THE MASS ABSORPTION AND MASS SCATTERING COEFFICIENTS FOR HOMOGENEOUS X RAYS OF WAVE-LENGTH BETWEEN 0.13 AND 1.05 ÅNGSTRÖM UNITS IN WATER, LITHIUM, CARBON, NITROGEN, OXYGEN, ALUMINUM, AND IRON.

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Synopsis.

I. *Present Status of the Problem of Absorption and Scattering of X Rays by Elements of Small Atomic Weight.*—The mass scattering and mass absorption coefficients are defined, and a brief summary is given of the results of previous absorption and scattering measurements in the regions of wave-lengths on the short wave-length side of the K radiation of the absorbing substance, in so far as they relate to the present investigation. The current views in regard to the mechanism of absorption and scattering, and some of the theoretical work which has been advanced to account for these phenomena, has been outlined with regard to their bearing on the work reported in this paper.

II. *Absorption and Scattering of Homogeneous X Rays by Water, Lithium, Carbon, Nitrogen, Oxygen, Aluminium, and Iron at various Wave-lengths within the Region Bounded by 0.13 and 1.05 Å. U.*—(1) The total absorption coefficient of homogeneous X rays obtained by passing the X rays from a Coolidge tube through a Bragg X ray spectrometer was measured for the above-named materials at various wave-lengths in the range indicated. The total mass absorption coefficient was found to be proportional to the cube of the wave-length of the X rays over certain regions, but for all the substances where the investigation was complete over the whole range mentioned, the constant of proportionality was different for different ranges of wave-length. (2) *Hydrogen* is apparently an exception to the above statement, for its total mass absorption coefficient appears to be proportional to the nine halves power of the wave-length of the X rays. (3) *The constant which multiplies the cube of the wave-length* to give the true mass absorption coefficient is found to be approximately proportional to the cube of the atomic number of the absorbing element, except for lithium, and this indicates that Moseley's law will not be found to hold for this latter element. (4) *The true mass absorption coefficient for iron* is apparently not proportional to the cube of the wave-length for waves between 0.70 and 1.05 Å. U. No explanation is suggested for this. (5) *The true mass absorption coefficient* for the elements, with the exception of iron, for wave-lengths less than 0.20 Å. U. are smaller than is to be expected from theory if the electron has a diameter of the order of 10^{-13} cm. Compton's modification of the expressions for these absorption coefficients accounts for these experimental results if we assign to the electron a diameter of 0.75 to 0.85×10^{-10} cm. (6) *The lack of this decrease in the coefficients for iron is* attributed to a shrinkage in the diameter of the electrons in the atoms on account of the closer packing of the electrons in this element. (7) *The mass scattering coefficient* of all the substances used is less than that given by

Thomson's theory, but no explanation has been found. (8) *The mass scattering coefficient for hydrogen* is approximately twice that for the other elements, and this is interpreted as further evidence that hydrogen has twice the number of scattering electrons per unit mass as the other elements. (9) *The mass scattering coefficient for iron* apparently increases for wave-lengths longer than 0.70 Å. U. It is suggested as an explanation of this fact that the distance between the scattering electrons in the iron atom is of the order of magnitude of 0.70 Å. U.

III. *Projects for Further Absorption and Scattering Measurements.*—(1) The scattering of X rays should be measured as a function of the angle of scattering. (2) The absorption and scattering of hydrogen should be measured directly.

I. PRESENT STATUS OF PROBLEM OF ABSORPTION AND SCATTERING OF X RAYS BY ELEMENTS OF SMALL ATOMIC WEIGHT.

THE investigation reported below was suggested to the writer by Dr. A. W. Hull of the General Electric Co., Schenectady, New York.

Several investigators have measured the absorption of X rays by various kinds of matter. Much of the earlier work was done with beams of X rays which had been filtered to ensure homogeneity of wave-length, but it appears that in many cases the X rays even after filtering were decidedly heterogeneous. The advent of the X-ray spectrometer has made it possible to obtain highly homogeneous beams of X rays. Experiment shows that X rays of a definite wave-length are absorbed according to an exponential law in passing through matter. If I_0 represents the intensity of a beam of homogeneous X rays as it enters an absorbing substance, and I the intensity of the beam after it has passed through such a thickness of the substance that the mass per square cm. penetrated is m, then the following equation expresses the relation between these quantities:

$$I = I_0 e^{-\frac{\mu}{d}m},$$

where d is the density of the substance and μ is called the total absorption coefficient for the substance. The quantity μ/d is usually called the "total" or "gross mass absorption coefficient."

The beam of X rays in passing through the absorbing matter loses its intensity in two distinct ways. First, the disturbance in passing over the electrons sets them in vibration, and this results in a reëmission of X radiation, by these disturbed electrons, of the same wave-length as that of the beam of X rays. This reëmitted radiation is called " scattered radiation," and it is thought that the electrons outside the nucleus of the atoms are responsible for this scattering, those electrons within the nucleus being held by such enormous forces that their effect, as well as that of the nucleus, is negligible as far as scattering is concerned.

