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$Ca^{48}(p,d)Ca^{47}$ and $Ca^{48}(p,t)Ca^{46}$ Reactions*

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This study of the $Ca^{48}(p,d)Ca^{47}$ reaction at a bombarding energy of 18 MeV finds evidence for a small admixture of neutrons from the $2p$ orbitals in the ground state of Ca^{48}. The summed spectroscopic factor for $2p$ pickup is 0.36. If the mixing is of zero-coupled pairs of $2p_{3/2}$ neutrons, the coefficient of this admixed term in the wave function is 0.42. The positions and spectroscopic factors for several states that appear to be due to d-wave or f-wave pickup are also given, and two apparent examples of j dependence are used to make spin assignments of $\frac{5}{2}^-$ in Ca^{47}. Angular distributions of tritons from the $Ca^{48}(p,t)Ca^{46}$ reaction are shown for seven states of Ca^{46}, but reliable spin determinations cannot be made, owing to the lack of structure in the observed distributions.

I. INTRODUCTION

RECENT experimental work has demonstrated that the nucleus Ca^{40} has a ground state that is not a simple $2s$-$1d$ closed shell.[1] There is some evidence from its level structure that Ca^{48}, with the neutron $f_{7/2}$ orbital filled, is a more nearly magic nucleus.[2,3] This would not be unexpected, since the configuration mixing in Ca^{40} is due to the presence of fairly low-lying deformed states.[4] The energies of two of these (Nilsson Nos. 13 and 14) decrease with deformation, but after these are filled with four neutrons the deformed states are found much higher. Consequently, we expect less mixing of the neutron states in Ca^{48} than in Ca^{40}.

In order to determine the content of the ground state of Ca^{48} the cross sections for single-neutron pickup via the $Ca^{48}(p,d)Ca^{47}$ reaction at 18 MeV have been studied. This reaction was investigated by Conlon, Bayman, and Kashy at a proton energy of 17.5 MeV.[5] Recent improvements in the Princeton FM cyclotron made it feasible to investigate the weaker states in the deuteron spectrum which could not be seen in the earlier experiment because of poor resolution and poor statistics. Comparison of the measured angular distributions to the predictions of the distorted-wave Born-approximation (DWBA) code JULIE then allows one to assign values of l (the orbital angular-momentum transfer), and C^2S_l (the spectroscopic factor) for states of Ca^{47} up to 4.76 MeV in excitation. The sums of these spectroscopic factors for $l=1$ and $l=3$ provide the amounts of $2p$ and $1f$ neutron strength in Ca^{48}. In a few cases the empirical j dependence of the angular distributions allows one to specify the amounts of $1f_{5/2}$ or $1f_{7/2}$ strength.

The Ca^{48} targets for this experiment were flakes of calcium oxide and carbonate formed by exposing to the atmosphere an enriched film of the evaporated metal about 0.8 mg/cm² thick. These flakes were mounted on thin Formvar and proved to be quite rugged.

Conventional dE/dX-E particle-identification techniques were used to identify the deuterons and tritons. The first counter (ΔE) was a 47-μ-thick silicon surface-barrier detector, and the second E' detector was 684 μ thick. A third detector was used to reject the high-energy protons that passed through the first two detectors. The sums of the ΔE and E' pulses were routed into various quadrants of a 4096-channel analyzer according to the value of the product $\Delta E\times(E'+\Delta E)$, which is proportional to the mass of the particle producing the pulse. Spectra due to the protons stopping in the first two detectors were also recorded so as to note if any strong proton peak would happen to feed into the much weaker deuteron spectrum.

A beam of 18-MeV protons was provided by the Princeton FM cyclotron. The exit slit of the energy analyzing magnet was 3 mm wide with a "wedge" of aluminum strips to compensate for the energy difference across the slit. The emerging protons were then focused by a quadrupole lens onto the target in a 20-in. scattering chamber. This slit provided a beam resolution of about 25 keV, instead of the 60 keV expected without compensation, and allowed beam currents of up to 60 nA to be obtained.[6]

All data were normalized by comparison to the elastic cross section at the 95° maximum, where a monitor counter was fixed during each run. The uncertainty in the relative cross sections is determined by the counting statistics in each peak, while all cross sections are subject to an over-all uncertainty in absolute normalization. The absolute differential cross section at 95° was determined by a study of the elastic scattering at small angles. At angles near 20° it has been observed at this laboratory that the ratio of the elastic scattering cross section to the Rutherford prediction is almost constant for nuclei near Ca^{48} at proton bombarding

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* Supported in part by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.

[1] J. C. Hiebert, E. Newman, and R. H. Bassel, Oak Ridge National Laboratory Report No. ORNL-3800, 1965, p. 16 (unpublished); C. Glashausser, M. Kondo, M. E. Rickey, and E. Rost, Phys. Letters **14**, 113 (1965); J. C. Hiebert, E. Newman, and R. H. Bassel, Phys. Rev. **154**, 898 (1967); E. Rost, *ibid.* **154**, 994 (1967).

[2] R. J. Peterson, Phys. Rev. **140**, 1479 (1965).

[3] A. M. Bernstein and E. P. Lippincott, Phys. Rev. Letters **17**, 321 (1966).

[4] W. E. Gerace and A. M. Green, Nucl. Phys. **A93**, 110 (1967).

[5] T. W. Conlon, B. F. Bayman, and E. Kashy, Phys. Rev. **144**, 941 (1966).

[6] This wedge was developed and installed by Professor R. E. Pollock and Dr. L. C. McIntyre.

energies near 17.5 MeV. By a consideration of the scatter in this ratio for the nearby nuclei and the uncertainties in the absolute cross sections for these nuclei, the uncertainty in the normalization of the present results is estimated to be ±12%. This normalization is consistent with that of Timbie,[7] who compared the elastic cross sections for Ca^{48} to the known cross sections for elastic proton scattering from F^{19} by the use of a CaF_2 target. A measurement of the total energy loss of Am^{241} α particles in passing through the flake target was used to measure a calcium target thickness after a correction for the C^{12} and O^{16} content of the target. The normalization calculated from this target thickness was also consistent with that used above.

II. THEORY FOR THE (p,d) REACTION

The DWBA code JULIE[8] was used to fit the data in order to extract the values of l, the orbital angular-momentum transfer, and the spectroscopic factor. For many of the weaker states seen in this work the angular distribution at back angles is flat and structureless. Nevertheless, it is believed that the most relevant part of the angular distribution is the region of the first maximum, and more emphasis was placed on fitting these points. For each case the extreme credible fits were used to assign uncertainties for the relative spectroscopic factors.

One would expect the sum of the spectroscopic factors for the pickup of $1f$ and $2p$ neutrons to be eight. The simplest shell model would lead one to expect that all of this strength would be found in the transition to the $\frac{7}{2}^-$ ground state of Ca^{47}. The fragmentation of the $1f$ strength and the presence of $2p$ neutron strength would be of relevance for improved theoretical models for Ca^{48}.

