

Analysis of the Nuclear Reaction Spectroscopy of ^{177}Hf , ^{179}Hf , and ^{181}Hf

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This study of the nuclear levels in ^{181}Hf , ^{179}Hf , and ^{177}Hf has utilized the reactions $^{180}\text{Hf}(d,p)^{181}\text{Hf}$, $^{180}\text{Hf}(d,t)^{179}\text{Hf}$, $^{178}\text{Hf}(d,t)^{177}\text{Hf}$, and $^{176}\text{Hf}(d,p)^{177}\text{Hf}$ at incident deuteron energies of 10–12 MeV. The emergent particles were analyzed in a broad-range magnetic spectrograph. The ground-state Q values were determined to be $^{180}\text{Hf}(d,p)$, $Q = 3475 \pm 10$ keV; $^{180}\text{Hf}(d,t)$, $Q = -1112 \pm 4$ keV; $^{178}\text{Hf}(d,t)$, $Q = -1364 \pm 9$ keV; and $^{176}\text{Hf}(d,p)$, $Q = 4150 \pm 7$ keV. Angular distributions were measured for the reactions $^{180}\text{Hf}(d,p)^{181}\text{Hf}$, $^{178}\text{Hf}(d,t)^{177}\text{Hf}$, and $^{176}\text{Hf}(d,p)^{177}\text{Hf}$, and interpreted with the aid of the distorted-wave Born approximation theory. l values have been assigned and spectroscopic factors extracted. The assignment of l values has resulted in the interpretation of the structure of ^{181}Hf and ^{177}Hf up to 2 MeV. The (d,t) study of ^{179}Hf has been used in conjunction with previously reported (d,p) data, and has resulted in an extension of the previous analysis. Nilsson states identified in one or more nuclei are the $\frac{7}{2}^+[633]$, $\frac{5}{2}^- [521]$, $\frac{5}{2}^- [512]$, $\frac{7}{2}^- [514]$, $\frac{9}{2}^+ [624]$, $\frac{3}{2}^- [510]$, $\frac{3}{2}^- [512]$, $\frac{7}{2}^- [503]$, $\frac{5}{2}^- [501]$, $\frac{5}{2}^- [503]$, and $\frac{1}{2}^+ [606]$. In addition a large number of $K = \frac{1}{2}$ and $\frac{3}{2}$ bands have been observed which do not have a simple interpretation in the Nilsson theory. While the low-lying (predominantly Nilsson) states agree quite well with the calculations of Soloviev, the higher states do not.

I. INTRODUCTION

IN recent years the Nilsson model¹ has been used quite successfully to explain the low-lying energy levels of deformed odd- A nuclei. It has, however, been recognized that the level structure above about 1 MeV becomes more complex than the simple Nilsson model predicts. The present work is an attempt to analyze in detail the structure of ^{177}Hf , ^{179}Hf , and ^{181}Hf up to an excitation energy of 2 MeV, utilizing the technique of charged-particle spectroscopy. Two major tools have been used in this experiment. The first is the use of both (d,p) and (d,t) reactions where possible;² the second is the use of angular distributions.

Ten- and 12-MeV deuterons from the Florida State University tandem Van de Graaff accelerator³ were used to initiate the reactions $^{176}\text{Hf}(d,p)^{177}\text{Hf}$, $^{178}\text{Hf}(d,t)^{177}\text{Hf}$, $^{180}\text{Hf}(d,t)^{179}\text{Hf}$, and $^{180}\text{Hf}(d,p)^{181}\text{Hf}$. The emergent particles were analyzed in a broad range Browne-Buechner magnetic spectrograph. This system has been described elsewhere.⁴ The data from the reaction $^{180}\text{Hf}(d,t)^{179}\text{Hf}$ were used in conjunction with the previously reported data of Vergnes⁵ from the reaction $^{178}\text{Hf}(d,p)^{179}\text{Hf}$. Angular distributions were measured for the reactions $^{176}\text{Hf}(d,p)^{177}\text{Hf}$, $^{178}\text{Hf}(d,t)^{177}\text{Hf}$, and $^{180}\text{Hf}(d,p)^{181}\text{Hf}$. The proper normalization of the cross sections in these data was obtained from elastic scatter-

ing measured at each angle. The measured angular distributions were compared with predictions of the distorted-wave Born-approximation (DWBA) code T-SALLY⁶ in order to extract l values and spectroscopic factors.

II. THEORETICAL CONSIDERATIONS

The characteristic feature of the level structure of a deformed odd- A nucleus is the presence of rotational bands built on intrinsic states. The energy of any level in the rotational band relative to the bandhead is given by the relation⁷

$$\Delta E = (\hbar^2/2\mathcal{I})\{I(I+1) - K(K+1) + \delta_{K,1/2}a[(-1)^{I+1/2}(I+\frac{1}{2})+1]\},$$

where I is the angular momentum of the level, K is the angular momentum of the bandhead, \mathcal{I} is the moment of inertia, and a is the decoupling parameter.

For the purposes of this paper the intrinsic states fall into two categories: (a) single-particle states, and (b) complex states. The single-particle states have been described by Nilsson.¹ Using Nilsson's wave functions, Satchler⁸ has derived cross sections for (d,p) and (d,t) reactions leading to single-particle states of an odd- A deformed nucleus, which, when corrected for pairing effects, are

$$(d,p) \quad d\sigma/d\omega = 2C_{j,i}^2 U^2 \Phi_l(\theta),$$

and

$$(d,t) \quad d\sigma/d\omega = (4/3)C_{j,i}^2 V^2 \Psi_l(\theta),$$

where $V^2 = 1 - U^2$ is the occupation probability, and the $\Phi_l(\theta)$ [$\Psi_l(\theta)$] are DWBA predictions of the (d,p) [(d,t)]

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¹ S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 29, No. 16 (1955).

² D. G. Burke, B. Zeidman, B. Elbek, B. Hersking, and M. Olesen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. (to be published).

³ Operation of the F. S. U. Tandem Accelerator Laboratory is supported in part by the U. S. Air Force Office of Scientific Research.

⁴ A. Kenefick and R. K. Sheline, Phys. Rev. 133, B25 (1964).

⁵ M. N. Vergnes and R. K. Sheline, Phys. Rev. 132, 1736 (1963).

⁶ R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240 (unpublished).

⁷ A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 27, No. 16 (1953).

⁸ G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

⁹ R. K. Sheline, W. N. Shelton, H. T. Motz, and R. E. Carter, Phys. Rev. 136, B351 (1964).

cross sections. The $C_{j,l}$'s are obtained from Nilsson's calculations.⁵ The second type of states, labeled here as complex states, include all states other than single-particle states. A particular complex state which has recently attracted much experimental⁹ and theoretical attention^{10,11} is the state formed by the coupling of a quasiparticle to a phonon, the "vibrational" state. The (d,p) or (d,t) cross sections to these complex states should be zero. The two types of states mix in an actual nucleus so that a given state may have components of both single-particle and complex states; thus the single-particle states are fragmented, the cross section to a predominantly single-particle state is smaller than the Satchler-Nilsson predictions, and the cross section to a predominantly complex state is nonzero. The relative cross sections to rotational levels built on a predominantly single-particle state should still agree with the Satchler-Nilsson predictions and provide a means of identifying the strongest components of these states. We will follow the convention of identifying the strongest component of the fragmented single-particle state as the Nilsson state.

III. EXPERIMENTAL TECHNIQUE AND DATA REDUCTION

The targets used in this experiment were prepared by vacuum evaporating separated isotopes,¹² obtained in the form HfO_2 from Oak Ridge, onto carbon backings of approximately $50 \mu\text{g}/\text{cm}^2$. The evaporation was carried out using an electron gun, producing targets of $100\text{--}200 \mu\text{g}/\text{cm}^2$. The deuteron beam for the reactions was obtained from the Florida State University tandem Van de Graaff accelerator. The protons or tritons were analyzed in a Browne-Buechner magnetic spectrograph using Kodak NTA nuclear track plates as the detector. The plates were counted in 0.5-mm strips on special scanning microscopes. A least-squares fitting routine was used,^{13,14} assuming the peak shape to be a skewed Gaussian, to calculate the area and position along the plates of the peaks observed. The energies of the particle groups were determined relative to lines from a ^{210}Po α source. In order to obtain the absolute cross section for each state observed, each exposure was accompanied by an elastic scattering measurement. The product of target thickness and solid angle determined from the elastic scattering, when corrected for the variation in solid angle along the focal curve, was used to calculate the correct cross section for each state observed. Absolute Q values were obtained by mathematically correcting the incident energy and spectrograph field so

that the correct Q values were obtained for two reference peaks in the data. Carbon and oxygen reference peaks were used in the (d,p) data, and elastic reference peaks were used in the (d,t) data.

IV. RELIABILITY OF THE DWBA ANALYSIS

In the present work, direct-reaction calculations are regarded as a potential spectroscopic tool rather than a subject for detailed investigation. The most important requirement is that the calculations permit the determination of l transfers. The T-SALLY predictions for different l transfers are compared with typical experimental angular distributions in Fig. 1. The different l -transfer curves are distinctive enough to assign l values unambiguously when the experimental data are good.

The spectroscopic factors obtained for different states are assumed to have the correct relative magnitudes, and the conclusions of the present work are based in part on this assumption. The absolute magnitudes may be incorrect since a spherical nucleus reaction calculation has been used to treat reactions involving deformed nuclei. The crucial factor as to the absolute magnitudes of spectroscopic factors is how well a spherical calculation can approximate a deformed calculation.

Nuclear deformations affect the bound-state form factor and wave functions for the entrance and exit channels. The bound-state problem has been investigated for deformed nuclei by Rost.¹⁵ A negligible effect on the stripping cross section due to nuclear distortions is predicted when the incident particle energy is approximately that of the Coulomb barrier. Thus for 10- or 12-MeV deuterons on the Hf isotopes, the spherical approximation is an acceptable one. Wave functions for the entrance and exit channels are a more serious problem. The correct wave functions should include inelastic excitations to rotational states. Wave functions of this type might be obtained from a coupled-channels cal-

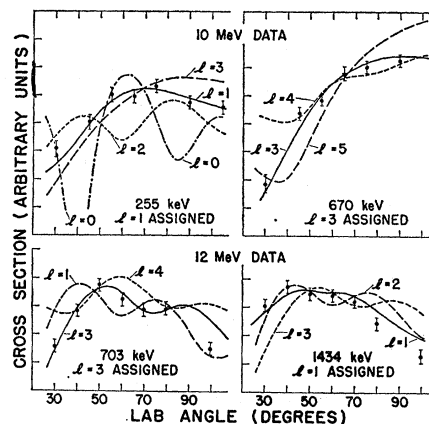


FIG. 1. Comparisons of T-SALLY calculations for different l transfers with data.

