Spectroscopic Factors for Two-Neutron Transfer Reactions in Vibrational Nuclei*

B. GYARMATI† AND J. SAWICKI
International Centre for Theoretical Physics, