough to guarantee positive definiteness in the presence of the linear σ term. In particular, none of the equations of motion or transformation properties depend on this consideration.[20]

By commuting the translation operators with the currents and using Eqs. (2.8), one easily establishes that

$$\partial_\mu V_a{}^\mu=0,\quad \partial_\mu A_a{}^\mu=f_\pi\mu^2\phi_a,\qquad (4.6a)$$
$$\partial_\mu V_\nu{}^a-\partial_\nu V_\mu{}^a=(1/2C)\epsilon^{abc}\{[V_\mu{}^b,V_\nu{}^c]_+ + [A_\mu{}^b,A_\nu{}^c]_+\},\qquad (4.6b)$$
$$\partial_\mu A_\nu{}^a-\partial_\nu A_\mu{}^a=(1/2C)\epsilon^{abc}\{[V_\mu{}^b,A_\nu{}^c]_+ + [A_\mu{}^b,V_\nu{}^c]_+\};\qquad (4.6c)$$

[16] Such as, e.g., the Lagrangian of B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968).

[17] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); J. Schwinger, Ann. Phys. (N. Y.) **2**, 407 (1957). A possible approach to nonconserved currents without the introduction of ϕ_a, σ is to put different coefficients in $\theta_{\mu\nu}$, keeping the same algebra, but this leads to trouble with Poincaré invariance.

[18] Note that although we call these densities ϕ_a, σ, no canonically conjugate variables are introduced. In fact, it is simple to show that the existence of ordinary momenta conjugate to ϕ_a, σ is inconsistent in the theory.

[19] J. Schwinger, Phys. Rev. **130**, 406 (1963); **130**, 800 (1963).

[20] When $\phi^2+\sigma^2$ is omitted, one need only start with the commutators involving σ with the currents: σ commutes with $V_\mu{}^a$, $A_\kappa{}^a$ and $[A_0{}^a(\mathbf{x}),\sigma(\mathbf{y})]\equiv iD^a(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y})$. Then $D^a(x)$ turns out, via the equations of motion, to be $\mu^2 f_\pi D_a(x)=\partial^\mu A_\mu{}^a(x)$. It would be interesting in this manner to use the Jacobi identity, etc., to find the most general form of the theory with the extra σ term.

that is, we have PCAC, but the "equations of motion" are unchanged. Note that, by virtue of these equations, ϕ^a and σ are observables (if $V_\mu{}^a$, $A_\mu{}^a$ are)—that is, the matrix elements of ϕ^a can be calculated from the matrix elements of $A_\mu{}^a$ through multiplication by the momentum difference of the two states. σ can then be calculated from the commutator of $A_0{}^a$ with ϕ^b. In this sense, one might say that ϕ^a and σ are not really independent coordinates in the theory.

By further commuting the translation operators with ϕ^a, σ we obtain the equations of motion of these fields

$$\begin{aligned} \partial_\mu\phi^a(x) &= (1/2C)\{\epsilon^{abc}[V_\mu{}^b,\phi^c]_+ - [A_\mu{}^a,\sigma]_+\}, \\ \partial_\mu\sigma(x) &= (1/2C)[A_\mu{}^a,\phi^a]_+. \end{aligned} \tag{4.7}$$

With the help of the equations of motion, it can also be established that all the coordinates transform under the Lorentz group as they should [see Eqs. (2.8)]:

$$\begin{aligned} [M_{\mu\nu},\phi^a(x)] &= -i(x_\mu\partial_\nu - x_\nu\partial_\mu)\phi^a(x), \\ [M_{\mu\nu},\sigma(x)] &= -i(x_\mu\partial_\nu - x_\nu\partial_\mu)\sigma(x), \\ [M_{\mu\nu},V_\rho{}^a(x)] &= -i(x_\mu\partial_\nu - x_\nu\partial_\mu)V_\rho{}^a \\ &\quad + i(g_{\nu\rho}V_\mu{}^a - g_{\mu\rho}V_\nu{}^a), \\ [M_{\mu\nu},A_\rho{}^a(x)] &= -i(x_\mu\partial_\nu - x_\nu\partial_\mu)A_\rho{}^a \\ &\quad + i(g_{\nu\rho}A_\mu{}^a - g_{\mu\rho}A_\nu{}^a). \end{aligned} \tag{4.8}$$