¹⁰ V. G. Soloviev and P. Vogel, Joint Institute for Nuclear Research Report, Dubna, 1966 (unpublished).

¹¹ D. Bes and Cho Yu-chung, Niels Bohr Institute Report, Copenhagen, 1965 (unpublished).

¹² R. A. Harlan, Ph.D. thesis, Florida State University, 1963 (unpublished).

¹³ R. H. Moore and R. K. Zeigler, Los Alamos Scientific Laboratory Report No. LA-2367 (unpublished).

¹⁴ C. Fred Moore and C. Watson (unpublished).

¹⁵ E. Rost, Phys. Rev. **154**, 994 (1967).

ulation. If an optical-model wave function is used to approximate a coupled-channels wave function (as is the case in the present work) the optical parameters may not be those which best describe elastic scattering. Iano *et al.*¹⁶ conclude that the inelastic contributions increase with increasing deformation, but that the scattered waves must have enough energy to penetrate the Coulomb barrier. The optical parameters used in the approximation in question must reflect both of these considerations. In the present work the ^{177}Hf data were taken with an incident deuteron energy of 12 MeV, and the ^{181}Hf data with an incident deuteron energy of 10 MeV. Moreover, since the Hf nuclei are at the upper end of this particular region of deformation, the deformation of ^{177}Hf is some 20% larger than the deformation of ^{181}Hf . In the present work, the deuteron parameters used for ^{177}Hf and ^{181}Hf are different; they are respectively sets A and B from the systematics of Perey. The parameters used were based on a comparison of T-SALLY calculations to the angular distributions for states of known spin in ^{177}Hf and ^{181}Hf . This difference of parameters may reflect the higher incident energy and deformation for the ^{177}Hf work. It is likely that a smoother variation in optical parameters could have been obtained by varying the parameters from the systematic values, but a variation procedure was regarded as more dangerous than the one used.

In the light of the uncertainties involved in the calculation, the spectroscopic factors obtained in the present work are regarded as just a convenient parameterization of the data. As will be seen later, however, the absolute magnitudes obtained are in good agreement with theory.

V. EXPERIMENTAL RESULTS AND INTERPRETATIONS

A. ^{181}Hf

The levels of ^{181}Hf were observed by the reaction $^{180}\text{Hf}(d,p)^{181}\text{Hf}$ at the angles 30° , 45° , 55° , 65° , 75° , 90° , and 105° , with an incident deuteron energy of 10 MeV. A typical spectrum is shown in Fig. 2. Levels observed at three or more angles are considered to be states in ^{181}Hf ; light impurities are eliminated by their kinematic shift, and heavy impurities, with the exception of tungsten, are considered unlikely because of the 99% enrichment of the target. Levels in tungsten were eliminated by comparison with the known tungsten (d,p) levels. The average ground-state Q value and average level energies to 2 MeV were determined. The measured ground-state Q value is 3475 ± 10 keV. Absolute cross sections were determined in order to obtain the angular distribution for each state.

Calculations of $\Phi_l(\theta)$ were performed for Q values of 3.5, 3.0, 2.5, and 2.0 MeV using the DWBA code

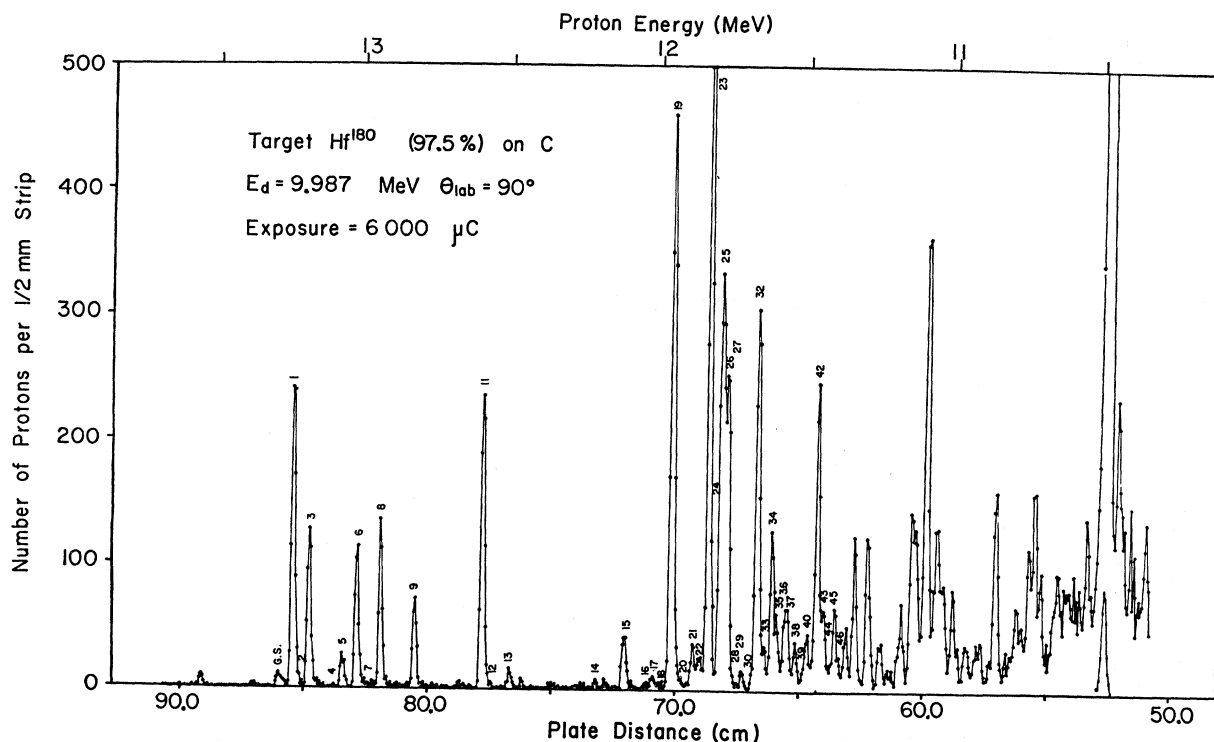


FIG. 2. Spectrum of the levels in ^{181}Hf populated by the reaction $^{180}\text{Hf}(d,p)^{181}\text{Hf}$ at 90° .

¹⁶ P. J. Iano and N. Austern, *Phys. Rev.* **151**, 853 (1966).

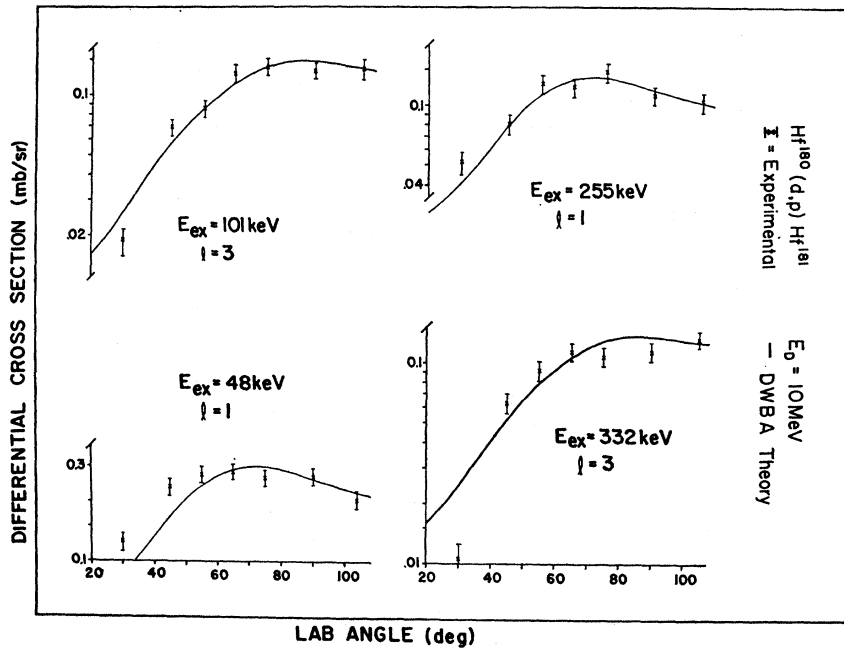


FIG. 3. Typical angular distribution of levels in ^{181}Hf compared with DWBA theory.

T-SALLY⁶ on the IBM 709 computer. The optical-model parameters used in this calculation were

	V	W	r_0	a	r'	a'	W'	r_c
Deuteron potential	104	0	1.15	0.81	1.34	0.68	48	1.15
Proton potential	55	0	1.25	0.65	1.25	0.47	60	1.25

These parameters were obtained from the systematic study of Perey.^{17,18} The predicted deuteron elastic scattering

was compared with the experimentally measured elastic scattering, and the agreement was found to be good.

Values of l were assigned to the states for which comparison of theoretical and experimental angular distributions allowed an assignment. Typical comparisons are shown in Figs. 3 and 4, where the theoretical curves have been normalized for best fits. No assignments can

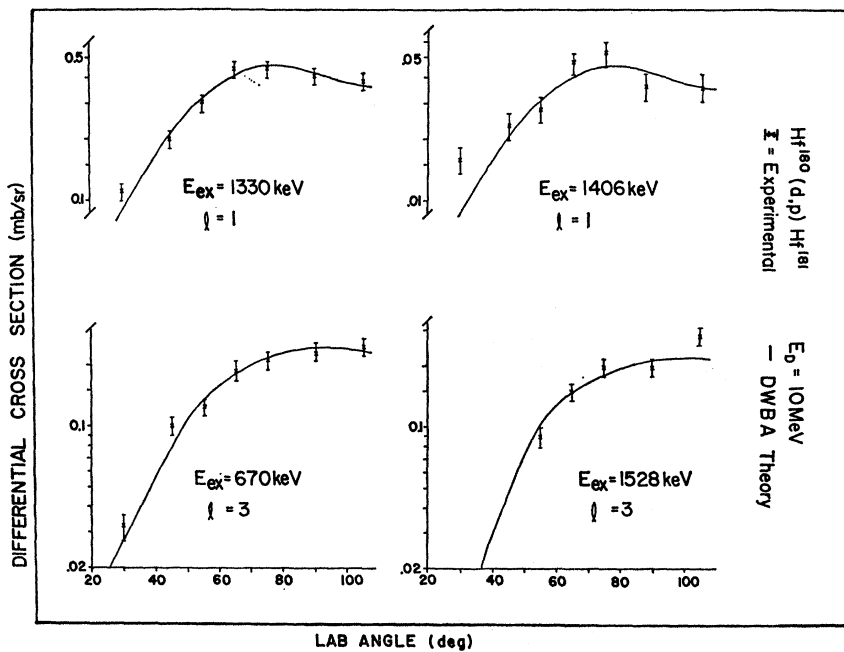


FIG. 4. Typical angular distributions of levels in ^{181}Hf compared with DWBA theory.

¹⁷ F. G. Perey, Phys. Rev. 131, 745 (1963).

¹⁸ C. M. Perey and G. G. Perey, Phys. Rev. 132, 755 (1963).

be made for some states; in some cases the statistics are poor; in other cases it is difficult to obtain the correct areas for individual peaks of a doublet even with a fitting routine. There are many states where only one l -value assignment is acceptable. There are in addition states for which more than one l value is possible, but a particular assignment is suggested. In these cases, the most probable l value is enclosed in parentheses.

The measured level energies and l values are given in Table I. The observed states have been grouped into rotational bands by matching the predicted sequence of l values with the proper energy spacings for a band with a given value of Ω . In some cases the l values of states in the vicinity are not known. In such cases one must rely on the dependability of the rotational energy calculations. In all cases reasonable values of the inertial and decoupling parameters were used. In two specific

TABLE I. Energy levels in ^{181}Hf .

Level No.	$E_{(\text{ex})}$ (keV)	σ (keV)	l value
0	0	3.6	(1)
1	48	3.3	1
2	68	2.8	4
3	101	3.2	3
4	170	3.3	
5	207	2.8	3
6	255	2.8	1
7	298	6.3	
8	332	3.3	3
9	445	2.6	3
10	471	3.8	
11	670	2.6	3
12	692	4.0	
13	760	3.8	
14	1063	3.7	1
15	1163	2.1	3
16	1241	3.0	
17	1267	2.0	(1)
18	1291	3.7	
19	1330	1.8	1
20	1366	1.7	
21	1406	2.0	1
22	1431	2.5	(3)
23	1459	1.4	1
24	1474	2.5	(1,2)
25	1502	1.8	1
26	1515	2.4	3
27	1528	1.3	3
28	1551	2.9	
29	1578	5.2	
30	1594	4.9	
31	1617	1.6	
32	1637	1.3	3
33	1662	2.1	(3)
34	1690	1.1	1
35	1711	2.1	(3)
36	1729	5.0	6
37	1745	2.6	1
38	1776	2.1	1
39	1779	2.1	
40	1817	2.9	(3)
41	1831	1.5	
42	1854	1.2	1
43	1878	3.1	3
44	1900	4.2	(3)
45	1920	2.5	
46	1935	4.1	
47	1964	2.0	5
48	1995	1.0	

TABLE II. Rotational parameters for ^{181}Hf .

$E_{(\text{ex})}$ (keV)	Assignment	$\hbar^2/2\mathcal{I}$ (keV)	a
0	$\frac{1}{2}^-$ [510]	13.3	0.120
68	$\frac{3}{2}^+$ [624]	9.26	
255	$\frac{3}{2}^-$ [512]	15.6	
670	$\frac{7}{2}^-$ [503]		
1063	$\frac{3}{2}^-$	20	
1267	$\frac{3}{2}^-$	20	
1330	$\frac{3}{2}^-$	20	
1406	$\frac{3}{2}^-$	15.7	0.125
1503	$\frac{3}{2}^-$ [501]	9.60	
1637	$\frac{3}{2}^-$ [503]	10.6	
1729	$\frac{1}{2}^+$ [606]		
1745	$\frac{1}{2}^-$	9.26	0.117
1799	$\frac{1}{2}^-$	11.57	0.58

cases, the 670-keV level ($l=3$) and the 1729-keV level ($l=6$), a single level was assigned a value of J^π without observing additional members of the band. This was possible because of specific predictions of Nilsson's calculations concerning cross sections and approximate energies.

As discussed earlier, the $C_{j,i}$'s for Nilsson single-particle states enable one to predict the relative intensities in the associated rotational bands. The order in which the Nilsson states should occur has also been predicted. On the basis of these predictions, asymptotic quantum numbers have been assigned to the rotational bands whose relative cross sections and locations agree with the Nilsson predictions. The resulting interpretation is shown in Fig. 5.

The rotational parameters calculated for this interpretation are given in Table II. Comparison of the experimental angular distribution and the theoretical angular distribution of $\Phi_l(\theta)$, by least-squares analysis, has resulted in the determination of $2C_{j,i}^2U^2$ for each level. The value of U^2 has been calculated theoretically. The experimental $C_{j,i}^2$'s for each level have thus been determined, and are presented in Table III. It should be emphasized that the absolute magnitude of the experimental $C_{j,i}$'s may be unreliable since they depend on the accuracy of the particular DWBA calculations. Their relative magnitudes, however, should be reliable. We have chosen to regard the numbers presented in Table III as a convenient parametrization of the data, using the method outlined above. The absolute magnitudes for the ground-state rotational band agree with Nilsson's predictions to within 5%.

The level structure below 1 MeV is rather well explained by the Nilsson model. There are three weak states at 471, 692, and 760 keV which are not assigned. The angular distributions for these weak states are not conclusive, but indicate they are probably positive-parity states. It is possible that they may be states from a higher shell. The 670-keV state is the strongest $l=3$ state in this region; it is assigned as $J^\pi = \frac{7}{2}^-$ because the

TABLE III. Experimental values of $C_{j,i}^2$ for levels of ^{181}Hf .

$E_{(ex)}$ (keV)	Assignment	Theory		Experimental values $C_{j,i}^2$						
		U^2	$C_{1/2,1}^2$	$C_{3/2,1}^2$	$C_{5/2,3}^2$	$C_{1/2,3}^2$	$C_{9/2,5}^2$	$C_{9/2,4}^2$	$C_{11/2,6}^2$	$C_{13/2,6}^2$
0	$\frac{1}{2}^-$ [510]	0.73	0.010	0.283	0.478	0.099				
68	$\frac{3}{2}^+$ [624]	0.21								
255	$\frac{3}{2}^-$ [512]	0.79		0.126	0.392	0.217				
670	$\frac{7}{2}^-$ [503]	0.85				0.575				
1063	$\frac{3}{2}^-$	~ 1		0.003	0.095	0.010				
1267	$\frac{3}{2}^-$	1		0.006	0.016	0.026				
1330	$\frac{3}{2}^-$	1		0.219	0.055	0.019				
1406	$\frac{1}{2}^-$	1	0.023	0.235	0.337	0.021				
1503	$\frac{3}{2}^-$ [501]	1		0.136	0.014	0.017				
1729	$\frac{1}{2}^+$ [606]	1								2.23
1637	$\frac{5}{2}^-$ [503]	1			0.526	0.072				
1745	$\frac{1}{2}^-$	1	0.027	0.014	0.062	0.028	1.57			
1799	$\frac{1}{2}^-$	1	0.005	0.093	0.103	0.185				

$\frac{7}{2}^-$ state of the $\frac{7}{2}^-$ [503] rotational band is predicted to have almost all the strength of the band.

In the region above 1 MeV the level structure is much

more complex. The Nilsson model predicts rotational bands built on the $\frac{1}{2}^+$ [615], $\frac{9}{2}^-$ [505], $\frac{3}{2}^-$ [501], $\frac{1}{2}^+$ [606], $\frac{5}{2}^-$ [503], and $\frac{1}{2}^-$ [501] intrinsic states. The

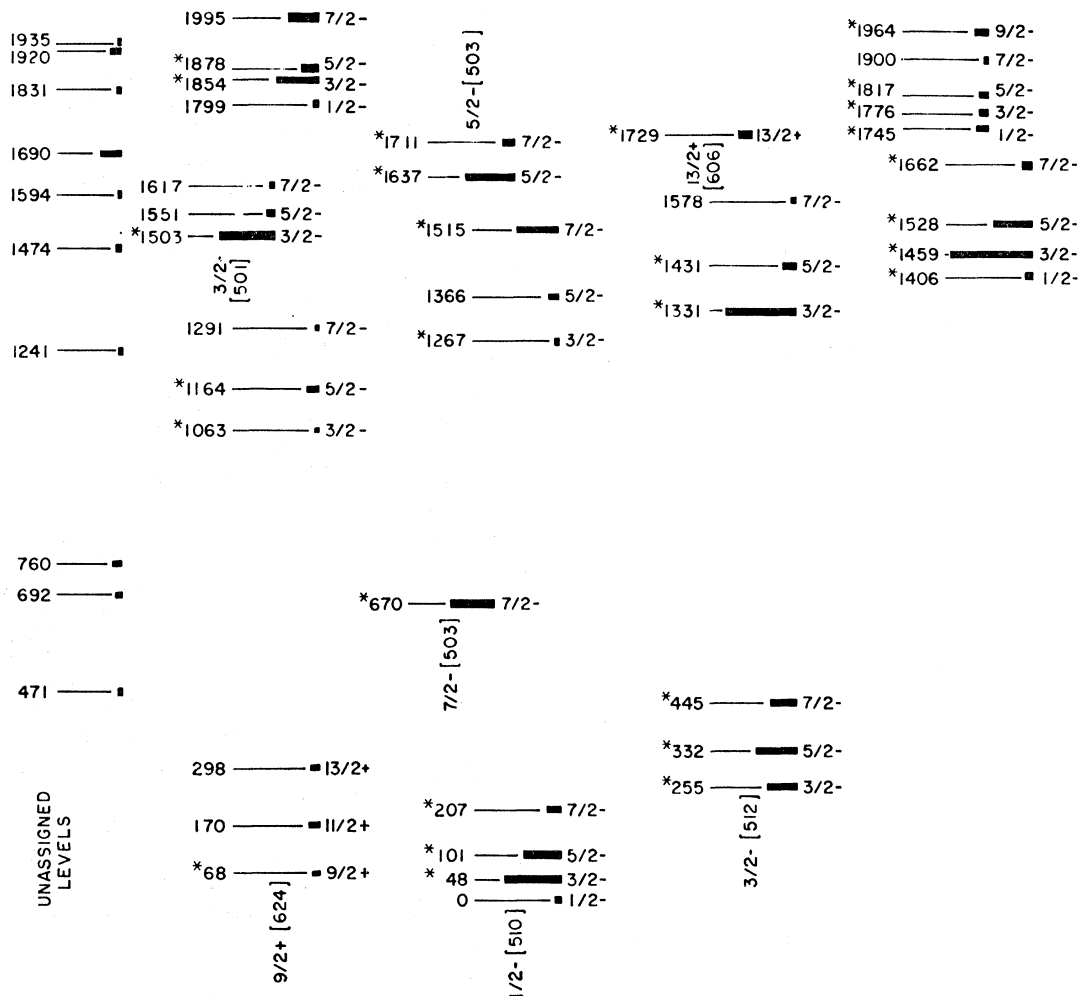


FIG. 5. Level scheme for ^{181}Hf showing assignments of rotational bands. The heavy line for each level is proportional to the (d,p) cross section of the level. An asterisk beside the level indicates that angular distribution of the protons from the (d,p) reaction has been measured and is in agreement with the spin and parity assigned. Nilsson assignments are indicated.

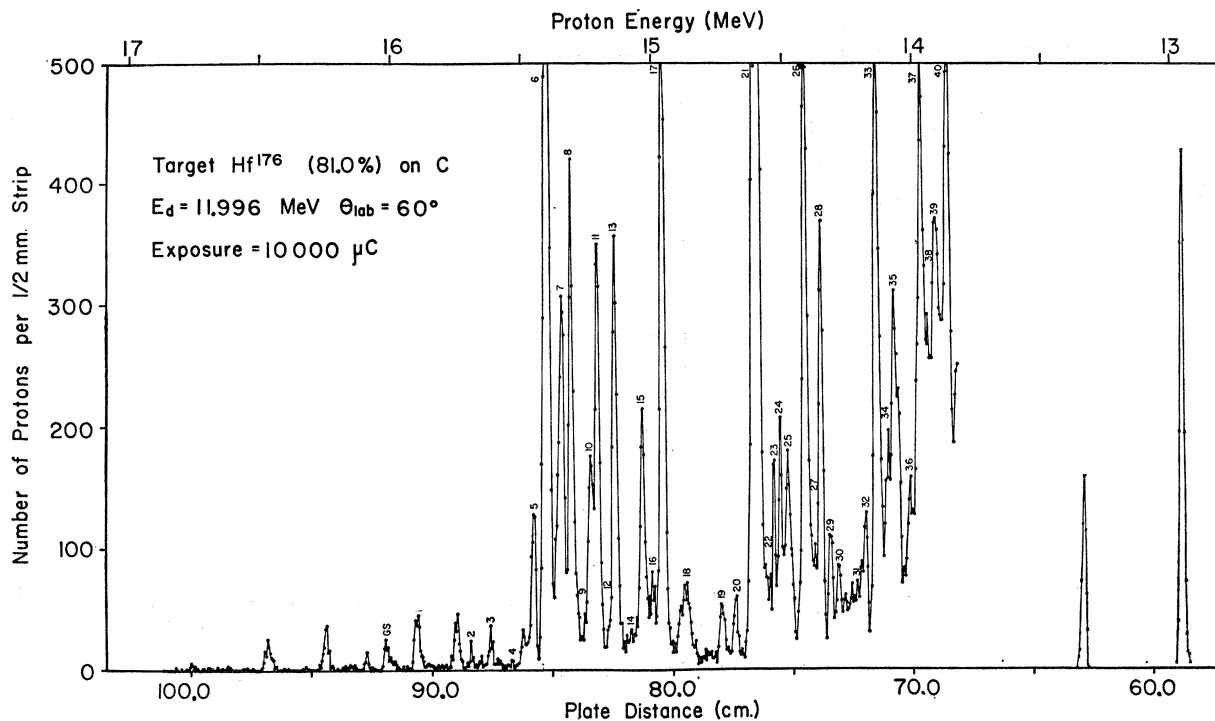


FIG. 6. Spectrum of the levels in ^{177}Hf as populated by the reaction $^{176}\text{Hf}(d,p)^{177}\text{Hf}$ at 60° .

first two bands are not seen, but since the predicted cross sections are small for all members of both bands this is not surprising. The 1729-keV state has a measured $l=6$, and has been assigned as the $\frac{1}{2}^2+$ member of the $\frac{1}{2}^3+$ [606] band. No other state in this band should be seen. Four $\frac{3}{2}^-$ bands, three $\frac{1}{2}^-$ bands, and one $\frac{5}{2}^-$ band are seen. On the basis of relative cross sections, the $\frac{3}{2}^-$ band at 1503 keV could be interpreted as the $\frac{3}{2}^-$ [501] band, and the $\frac{5}{2}^-$ band at 1637 keV as the $\frac{5}{2}^-$ [503] band. The absolute cross sections, however, are too small. None of the observed $\frac{1}{2}^-$ bands can be interpreted as the $\frac{1}{2}^-$ [501] band. Thus it must be concluded that this region of the level structure cannot be successfully interpreted in terms of the Nilsson model. These unidentified states are classified as complex states, and will be discussed later.

B. ^{177}Hf

The levels of ^{177}Hf were studied by the reaction $^{176}\text{Hf}(d,p)^{177}\text{Hf}$ and $^{178}\text{Hf}(d,t)^{177}\text{Hf}$ at the common angles 30° , 40° , 50° , 60° , 70° , 80° , and 100° with an incident deuteron energy of 12 MeV. Typical spectra for the (d,p) and (d,t) reactions are shown in Figs. 6 and 7, respectively.

Peaks in the spectra corresponding to light impurities were identified by their kinematic shifts. The 96% isotopic enrichment of the ^{178}Hf target eliminated the probability of heavy impurities, but the ^{176}Hf target, with only 81% isotopic enrichment, contained other Hf isotopes. The levels in ^{181}Hf and ^{179}Hf have been identified in the present work, and an exposure of the

reaction of $^{177}\text{Hf}(d,p)^{178}\text{Hf}$ was taken to identify the strong states of ^{178}Hf . This information was used to identify impurity peaks in the $^{176}\text{Hf}(d,p)^{177}\text{Hf}$ spectra. The average ground-state Q value was measured to be 4150 ± 7 keV for the reaction $^{176}\text{Hf}(d,p)^{177}\text{Hf}$, and -1364 ± 9 keV for the reaction $^{178}\text{Hf}(d,t)^{177}\text{Hf}$. Angular distributions for each state were determined for both (d,p) and (d,t) reactions.

The $\Phi_l(\theta)$ for the (d,p) reactions were computed for Q values of 4.14, 3.64, 3.14, 2.64, and 2.14 MeV using the DWBA code T-SALLY.⁶ The optical-model parameters used in this calculation were

	V	W	r_0	a	r'	a'	W'	r_c
Deuteron potential	80	0	1.15	0.87	1.37	0.70	48	1.25
Proton potential	55	0	1.25	0.65	1.25	0.47	60	1.25

A lower cutoff radius of 4 F was used in the calculation.

The $\psi_l(\theta)$ for the (d,t) reaction were obtained from Satchler, who used the optical-model parameters

	V	W	r_0	a	r'	a'	W'	r_c
Deuteron potential	77	0	1.24	0.84	1.37	0.93	45	1.25
Triton potential	100	14	1.07	0.854	1.70	0.73	0	1.40

These $\psi_l(\theta)$ were calculated for Q values of 0 and -3 MeV. Interpolations of the $\psi_l(\theta)$ to different Q values were made assuming an exponential dependence.

The (d,p) angular distributions were used to determine l values for the states where possible. Typical comparisons of the experimental and theoretical curves are shown in Figs. 8 and 9. The (d,t) angular distributions are relatively insensitive to the l values, as is seen in

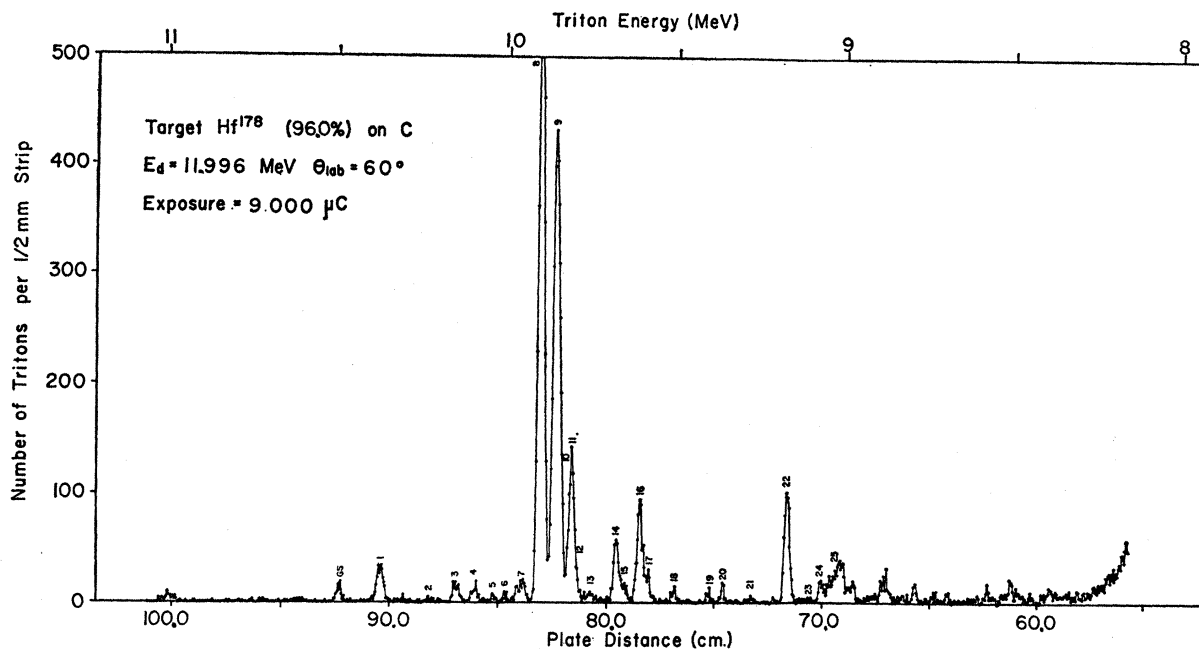


Fig. 7. Spectrum of the levels in ^{177}Hf as populated by the reaction $^{178}\text{Hf}(d,t)^{177}\text{Hf}$ at 60° .

Fig. 10, but the agreement between the experiment and theory is good and allows the extraction of spectroscopic factors for the (d,t) states when the l values are known. The measured level energies and l values for the (d,p) reaction are given in Table IV, and the measured level energies for the (d,t) reaction in Table V. Although many states are seen in both reactions, there are some states seen in the (d,t) reaction exclusively and some seen exclusively in the (d,p) reaction. This is clearly understood

from consideration of the respective reaction mechanisms; pickup reactions selectively populate hole states, while stripping reactions selectively populate particle states.

