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Matrix Elements of the Spin-Spin Interaction for the f^3 Electron Configuration

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The spin-spin matrix elements for the f^3 electron configuration are computed by a method developed by Innes and checked by another method developed by Trees. A few irregularities have been noted.

I. INTRODUCTION

ENERGY-LEVEL structures showing deviations from the Landé interval rule for complex spectra can be explained in part by the consideration of the Heisenberg[1] spin-spin interaction operator. Araki[2] and Trees[3] suggest that this interaction may be very appreciable in discussing the spectra of atoms that have large atomic numbers.

The spin-spin interaction for the f^3 electron configuration has been computed by two independent methods, a recurring tensor method developed by Trees,[3] and a method developed by Innes[4] which expresses the submatrix element directly in the $l^n\alpha vSLJM$ scheme.

II. MATRIX ELEMENTS OF THE SPIN-SPIN INTERACTION

From the tensor equation in Racah,[5] Eq. (38), it can be shown that the spin-spin interaction is given by

$$(l^n\alpha SLJM|H_m{}^{\mathrm{II}}|l^n\alpha'S'L'JM) = (-)^{S+L'-J}W(SLS'L';J2) \times(l^n\alpha SL\|T^{(22)}\|l^n\alpha'S'L'), \quad (1)$$

where $(l^n\alpha SL\|T^{(22)}\|l^n\alpha'S'L')$ is given by Eq. (9) in Ref. 3 when $p=q=2$; namely,

$$(l^n\alpha SL\|T^{(22)}\|l^n\alpha'S'L') = (-)^{L+S-l+7/2}[n/(n-2)][(2S+1)(2L+1)(2S'+1)(2L'+1)]^{1/2} \times\sum_{\alpha_1S_1L_1,\alpha_2S_2L_2}(-)^{L_2+S_2}(l^n\alpha SL\{|l^{n-1}(\alpha_1S_1L_1)lSL(l^{n-1}\alpha_1S_1L_1\|T^{(22)}\|l^{n-1}\alpha_2S_2L_2) \times l^{n-1}(\alpha_2S_2L_2)lS'L'|\}l^n\alpha'S'L')W(S_1SS_2S';\tfrac{1}{2}2)W(L_1LL_2L';l2). \quad (2)$$

This submatrix element is the coefficient of M^t in Marvin's[6] notation.

Innes[4] in Eq. (18) expresses this submatrix element directly in the $l^n\alpha vSLJM$ scheme so that the coefficient of M^t is given by

$$(l^n\alpha vSL\|\mu_t\|l^n\alpha'vS'L') = -2(5)^{1/2}[(2t+5)!/(2t)!]^{1/2}(l\|C^{(t+2)}\|l)(l\|C^{(t)}\|l)\sum_{\alpha''S''L''}(l^n\alpha vSL\|V^{(1,t+2)}\|l^n\alpha''vS''L'') \times(l^n\alpha''vS''L''\|V^{(1,t)}\|l^n\alpha'vS'L')W(1S1S';S''2)W(t+2LtL';L''2). \quad (3)$$

[1] W. Heisenberg, Z. Physik 39, 499 (1926).
[2] G. Araki, Progr. Theoret. Phys. (Kyoto) 3, 152 (1948).
[3] R. E. Trees, Phys. Rev. 82, 683 (1951).
[4] F. R. Innes, Phys. Rev. 91, 31 (1953).
[5] G. Racah, Phys. Rev. 62, 438 (1942).
[6] H. H. Marvin, Phys. Rev. 71, 102 (1947).

For f electrons, $l=f=3$, so that in Eq. (3)

$$(f\|C^{(t)}\|f)=-7V(fft;000);\quad V(fft;000)=0\quad(2f+t,\ \text{odd}),$$

$$V(fft;000)=(-)^g\left[\frac{(2f-t)!(t!)^2}{(2f+t+1)!}\right]^{1/2}\frac{g!}{[(g-f)!]^2(g-t)!}\quad(2f+t=2g,\ \text{even}).\tag{4}$$