Useful identities in the computation of the axial-vector current transformation properties are

$$\begin{aligned} [\theta_{0i}(\mathbf{x}),A_0{}^a(\mathbf{y})] &= iA_0{}^a(\mathbf{x})\partial_i{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y}), \\ [\theta_{00}(\mathbf{x}),A_0{}^a(\mathbf{y})] &= -if_\pi\mu^2\phi^a(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}) \\ &\quad + iA_j{}^a(\mathbf{x})\partial_j{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y}). \end{aligned} \tag{4.9}$$

Finally, the commutation relations of the generators of the Poincaré group come out as they should:

$$\begin{aligned} [P_\mu,P_\nu] &= 0, \quad [M_{\mu\nu},P_\rho] = +i(g_{\nu\rho}P_\mu - g_{\mu\rho}P_\nu), \\ [M_{\mu\nu},M_{\rho\kappa}] &= i(g_{\nu\rho}M_{\mu\kappa} + g_{\mu\kappa}M_{\nu\rho} \\ &\quad - g_{\nu\kappa}M_{\mu\rho} - g_{\mu\rho}M_{\nu\kappa}). \end{aligned} \tag{4.10}$$

Note also that the theory could be rewritten (in a nonlinear fashion) in terms of the currents (and their time derivatives) alone: The pseudoscalar density can be written as the divergence of the axial-vector current and the scalar density can be expressed as a function of ϕ^a through $\sigma^2+\phi^2=$const. Moreover, of course, no necessary particle content is implied for any of the densities in the model, in particular the scalar density.

PCAC in $SU(3)\otimes SU(3)$

To include PCAC at the level of $SU(3)\otimes SU(3)$, a possible approach is to introduce nonets of currents along with nonets of scalar and pseudoscalar densities, as one would obtain in the limit of a nonet σ model coupled to Yang-Mills fields. The relevant commutation relations[21] are (a, b, c run from 1 to 9)

$$\begin{aligned} [V_0{}^a(\mathbf{x}),\phi^b(\mathbf{y})] &= if^{abc}\phi^c(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}), \\ [V_0{}^a(\mathbf{x}),\sigma^b(\mathbf{y})] &= if^{abc}\sigma^c(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}), \\ [A_0{}^a(\mathbf{x}),\phi^b(\mathbf{y})] &= id^{abc}\sigma^c(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}), \\ [A_0{}^a(\mathbf{x}),\sigma^b(\mathbf{y})] &= -id^{abc}\phi^c(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}), \\ f^{9ab} &= 0, \quad d^{9ab} = (\tfrac{2}{3})^{1/2}\delta^{ab}, \end{aligned} \tag{4.11}$$

together with the usual algebra of fields among the currents. (As usual, pseudoscalar and scalar densities commute with spatial currents.) All other commutators, in particular those among ϕ^a, σ^b, vanish. This algebra is to be taken with the stress-energy-momentum tensor

$$\theta_{\mu\nu} = \theta_{\mu\nu}{}^S + g_{\mu\nu}\{\sigma^a\sigma^a + \phi^a\phi^a - (\tfrac{3}{2})^{1/2}f_\pi\mu^2\sigma^9 + \tfrac{3}{8}f_\pi{}^2\mu^2\}, \tag{4.12}$$

where all summations go from 1 to 9. The positivity of the Hamiltonian is seen through the transformation

$$\sigma_9 = \sigma_9' + (\tfrac{3}{2})^{1/2}\times\tfrac{1}{2}f_\pi\mu^2. \tag{4.13}$$