Neutrons may also be picked out of the deeper $2s$ and $1d$ orbitals. In order to preserve the isospin quantum number the wave function of the lowest $(T=\frac{7}{2})$ $d_{3/2}$ hole state of Ca^{47} must be written as

$$
\begin{aligned}
|\tfrac{3}{2}^+ T=T_Z=\tfrac{7}{2}\rangle
&= (8/9)^{1/2}\{d_{3/2}^{-1}T=\tfrac{1}{2},\, T_Z=-\tfrac{1}{2}\}\{f_{7/2}^{8}T=4,\, T_Z=4\} \\
&\quad -(\tfrac{1}{9})^{1/2}\{d_{3/2}^{-1}T=\tfrac{1}{2},\, T_Z=\tfrac{1}{2}\}\{f_{7/2}^{8}T=4,\, T_Z=3\}.
\end{aligned}
$$

The spectroscopic factor to this state is then expected to be 32/9, rather than 4. The remaining strength would be in the transition to the analog to the K^{47} first excited state, which would lie at an excitation energy near 13.8 MeV in Ca^{47}.[9] Similarly, the spectroscopic factor to the $T=\frac{7}{2}$, $2s_{1/2}$ hole state is expected to be 16/9, and that to the $T=\frac{7}{2}$, $d_{5/2}$ hole state is expected to be 48/9. The spectroscopic strength of these hole states may be scattered over several states of Ca^{47}, and the sum of the observed spectroscopic factors should then be compared to the above predictions.[10]

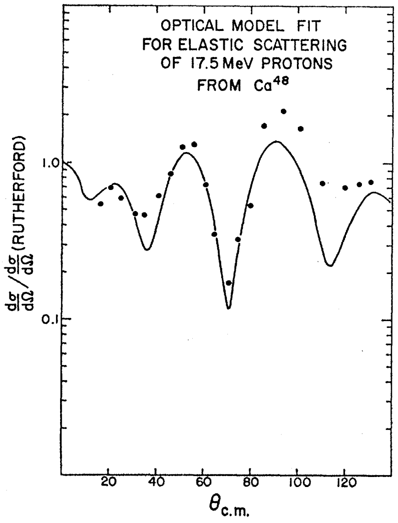

FIG. 1. The angular distribution for the elastic scattering of 17.5-MeV protons from Ca^{48} is presented as the ratio to the Rutherford prediction. The solid curve is the optical-model prediction obtained after a computer search for the best fit. The parameters are listed in Table I.

In order to obtain reliable optical-model parameters for the DWBA analysis, the elastic scattering of 17.5-MeV protons from Ca^{48} was first studied. In Fig. 1, we show the observed data (presented as the ratio to the Rutherford cross section) and the fit obtained with the parameters shown in Table I. The real potential is a bit shallower and larger than that suggested by Perey.[11] The deuteron potential was taken to be an average of those measured by the elastic scattering of 12-MeV deuterons from Ca^{48} [12] and 10-MeV deuterons from Ca^{46}.[13] The parameters are listed in Table I. The wave function for the picked-up neutron was generated from a Saxon-Woods potential with radius $1.25A^{1/3}$ F, diffusivity 0.65 F, and a spin-orbit strength of 25 times the Thomas value.

TABLE I. Here we list the optical-model parameters used by the code JULIE to produce the DWBA fits shown in the figures. First are given the parameters of the real well, then the spin orbit (SO) strength for a surface derivative shape with the same parameters as the real well. An independent surface derivative absorption was used, with the listed parameters. A Coulomb radius of $1.24A^{1/3}$ F was used.

	V (MeV)	r_0 (F)	a (F)	V_{so} (MeV)	r_0' (F)	a' (F)	W' (MeV)
Proton	42.9	1.34	0.76	7.50	1.25	0.50	64.0
Deuteron	116	1.00	0.78	0	1.39	0.62	57.2

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[7] J. P. Timbie, thesis, Princeton University, 1966 (unpublished); G. M. Crawley, Ph.D. thesis, Princeton University, 1965 (unpublished).

[8] The program for the DWBA code JULIE was kindly provided by R. H. Bassel, R. M. Drisko, and G. R. Satchler.

[9] R. Sherr, Phys. Letters **24B**, 321 (1967); J. H. Bjerregaard *et al.*, *ibid.* **24B**, 568 (1967).

[10] R. K. Bansal and J. B. French, Phys. Letters **11**, 145 (1964).

[11] F. G. Perey, Phys. Rev. **131**, 745 (1963).

[12] A. Marinov, L. L. Lee, and J. P. Schiffer, Phys. Rev. **145**, 852 (1966).

[13] T. A. Belote, J. H. Bjerregaard, O. Hansen, and G. R. Satchler, Phys. Rev. **138**, B1067 (1965).

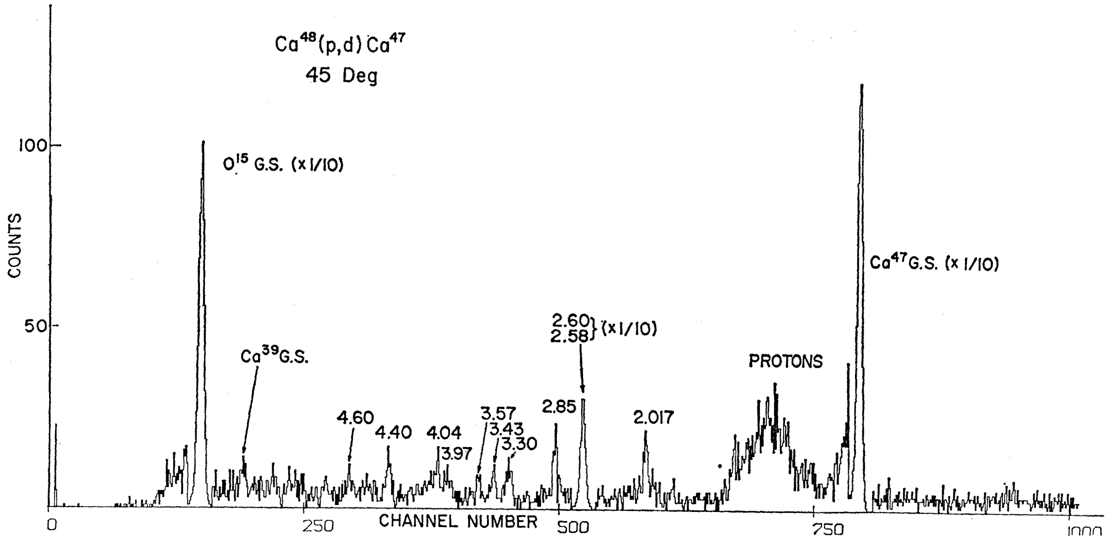

FIG. 2. The levels of Ca^{47} discussed in the text are indicated on this spectrum of the deuterons observed from the $Ca^{48}(p,d)Ca^{47}$ reaction.

The prescription for determining the depth of the potential that binds the $1f_{7/2}$ neutrons is straightforward. The physical separation energy (9.940 MeV) of the last neutron is provided for the DWBA code JULIE, and the depth of the potential is automatically varied to give the correct separation energy. The wave function of the neutron bound in this well is then used as the form factor for the computation. This separation-energy method cannot be used to generate the wave functions for the $2p_{3/2}$ neutrons, because the $2p_{3/2}$ single-particle energy is known from stripping reactions (see below) to lie 2.0 MeV above the $1f_{7/2}$ energy. The effective binding scheme would be to use a separation energy of 7.940 MeV to generate the $2p_{3/2}$ wave function. Another method to determine the $2p_{3/2}$ wave function would be to demand that the $2p_{3/2}$ neutrons be bound in the same well as was found for the $1f_{7/2}$ ground-state transition. A separation energy of 6.440 MeV is required to bind the $2p_{3/2}$ neutrons in this common well. Only the separation-energy method provides internal wave functions that connect smoothly with the free-particle external wave functions. The other schemes then cannot be complete. Unfortunately, the spectroscopic factors C^2S_1 are very sensitive to the choice of binding. The results of all three methods of analysis will be given. This discussion also holds, of course, for the pickup of $1f_{5/2}$ neutrons from Ca^{48}.