When $n=3$,

$$(f^3\alpha vSL\|V^{(1,t)}\|f^3\alpha'v'S'L')=(-)^{t-5/2-S'-L'}[3(6)^{1/2}/2][(2S+1)(2L+1)(2S'+1)(2L'+1)]^{1/2}$$
$$\times\sum_{S_1L_1}(f^3\alpha vSL\{|f^2(S_1L_1)fSLf^2(S_1L_1)fS'L'|\}f^3\alpha'v'S'L')W(\tfrac{1}{2}S\tfrac{1}{2}S';S_11)W(3L3L';L_1t),\tag{5}$$

and when $n=2$ the coefficients of fractional parentage are unity so that

$$(f^2SL\|V^{(1,t)}\|f^2S'L')=(-)^{t+1-S'-L'}6^{1/2}[(2S+1)(2L+1)(2S'+1)(2L'+1)]^{1/2}W(\tfrac{1}{2}S\tfrac{1}{2}S';S_11)W(3L3L';L_1t).\tag{6}$$

W is a Racah coefficient given in Ref. 7. Since t is restricted to the even numbers ranging from 0 to $2l-2$, we have, for f electrons, $t=0$, 2, 4. The spin-spin interaction is diagonal in v, as this interaction breaks up into products of one-particle tensors of odd degree and these are known to be diagonal in v.

The submatrix element in Eq. (3) was checked by Trees's recurring tensor method [see Eq. (2)] for $l=f$ and $n=3$; namely,

$$(f^3\alpha SL\|T^{(22)}\|f^3\alpha'S'L')=(-)^{L+S+1/2}3[(2S+1)(2L+1)(2S'+1)(2L'+1)]^{1/2}\sum_{S_1L_1,S_2L_2}(f^3\alpha SL\{|f^2(S_1L_1)fSL$$
$$\times(f^2S_1L_1\|T^{(22)}\|f^2S_2L_2)f^2(S_2L_2)fS'L'|\}f^3\alpha'S'L')W(S_1SS_2S';\tfrac{1}{2}2)W(L_1LL_2L';32),\tag{7}$$

where $(f^2S_1L_1\|T^{(22)}\|f^2S_2L_2)$ is evaluated by Eq. (3) when $l=f$ and $n=2$; namely,

$$(f^2S_1L_1\|T^{(22)}\|f^2S_2L_2)=-2(5)^{1/2}[(2t+5)!/(2t)!]^{1/2}(f\|C^{(t+2)}\|f)(f\|C^{(t)}\|f)$$
$$\times\sum_{S''L''}(f^2S_1L_1\|V^{(1,t+2)}\|f^2S''L'')(f^2S''L''\|V^{(1,t)}\|f^2S_2L_2)W(1S_11S_2;S''2)W(t+2L_1tL_2;L''2),\tag{8}$$

where use is made of Eq. (6).

Table I gives the spin-spin interaction of the f^3 electron configuration for the various J levels, where the following definitions for the various integrals M^t are used:

$$M_0=M^0/15,\quad M_2=M^2/225,\quad M_4=M^4/1089.$$

The common multiple of each interaction is placed at the head of the column of the interaction with the values of the coefficients of M_0, M_2, and M_4 directly below. For example, for $J=\frac{1}{2}$ the spin-spin interaction gives

$${}_3{}^2P_{11}-{}_3{}^4D_{20}=3(10)^{1/2}(M_0+90M_2-990M_4).$$

It can be seen from this table that in most cases the value of the coefficient of M_t increases as the value of t increases. The exceptional cases are given in Table II with respect to the J values. These irregularities are due to the linear combination of the coefficients of fractional parentage used in Eq. (2).

[7] T. Ishidzu *et al.*, *Tables of the Racah Coefficients* (Pan-Pacific Press, Tokyo, 1960).

TABLE I. Spin-spin interaction of f^3 electrons for various J levels.

$J=\frac{1}{2}$

		$_3{}^2P_{11}$	$_3{}^4D_{20}$
$_3{}^2P_{11}$			$3(10)^{1/2}$
	M_0	0	1
	M_2	0	90
	M_4	0	−990
$_3{}^4D_{20}$		$3(10)^{1/2}$	2
	M_0	1	34
	M_2	90	−690
	M_4	−990	−4785