The complementary (d,t) and (d,p) reactions are a very useful tool for interpretation of the spectra.² The level scheme has been interpreted in terms of rotational bands built on intrinsic states. In addition, the relative magnitudes of the (d,p) and (d,t) cross sections clearly

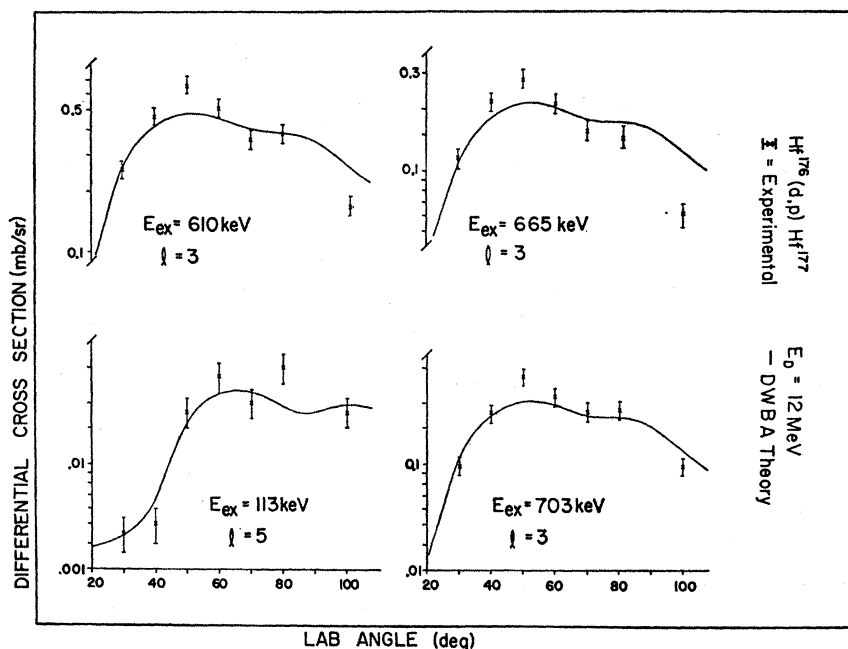
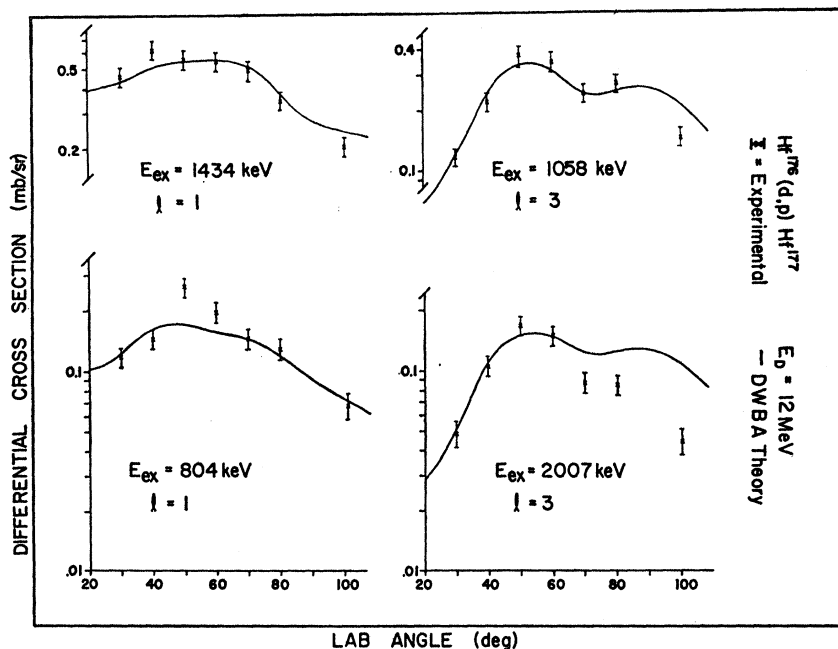


Fig. 8. Typical angular distributions for the reaction $^{178}\text{Hf}(d,p)^{177}\text{Hf}$ compared with DWBA theory.

FIG. 9. Typical angular distributions for the reaction $^{178}\text{Hf}(d,p)^{177}\text{Hf}$ compared with DWBA theory.



label the intrinsic states as particle or hole states, and result in a more positive identification of Nilsson states. The interpretation of the states in ^{177}Hf is shown in Fig. 11. The rotational parameters corresponding to this interpretation are given in Table VI. The quantity $C_{j,i}^2 U^2$ has been evaluated for the states seen in the (d,p) reactions; in addition, the quantity $C_{j,i}^2 V^2$ has been evaluated for the states seen in the (d,t) reaction. The ratio of these quantities for a given state gives the value of U^2/V^2 for the state, and use of the relationship

$U^2 + V^2 = 1$ has resulted in the determination of U^2 and of V^2 . The experimental $C_{j,i}^2$ for each state can then be obtained. Table VII gives the results of this analysis.

The ground-state rotational band has been assigned as the $\frac{7}{2}^- [514]$, in agreement with Nilsson's predictions. The first-excited-particle state is predicted to be the $\frac{9}{2}^+ [624]$, and the 324- and 433-keV states have been assigned as the $\frac{3}{2}^+$ and $\frac{1}{2}^+$ members of this band. The $\frac{1}{2}^+$ member of the band should lie at 556 keV according to the rotation-energy formula. This state is assumed to

FIG. 10. Typical angular distributions for the reaction $^{178}\text{Hf}(d,t)^{177}\text{Hf}$ compared with DWBA theory.

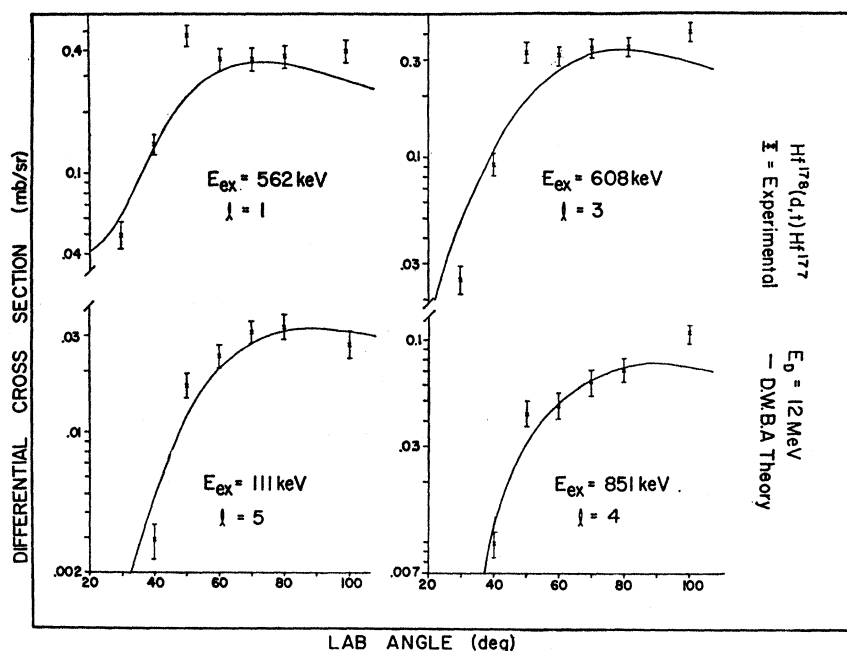


TABLE IV. Energy levels in ^{177}Hf by (d,p) reaction.

Level No.	$E_{\text{(ex)}}$ (keV)	σ (keV)	l value
0	0	5.5	(3)
1	111	6.9	5
2	319	3.1	
3	390	4.6	
4	432	3.0	
5	556	4.9	1
6	610	4.7	3
7	665	4.9	3
8	703	4.6	3
9	739	6.9	(1)
10	777	4.6	3
11	804	4.4	1
12	839	6.6	
13	872	4.6	3
14	919	4.8	(1)
15	979	3.4	3
16	1016	4.6	3
17	1058	4.2	3
18	1294	4.7	1
19	1352	6.6	
20	1434	4.7	1
21	1475	5.1	
22	1502	3.7	(1)
23	1535	4.8	3
24	1565	5.8	(3)
25	1634	3.9	1
26	1666	5.7	(1)
27	1701	4.6	1
28	1743	5.3	
29	1779	2.8	3
30	1847	4.0	
31	1882	5.3	1
32	1932	5.2	1
33	1969	6.0	3
34	2007	9.4	3
35	2071	4.5	3
36	2114	6.2	3

be unresolved from the $l=1$ state assigned as the bandhead of the $\frac{1}{2}^-$ [521] band. While the cross sections of the states in these first two bands are too low to allow definite assignments of l values to all but the 11-keV

TABLE V. Energy levels in ^{177}Hf by (d,t) reaction.

Level No.	$E_{\text{(ex)}}$ (keV)	σ (keV)
0	0	8.7
1	111	2.2
2	251	3.5
3	325	1.5
4	375	2.8
5	433	2.4
6	459	2.7
7	504	5.6
8	562	2.5
9	607	1.5
10	624	4.6
11	652	3.6
12	664	2.9
13	705	4.4
14	782	2.1
15	806	2.7
16	851	2.5
17	875	2.5
18	1059	0.8
19	1101	1.2
20	1296	1.0
21	1352	3.4

state, the energy spacing and relative (d,p) - (d,t) cross sections support these assignments.

There are three weak states unassigned in this energy region, the 375- and 459-keV states seen only in the (d,t) reaction, and the 390-keV state seen only in the (d,p) reaction.

The state at 608 keV is the strongest state in both the (d,t) and (d,p) spectra. The (d,p) angular distribution indicates $l=3$ (see Fig. 8). The strong (d,t) cross section suggests that it is the $\frac{7}{2}^-$ member of the $\frac{5}{2}^-$ [512] band, with the $\frac{5}{2}^-$ bandhead at 504 keV. West¹⁹ has identified a state at 605.5 keV as $\frac{7}{2}^-$, $\frac{5}{2}^-$ [512] in the electron capture decay of ^{177}Ta . The (d,p) cross section is then too strong by a factor of 2. This suggests that the state is a doublet. The interpretation most consistent with the data is that the doublet contains the $\frac{7}{2}^-$, $\frac{5}{2}^-$ [512] and $\frac{3}{2}^-$, $\frac{1}{2}^-$ [510]. The flat behavior of an $l=1$ angular distribution would not alter the distinctive $l=3$ shape significantly. Namenson²⁰ observed a 5762-keV primary

TABLE VI. Rotational parameters for ^{177}Hf .