No cutoffs were employed, because the resulting DWBA predictions lie even further below the data for the ground-state transition than does the prediction with no cutoff (see Fig. 3).

It has been observed that the value of the total angular momentum transfer J, as well as the value of the orbital angular momentum transfer l, has an effect on the shape of the experimental angular distributions. This has been observed both for the (p,d) reaction[14] and its inverse, the (d,p) reaction.[15] This j dependence sometimes makes it possible to assign the spin of the final nuclear states. An apparent example of this effect will be given in the next section, where the detailed results of the $Ca^{48}(p,d)Ca^{47}$ reaction will be discussed.

III. RESULTS FOR THE $Ca^{48}(p,d)Ca^{47}$ REACTION

In order to identify states of Ca^{47}, Q values from the 1964 mass table[16] were used, with calibration points being taken as the Ca^{47} ground state, the Ca^{46} ground state (triton spectra were taken simultaneously), and the O^{15} ground state. States of C^{12} from the 1% C^{13} content of the carbon in the target and the Ca^{43} ground state from the 2% Ca^{44} content of the target were also seen and provided consistency checks for the energy calibration. Within the accuracy of this work in determining energies (15 keV), no discrepancies with the published nuclear masses were found.[16,17]

A typical deuteron spectrum is shown in Fig. 2, with an energy resolution of 50 keV full width at half-maximum (FWHM). The best resolution obtained was 35 keV. The two strong peaks are the Ca^{47} ground-state and the $\frac{3}{2}+$-$\frac{1}{2}+$ hole-state doublet at excitations of 2.580 and 2.600 MeV. These states were seen and discussed by Conlon, Kashy, and Bayman[5] and by Kavaloski *et al.*[18] Many states of Ca^{47} have also been

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[14] C. A. Whitten, E. Kashy, and J. P. Schiffer, Nucl. Phys. **86**, 307 (1967).

[15] L. L. Lee and J. P. Schiffer, Phys. Rev. **136**, B405 (1964).

[16] J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, Nucl. Phys. **67**, 32 (1965).

[17] J. R. Erskine, A. Marinov, and J. P. Schiffer, Phys. Rev. **142**, 633 (1966).

[18] C. D. Kavaloski, G. Bassani, and J. R. Maxwell, in *Proceedings of the Congrès International de Physique Nucleaires*, edited by P. Gugenberger (Centre National de la Recherche Scientifique, Paris, 1964), p. 487.

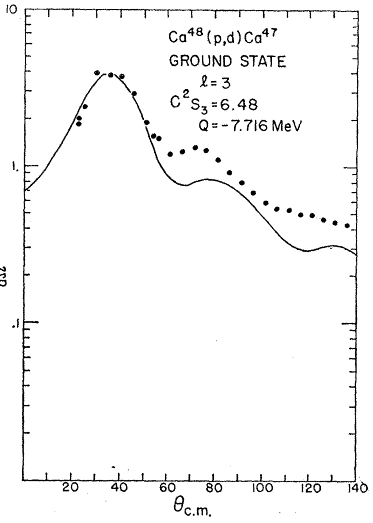

FIG. 3. The angular distribution of the deuterons populating the $\frac{7}{2}^-$ ground state of Ca^{47} is compared to the DWBA prediction.

seen from the Ca^{46}(d,p)Ca^{47} reaction, and some l values assigned.[19,20]

In Fig. 3 we show the data and DWBA fit for the ground-state pickup. The fit to the forward-angle data is quite good, and the lack of structure at larger angles is consistent with previous studies of the j dependence of pickup angular distributions. The measured spectroscopic factor is 6.48, subject to the 12% uncertainty in normalization mentioned above. Conlon, Bayman, and Kashy obtain a ground-state spectroscopic factor of 6.3.[5]

The first excited state of Ca^{47} seen in the (d,p) studies is a $\frac{3}{2}^-$ state at 2.017 MeV; the spin assignment comes from the observed j dependence.[19,20] In the present work this state is clearly seen, and the angular distribution shown in Fig. 4 is measured. The DWBA fit obtained with the separation-energy prescription for generating the neutron wave function is poor, even at the small-angle maximum. If the stripping data had not been available no l value would be assigned from this data. The poor fit to the data provides a spectroscopic factor equal to 0.05, with a 25% relative uncertainty. The effective binding method provides a value of C^2S equal to 0.04, and the common well method provides 0.025. These values of this spectroscopic factor will be discussed below.

The second and third excited states of Ca^{47} are the $1d_{3/2}$ and $2s_{1/2}$ hole states at 2.580 and 2.600 MeV. These could not be resolved in the present experiment, but it was observed that the 2.600-MeV member was the more strongly excited. In Fig. 5 the data for exciting the doublet are compared to the DWBA predictions for $l=0$ and $l=2$ pickup alone and to the best-fit sum of these shares. This best fit gives spectroscopic factors of 0.86 for the $l=2$ state and 5.0 for the $l=0$ state. The first of these numbers agrees with the result of Conlon, Kashy, and Bayman[5] but the $l=0$ spectroscopic factor is impossibly large; we expect a value of 1.78 at the most. In Fig. 5 we also show the angular distribution obtained using the expected values of the spectroscopic factors, 1.78 for the $l=0$ and 3.56 for the $l=2$. The fit at back angles is improved somewhat and the magnitude of the prediction is correct, but the strong dip near 25° is completely lost. The shape of the observed angular distribution must be due predominantly to the $l=0$ pickup. The spectroscopic factor (5.0) for the $l=0$ transition is obtained from fitting the 35° maximum in the angular distribution; this is in contrast to the standard practice used for the other states of placing the emphasis on the fit to the first maximum. If the shape predicted by the DWBA were wrong, and the second peak were larger, then a smaller spectroscopic factor would result. The fit to the second maximum for the ground-state angular distribution is noted to be poor. It would be difficult to place any trust in a DWBA calculation with parameters varied specifically in order to fit the angular distribution to this unresolved doublet, so no attempt was made.

It must be concluded that the spectroscopic factors for these hole states cannot be measured from the present work. The $d_{3/2}$ spectroscopic factor might very well be larger than the value of 0.86 quoted above.

The fourth and fifth excited states of Ca^{47} lie at excitation energies of 2.849 and 2.874 MeV.[19,20] The stripping to both states proceeds by the transfer of a $2p$ neutron, and the j dependence of the data indicates spins of $\frac{1}{2}^-$ for both states. This close pair of states could not be resolved in the present work, but most of the cross section was to a state near 2.85 MeV. The data are

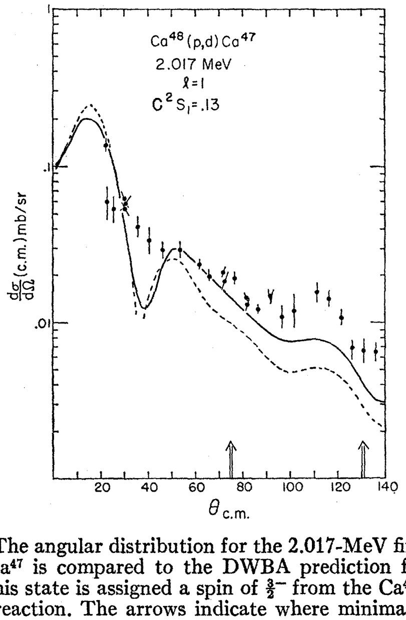

FIG. 4. The angular distribution for the 2.017-MeV first excited state of Ca^{47} is compared to the DWBA prediction for a $2p_{3/2}$ pickup. This state is assigned a spin of $\frac{3}{2}^-$ from the Ca^{46}(d,p)Ca^{47} stripping reaction. The arrows indicate where minima would be expected for a $2p_{1/2}$ pickup. (See Refs. 14 and 21.) The dashed line shows the shape of the prediction using the common well scheme described in the text.