$J=\frac{3}{2}$

		$_3{}^4S_{00}$	$_3{}^2P_{11}$	$_3{}^2D_{20}$	$_3{}^2D_{21}$	$_3{}^4D_{20}$	$_3{}^4F_{10}$
$_3{}^4S_{00}$				$\frac{2(210)^{1/2}}{7}$	$\frac{90(770)^{1/2}}{77}$	$4(30)^{1/2}$	
	M_0	0	0	7	0	−2	0
	M_2	0	0	−300	13	−30	0
	M_4	0	0	2310	−77	825	0
$_3{}^2P_{11}$						$3(5)^{1/2}$	$3(30)^{1/2}$
	M_0	0	0	0	0	−1	2
	M_2	0	0	0	0	−90	−95
	M_4	0	0	0	0	990	715
$_3{}^2D_{20}$		$\frac{2(210)^{1/2}}{7}$				$\frac{(7)^{1/2}}{7}$	$\frac{(42)^{1/2}}{7}$
	M_0	7	0	0	0	67	16
	M_2	−300	0	0	0	30	465
	M_4	2310	0	0	0	660	−10 065
$_3{}^2D_{21}$		$\frac{90(770)^{1/2}}{77}$				$\frac{6(231)^{1/2}}{77}$	$\frac{9(154)^{1/2}}{7}$
	M_0	0	0	0	0	11	1
	M_2	13	0	0	0	−635	40
	M_4	−77	0	0	0	2200	−185
$_3{}^4D_{20}$		$4(30)^{1/2}$	$3(5)^{1/2}$	$\frac{(7)^{1/2}}{7}$	$\frac{6(231)^{1/2}}{77}$		$2(6)^{1/2}$
	M_0	−2	−1	67	11	0	8
	M_2	−30	−90	30	−635	0	−105
	M_4	825	990	660	2200	0	1650
$_3{}^4F_{10}$			$3(30)^{1/2}$	$\frac{(42)^{1/2}}{7}$	$\frac{9(154)^{1/2}}{7}$	$2(6)^{1/2}$	12
	M_0	0	2	16	1	8	−2
	M_2	0	−95	465	40	−105	−30
	M_4	0	715	−10 065	−185	1650	825

$J=\frac{5}{2}$

		$_3{}^2D_{20}$	$_3{}^2D_{21}$	$_3{}^4D_{20}$	$_1{}^2F_{10}$	$_3{}^2F_{21}$	$_3{}^4F_{10}$	$_3{}^4G_{20}$
$_3{}^2D_{20}$				$\frac{(2)^{1/2}}{7}$			$\frac{(42)^{1/2}}{7}$	$\frac{(110)^{1/2}}{77}$
	M_0	0	0	−67	0	0	−16	44
	M_2	0	0	−30	0	0	−465	155
	M_4	0	0	−660	0	0	10 065	60 005
$_3{}^2D_{21}$				$\frac{6(66)^{1/2}}{77}$			$\frac{9(154)^{1/2}}{7}$	$\frac{(30)^{1/2}}{77}$
	M_0	0	0	−11	0	0	−1	341
	M_2	0	0	635	0	0	−40	14 860
	M_4	0	0	−2200	0	0	185	27 775

TABLE I. (*continued*).

$J=\frac{5}{2}$

		${}_3{}^2D_{20}$	${}_3{}^2D_{21}$	${}_3{}^4D_{20}$	${}_1{}^2F_{10}$	${}_3{}^2F_{21}$	${}_3{}^4F_{10}$	${}_3{}^4G_{20}$
		$\frac{(2)^{1/2}}{7}$	$\frac{6(66)^{1/2}}{77}$	$\frac{10}{7}$		$\frac{2(2310)^{1/2}}{77}$	$\frac{4(21)^{1/2}}{7}$	$\frac{4(55)^{1/2}}{7}$
${}_3{}^4D_{20}$	M_0	−67	−11	−34	0	22	8	8
	M_2	−30	635	690	0	705	−105	95
	M_4	−660	−2200	4785	0	165	1650	−535
${}_1{}^2F_{10}$	M_0	0	0	0	0	0	0	0
	M_2	0	0	0	0	0	0	0
	M_4	0	0	0	0	0	0	0
				$\frac{2(2310)^{1/2}}{77}$			$\frac{3(110)^{1/2}}{22}$	$\frac{5(42)^{1/2}}{154}$
${}_3{}^2F_{21}$	M_0	0	0	22	0	0	−11	33
	M_2	0	0	705	0	0	210	−3 790
	M_4	0	0	165	0	0	5115	−30 745
		$\frac{(42)^{1/2}}{7}$	$\frac{9(154)^{1/2}}{7}$	$\frac{4(21)^{1/2}}{7}$		$\frac{3(110)^{1/2}}{22}$	3	$\frac{5(1155)^{1/2}}{77}$
${}_3{}^4F_{10}$	M_0	−16	−1	8	0	−11	2	22
	M_2	−465	−40	−105	0	210	30	−630
	M_4	10 065	185	1650	0	5115	−825	2475
		$\frac{(110)^{1/2}}{77}$	$\frac{(30)^{1/2}}{77}$	$\frac{4(55)^{1/2}}{7}$		$\frac{5(42)^{1/2}}{154}$	$\frac{5(1155)^{1/2}}{77}$	$\frac{5}{77}$
${}_3{}^4G_{20}$	M_0	44	341	8	0	33	22	−22
	M_2	155	14 860	95	0	−3 790	−630	−14 970
	M_4	60 005	27 775	−535	0	−30 745	2475	−14 025