$E_{\text{(ex)}}$ (keV)	Assignment	$\hbar^2/2\mathcal{I}$ (keV)	a
0	$\frac{7}{2}^-$ [514]	12.4	
324	$\frac{9}{2}^+$ [624]	9.9	
504	$\frac{5}{2}^-$ [512]	14.9	
562	$\frac{1}{2}^-$ [521]	12.7	0.46
567	$\frac{1}{2}^-$ [510]	12.5	0.09
804	$\frac{3}{2}^-$ [512]	14.8	
851	$\frac{7}{2}^+$ [633]	10.41	
1058	$\frac{7}{2}^-$ [503]		
1434	$\frac{3}{2}^-$ [501]	8.4	
1502	$\frac{3}{2}^-$	12.6	
1634	$\frac{1}{2}^-$	18.9	0.180
1666	$\frac{3}{2}^-$	15.4	
1882	$\frac{1}{2}^-$	12.0	0.391

transition in the reaction $^{176}\text{Hf}(n,\gamma)^{177}\text{Hf}$. This transition is expected to populate selectively states of spin $\frac{1}{2}$ or $\frac{3}{2}$. On the basis of our Q value, this transition populates the 608-keV state, supporting the present interpretation. The 665-, 703-, and 839-keV states are then assigned as the $\frac{5}{2}^-$, $\frac{7}{2}^-$, and $\frac{9}{2}^-$ members of the $\frac{1}{2}^-$ [510] band. Using these states to extract the parameters $\hbar^2/2\mathcal{I}=12.5$ keV and $a=0.09$, the $\frac{1}{2}^-$ bandhead should lie at 567 keV and could be obscured by the bandhead of the $\frac{1}{2}^-$ [521] band. The weak states at 739, 919, and 1016 keV are unassigned.

The remaining states below 1200 keV are explained quite well by the Nilsson model. The $\frac{7}{2}^+$ and $\frac{1}{2}^+$ states of the $\frac{7}{2}^+$ [633] rotational band are predicted to have very low cross sections, and their absence from the observed levels is not unexpected.

The only unassigned states between 1 MeV and 2

¹⁹ H. I. West, Jr., L. G. Mann, and R. J. Nagle, Phys. Rev. **124**, 527 (1961).

²⁰ A. I. Namenson and H. H. Bolotin, Phys. Rev. **158**, 1206 (1967).

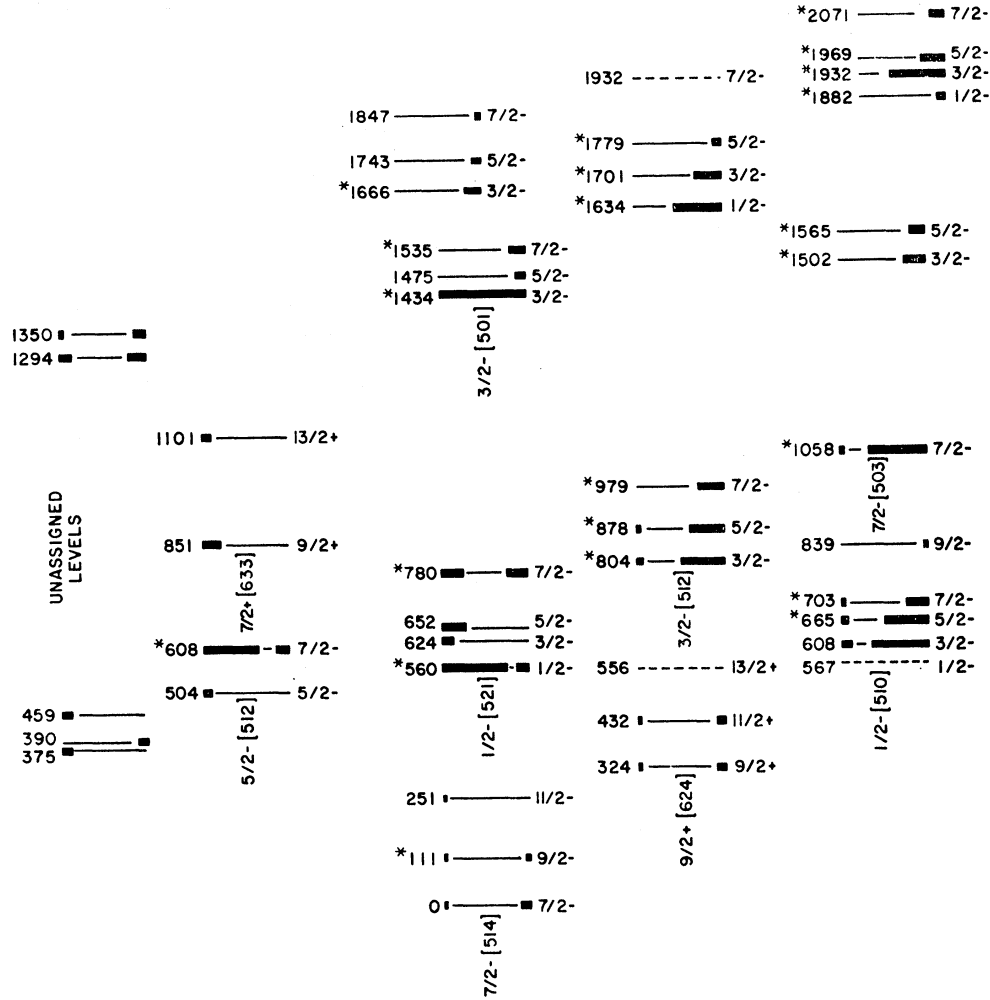


FIG. 11. Level scheme for ^{177}Hf showing assignments of rotational bands. The heavy lines for each level are proportional to the (d, i) and (d, p) cross sections for the level. An asterisk beside the level indicates that angular distribution of the protons from the (d, p) reaction has been measured and is in agreement with the spin and parity assigned. Nilsson assignments are indicated.

MeV are the 1294- and 1352-keV states. The 1294-keV state has $l=1$, and it is possible that the two states may be members of a $\frac{3}{2}^-$ or $\frac{1}{2}^-$ rotational band. The absence of other states close by makes a definite assignment impossible. The $\frac{3}{2}^-$ band at 1434 keV has been identified as the

TABLE VII. Experimental values of U^2 and $C_{j,i}^2$ for levels of ^{177}Hf .

$E_{(ex)}$ (keV)	Assignment	Expt. U^2	Experimental values of $C_{j,i}^2$							
			$C_{1/2,1}^2$	$C_{3/2,1}^2$	$C_{5/2,3}^2$	$C_{7/2,3}^2$	$C_{9/2,5}^2$	$C_{11/2,5}^2$	$C_{9/2,4}^2$	$C_{11/2,8}^2$
0	$\frac{7}{2}^- [514]$	0.427				0.048	0.700	0.065		
324	$\frac{9}{2}^+ [624]$	0.706							0.092	0.148
504	$\frac{5}{2}^- [512]$	~ 0.3			~ 0.04	~ 0.7				
562	$\frac{3}{2}^- [521]$	0.10	0.378	0.041	0.218	0.114				
567	$\frac{1}{2}^- [510]$	0.921	?	~ 0.2	0.427	0.418				
804	$\frac{3}{2}^- [512]$	0.93		0.110	0.370	0.232				
1058	$\frac{7}{2}^- [503]$	0.94				0.542				
851	$\frac{7}{2}^+ [633]$	0.01							0.40	
1434	$\frac{3}{2}^- [501]$	1		0.261	0.039	0.100				
1502	$\frac{3}{2}^-$	1		0.029	0.083					
1634	$\frac{3}{2}^-$	1	0.109	0.061	0.022					
1666	$\frac{3}{2}^-$	1		0.013	0.039	0.010				
1882	$\frac{1}{2}^-$	1	0.018	0.103	0.082	0.083				

$\frac{3}{2}^-$ [501] band. The other states below 2 MeV have been assigned to two $\frac{3}{2}^-$ bands and two $\frac{1}{2}^-$ bands. These bands cannot be interpreted as Nilsson bands. They will be discussed later as complex states.

C. ^{179}Hf

The levels of ^{179}Hf have been studied by the reaction $^{180}\text{Hf}(d,t)^{179}\text{Hf}$ at 50° , 65° , and 90° , with an incident deuteron energy of 12 MeV. ^{179}Hf has previously been studied in this laboratory by Vergnes,⁵ and it was believed that the additional information obtainable from the (d,t) reactions justified its inclusion in the present work. Absolute cross sections were measured for the (d,t) reactions, but complete angular distributions were not performed, since they were not undertaken by Vergnes. Typical spectra for the (d,t) and (d,p) reactions are shown in Figs. 12 and 13.

Short (d,p) exposures were taken at 65° and 90° in order to properly normalize the corresponding Vergnes data, so that in spite of the absence of angular distributions, absolute cross sections are known for the (d,p) levels.

The ground-state Q value for the (d,t) reaction was measured to be -1112 ± 4 keV. The average level energies observed in the (d,t) reaction are given in Table

VIII. The level energies previously determined by Vergnes are included for comparison.

The interpretation of the level structure indicates the value of the complementary (d,t) and (d,p) reactions. Vergnes has previously assigned the states at 0, 122, and 268 keV to the $\frac{3}{2}^+$ [624] band, the states at 217 and 366 keV to the $\frac{7}{2}^-$ [514] band, and the states at 373, 421, 476, and 582 keV to the $\frac{1}{2}^-$ [510] band. These assignments are in agreement with the present work. An additional state at 483 keV is observed in the (d,t) reaction whose energy suggests that it is the $\frac{1}{2}^-$ member of the $\frac{7}{2}^-$ [514] band.