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[19] J. H. Bjerregaard, O. Hansen, and G. Sidenius, Phys. Rev 138, B1067 (1965).

[20] T. A. Belote, H. Y. Chen, O. Hansen, and J. Rapaport, Phys. Rev. 142, 624 (1966).

shown in Fig. 6, where they are compared to the DWBA prediction for an $l=3$ pickup as well as the $l=1$ prediction. The $l=3$ fit is in contradiction to the conclusions reached in the $Ca^{46}(d,p)Ca^{47}$ work. There, the 2.849-MeV state was quite weakly excited, with a stripping spectroscopic factor about one-fifth of that to the 2.874-MeV state. As may be seen in Fig. 6, it is not possible for the $l=1$ pickup prediction to fit the present pickup data near 40°. At angles beyond 60° the data show more structure than does the DWBA prediction for $l=3$. There are two simple ways to produce this. The pickup to a $p_{1/2}$ state would be expected to be small near 40° and add to the back-angle data, with minima near 75° and 120° expected from the j-dependence.[21] If this $p_{1/2}$ pickup strength to the unresolved 2.874-MeV state is present, the spectroscopic factor must be less than $C^2S_1=0.02$. It has also been noted that for $l=3$ pickup, the angular distributions to $\frac{5}{2}^-$ states show more structure at back angles than do either the DWBA predictions or the data to $\frac{7}{2}^-$ states.[14,22] In Fig. 6 we show the ratio of the differential cross sections for the 2.85-MeV doublet to those for the ground state, a known $\frac{7}{2}^-$ transition. The observed structure is similar to that found for pickup transitions to states of known spins $\frac{5}{2}^-$ and $\frac{7}{2}^-$.[22] If this cross section is due to the pickup of an $f_{5/2}$ neutron, the spectroscopic factor is $C^2S_3=0.08\pm25\%$. If an $f_{7/2}$ neutron is being picked up, the spectroscopic factor would be $C^2S_3=0.10\pm25\%$. These numbers are obtained from the fit to the 40° maximum and are not sensitive to the presence of any $l=1$ pickup. Possibly three states, two with spins $\frac{1}{2}^-$ or $\frac{3}{2}^-$, and one with spins $\frac{5}{2}^-$ or $\frac{7}{2}^-$, are present.

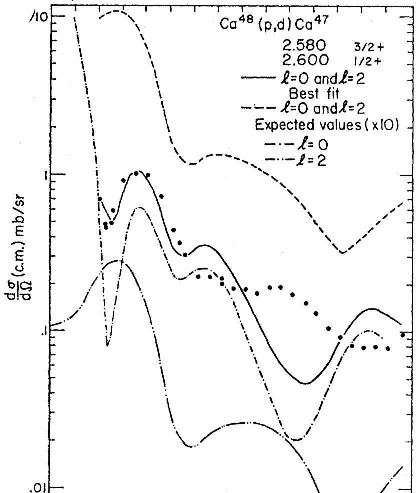

FIG. 5. The angular distribution to the unresolved $d_{3/2}$ and $2s_{1/2}$ hole states is compared to the DWBA prediction for a pure $l=0$ pickup and a pure $l=2$ pickup. The solid curve is the best-fit combination of these curves, and provides spectroscopic factors of 0.86 for the $l=2$ transition and 5.0 for the $l=0$. The expected values of the spectroscopic factors (3.56 for $l=2$ and 1.78 for $l=0$) were used to generate the dashed curve.

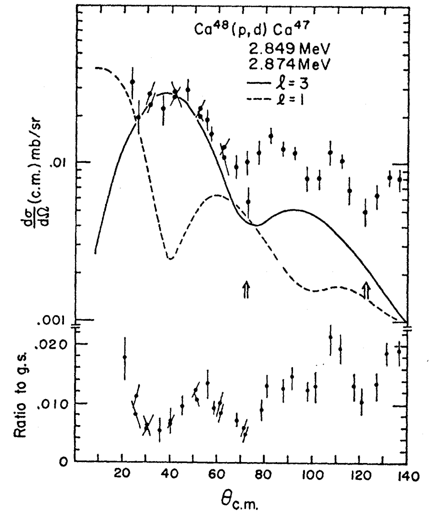

FIG. 6. The angular distribution for exciting an unresolved doublet at 2.849–2.874 MeV is shown. Both states are assigned $l=1$ from stripping experiments, but the DWBA curves shown indicate an $l=3$ assignment. The arrows indicate the positions where minima are expected for a $2p_{1/2}$ pickup. The ratio of this data to the ground-state cross sections is shown at the bottom; the observed structure is reminiscent of that for pickup to known $\frac{5}{2}^-$ states.

The sixth state of Ca^{47} seen in the $Ca^{46}(d,p)$ reaction lies at 3.296 MeV.[19,20] In Fig. 7 we show the weak cross sections seen for this state in the present experiment. The data at angles below 60° are not inconsistent with the $l=1$ assignment made in the stripping experiment.[20] There is little structure at back angles and it is not clear that a direct reaction analysis is possible, but a straightforward comparison to the DWBA prediction (with the separation-energy prescription) provides a spectroscopic factor equal to $C^2S_1=0.08\pm25\%$. A $p_{3/2}$ pickup is presumed from the bump seen near 100° in the stripping data.[15,20]

At 3.425 MeV we find a state whose angular distribution is shown in Fig. 8. The comparison is made to the DWBA prediction for $l=3$ pickup. Again there is extra structure in the back-angle data: This is emphasized by showing the ratio of the 3.425-MeV data to the ground-state data. This structure is again similar to that seen for the ratio of cross sections to known $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states[22] and is almost identical to that for the 2.85-MeV state. If the transition is considered as the pickup of an $f_{5/2}$ neutron the spectroscopic factor would be $C^2S_3=0.08\pm20\%$. The $Ca^{46}(d,p)Ca^{47}$ stripping data to this state show a nonstripping angular distribution.[19,20]

In Fig. 7 we show the data for the pickup reaction to a state of Ca^{47} at 3.565 ± 0.020 MeV and the DWBA prediction for an $l=1$ pickup. The fit is not very convincing, but the minimum near 40° provides some

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[21] L. C. McIntyre, Phys. Rev. **152**, 1013 (1966); C. A. Whitten, *ibid.* **156**, 1228 (1967).

[22] C. Glashausser and M. E. Rickey, Phys. Rev. **154**, 1033 (1967).

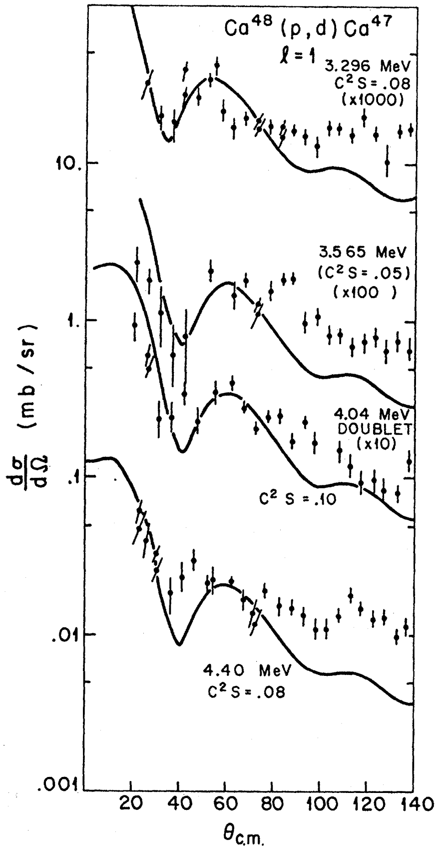

FIG. 7. The data for populating states of Ca^{47} by $l=1$ transitions are shown for four levels. The solid curve is the DWBA prediction using the separation-energy scheme to define the neutron well. These are assumed to be $2p_{3/2}$ pickups.