Second, if the beam of X rays is of shorter wave-length than the characteristic radiation of the absorbing substance, electrons are ejected from the atoms of the absorbing matter by the X rays, and these ejected electrons carry with them the energy which they took from the beam of X rays. Later on when another electron falls into the atom to take the place of the ejected one there is emitted a train of X waves with the characteristic frequency of the absorbing substance. In this way the beam of X rays loses energy by "fluorescent absorption" or "true absorption," and the absorbing substance at the same time emits "characteristic radiation." Accordingly the total or gross mass absorption coefficient is usually considered in two parts as follows:

$$\frac{\mu}{d} = \frac{\mu_0}{d} + \frac{\sigma}{d},$$

where μ_0/d is that part which represents the "true absorption" and is called the "true" or "fluorescent mass absorption coefficient," and σ/d is called the "mass scattering coefficient."

Now it has generally been found for regions of wave-length removed from the K and L radiation of the absorbing substance, that the values of the total mass absorption coefficient when plotted against the cube of the wave-length, fall very closely on a straight line which cuts the μ/d axis at some point between 0.10 and 0.20 according to the substance. This is usually interpreted by saying that the true mass absorption coefficient over this region is proportional to the cube of the wave-length, while the mass scattering coefficient is independent of the wave-length and is given by the intercept on the μ/d axis by the straight line. The expression for the total absorption coefficient may then be written

$$\frac{\mu}{d} = K\lambda^3 + \frac{\sigma}{d}.$$

K is a constant for a given absorbing substance and has been found to be roughly proportional to the cube of its atomic number.

If the mechanism of scattering is as simple as outlined above we should expect the mass scattering coefficient to be very nearly the same for all substances and for all wave-lengths so long as the wave-length of the X rays used are long compared to the dimensions of the electron, and short compared to the distance between the electrons. For, equal masses of all substances are supposed to contain very nearly equal numbers of scattering electrons, and if the X rays are long compared to the dimenssion of the electron then there can be no interference between the portion of scattered radiation from the different parts of the electron. If the wave length of the X rays is comparable to the dimensions of the

electron then this latter statement would no longer hold true and we should expect the mass scattering coefficient to be a function of the wave-length. If the wave-length of the X rays is short compared to the distance between the electrons, then we should expect the electrons to scatter independently of one another, so that as long as this condition is satisfied the distance between the electrons in the atom would have no effect on the amount of scattered radiation. If the wave-length is comparable to the distance between the electrons then the electrons will no longer scatter independently and the amount of scattered radiation will be greater than when they scattered independently. In this latter case there is said to be "excess scattering."

J. J. Thomson,[1] on the basis of the electromagnetic theory and the two further assumptions that the electrons are small compared to the wave-length of the X rays, and that the X rays are short compared to the distance between the scattering electrons, has derived an expression for the amount of scattered radiation which for the purposes of the present discussion may be written in the following form:

$$\frac{\sigma}{d} = 0.4019 \frac{N}{A},$$

where N is the atomic number and A is the atomic weight of the absorbing element.

It is not so easy to account for the view that the true mass absorption coefficient is proportional to the cube of the wave-length. In the same place in which the expression for the scattering was deduced, J. J. Thomson has also deduced an expression for the true absorption, which gives the true mass absorption coefficient proportional to the cube of the wave-length. The additional assumptions made as to the nature of the X ray, however, are rather arbitrary. A. H. Compton[2] has shown by introducing Moseley's law into this expression of Thomson's, (M oseley's law being that the wave-length of a given type of characteristic radiation is proportional to the square of the atomic number), that the "atomic absorption coefficient" is approximately proportional to the fourth power of the atomic number. Calculating from this " atomic absorption coefficient" the true mass absorption coefficient, it appears from this modified expression of Thomson's that the true mass absorption coefficient should be approximately proportional to the cube of the atomic number. But while Moseley's law is known to hold very well for the elements of high atomic number, it does not hold so well for the lighter elements, and consequently we might expect to find the value of

[1] J. J. Thomson, Conduction of Electricity through Gasses, 2d ed., p. 325.
[2] A. H. Compton, PHYS. REV., Vol. 14, Sept., 1919, p. 247.

K for the elements used in the present investigation only roughly proportional to the cube of the atomic number.

The usually accepted diameter of the electron, is 1.9×10^{-13} cm., which is calculated on the assumptions that the mass of an electron is entirely electromagnetic mass, and that the charge of an electron is a uniform spherical surface distribution. In accordance with this and the views outlined above we should expect the mass scattering coefficient for all X rays producible by ordinary methods, and even for all γ rays, except possibly the very shortest ones about which there still exists uncertainty in regard to the limiting shortness, to be independent of the wave-length so long as the wave-length is not long enough to give rise to "excess scattering." But experiment does not confirm this conclusion.

For very short X rays, and γ rays (below 0.20 Å.U.) the total mass absorption coefficient becomes much smaller for elements of small atomic weight than the calculated value for the mass scattering coefficient alone. In order to explain this marked decrease in the total mass absorption coefficient of very short wave-lengths A. H. Compton[1] has modified the expression given by Thomson, to take account of the relative size of the electron and the wave-length of the X rays. In his papers, Compton considers three types of electron, a rigid spherical electron, a deformable spherical electron, and a deformable ring electron, the latter of which he regards as the most suitable for the purpose in hand. The calculation for the ring electron is only approximate on account of certain assumptions made in order to render the calculation manageable. His expression for the total mass absorption coefficient may be written in the following form

$$\frac{\mu}{d} = \varphi K \lambda^3 + \theta \frac{\sigma}{d},$$

where φ and θ are both functions of the ratio of the wave-length of the X rays to the diameter of the electron. The values of φ and θ lie between 0 and 1 and approach 1 as the wave-length becomes large compared to the diameter of the electron. The other quantities have the same meaning as before. Taking the only available data at very short wave-lengths, which was on aluminium, Compton concluded that if the electron was a ring 1.85×10^{-10} cm. diameter, the marked decrease in the total mass absorption coefficient at very short wave-lengths could be accounted for. For light elements the scattering for very short wave-lengths is comparable to or even greater than the true mass absorption coefficient. Consequently it would seem advisable, in obtaining information to test the validity of Compton's equation, to work with the elements of small atomic number.

[1] A. H. Compton, Phys. Rev., Vol. 14, July, 1919, p. 20; Sept., 1919, p. 247.

Barkla and White[1] found an abnormally great mass absorption coefficient for aluminum at 0.37 Å. U. and concluded from this that aluminum should give off a characteristic radiation in this region, which they called "J" radiation. Other observers[2] failed to confirm the work or conclusions of Barkla and White. It appeared that an investigation of some of the other light elements in the region from 0.10 to 1.0 Å. U. would be of interest from the standpoint of a possible characteristic absorption on the short wave-length side of the K radiation of the absorbing substance.

It is also of interest to know whether the true mass absorption coefficient is proportional to the cube of the wave-length for some of the lightest elements, for if some of these lighter elements should absorb according to another law of simple expression, we should have a new point of view in regard to the mechanism of the true absorption.

II. ABSORPTION AND SCATTERING OF HOMOGENEOUS X RAYS BY WATER, LITHIUM, CARBON, NITROGEN, OXYGEN, ALUMINUM, AND IRON, AT VARIOUS WAVE-LENGTHS WITHIN THE REGION BOUNDED BY 0.13 AND 1.05 Å. U.

During the larger portion of this work the source of X rays was a water-cooled tungsten button anode Coolidge tube. The X rays from the tube were passed into a Bragg X-ray spectrometer constructed by Dr. A. W. Hull[3] and used by him sometime ago. For a description of the spectrometer see F. K. Richtmyer and Kerr Grant.[4] The tube was frequently operated continuously for five hours with one kilowatt input to the tube. For the greater part of the time the regulation of the voltage across the tube and the current through it left nothing to be desired in this direction. A 1,000-volt generator with an automatic voltage regulator supplied the power to the transformer and there was an adjustable inductive regulator in the primary of the transformer, so that any progressive change in the operating conditions could be adjusted. In order to still further render the results free of error owing to slight changes in the intensity of the X ray beam, a set of observations was spread over several minutes, alternating the intensity determinations with and without the absorbing screen in such a manner that the mean intensity without the absorbing screen corresponded to the same time as that with the screen. In order to calculate the mass absorption

[1] Barkla and White, Phil. Mag., Vol. 34, October, 1917, p. 270.
[2] Hull and Rice, PHYS. REV., Vol. 8, September, 1916, p. 326.
    Duane and Shimizu, PHYS. REV., Vol. 13, Apr., 1919, p. 288; Vol. 14, Nov., 1919, p. 389.
    Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.
[3] Hull and Rice, PHYS. REV., Vol. 8, Sept., 1916, p. 326.
[4] Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.

coefficient it is sufficient to know the ratio of the intensity of the beam after passing through the absorbing screen, to its intensity without the screen. The time required for the gold leaf to move over the same distance on the scale of the electroscope, was determined for the beam of X rays with and without the absorbing screen, and from these time intervals the corresponding rates of motion of the gold leaf were calculated and corrected for the natural leak of the instrument. These corrected rates of motion of the gold leaf were taken as proportional to the intensities of the X ray beam. Since the beam of X rays was homogeneous, this latter assumption is justified.

The zero reading of the instrument was determined several times during the course of the investigation by determining the position of three of the L lines of the tungsten spectrum. With the tube in any one adjustment the zero determinations always agreed to within o.oi degree, and this is as close as an individual setting of the instrument could be made.

With the exception of lithium all the substances used as absorbers were of very high purity, all the substances having been especially selected for this work.

With solid absorbing substances the areas and masses of the screens were determined accurately, and the mass absorption coefficients determined from the mass per square centimeter of the absorbing screen. In order to avoid any error arising from lack of homogeneity of the absorbing screens, the screens were moved along during a set of observations so that the beam of X rays passed through it in eight or ten different places.

For the work on water, cells with parallel sides were made by cutting slots in plate glass of appropriate thickness, and then cementing on the two sides of these slots very thin aluminum on microscope cover glasses. The intensity of the beam of X rays was determined after passing through the empty cell, and then after passing through the cell filled with distilled water.

Liquid oxygen was made by .condensing electrolytic oxygen gas at atmospheric pressure in a large test tube emersed in liquid air. A special cylindrical dewar flask was made of very thin lime glass for holding the liquid oxygen. A screen of aluminum was made up by trial that had nearly the same total absorption as did the empty dewar flask. In order to determine the absorption coefficient of liquid oxygen at a given wave-length, three sets of observations were made at this wavelength as follows: (1) The ratio of the intensities of the beam after passing through the aluminum screen, and after passing through the empty flask. (2) The ratio of the intensities of the beam after passing

through the aluminum screen and after passing through the flask filled with liquid oxygen. (3) The ratio of the intensities of the beam after passing through the aluminum screen, and after passing through the flask filled with water. It is hardly necessary to remark that the dewar flask was so mounted that it could always be placed in the same relative position with respect to the beam of X rays. The same procedure was followed in working with liquid nitrogen. The liquid nitrogen was made by condensing gaseous nitrogen at atmospheric pressure in a test tube immersed in liquid air boiling at reduced pressure. The densities of liquid oxygen and nitrogen were taken as 1.14 and 0.80 respectively.

The lithium used was made for this work by Mr. J. C. O'Brien of the research laboratory and was obtained by electrolyzing molten lithium chloride. Considerable difficulty was experienced in obtaining pure lithium. This was evidenced by the fact that different specimens of lithium gave widely different values for the absorption coefficient even at the same wave-length. Finally the sample that gave the smallest absorption coefficient was chosen, and the data given was obtained with this sample. To protect the lithium from oxidation the freshly cut surfaces were enclosed in very thin sheet aluminum. To all outward appearances the lithium used was very pure. The salt was marked chemically pure, and the metal was remelted and forced through a small hole under melted paraffin. This fact, that different samples of lithium, all apparently homogeneous and pure, would give values for the mass absorption coefficient differing by 50 per cent. is a marked illustration of how an exceedingly small amount of an element of high atomic weight might easily be detected and measured when mixed with an element of very low atomic weight. Even if the lithium used in this work is impure enough to affect seriously the accuracy of the determinations of the true mass absorption coefficients, it is probable that the mass scattering coefficient is very closely correct. It would take a relatively large amount of any impurity to affect seriously the mass scattering coefficient, since this is so nearly the same for all substances.

The observed values of the gross mass absorption coefficient for the various substances are given in Table I., while the curves show this data graphically for the purposes of the discussion that is to follow.

An inspection of the curves brings out the following outstanding features:

1. The values of the total mass absorption coefficient when plotted against the wave-length of the X rays lie very closely to a smooth curve.

2. The values of the total mass absorption coefficient when plotted against the cube of the wave-length of the X rays lie very closely to straight lines in certain regions.

TABLE I.

Total Mass Absorption Coefficients.

$$\frac{\mu}{d} = K\lambda^3 + \frac{\sigma}{d}.$$

Wave Length Ångström Units.	Lithium.	Carbon.	Nitrogen.	Water.	Oxygen.	Aluminum.	Iron.
.105		.146					
.130		.156		.165	.149	.179	.437
.154		.165	.165	.183	.165	.204	.620
.179		.166		.197	.176	.242	.864
.203			.178				
.210		.176		.204	.187	.288	1.16
.228		.178		.213			
.241		.185		.213		.332	
.252		.188	.195				
.265		.191		.228	.214	.403	2.12
.290		.200				.486	
.301			.225				
.309				.251			
.314	.176				.250		
.319						.577	
.324		.206					3.76
.350			.251				
.373	.195	.229		.301	.302	.825	5.66
.422	.218	.250		.360	.365	1.11	8.12
.471	.228	.280		.444	.440	1.60	11.22
.520	.256	.321		.524	.529	2.03	15.50
.569	.286	.365		.629	.619	2.60	19.40
.618	.318	.413		.759	.802	3.31	24.50
.666	.361	.477		.910	.980	4.08	31.05
.715	.422	.551		1.073	1.126	5.17	37.14
.764		.636		1.310	1.337	6.30	45.65
.812		.729		1.530	1.560	7.58	52.35
.861		.838		1.80		8.96	60.60
.910		.959		2.06		10.38	70.30
.958		1.119		2.43		12.19	79.63
1.006		1.311		2.74		14.03	91.70
1.054		1.470		3.19		16.55	

3. There is no abrupt decrease in the absorption coefficient with increasing wave-length over the region investigated, such as occurs in the region of the K and L characteristic radiation of the absorbing substance.