The 617-keV state is the strongest state in the (d,t) spectra, and is weak in the (d,p) spectra, indicating a hole state. The 517-keV state is not observed in the (d,p) reaction, and the relative energy of these two states suggests the $\frac{5}{2}^-$ [512] band. The cross section of the 617-keV state is too large by a factor of 2. Kramer *et al.*²¹ observe a 239.1-keV internal-conversion transition from a state at 614.1 keV to the 375-keV state, which is the bandhead of the $\frac{1}{2}^-$ [510] band. If the 614.1-keV state has spin $\frac{7}{2}^-$, the transition must be $M3$, which is extremely unlikely. We infer that our 617-keV peak is a doublet containing strength from a low-spin state. If one assumes that this low-spin state is $J^\pi = \frac{1}{2}^-$, then the 614-, 680-, 701-, and 853-keV states have the proper

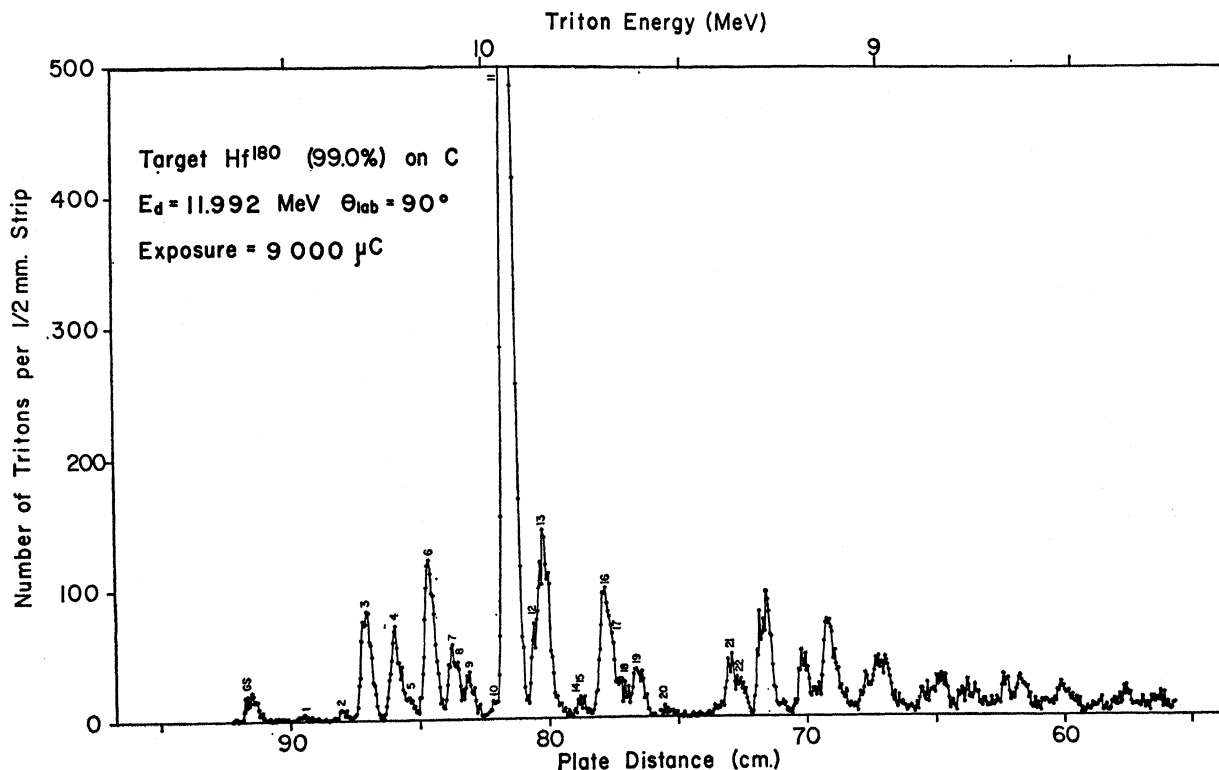


FIG. 12. Spectrum of the levels in ^{179}Hf as populated by the reaction $^{180}\text{Hf}(d,t)^{179}\text{Hf}$ at 90° .

²¹ N. D. Kramer and P. T. Prokof'ev, *Yadern. Fiz.* 4, 228 (1966) [English transl.: *Soviet J. Nucl. Phys.* 4, 165 (1967)].

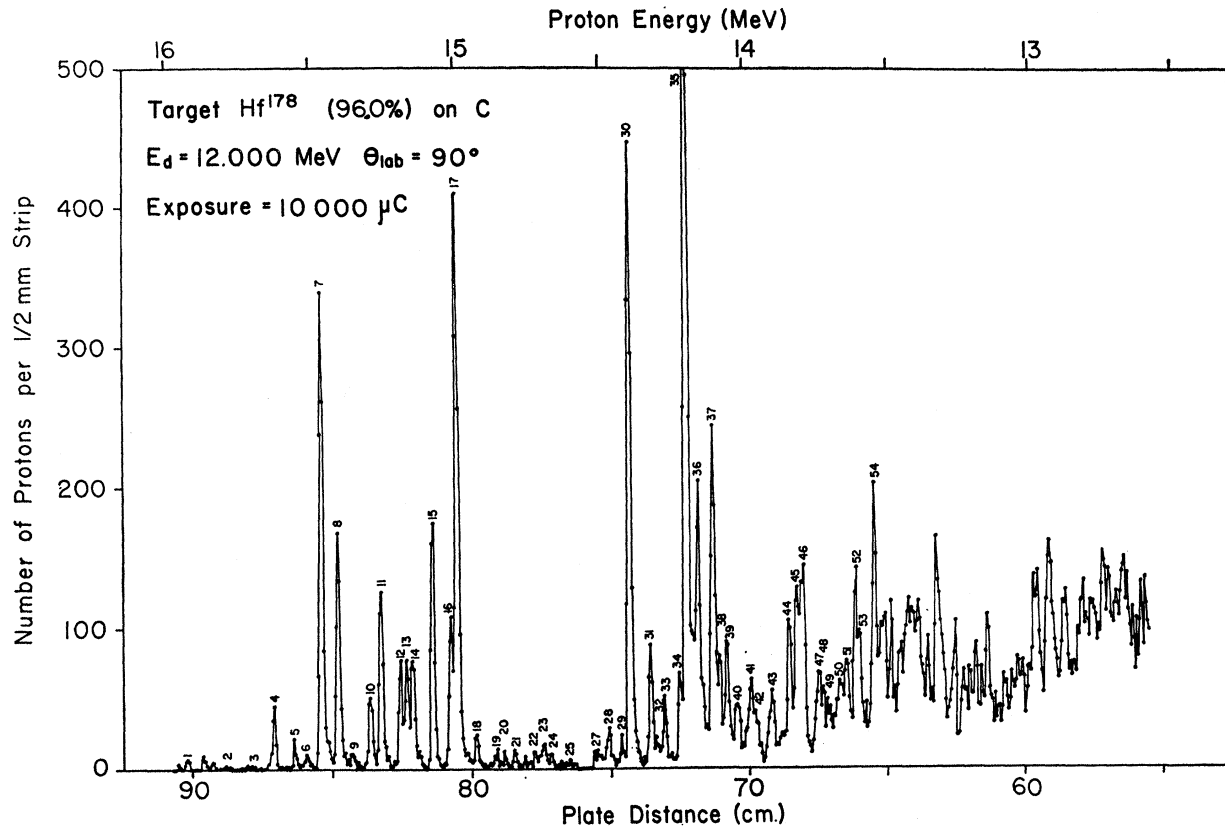


FIG. 13. Spectrum of the levels in ^{179}Hf as populated by the reaction $^{178}\text{Hf}(d,p)^{179}\text{Hf}$ at 90° .

cross sections and energies to be the $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ members of the $\frac{1}{2}^-$ [521] band.

The state reported by Vergnes at 721 keV is not seen in the (d,t) reaction. It is therefore assumed to be the bandhead of the $\frac{3}{2}^-$ [512] band, predicted to be a particle state. The next strong (d,p) state, which is seen weakly in the (d,t) reaction, is the 790-keV state, which is assumed to be the $\frac{5}{2}^-$ member of the $\frac{3}{2}^-$ [512] band. On the basis of these assignments the $\frac{7}{2}^-$ member is predicted by the rotational-energy formula to lie at 884 keV. While Vergnes does not report a state at this energy, there is evidence for a weak state on the back edge of the strong 874-keV (d,p) state which could be the 884-keV state in question.

The strong 874-keV state is seen weakly in the (d,t) reaction. It is therefore assigned as the $\frac{7}{2}^-$ member of the $\frac{3}{2}^-$ [503] band. This interpretation is shown in Fig. 14.

The additional information obtained from the (d,t) reaction has resulted in the assignment of four additional rotational bands which would not be assigned on the basis of the previous (d,p) work alone.

The rotational parameters resulting from this interpretation are given in Table IX. Following the method outlined in the discussion of ^{177}Hf , the experimental U^2 , V^2 , and $C_{j,i}^2$ have been calculated for each state, and

are given in Table X. However, the lack of angular distributions introduced more uncertainty in the calculations than in the cases of ^{177}Hf and ^{179}Hf .

TABLE VIII. Energy levels in ^{179}Hf .

Level No.	$E_{(\text{ex})}(d,t)$ (keV)	σ (keV)	$E_{(\text{ex})}(d,p)$ (keV)
0	0	2.9	0
1	216	0.6	122
2	268	1.9	268
3	339	0.9	336
4	376	1.1	373
5	422	1.2	421
6	471	1.5	476
7	483	4.6	
8	517	2.2	
9	579	2.3	582
10	617	0.9	616
11	681	1.1	679
12	701	1.8	702
			721
13	794	4.2	789
14	853	0.75	851
15	875	1.0	871
16	902	3.6	
17	937	2.4	937
18	1010	3.1	1004
			1031
			1065
19	1167	0.6	1168
20	1201	0.9	1192

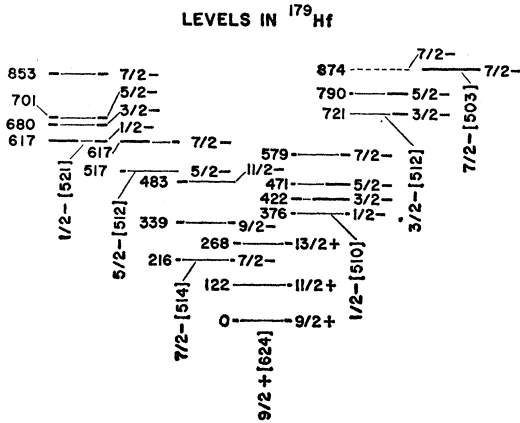


FIG. 14. Level scheme for ^{179}Hf showing assignments of rotational bands. The heavy lines for each level are proportional to the (d,t) or (d,p) cross section of the level.

D. Complex States

In ^{177}Hf and ^{181}Hf , a large number of rotational bands have been observed which do not resemble Nilsson's predictions; they have been classified as complex states. It must be pointed out from the beginning that our assignments, while based on l values and energy spacings, are uncertain because of the high-level density and the lack of systematic knowledge of this excitation region. On the other hand, almost all of the levels observed fit into our interpretation, and it seems to be highly consistent with the data.

The large number of low K rotational bands cannot be easily explained. We believe these bands, observed in the (d,p) reaction through single-particle components in their wave functions, to be partially vibrational in character. In a previous work,²² we have discussed a possible explanation of these bands as γ , β , or pairing vibrations based on low-lying Nilsson states. This explanation seems reasonable since these vibrations are predicted to be the basis of the lowest-lying complex states. We further suggested that the total cross section below 2 MeV is consistent with the identification of these states as fragments of the observed Nilsson states, i.e., the total cross section below 2 MeV is 97% of that

TABLE IX. Rotational parameters for ^{179}Hf .