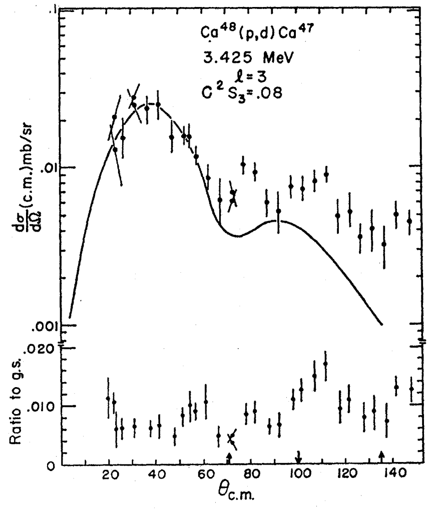

FIG. 8. The angular distribution for exciting the 3.425-MeV state of Ca^{47} is compared to the DWBA prediction for an $l=3$ pickup. The extra structure at back angles is emphasized by also showing the ratio of the data to the ground-state cross sections. Also shown are the locations of maxima or minima in the ratio found by Glashausser and Rickey (see Ref. 22) for an Fe^{56} target.

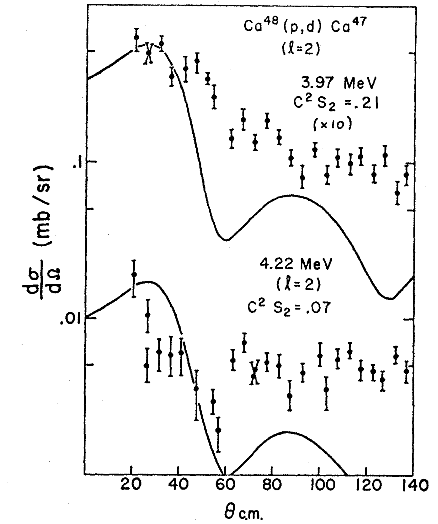

FIG. 9. The data for populating two states of Ca^{47} are compared to the DWBA predictions for the pickup of a $1d_{3/2}$ neutron. The fits are not good.

evidence for an $l=1$ transfer, with a spectroscopic factor equal to 0.05±30%. This state is not reported from the $Ca^{46}(d,p)$ stripping studies.

A broad peak of several states is seen near 3.9 MeV. At least three states are present, centered near 3.88 MeV, at 3.97±0.02 MeV, and near 4.04 MeV. The data to the region near 3.88 MeV did not show a stripping pattern, and the maximum differential cross section is 0.028 mb/sr. A peak near the middle of this group was large enough to be reasonably well resolved. In Fig. 9 we exhibit the angular distribution to this 3.97-MeV state. The comparison to the DWBA prediction for a

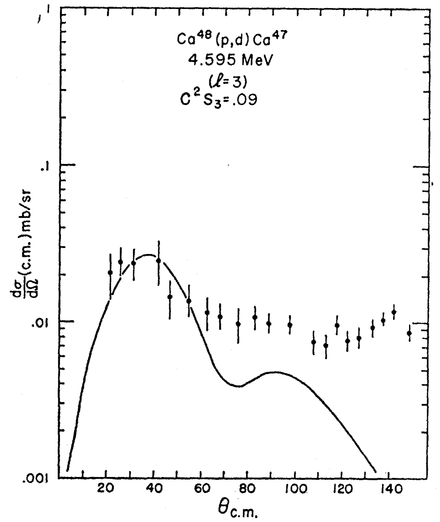

FIG. 10. The angular distribution to a state of Ca^{47} at 4.595 MeV is compared to the DWBA prediction for the pickup of an $f_{7/2}$ neutron.

$d_{3/2}$ pickup provides a spectroscopic factor equal to 0.21±20%.

The $Ca^{46}(d,p)Ca^{47}$ reaction populates states at 4.019 and 4.057 MeV; both are assigned $l=1$, and the j dependence suggests spins of $\frac{1}{2}^-$ for the lower and $\frac{3}{2}^-$ for the upper. Indeed, Fig. 7 indicates that an $l=1$ pickup is consistent with the present data, with a spectroscopic factor for the unresolved doublet equal to 0.10±30% (for the pickup of a $p_{3/2}$ neutron).

Another weak state is seen at 4.22±0.02 MeV. The data and the comparison of the DWBA prediction for an $l=2$ pickup are shown in Fig. 9. The drop in the angular distribution from 20° to 60° agrees with the prediction, but at larger angles the cross sections are too large. If this state is populated by a direct-pickup process, an $l=2$ assignment would be made, with a spectroscopic factor $C^2S_2=0.07\pm50\%$. This state is not reported from the stripping reaction on Ca^{46}.[19,20]

In Fig. 7 we also show the data for populating a state of Ca^{47} at 4.40 MeV. The $l=1$ assignment made from the DWBA fit shown is in agreement with the results of the $Ca^{46}(d,p)Ca^{47}$ stripping reaction.[19,20] A spectroscopic factor equal to 0.08±20% is found for the pickup of a $p_{3/2}$ neutron. Again the back-angle data appear too high and structureless when compared to the DWBA prediction, providing evidence for an excitation by other than a simple direct-pickup reaction.

At 4.595±0.01 MeV we find a state providing the angular distribution shown in Fig. 10. The DWBA prediction for an $f_{7/2}$ pickup is also shown. There is some evidence for a fit to the maximum near 35°, but again the direct nature of the excitation is in doubt. The small-angle data are fit with a spectroscopic factor $C^2S_3=0.09\pm30\%$.

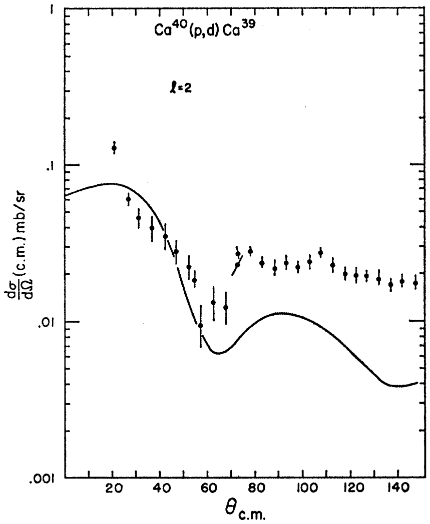

FIG. 11. The angular distribution for the $Ca^{40}(p,d)Ca^{39}$ transition is compared to the DWBA prediction for the pickup of a $1d_{3/2}$ neutron. The Ca^{48} optical-model parameters are used for this DWBA prediction.

TABLE II. Ca^{47}. The results of the present experiment are summarized here. The excitation energies and largest observed differential cross sections (at any angle) are listed, as well as the angular-momentum transfer and spectroscopic factors obtained from the (p,d) data with the aid of the DWBA predictions. The listed spectroscopic factors are obtained with the separation-energy (S.E.) method described in the text, and with the common well (C.W.) scheme. The known spins of the observed states are listed where known.