$J=\frac{7}{2}$

		${}_3{}^4D_{20}$	${}_1{}^2F_{10}$	${}_3{}^2F_{21}$	${}_3{}^4F_{10}$	${}_3{}^2G_{20}$	${}_3{}^2G_{21}$	${}_3{}^4G_{20}$
		$\frac{4}{7}$		$\frac{(1155)^{1/2}}{77}$	$\frac{3(70)^{1/2}}{7}$	$\frac{2(330)^{1/2}}{77}$	$\frac{15(13)^{1/2}}{91}$	$\frac{5(66)^{1/2}}{7}$
${}_3{}^4D_{20}$	M_0	34	0	−22	−8	−11	−26	8
	M_2	−690	0	−705	105	985	−2 015	95
	M_4	−4785	0	−165	−1650	−21 065	−10 835	−535
${}_1{}^2F_{10}$	M_1	0	0	0	0	0	0	0
	M_2	0	0	0	0	0	0	0
	M_4	0	0	0	0	0	0	0
		$\frac{(1155)^{1/2}}{77}$			$\frac{5(66)^{1/2}}{44}$			$\frac{9(70)^{1/2}}{308}$
${}_3{}^2F_{21}$	M_0	−22	0	0	11	0	0	−33
	M_2	−705	0	0	−210	0	0	3 790
	M_4	−165	0	0	−5115	0	0	30 745
		$\frac{3(70)^{1/2}}{7}$		$\frac{5(66)^{1/2}}{44}$	10	$\frac{10(231)^{1/2}}{77}$	$\frac{3(910)^{1/2}}{364}$	$\frac{2(1155)^{1/2}}{77}$
${}_3{}^4F_{10}$	M_0	−8	0	11	2	−11	13	22
	M_2	105	0	−210	30	−405	−4030	−630
	M_4	−1650	0	−5115	−825	7425	−6325	2475
		$\frac{2(330)^{1/2}}{77}$			$\frac{10(231)^{1/2}}{77}$			$\frac{10(5)^{1/2}}{77}$
${}_3{}^2G_{20}$	M_0	−11	0	0	−11	0	0	−11
	M_2	985	0	0	−405	0	0	1083
	M_4	−21 065	0	0	7425	0	0	3729

TABLE I. (*continued*).

$J=\frac{7}{2}$

		$_3{}^4D_{20}$	$_1{}^2F_{10}$	$_3{}^2F_{21}$	$_3{}^4F_{10}$	$_3{}^2G_{20}$	$_3{}^2G_{21}$	$_3{}^4G_{20}$
		$\frac{15(13)^{1/2}}{91}$			$\frac{3(910)^{1/2}}{364}$			$\frac{(858)^{1/2}}{4004}$
$_3{}^2G_{21}$	M_0	−26	0	0	13	0	0	4 433
	M_2	−2 015	0	0	−4030	0	0	1 170
	M_4	−10 835	0	0	−6325	0	0	−102 465
		$\frac{5(66)^{1/2}}{7}$		$\frac{9(70)^{1/2}}{308}$	$\frac{2(1155)^{1/2}}{77}$	$\frac{10(5)^{1/2}}{77}$	$\frac{(858)^{1/2}}{4004}$	$\frac{2}{77}$
$_3{}^4G_{20}$	M_0	8	0	−33	22	−11	4 433	22
	M_2	95	0	3 790	−630	1083	1 170	14 970
	M_4	−535	0	30 745	2475	3729	−102 465	14 025

