REFLECTION OF RADIATION FROM AN INFINITE SERIES OF EQUALLY SPACED PLANES.

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SYNOPSIS.

Reflection of Radiation from an Infinite Series of Equally Spaced Planes.—If the fractions of the incident radiation upon any plane which are absorbed, reflected and transmitted by that plane are constant and denoted by a, r and t respectively, and if all possible internal reflections are taken account of, the *coefficient of reflection* is shown to equal $(1/2r)\{1 - t^2 - r^2 - \sqrt{[1 - 2t^2 - r^2 + (t^2 - r^2)^2]}\}$. When both a and r are very small, as in the case of the *reflection of X-rays from a crystal surface*, the expression becomes $[1 + a/r - \sqrt{(a^2/r^2 + 2a/r)}]$; thus the proportion reflected depends only on the ratio of absorption to reflection per plane.

THE surprisingly large value of the reflection of X-rays from calcite (about 45 per cent., obtained by Bergen Davis and W. M. Stempel, an account of which appears in this number of the PHYSICAL REVIEW, suggests the problem of investigating the reflection that might be expected from a system of equally spaced reflecting planes.

No special assumptions are made as to the physical nature of the radiation or of the reflecting medium other than that it consists of an indefinite number of equally spaced planes. As the radiation passes through each plane a minute fraction r is reflected and a minute fraction a is absorbed. The amount transmitted is $t = 1 - r - a$. The fraction reflected per plane is quite small being of the order perhaps of 10^{-5} to 10^{-6}.

In the case of a perfect crystal the absorption per plane is probably much less than the reflection[1] for the particular wave-length for which the reflection is a maximum for any given angle. The percentage of transmission, reflection and absorption is assumed to be the same for all planes.

The final emergent (front reflecting face) ray from a system of planes is made up of parts, each of which has been reflected an *odd* number of times and transmitted through an *even* number of planes.

Assume the intensity of the ray incident at the first plane to be unity. Then the intensity of a ray after $2j - 1$ reflections and $2i$ transmissions is

$$I = t^{2i} r^{2j-1}. \tag{1}$$

Let C_{ij} be the number of ways in which it is possible for a ray to be

[1] C. G. Darwin, Phil. Mag., April, 1914.

transmitted $2i$ times and reflected $2j - 1$ times. Then the total amount reflected is

$$R = \sum_{ij=1}^{\infty} C_{ij} t^{2i} r^{2j-1}. \tag{2}$$

The values of C_{ij} were computed directly for all values of i and j up to 5 and were seen to be given by the following formula.

$$C_{ij} = \frac{(i+j-2)!\,(i+j-1)!}{(i-1)!\,i!\,(j-1)!\,j!} = \frac{i(i+1)^2(i+2)^2\cdots(i+j-2)^2(i+j-1)}{(j-1)!\,j!}. \tag{3}$$

This formula has been checked by the independent computation of C_{ij} for $1 = 8$ and $j = 7$.

For the case when $j = 1$, $C_{i_1} = 1$, the contribution of a single reflection from all planes is

$$\sum_{i=1}^{\infty} (C_{i1} t^{2i}) r = \frac{r}{1 - t^2} = \frac{1}{2}\left(\frac{1}{1 + a/r}\right). \tag{4}$$

The sum appearing in the formula for R Equation (2) has been found by T. H. Gronwall to be

$$\frac{1}{2r}[1 - t^2 - r^2 \sqrt{1 - 2t^2 - 2r^2 + (t^2 - r^2)^2}]. \tag{5}$$

For the values of a, r, t which we are considering, this is equal to

$$R = \frac{a}{r} + 1 - \sqrt{\frac{a^2}{r^2} + \frac{2a}{r}}. \tag{6}$$

Thus the total reflection depends only on the ratio of absorption to reflection at each plane. As r is so small it might be expected that the terms in r^3, r^5, etc., might be neglected, that is, that the contributions of repeated internal reflections might be neglected. This however, is not the case. The number of possible paths increases so rapidly with the number of reflections that for probable values of a/r, the repeated reflections contribute a considerable portion of the total energy reflected.

The following table is computed from formula (6) for various values of the ratio a/r.

Absorption per Plane / Reflection per Plane.	0	.02	.05	.1	.2	.3	.4	.5	1	2	3	10
Per cent. reflection	100	82	73	64	54	47	42	38	27	17	13	5

Columbia University,
January, 1921.