Ex. energy (MeV)	l	$\sigma_{\max}$ (mb/sr)	C^2S S.E.	C^2S C.W.	Known spin
0	3	4.0	6.5		$\frac{7}{2}^-$
2.017	1	0.13	0.05	0.025	$\frac{3}{2}^-$
2.58	2	1.0			$\frac{3}{2}^+$
2.60	0				$\frac{1}{2}^+$
2.85	(3)	0.03	0.08	0.02	$\frac{3}{2}^-$, $\frac{1}{2}^-$ nearby
3.296	1	0.04	0.08	0.04	$l=1$
3.425	3	0.027	0.08	0.02	
3.565	(1)	0.022	0.05	0.025	
3.97	(2)	0.060	0.21		
4.04	1	0.09	0.10	0.05	$l=1$ doublet
4.22	(2)	0.016	0.07		
4.40	1	0.060	0.08	0.04	$l=1$
4.59	(3)	0.027	0.09		

A flat angular distribution is measured for the pickup transition to a state at 4.76±0.03 MeV in Ca^{47}. The maximum differential cross section is 0.020 mb/sr. This is probably the state seen at 4.785 MeV in the $Ca^{46}(d,p)Ca^{47}$ reaction.[19,20] There, an $l=3$ stripping pattern was measured, and by a consideration of the summed spectroscopic factors an $f_{5/2}$ transition was assigned.

The $Ca^{40}(p,d)Ca^{39}$ transition to the ground state of Ca^{39} was also seen. In Fig. 11 we show the observed angular distribution and the DWBA prediction for a $d_{3/2}$ pickup. A spectroscopic factor equal to 4.6±30% was calculated with the Ca^{48} DWBA parameters. This peak provided a further check on the energy calibration for the excited states of Ca^{47}.

In Table II we summarize the data obtained from the present $Ca^{48}(p,d)Ca^{47}$ experiment, and the results are compared to other relevant data. The spectroscopic factors for transitions identified as $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $d_{3/2}$, and $s_{1/2}$ neutron pickups are listed, as are the total strengths for $l=1$ and 3.

As an example of the meaning of the present results let us write a model wave function for the neutrons in the Ca^{48} ground state of the form

$$\alpha(f_{7/2})_0{}^8+\beta(f_{7/2})_0{}^6(p_{3/2})_0{}^2+\gamma(f_{7/2})_0{}^6(f_{5/2})_0{}^2.$$

The spectroscopic factors obtained with the separation-energy method described previously then imply the values of the coefficients:

$$\alpha^2=0.62\pm0.07=(0.79\pm0.04)^2,$$
$$\beta^2=0.18\pm0.03=(0.42\pm0.03)^2,$$
$$\gamma^2=0.08\pm0.02=(0.29\pm0.03)^2.$$

The quoted uncertainties are relative only, and are subject to a 12% uncertainty in the normalization. We may check the relative completeness of our model wave

TABLE III. Ca^{46}. The results of the $Ca^{48}(p,t)Ca^{46}$ studies are summarized in this table. The maximum cross sections at any angle are listed in mb/sr.

Ex. energy (MeV)	L	$\sigma_{\max}$ (mb/sr)	Known spin
0	0	0.29	0^+
1.347	2	0.22	2^+
2.42	(0)	0.03	0^+
2.58		0.07	4^+
2.97		0.05	
3.03			2^+
3.61		0.054	3^-
3.85		~0.01	
4.23		~0.01	
4.44		0.070	$2^+, 3^-$
4.74		~0.01	

function from

$$\alpha^2+\beta^2+\gamma^2=0.88\pm0.08.$$

This normalization is within the experimental uncertainty.

The results of this experiment are surprising in that they indicate that Ca^{48} is not completely a closed-shell nucleus, but contains appreciable admixtures from the $2p_{3/2}$ and $1f_{5/2}$ orbitals.

The importance of this conclusion warrants a closer examination of the orbital angular momenta and spectroscopic factors assigned in this work. The critical measurements are those for the $l=1$ neutron pickups. Four of the five $l=1$ transitions found here proceed to states excited in the $Ca^{46}(d,p)Ca^{47}$ reaction, where large cross sections were measured and good DWBA fits were obtained.[19,20] In each case the stripping analysis agrees with the present assignments of the transferred angular momentum. The one new $l=1$ state (at 3.565 MeV) provides only 13% of the total $l=1$ pickup strength, and a complete misassignment would change the value of β^2 by only 13%.

Many of the weak angular distributions seen to these states in the present work show a nondirect behavior at large angles, indicating that the fits are overestimating the direct spectroscopic factor. If the difference of the average back-angle differential cross sections from the DWBA prediction is subtracted from the small-angle maximum, the total $l=1$ spectroscopic factor would change from 0.36 ± 0.04 to 0.30 ± 0.04 and β would be reduced only to 0.39. The completeness check becomes

$$\alpha^2+\beta^2+\gamma^2=0.75\pm0.07.$$

There are five other states of Ca^{47} populated by $l=1$ transfers in the $Ca^{46}(d,p)Ca^{47}$ reaction that are not seen in the present work.[19,20] At least two of these (at 2.849 and 2.874 MeV) are assigned spins $\frac{1}{2}^-$ and might be expected to be weak in the pickup data.

If the Ca^{48} ground state does not contain a filled $f_{7/2}$ neutron shell, then the $Ca^{48}(d,p)Ca^{49}$ reaction should populate some $\frac{7}{2}^-$ states of Ca^{49}. Indeed, studies of this reaction find that the transition to the 3.595-MeV state of Ca^{49} does not have the same angular distribution as that to the strong 4.005-MeV state, although both show $l=3$ patterns.[23] If the latter state is interpreted as being an $f_{5/2}$ transition, then perhaps the former is an $f_{7/2}$ transition. The spectroscopic factor of 0.66 provides a value of $\beta^2+\gamma^2=0.33$, which should be compared to the value of 0.26 ± 0.05 measured in the present work. This would be considered close enough to be in agreement with the extent of the mixing of the $2p_{3/2}$ orbital into Ca^{48} if this 3.595-MeV state of Ca^{49} did in fact have spin $\frac{7}{2}^-$. The stripping of neutrons into the nonclosed $f_{7/2}$ shell of Ca^{48} would seem to be the best check of the present conclusions.

The above discussion has used the results of the conventional separation-energy scheme to generate the wave functions for the $2p_{3/2}$ neutrons. The effective binding scheme provides $l=1$ spectroscopic factors smaller by a factor of 0.7, and the results of the common well scheme are smaller by a factor 0.5. The coefficients of the model wave function obtained from the latter results are

$$\alpha^2=0.69\ =(0.83)^2,$$
$$\beta^2=0.087=(0.30)^2,$$
$$\gamma^2=0.022=(0.15)^2,$$

and

$$\alpha^2+\beta^2+\gamma^2=0.80.$$

The admixture of neutrons from the $f_{5/2}$ orbital seems to be much less than that from the $2p_{3/2}$ orbital. The empirical phenomenon of j dependence was used to assign $\frac{5}{2}^-$ levels. One of these is particularly dubious, since the 2.85-MeV state seen in the present work corresponds to a state populated by an $l=1$ neutron transfer in the $Ca^{46}(d,p)$ work. The evidence for $f_{5/2}$ mixing is not nearly so striking as is the evidence of $2p_{3/2}$ mixing.

The spectroscopic factors for exciting the $d_{3/2}$ and $2s_{1/2}$ hole states at 2.58 and 2.60 MeV cannot be determined from the present work. Further $d_{3/2}$ neutron strength is found at 3.97 MeV and possibly 4.22 MeV, with a total spectroscopic factor of 0.28. In studies of the $Ca^{44}(p,d)$-Ca^{43} reaction, five $l=2$ states are found below 3 MeV, but most of the strength lies in the lowest state at 0.993 MeV.[24] In Ca^{43} the sum of the spectroscopic factors to all of the $l=2$ transitions seen is 1.14,[5] while a strength of 3.56 is expected. No further strong candidates for $l=2$ pickup states are seen below 5 MeV. The fragmentation of the $d_{3/2}$ strength appears to be less for Ca^{47} than for Ca^{43}.