$J=\frac{9}{2}$

		$_3{}^4F_{10}$	$_3{}^2G_{20}$	$_3{}^2G_{21}$	$_3{}^4G_{20}$	$_3{}^2H_{11}$	$_3{}^2H_{21}$	$_3{}^4I_{20}$
		5	$\frac{4(231)^{1/2}}{77}$	$\frac{3(910)^{1/2}}{910}$	$(3)^{1/2}$	$\frac{(22)^{1/2}}{11}$	$\frac{(10\,010)^{1/2}}{715}$	
$_3{}^4F_{10}$	M_0	−2	11	−13	−22	−44	143	0
	M_2	−30	405	4030	630	1 635	−780	0
	M_4	825	−7425	6325	−2475	−13 035	−15 675	0
		$\frac{4(231)^{1/2}}{77}$			$\frac{20(77)^{1/2}}{847}$			$\frac{4(3003)^{1/2}}{11\,011}$
$_3{}^2G_{20}$	M_0	11	0	0	11	0	0	1 001
	M_2	405	0	0	−1083	0	0	−29 770
	M_4	−7425	0	0	−3729	0	0	107 030
		$\frac{3(910)^{1/2}}{910}$			$\frac{(2730)^{1/2}}{10\,010}$			$\frac{6(70)^{1/2}}{5005}$
$_3{}^2G_{21}$	M_0	−13	0	0	−4 433	0	0	−2 002
	M_2	4030	0	0	−1 170	0	0	−44 005
	M_4	6325	0	0	102 465	0	0	544 775
		$(3)^{1/2}$	$\frac{20(77)^{1/2}}{847}$	$\frac{(2730)^{1/2}}{10\,010}$	$\frac{7}{121}$	$\frac{3(66)^{1/2}}{121}$	$\frac{3(30\,030)^{1/2}}{7865}$	$\frac{8(39)^{1/2}}{1573}$
$_3{}^4G_{20}$	M_0	−22	11	−4 433	22	−88	−143	−1 144
	M_2	630	−1083	−1 170	14 970	−3 825	12 480	18 785
	M_4	−2475	−3729	102 465	14 025	62 865	1 815	113 465
		$\frac{(22)^{1/2}}{11}$			$\frac{3(66)^{1/2}}{121}$			$\frac{7(286)^{1/2}}{121}$
$_3{}^2H_{11}$	M_0	−44	0	0	−88	0	0	11
	M_2	1 635	0	0	−3 825	0	0	210
	M_4	−13 035	0	0	62 865	0	0	−6270
		$\frac{(10\,010)^{1/2}}{715}$			$\frac{3(30\,030)^{1/2}}{7865}$			$\frac{2(770)^{1/2}}{7865}$
$_3{}^2H_{21}$	M_0	143	0	0	−143	0	0	−2 431
	M_2	−780	0	0	12 480	0	0	115 635
	M_4	−15 675	0	0	1 815	0	0	145 200
			$\frac{4(3003)^{1/2}}{11\,011}$	$\frac{6(70)^{1/2}}{5005}$	$\frac{8(39)^{1/2}}{1573}$	$\frac{7(286)^{1/2}}{121}$	$\frac{2(770)^{1/2}}{7865}$	$\frac{70}{1573}$
$_3{}^4I_{20}$	M_0	0	1 001	−2 002	−1 144	11	−2 431	286
	M_2	0	−29 770	−44 005	18 785	210	115 635	−11 310
	M_4	0	107 030	544 775	113 465	−6270	145 200	−122 595

TABLE I. (*continued*).