The resulting divergence conditions are

$$\partial_\mu V_a{}^\mu = 0, \quad \partial_\mu A_a{}^\mu = \mu^2 f_\pi\phi_a, \tag{4.14}$$

whereas the equations of motion are

$$\begin{aligned} \partial_\mu\sigma_a &= (1/2C)\{d^{abc}[A_\mu{}^b,\phi_c]_+ + f^{abc}[V_\mu{}^b,\sigma_c]_+\}, \\ \partial_\mu\phi_a &= (1/2C)\{f^{abc}[V_\mu{}^b,\phi_c]_+ - d^{abc}[A_\mu{}^b,\sigma_c]_+\}, \end{aligned} \tag{4.15}$$

plus the usual Eqs. (4.6b) and (4.6c) for the currents themselves. All other properties (Poincaré invariance, etc.) work out correctly. Again of course the ϕ and σ densities are not completely independent and there is no necessary particle content to the densities, particularly the σ densities.

Note that because $f^{9ab}=0$, then $V_\mu{}^9$ (the baryon-number current) satisfies

$$\partial_\mu V_\nu{}^9 - \partial_\nu V_\mu{}^9 = 0, \tag{4.16}$$

and because it has no divergence, then $V_9{}^\mu = \partial^\mu\Lambda(x)$, $\Box^2\Lambda(x)=0$; that is, for $\Lambda\neq 0$, there is a zero-mass scalar, $SU(3)$-scalar particle in the theory. One can avoid this by simply omitting $V_\mu{}^9$ everywhere in the theory. This is entirely consistent because $V_\mu{}^9$ commutes with all the operators in the theory except itself.[22] If desired, one can also omit $A_\mu{}^9$ (which has no curl) entirely as well. In this case the summations in $\theta_{\mu\nu}$ go from 1 to 8 (for the spin-one currents) and the algebra is that of Eq. (4.11) (only from 1 to 8) plus

$$\begin{aligned} [A_0{}^a(\mathbf{x}),\phi^9(\mathbf{y})] &= i(\tfrac{2}{3})^{1/2}\sigma^a(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}), \\ [A_0{}^a(\mathbf{x}),\sigma^9(\mathbf{y})] &= -i(\tfrac{2}{3})^{1/2}\phi^a(\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{y}), \end{aligned} \tag{4.17}$$

with all other commutators vanishing. The equations

[21] These commutation relations also follow in the model of M. Lévy, Nuovo Cimento 52A, 23 (1967).

[22] It is of course just this fact that makes the baryon-number current trivial in this theory. Although having no $V_\mu{}^9$ is not obviously a defect, it would be interesting to discover a theory with nontrivial $V_\mu{}^9$.

of motion are just the usual ones from 1 to 8, plus

$$\partial_\mu \sigma^9 = (1/2C)(\tfrac{2}{3})^{1/2}[A_\mu{}^a, \phi^a]_+ ,$$
$$\partial_\mu \phi^9 = -(1/2C)(\tfrac{2}{3})^{1/2}[A_\mu{}^a, \sigma^a]_+ , \quad (4.18)$$

with summations running from 1 to 8. One cannot drop ϕ^9 (or σ^9) without violating a Jacobi identity.

$SU(3)$ Breakdown

To accomplish this we can simply add a term (to $\theta_{\mu\nu}$) which transforms like the eighth component of the octet[23]

$$\Delta\theta_{\mu\nu} = g_{\mu\nu}\{-\lambda\sigma_8 + \tfrac{1}{4}\lambda^2\} , \quad (4.19)$$

which leaves $\theta_{00} \geqq 0$ ($\sigma_8 = \sigma_8' + \frac{1}{2}\lambda$). Then

$$\partial_\mu V_a{}^\mu = -\lambda f^{a8c}\sigma^c , \quad \partial_\mu A_a{}^\mu = f_\pi \mu^2 \phi_a + \lambda d^{a8c}\phi^c , \quad (4.20)$$

that is, $V_{4,5,6,7}{}^\mu$ are not conserved. The (curl) equations of motion for all the densities are unchanged.

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We understand that Professor C. Sommerfield has independently derived the extension to PCAC. One of the authors (MBH) acknowledges an interesting conversation with Professor M. Vaughn.