IV. $Ca^{48}(p,t)Ca^{46}$ REACTION

Triton spectra were obtained simultaneously with the deuteron data discussed in Sec. III. In Fig. 12 we show a sample spectrum, with the excitation energies of the

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[24] S. Smith and A. M. Bernstein, Bull. Am. Phys. Soc. **12**, 93 (1967).

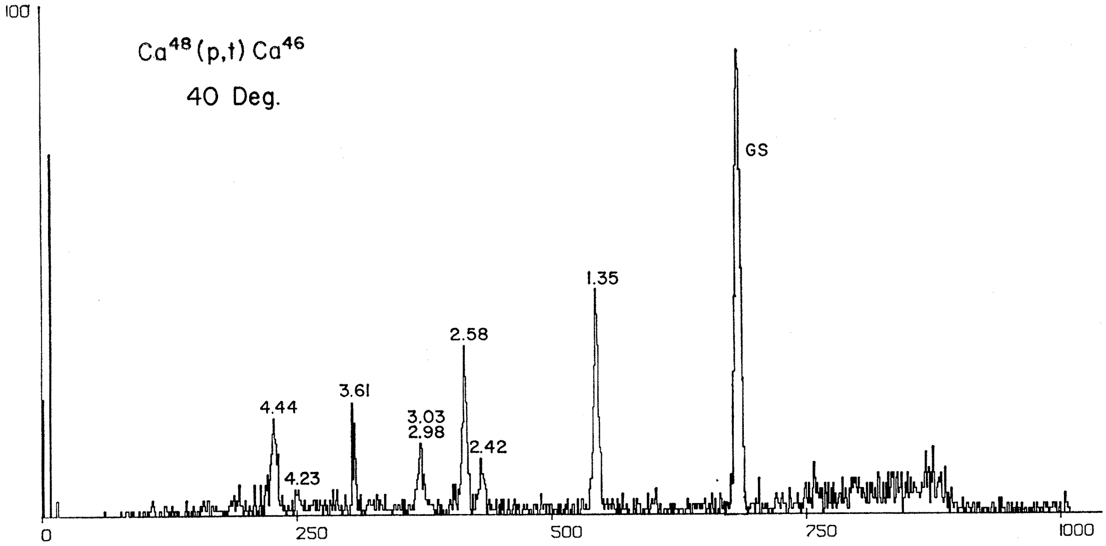

FIG. 12. This spectrum of the tritons from the $Ca^{48}(p,t)Ca^{46}$ reaction shows the levels of Ca^{46} discussed in the text.

states of Ca^{46} labeled. Previous studies of the level scheme of Ca^{46} include the inelastic scattering of 10-MeV deuterons and 7-MeV protons,[13] the $Ca^{44}(t,p)Ca^{46}$ reaction,[25] and the investigation of the decay scheme of the K^{46}.[26]

The data for populating the ground state of Ca^{46} by the (p,t) reaction are shown in Fig. 13. This highly structured angular distribution looks much like those for other $L=0$ (p,t) transitions between 0^+ ground states.[21,27] Figure 14 exhibits the data for exciting the 1.35-MeV first excited state of Ca^{46}. The angular distribution is similar to those for known $L=2$ transitions[21]; this is expected from the 2^+ spin assignment made for this state from previous work.[13,25,26]

The second excited state of Ca^{46} at 2.42 MeV is quite weakly excited. The minima seen in the angular distribution shown in Fig. 15 are seen to coincide with those for the ground-state $L=0$ transition. The ratio of the

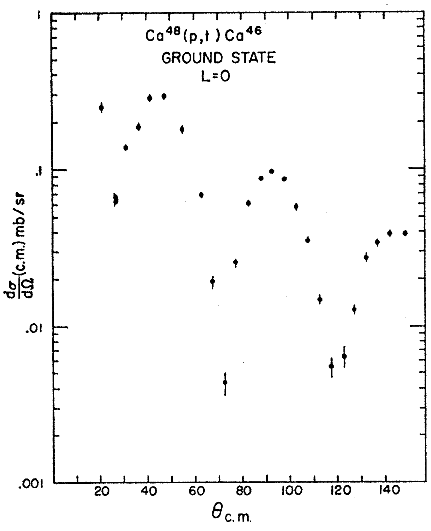

FIG. 13. This angular distribution for the tritons from the $Ca^{48}(p,t)Ca^{46}$ reaction populating the ground state of Ca^{46} shows the characteristic oscillatory pattern observed for other $L=0$ transitions.

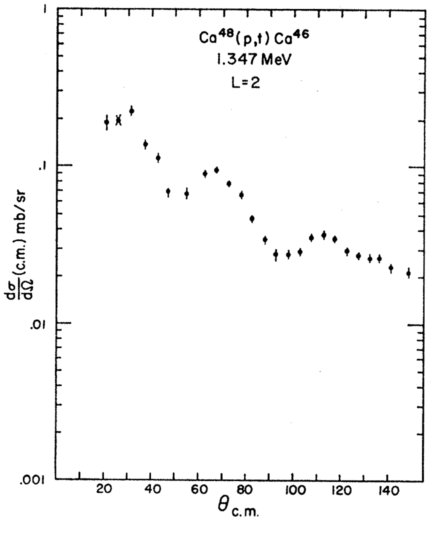

FIG. 14. The angular distribution to the 1.346-MeV state of Ca^{46} is shown in this figure. The observed structure is similar to that observed for $L=2$ transitions.

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[25] D. C. Williams, J. D. Knight, and W. T. Leland (to be published).

[26] B. Parsa and G. E. Gordon, Phys. Letters 23, 269 (1966).

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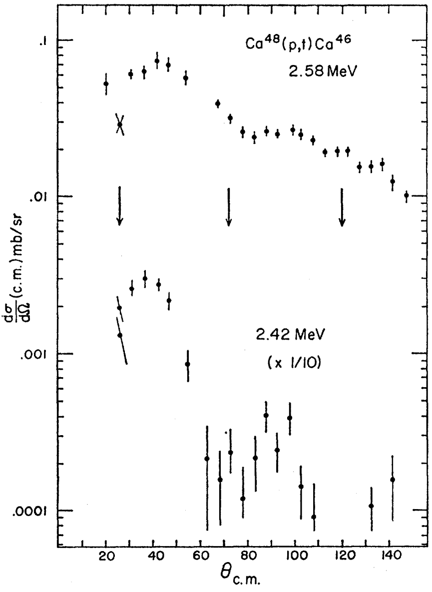

FIG. 15. The angular distributions for populating the states of Ca^{46} at 2.42 MeV and 2.58 MeV are shown in this figure. The arrows indicate the locations of the minima in the ground-state $L=0$ angular distribution.

90° maximum to the 40° maximum is about one-tenth, whereas the ratio of these two maxima for the ground-state transition is one-third. A very tentative 0$^+$ assignment is made for the 2.42-MeV state on the basis of the similarity of the angular distribution to that for the ground state. This spin assignment is in agreement with the work of Williams *et al.*[25]

A state at 2.58 MeV was more strongly excited, but the angular distribution shown in Fig. 15 bears no resemblance to either of the known shapes for $L=0$ or $L=2$ two-neutron pickups. The results of the Ca^{44}(t,p)-Ca^{46} experiment indicate a spin of 4$^+$ for this state.[25] One might expect the centrifugal barrier to inhibit the excitation of 4$^+$ states relative to 0$^+$ states at the bombarding energy used here, but in Fig. 15 it is seen that it is the probable 0$^+$ state that is three times weaker than the probable 4$^+$ state.