$E_{\text{(ex)}} \text{ (keV)}$	Assignment	$\hbar^2/2\mathcal{I} \text{ (keV)}$	a
0	$\frac{9}{2}^+ [624]$	11.1	
216	$\frac{7}{2}^- [514]$	13.5	
376	$\frac{1}{2}^- [510]$	12.6	0.21
517	$\frac{5}{2}^- [512]$	14.3	
617	$\frac{1}{2}^- [521]$	12.6	0.67
721	$\frac{3}{2}^- [512]$	13.8	
874	$\frac{7}{2}^- [503]$		

predicted for the observed Nilsson states alone. However, the possibility of detailed interpretation of these bands in terms of specific γ , β , or pairing vibrations seems impossible at this time. In particular, there is a large amount of cross section in the $\frac{1}{2}^-$ bands. The only $\frac{1}{2}^-$ Nilsson states that we have been able to identify are the $\frac{1}{2}^- [510]$ and the $\frac{1}{2}^- [521]$. These are very pure Nilsson states in ^{181}Hf and ^{177}Hf , hence the complex $\frac{1}{2}^-$ states cannot contain much of their strength. Furthermore, the calculations of Bes¹¹ and of Soloviev,¹⁰ who have independently treated vibrational states in odd- A nuclei, indicate that normally the lowest fragment of a Nilsson state contains the predominant strength of the state. We cannot identify any of the complex $\frac{1}{2}^-$ states as the $\frac{1}{2}^- [501]$ Nilsson state or any of the $N=7$, $\frac{1}{2}^-$ states; in fact, two of them in ^{181}Hf and one in ^{177}Hf resemble the $\frac{1}{2}^- [510]$ band. Thus we cannot account for the $\frac{1}{2}^-$ strength in the complex states.

Soloviev²³ has calculated the energies and character of states up to 1750 and 1900 keV in ^{177}Hf and ^{181}Hf , respectively. His calculations for the states which are predominantly Nilsson states agree quite well with our data, as is seen in Table XI. His results for the complex states do not agree with our experimental data (Table XI). Our data suggest that there is something missing in the calculations and the assumptions on which they are based.

Thus it seems that the level structure between 1 and 2 MeV in both ^{181}Hf and ^{177}Hf is more complicated than either a simple vibrational interpretation or Soloviev's calculations predict. Further experimental work is needed. The approaches which seem most feasible are

TABLE X. Experimental values of U^2 and $C_{j,l}^2$ for levels of ^{179}Hf .

$E_{\text{(ex)}} \text{ (keV)}$	Assignment	Expt. U^2	Experimental values of $C_{j,l}^2$							
			$C_{1/2,1}^2$	$C_{3/2,1}^2$	$C_{5/2,3}^2$	$C_{7/2,3}^2$	$C_{9/2,5}^2$	$C_{11/2,5}^2$	$C_{9/2,4}^2$	$C_{11/2,6}^2$
0	$\frac{9}{2}^+ [624]$	0.382							0.102	0.030
216	$\frac{7}{2}^- [514]$	0.365				0.0219	0.799	1.28		
376	$\frac{1}{2}^- [510]$	0.877	0.011	0.354	0.434	0.137				
517	$\frac{5}{2}^- [512]$	0.157			0.046	~ 0.7				
617	$\frac{1}{2}^- [521]$	0.157	~ 0.25	0.045	0.258	0.18				
721	$\frac{3}{2}^- [512]$	0.66		0.097	0.45					
874	$\frac{7}{2}^- [503]$	0.94				0.713				

²² F. A. Rickey and R. K. Sheline, Phys. Rev. Letters 17, 1057 (1966).

²³ V. G. Soloviev, P. Vogel, and G. Jungklaussen, Joint Institute for Nuclear Research Report, Dubna, 1966 (unpublished).

the use of reactions which selectively populate vibrational states or the use of a technique which allows one to study the decay of vibrational states. One of us is investigating both the use of the (t,p) reaction to study the levels in ^{181}Hf and the decay of analogs of the states in ^{181}Hf to states seen in the reaction $^{180}\text{Hf}(p,p')^{180}\text{Hf}$. We hope that these methods will lead to a more complete interpretation of the results presented here.

VI. SUMMARY AND CONCLUSIONS

The levels in ^{181}Hf up to 2 MeV have been studied by the reaction $^{180}\text{Hf}(d,p)^{181}\text{Hf}$. Forty-nine states were seen, and angular distributions allowed assignments of l values to 29 of these states. The combination of known l values and rotational-energy systematics have made possible the classification of 39 states into 13 rotational bands. The $\frac{3}{2}^+[624]$, $\frac{1}{2}^- [510]$, $\frac{3}{2}^- [512]$, $\frac{7}{2}^- [503]$, and $\frac{1}{2}^{3,+}[606]$ Nilsson states have been identified, and tentative identification made of the $\frac{3}{2}^- [501]$ and $\frac{5}{2}^- [503]$ Nilsson states. Three $\frac{3}{2}^-$ rotational bands and three $\frac{1}{2}^-$ rotational bands cannot be identified by the Nilsson model.

The levels in ^{177}Hf up to 2 MeV have been studied by the complementary reactions $^{176}\text{Hf}(d,p)^{177}\text{Hf}$ and $^{178}\text{Hf}(d,t)^{177}\text{Hf}$. Forty-six states have been observed, and angular distributions allowed the assignment of l values to 26 of these states. The consideration of l values, rotational-energy systematics, and relative (d,p) - (d,t) cross sections has made possible the classification of 39 states into 15 rotational bands. The $\frac{7}{2}^+[633]$, $\frac{1}{2}^- [521]$, $\frac{5}{2}^- [512]$, $\frac{7}{2}^- [514]$, $\frac{3}{2}^+[624]$, $\frac{1}{2}^- [510]$, $\frac{3}{2}^- [512]$, $\frac{7}{2}^- [503]$, and $\frac{3}{2}^- [501]$ Nilsson states have been identified. Two additional $\frac{1}{2}^-$ bands and two additional $\frac{3}{2}^-$ bands have been observed at higher excitation energies which cannot be identified by the Nilsson model.

Levels in ^{179}Hf up to 1 MeV have been studied by the reaction $^{180}\text{Hf}(d,t)^{179}\text{Hf}$, and were used in conjunction with the levels studied by the reaction $^{178}\text{Hf}(d,p)^{179}\text{Hf}$ in the previous work of Vergnes. The value of the complementary (d,t) and (d,p) reactions has been demonstrated by the identification of the $\frac{1}{2}^- [521]$, $\frac{5}{2}^- [512]$, $\frac{3}{2}^- [512]$, and $\frac{7}{2}^- [503]$ bands which had not been identified previously.

While we believe the complex states seen in ^{177}Hf and ^{181}Hf to be vibrational in nature, no detailed information about their exact nature is available. More experimental and theoretical work is needed to classify them further.

TABLE XI. Comparison of experiment with Soloviev's calculations.

Energy (keV)		Structure		
$K\pi$	Expt. Calc	Expt.	Calc	
^{177}Hf				
$\frac{7}{2}^-$	0 0	$\frac{7}{2}^- [514]$	80%	$\frac{7}{2}^- [514]$ 96%
$\frac{3}{2}^-$	504 230	$\frac{3}{2}^- [514]$	94%	$\frac{3}{2}^- [514]$ 97%
$\frac{3}{2}^+$	324 440	$\frac{3}{2}^+ [624]$	100%	$\frac{3}{2}^+ [624]$ 99.5%
$\frac{1}{2}^-$	567 600	$\frac{1}{2}^- [510]$	$\sim 102\% + ?$	$\frac{1}{2}^- [510]$ 80% + ...
$\frac{1}{2}^-$	560 720	$\frac{1}{2}^- [521]$	96%	$\frac{1}{2}^- [521]$ 90% + ...
$\frac{3}{2}^-$	804 750	$\frac{3}{2}^- [512]$	82.5%	$\frac{3}{2}^- [512]$ 68% + ...
$\frac{7}{2}^-$	1058 1100	$\frac{7}{2}^- [503]$	59%	$\frac{7}{2}^- [503]$ 82% + ...
$\frac{3}{2}^-$	1434 1700	$\frac{3}{2}^- [501]$	41%	$\frac{3}{2}^- [501]$ 26% + ...
$\frac{3}{2}^-$	1502 1660	$\frac{3}{2}^- [?]$	10%	$\frac{3}{2}^- [512]$ 11% + γ on $\frac{1}{2}^- [501]$ and $\frac{7}{2}^- [514]$
$\frac{1}{2}^-$	1634 1550	$\frac{1}{2}^- [?]$	18%	$\frac{1}{2}^- [510]$ 4% + γ on $\frac{5}{2}^- [512]$
$\frac{3}{2}^-$	1666 1750	$\frac{3}{2}^- [?]$	5%	$\frac{3}{2}^- [501]$ 30% + γ on and $\frac{1}{2}^- [501]$
$\frac{1}{2}^-$	1882 ...	$\frac{1}{2}^- [?]$	28%	
^{181}Hf				
$\frac{1}{2}^-$	0 0	$\frac{1}{2}^- [501]$	97%	$\frac{1}{2}^- [501]$ 94%
$\frac{3}{2}^-$	255 290	$\frac{3}{2}^- [512]$	83%	$\frac{3}{2}^- [512]$ 91%
$\frac{7}{2}^-$	670 610	$\frac{7}{2}^- [503]$	63%	$\frac{7}{2}^- [503]$ 92%
$\frac{3}{2}^-$	1063 ...	$\frac{3}{2}^- [?]$	11%	...
$\frac{3}{2}^-$	1267 ...	$\frac{3}{2}^- [?]$	5%	...
$\frac{3}{2}^-$	1330 ...	$\frac{3}{2}^- [?]$	29%	...
$\frac{1}{2}^-$	1406 ...	$\frac{1}{2}^- [?]$	62%	...
$\frac{3}{2}^-$	1503 800	$\frac{3}{2}^- [501]$	17%	$\frac{3}{2}^- [501]$ 90%
$\frac{5}{2}^-$	1637 1650	$\frac{5}{2}^- [503]$	60%	$\frac{5}{2}^- [503]$ 36%
$\frac{1}{2}^-$	1745 ...	$\frac{1}{2}^- [?]$	13%	...
$\frac{1}{2}^-$	1799 ...	$\frac{1}{2}^- [?]$	38%	...

The value of angular distributions in the study of deformed nuclei is clearly indicated. The interpretation of the region between 1 and 2 MeV in ^{177}Hf and ^{181}Hf has been a direct result of their use, and they have made the extraction of spectroscopic factors more reliable.

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