[23] Alternately, one could introduce a term in $\theta_{\mu\nu}$ of the form $g_{\mu\nu}d^{8ab}\phi_a\phi_b$. An attempt to add terms of the form $(1/2C)d^{8ab} \times \{[V_\mu{}^a, V_\nu{}^b]_+ - g_{\mu\nu}V_\lambda{}^a V_b{}^\lambda\}$ breaks Poincaré invariance. One could also break $SU(3)$ via an elementary fifth interaction [see Y. Ne'eman, Phys. Rev. **134**, B1355 (1964)] in a hybrid theory (only the 9th meson canonical). On the other hand, a pure theory ($\theta_{\mu\nu}{}^S(\hat{V}^8)$, $\hat{V}^8 = V^8 + \lambda V^9$, $[V_0{}^9(\mathbf{x}), V_i{}^9(\mathbf{y})] = iC\partial_i{}^x\delta^{(3)}(\mathbf{x}-\mathbf{y})$, V^9 commuting with all other coordinates) violates translational invariance.

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Chiral Dynamics of Octet Baryons and Nonleptonic Decays of Hyperons

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Chiral dynamics of the octet baryons is discussed, and a phenomenological chiral-invariant Lagrangian is derived. It is noted that in a chiral-invariant Lagrangian for the octet baryons, the structure of the axial-vector currents and the pattern of baryon-meson couplings are uniquely characterized by two free parameters, corresponding to G_A/G_V and the D/F ratio of the axial-vector currents. Nonleptonic hyperon decays are discussed from the chiral-dynamics point of view. A discussion is given on the transformation properties of the nonleptonic weak interactions under chiral transformations. Conditions under which the usual current-algebra analyses are valid are elaborated.

I. INTRODUCTION

In the present paper, we give a discussion on the construction of a phenomenological chiral-dynamics Lagrangian[1] for the octet baryons, and consider nonleptonic decays of hyperons from the point of view of chiral dynamics. Our views on chiral dynamics have been expressed previously,[2] and the construction of the chiral-dynamics Lagrangian we present here is a direct generalization of the material presented in Chap. IX of Ref. 2.

The construction of the chiral-dynamics Lagrangian for the nonet pseudoscalar mesons has been discussed by Cronin,[3] and will be discussed from a slightly different point of view by us in a separate communication.[4] The Lagrangian for the nonet pseudoscalar mesons is chiral $SU(3)\times SU(3)$ invariant, save for the mass terms for mesons, which make the hypothesis of partially conserved axial-vector current [(PCAC): identity of the divergence of the axial-vector current and the pion field in the sense of perturbative Lagrangian field theory] exactly satisfied for isospin axial-vector currents. To this we superimpose the chiral-invariant Lagrangian for the octet baryons, which then ensures the chiral $SU(3)\times SU(3)$ structure of vector and axial-vector currents, and PCAC for the isotopic-spin axial-vector currents.[5]

Chiral dynamics of the octet baryons is discussed in the next section. We construct a model of the octet

[1] S. Weinberg, Phys. Rev. Letters **18**, 188 (1967); J. Schwinger, Phys. Letters **24B**, 473 (1967); W. A. Bardeen and B. W. Lee (to be published); L. S. Brown, Phys. Rev. **163**, 1802 (1967); J. Wess and B. Zumino, *ibid.* **163**, 1727 (1967); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968), and references therein.

[2] W. A. Bardeen and B. W. Lee, Ref. 1.

[3] J. A. Cronin, University of Chicago, thesis, 1967 (unpublished).

[4] W. A. Bardeen, B. W. Lee, and D. Majumdar (unpublished).

[5] Chiral-dynamics Lagrangian for the nonet pseudoscalar mesons is in some sense unique, once the commutation relation between the axial current and and its divergence is specified. This statement is exactly true for chiral $SU(2)\times SU(2)$. See Ref. 2 and L. S. Brown, Ref. 1. I understand that this point with respect to $SU(3)\times SU(3)$ and the uniqueness of baryon dynamics will be discussed in a forthcoming paper by J. Wess and B. Zumino. I wish to express my thanks to Professor Zumino for this information. See also related discussion by M. Lévy, Nuovo Cimento **52**, 23 (1967).