$J=\frac{11}{2}$

		$_3{}^4G_{20}$	$_3{}^2H_{11}$	$_3{}^2H_{21}$	$_3{}^2I_{20}$	$_3{}^4I_{20}$
		$\frac{4}{121}$	$\frac{(110)^{1/2}}{121}$	$\frac{(2002)^{1/2}}{1573}$	$\frac{(10\,010)^{1/2}}{1573}$	$\frac{2(910)^{1/2}}{1573}$
$_3{}^4G_{20}$	M_0	−22	88	143	−286	−1 144
	M_2	−14 970	3 825	−12 480	−3 445	18 785
	M_4	−14 025	−62 825	−1 815	40 205	113 465
		$\frac{(110)^{1/2}}{121}$				$\frac{5(1001)^{1/2}}{121}$
$_3{}^2H_{11}$	M_0	88	0	0	0	−11
	M_2	3 825	0	0	0	−210
	M_4	−62 865	0	0	0	6270
		$\frac{(2002)^{1/2}}{1573}$				$\frac{2(55)^{1/2}}{1573}$
$_3{}^2H_{21}$	M_0	143	0	0	0	2 431
	M_2	−12 480	0	0	0	−115 635
	M_4	−1 815	0	0	0	−145 200
		$\frac{(10\,010)^{1/2}}{1573}$				$\frac{5(11)^{1/2}}{1573}$
$_3{}^2I_{20}$	M_0	−286	0	0	0	−1 859
	M_2	−3 445	0	0	0	117 390
	M_4	40 205	0	0	0	−642 840
		$\frac{2(910)^{1/2}}{1573}$	$\frac{5(1001)^{1/2}}{121}$	$\frac{2(55)^{1/2}}{1573}$	$\frac{5(11)^{1/2}}{1573}$	$\frac{40}{1573}$
$_3{}^4I_{20}$	M_0	−1 144	−11	2 431	−1 859	−286
	M_2	18 785	−210	−115 635	117 390	11 310
	M_4	113 465	6270	−145 200	−642 840	122 595

$J=\frac{13}{2}$

		$_3{}^2I_{20}$	$_3{}^4I_{20}$	$_3{}^2K_{21}$
			$\frac{(70)^{1/2}}{1001}$	
$_3{}^2I_{20}$	M_0	0	1 859	0
	M_2	0	−117 390	0
	M_4	0	642 840	0
		$\frac{(70)^{1/2}}{1001}$	$\frac{6}{143}$	$\frac{60(14)^{1/2}}{1001}$
$_3{}^4I_{20}$	M_0	1 859	−286	143
	M_2	−117 390	11 310	−5 460
	M_4	642 840	122 595	−10 131
			$\frac{60(14)^{1/2}}{1001}$	
$_3{}^2K_{21}$	M_0	0	143	0
	M_2	0	−5 460	0
	M_4	0	−10 131	0

$J=\frac{15}{2}$

		$_3{}^4I_{20}$	$_3{}^2K_{21}$	$_3{}^2L_{21}$
		$\frac{4}{143}$	$\frac{15}{143}$	$\frac{(255)^{1/2}}{143}$
$_3{}^4I_{20}$	M_0	286	−143	143
	M_2	−11 310	5 460	−780
	M_4	−122 595	10 131	−15 675

TABLE I. (*continued*).

$J=\frac{15}{2}$

		$_3{}^4I_{20}$	$_3{}^2K_{21}$	$_3{}^2L_{21}$
		$\frac{15}{143}$		
$_3{}^2K_{21}$	M_0	−143	0	0
	M_2	5 460	0	0
	M_4	10 131	0	0
		$\frac{(255)^{1/2}}{143}$		
$_3{}^2L_{21}$	M_0	143	0	0
	M_2	−780	0	0
	M_4	−15 675	0	0

$J=17/2$

		$_3{}^2L_{21}$
$_3{}^2L_{21}$	M_0	0
	M_2	0
	M_4	0

TABLE II. Exceptional cases of the spin-spin interaction for f^3 electrons.

J	Spin-spin configuration	J	Spin-spin configuration
$\frac{3}{2}$	$_3{}^2D_{20}-{}_3{}^4D_{20}$	$\frac{9}{2}$	$_3{}^2G_{21}-{}_3{}^4G_{20}$
			$_3{}^4G_{20}-{}_3{}^4G_{20}$
$\frac{5}{2}$	$_3{}^2D_{20}-{}_3{}^4D_{20}$		$_3{}^4G_{20}-{}_3{}^2H_{21}$
	$_3{}^4D_{20}-{}_3{}^2F_{21}$		
	$_3{}^4G_{20}-{}_3{}^4G_{20}$	$\frac{11}{2}$	$_3{}^4G_{20}-{}_3{}^4G_{20}$
			$_3{}^4G_{20}-{}_3{}^2H_{21}$
$\frac{7}{2}$	$_3{}^4D_{20}-{}_3{}^2F_{21}$		
	$_3{}^2G_{21}-{}_3{}^4G_{20}$		
	$_3{}^4G_{20}-{}_3{}^4G_{20}$		

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