4. For carbon, oxygen, water and aluminum the total mass absorption coefficient at wave-lengths below 0.20 A. U. decreases much more rapidly than is to be expected from the expression $\mu/d = K\lambda^3 + (\sigma/d)$.

5. The peculiarity mentioned in (4) is not observed for iron.

6. The $(\mu/d, \lambda^3)$ line for carbon and water bends up at 0.85 and 0.60 Å. U. respectively, becoming straight again with a greater slope for longer wave-lengths.

7. For iron the $(\mu/d, \lambda^3)$ line bends down at 0.70 becoming straight again for longer wave-lengths.

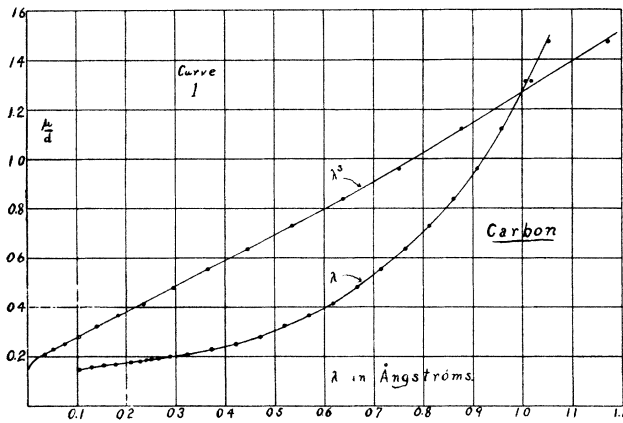These features will be discussed in the order mentioned:

Fig. 1.

1. It is seen that most of the observed values deviate from the curves by less than 1 per cent. or 2 per cent. Consequently we may look upon the values read off from the curves as correct for these particular samples
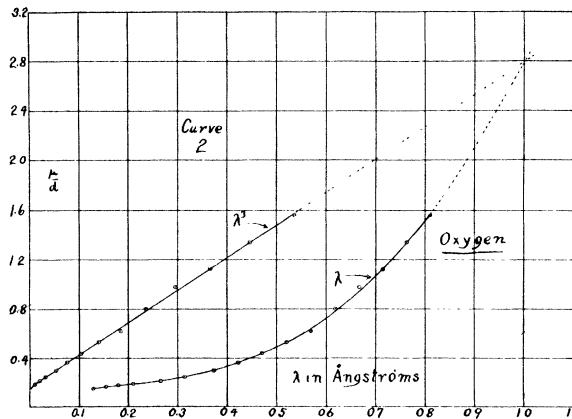
Fig. 2.

to much better than 1 per cent. as far as they may be affected by subjective error. This does not apply to the points at the extreme ends of the curves where the values read from the curves may be in error by as

much as 2 per cent. or 3 per cent.   It is believed that there are no con-
sistent instrumental errors as large as 1 per cent.

2. The relation $\mu/d = K\lambda^3 + (\sigma/d)$ is confirmed over certain regions.
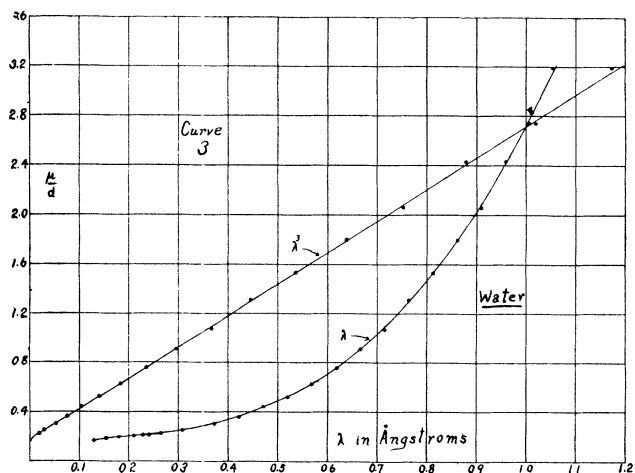

Fig. 3.

All of the curves have a straight portion beginning in the neighborhood
of 0.20 Å. U. and extending to 0.50 Å. U. or beyond.   For the purpose
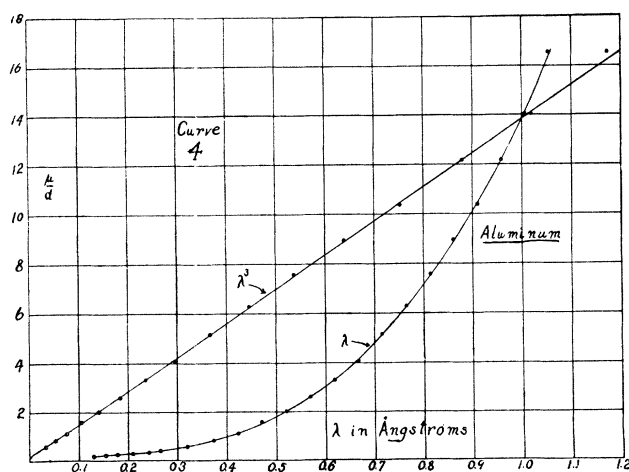of comparing the applicability of this expression to the absorption and