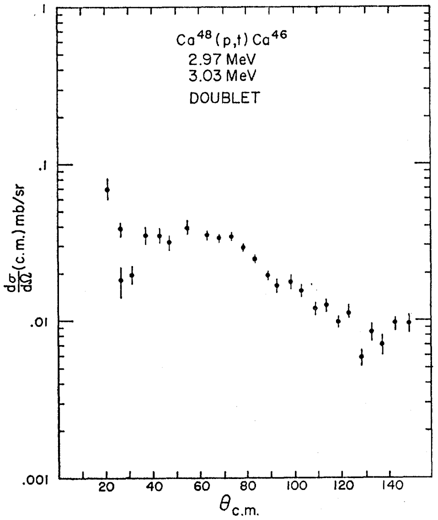

FIG. 16. This figure exhibits the data for populating an unresolved doublet at 2.97 and 3.03 MeV in Ca^{46}.

A pair of states in Ca^{46} was observed at 2.98±0.02 and 3.03±0.02 MeV in the present experiment. These could not be resolved completely, but were equally strongly excited at those angles (near 40°) where the experimental resolution was best. At small angles the 2.98-MeV state was the stronger. The angular distribution shown in Fig. 14 exhibits little structure except for a dip near 30° and a rise at smaller angles, presumably mainly due to the 2.98-MeV excitation.

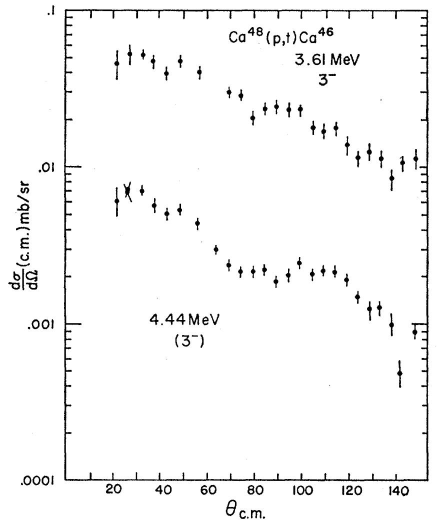

FIG. 17. The angular distributions of the tritons populating states of Ca^{46} at 3.61 and 4.44 MeV are shown here. A 3$^-$ state is known at 3.61 MeV, and a 2$^+$ and 3$^-$ state are reported near 4.44 MeV.

Two 3$^-$ states at 3.614 and 4.434 MeV were found in the inelastic scattering of 10-MeV deuterons from Ca^{46}.[13] Two states at 3.61±0.02 and 4.44±0.03 MeV show up strongly in the triton spectrum shown in Fig. 12, and it is tempting to identify these with the reported 3$^-$ states. The study of the Ca^{44}(t,p)Ca^{46} reaction, however, finds a 2$^+$ state at 4.433 MeV. The angular distributions obtained in the present work are shown in Fig. 15, where it may be seen that both are quite structureless. The magnitudes of the cross sections are nearly equal, and there is some similarity in the two angular distributions. Because of this similarity we conclude that a 3$^-$ state is being excited at 4.44 MeV. The two 3$^-$ states were equally strongly populated in inelastic scattering,[13] and it is interesting that this is also true in the (p,t) reaction.

Weaker transitions were found to states of Ca^{46} at 3.85±0.02, 4.23±0.02, and 4.74±0.03 MeV. No angular distributions were obtained for these transitions and no cross sections larger than 20 μb/sr were measured.

V. SUMMARY

The (p,d) neutron pickup reaction on Ca^{48} is found to weakly populate several states of Ca^{47} that are known to have spins $\frac{1}{2}^-$ or $\frac{3}{2}^-$ from studies of the $Ca^{46}(d,p)Ca^{47}$ reaction. The pickup of neutrons leading to these states is evidence for the presence of neutrons from the $2p$ orbitals mixed into the ground state of Ca^{48}. A DWBA analysis of the present results yields a total strength of about 0.4 such neutrons in Ca^{48}. Some evidence is also found for a smaller admixture of $f_{5/2}$ neutrons. When the measured spectroscopic factors are used to construct a model wave function for Ca^{48}, we find that the amplitudes for $2p_{3/2}$ and $1f_{5/2}$ admixtures are about one-half and one-third, respectively, of the dominant closed-shell amplitude. The $2s_{1/2}$ and $1d_{3/2}$ hole states of Ca^{47} formed an unresolved doublet in the observed deuteron spectra, and no meaningful statements could be made about the spectroscopic factor for these two states.

Several energy levels of Ca^{46} were populated by the two-neutron pickup reaction $Ca^{48}(p,t)Ca^{46}$. The conclusions drawn from the measured angular distributions are consistent with previous assignments for the energy levels of Ca^{46}.

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Nuclear-Structure Studies of the Chromium Isotopes: The $Cr^{50}(p,p')Cr^{50}$ and $Cr^{50}(d,p)Cr^{51}$ Reactions*

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The $Cr^{50}(p,p')Cr^{50}$ and $Cr^{50}(d,p)Cr^{51}$ reactions have been studied using 7.5-MeV protons and deuterons from the MIT-ONR electrostatic generator and the MIT multiple-gap spectrograph to analyze the reaction protons. Fifteen levels in Cr^{50} up to an excitation energy of 3.929 MeV and 156 levels in Cr^{51} up to an excitation energy of 7.931 MeV were observed. The ground-state Q value for the (d,p) reaction was measured as 7.041 ± 0.006 MeV. A zero-range, distorted-wave Born-approximation (DWBA) analysis was used to obtain values for the orbital angular momentum of the transferred neutrons and transition strengths for 52 levels. Three optical-model potentials for the deuteron were used in the DWBA analysis, each giving satisfactory fits to both elastic scattering and (d,p) data. The results are compared with shell-model, sum-rule limits, and with the level structure of other $N=27$ nuclei (Ca^{47} and Ti^{49}).

I. INTRODUCTION

THIS article is a report of an investigation of the level structure of Cr^{50} and Cr^{51} by means of the $Cr^{50}(p,p')Cr^{50}$ and $Cr^{50}(d,p)Cr^{51}$ reactions at an incident energy of 7.5 MeV. Fifteen levels in Cr^{50} up to 3.929-MeV excitation and 156 levels in Cr^{51} up to 7.931-MeV excitation are reported. Fifty-two of the latter have angular distributions characteristic of the (d,p) stripping process. A distorted-wave Born-approximation (DWBA) analysis was used to extract transition strengths $(2J+1)S$ and values for the orbital angular momentum l_n of the transferred neutrons. Three sets of optical-model parameters were tried in the DWBA analysis.

Nuclear-structure studies of Cr^{50} and Cr^{51} have been reported from several sources and involve different reactions and experimental techniques. The most recent detailed study of energy-level positions has been made by Macgregor and Brown,[1] using the (p,p') and (d,p) reactions on Cr^{50}. They report 84 levels in Cr^{50} up to an excitation energy of 6.376 MeV and 106 levels in Cr^{51} up to 7.661 MeV. Twin and Willmott[2] have made $(p,p\gamma)$ coincidence studies for individual proton groups from Cr^{50} and have determined the decay modes of levels at 3.96 and 3.81 MeV (both spin 2) and have confirmed the spin assignments for levels at 0.79 and 1.90 MeV

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