Fig. 4.

scattering of the substances it would seem reasonable to select this por-
tion of each of the curves.   Consequently we shall calculate K for each

substance from the straight portion of each curve immediately on the
long wave-length side of 0.20 Å. U., while the mass scattering coefficients
for wave-lengths long compared to the diameter of the electron are to
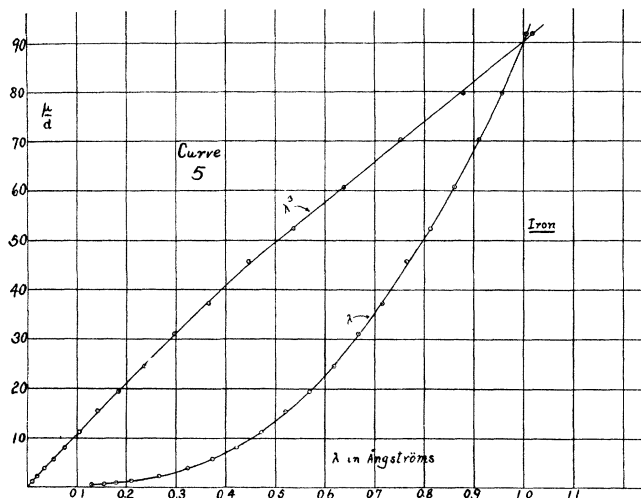be obtained by finding the intercept of this portion of the curve produced,

Fig. 5.

with the μ/d axis. The mass scattering coefficient determined in this
way, and its value calculated from Thomson's expression are given in
Table II.

TABLE II.

Element.	$\frac{\sigma}{d}$ Observed.	$\frac{\sigma}{d}$ Calculated.	Λ	$\frac{\Lambda}{N^3} \, \text{10}^3$.
Hydrogen	.309	.3987		
Lithium	.157	.1737	0.702	26.0
Carbon	.175	.2009	1.035	4.79
Nitrogen	.168	.2008	1.98	5.77
Oxygen	.165	.2010	2.644	5.20
Aluminum	.173	.1928	12.54	5.71
Iron	.18	.1871	106.	6.03

This table shows that the observed mass scattering coefficient is con-
siderably smaller than that calculated from Thomson's expression. It
should be clearly noted that the observed σ/d given in Table II. is the
intercept of the straight portion of the curves from 0.20 to about 0.40
Å. U. For most of the substances the intercept of the curve for longer
wave-lengths is different from that given in the table. For instance,
the intercept for aluminum between 0.50 and 1.10 Å. U. is 0.12, which

agrees with the value of σ/d found by Hull and Rice,[1] and Richtmyer and Grant.[2]

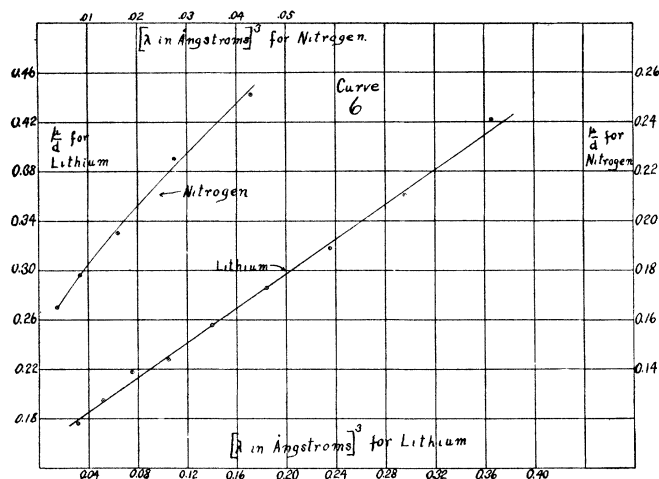It is interesting to note in this connection that the mass scattering

Fig. 6.

coefficient for hydrogen, calculated from the water and oxygen data is approximately twice that of the other elements. This may be inter-
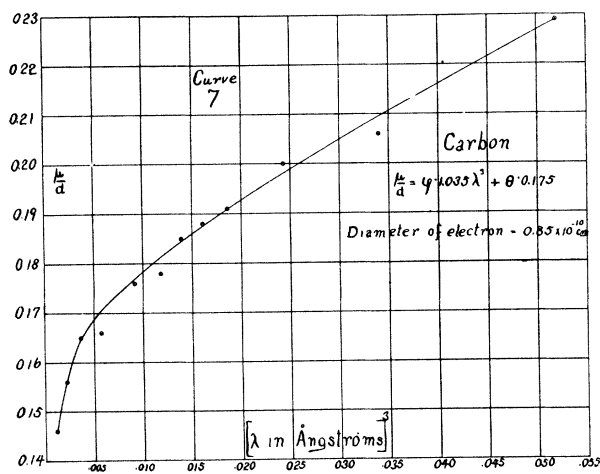

Fig. 7.

preted as further evidence that the number of electrons per unit mass for hydrogen is twice that for the other elements.

[1] Hull and Rice, PHYS. REV., Vol. 8, Sept., 1916, p. 326.

[2] Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.

It is very puzzling why the observed value of the mass scattering coef-
ficient should be less than the value calculated from Thomson's expres-
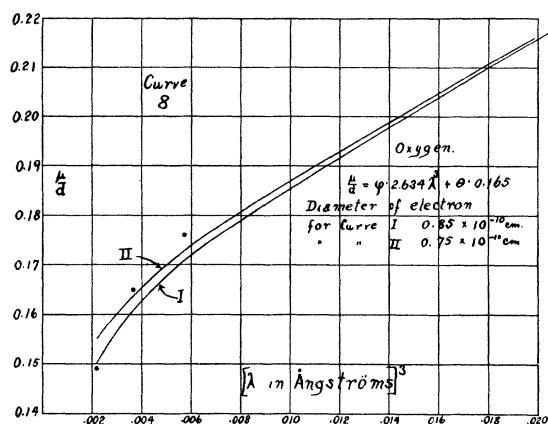sion. It was thought at first that possibly enough of the scattered

Fig. 8.

radiation entered the ionization chamber to account for this discrepancy,
but a calculation showed that a great deal less than 0.1 per cent. of this
radiation entered the ionization chamber, even in the most favorable
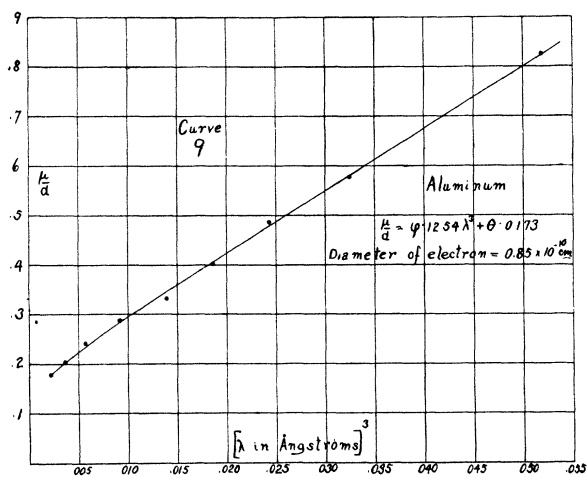

Fig. 9.

case. It is hardly conceivable that the mass of the electrons inside the
atom are sufficiently different from the value found for electrons outside
the atom to account for this decreased scattering. In fact a calculation
shows that if such an increase of mass were due to the velocity of the

electrons in the atom, they would have speeds of the order of 0.4 the velocity of light. It appears as if one of three alternatives must be invoked to account for the discrepancy: (1) The method used does not give the correct value of the mass scattering coefficient. (2) The number of scattering electrons is not strictly proportional to the atomic number. (3) The scattering electrons do not all scatter alike, and there is some other factor which affects the scattering of an electron which has not been considered in the deduction of Thomson's expression.

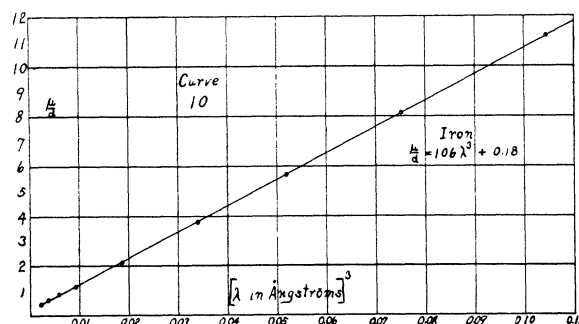The value of K calculated in the manner just mentioned is also given

Fig. 10.

in Table II., together with the quotient of this quantity by the cube of the atomic number. For the last five elements in the table this ratio is seen to be roughly constant. For lithium however the value of K is far from what would be expected if Moseley's law applied to it. This last fact is in agreement with a prediction by R. A. Millikan.[1]

3. The absence of any characteristic absorption in the region investigated confirms the work of other investigators[2] in the conclusion that these substances have no characteristic radiation in this region.

4. In order to apply Compton's theory of an electron of diameter of the order of 10^{-10} cm. it is here assumed that the value of σ/d in the equation $(\mu/d = \varphi K\lambda^3 + \theta(\sigma/d)$ is that given as the observed value in Table II. θ as used here is Compton's σ/σ_0, and its value as well as that of φ is taken from the curves on page 254 of Compton's paper.[3] The short wave-length data for carbon, oxygen, aluminum, and iron, are shown in Curves 7, 8, 9, and 10, and the full curves shown, except that for iron, are calculated from the above equation using the values for the

[1] R. A. Millikan, The Electron, 1st ed., p. 212.

[2] Hull and Rice, PHYS. REV., Vol. 8, Sept., 1916, p. 326.
     Duane and Shimizu, PHYS. REV., Vol. 13, Apr., 1919, p. 288; Vol. 14, Nov., 1919, p. 389.
     Richtmyer and Grant, PHYS. REV., Vol. 15, June, 1920, p. 547.

[3] A. H. Compton, PHYS. REV., Vol. 14, Sept., 1919, p. 254.

diameter of the electron shown on the curves. It appears that the data for carbon and aluminum are satisfactorily satisfied by assuming an electron whose diameter is 0.85×10^{-10} cm., while it is necessary to assume a still smaller electron namely 0.75×10^{-10} cm. to fit the oxygen data even approximately. These values are somewhat smaller than that given by Compton, namely 1.85×10^{-10} cm. Of course if the value of the mass scattering coefficient given by Thomson's equation is used, a larger electron would give the observed scattering at the very shortest wave-lengths, but then the curve would not fit the data satisfactorily at longer wave-lengths. No success attended the attempt to fit the data in this manner.

5. Since the electrons in the atoms are more closely packed the greater the atomic number, it is not inconceivable, that the electrons may alter in size and be smaller when they are more closely packed. With smaller electrons the wave-length at which φ and θ would be appreciably different from 1 would be shorter, and consequently we should expect the straight line relation to hold for shorter wave-lengths for the elements of high atomic weight than for elements of low atomic weight. The fact that the curve for iron remains straight even for the shortest wave-lengths investigated might be accounted for in this way.

6. The rise in the curves for water and carbon at long wave-lengths is too great to be accounted for by experimental error; apparently the departure from the straight line indicates that the absorption coefficient varies with the wave-length to a greater power than the third at these longer wave-lengths. The data on oxygen was not carried far enough to determine whether or not the same is true for it. A calculation of the absorption for hydrogen from that by water and oxygen indicated that the absorption of hydrogen varies as the wave-length to the nine halves power. The data on water and oxygen are not sufficiently accurate however to establish this relation, and it will be interesting to measure the absorption of hydrogen directly as a function of the wave-length. It is to be noted that the rise of the curve at long wave-lengths cannot be accounted for by an admixture of an impurity of relatively high atomic weight if we assume that the absorption of both the substance and impurity are proportional to the cube of the wave-length. In this case the curve would still have the same slope for short and long wave-lengths.

7. The curve for iron indicates that beyond 0.50 Ångström the true absorption coefficient is proportional to the wave-length to a smaller power than the cube, and at the same time it appears that the scattering must also be a function of the wave-length unless the true absorption coefficient is a more complicated function of the wave-length than

$\mu_0/d = K\lambda^n$. Since the electrons are more closely packed in the atoms of the elements of high atomic weight we should expect to find the scattering become a function of the wave-length at shorter wave-lengths for them, than for the elements of smaller atomic weight.

III. Conclusions and Projects for New Work on the Scattering and Absorption of X Rays.

It would appear that the main conclusions to be gathered at present from this investigation are as follows:

1. The total mass absorption coefficient for a number of the elements of small atomic number (the atomic numbers lying between 3 and 26) are proportional to the cube of the wave-length of the X rays over only limited ranges of wave-length.

2. The constant K which occurs in the usually accepted equation for the total mass absorption coefficient is only approximately proportional to the cube of the atomic number. In particular, for lithium, this discrepancy is marked, and indicates that Moseley's law does not apply to this element.

3. The total mass absorption coefficient for hydrogen appears not to be proportional to the cube of the wave-length, but to the nine halves power of the wave-length. This conclusion, however, is not certain on account of the uncertainty involved in calculating the absorption of hydrogen from that of water and oxygen.

4. The mass scattering coefficients for aluminum and the elements of smaller atomic number is considerably less than that given by the expression deduced by J. J. Thomson for this quantity on the electromagnetic theory, and there is at present no adequate account to be given for this discrepancy.

5. The mass scattering coefficient for hydrogen is approximately twice that for the other elements, and this is to be regarded as further evidence that the hydrogen atom contains twice as many scattering electrons per unit mass as the other atoms.

6. The total mass absorption coefficient for aluminum and the elements of smaller atomic weight below wave-length 0.20 Å. U. is considerably less than would be expected from that at longer wave-lengths if the electron is small compared to these short wave-lengths. This decrease in the mass absorption coefficient is accounted for quantitatively on Compton's theory by assuming that the diameter of the electron is from 0.75×10^{-10} cm. to 0.85×10^{-10} cm. in diameter.

It would seem advisable to measure the scattering of some of these elements independently of the absorption. The distribution of the

scattered radiation as a function of the angle of scattering from the incident beam, as well as the mass scattering coefficient, has an important bearing on the size and shape of the electron, and the writer plans to attempt some experimental work of this nature in the near future. It would also be interesting to measure directly the absorption coefficient in hydrogen as a function of the wave-length, in order to determine the relation between these quantities. If this relation is different for hydrogen than for the other light elements, as is indicated in this investigation, this fact ought to throw new light on the mechanism of absorption of X rays by matter.

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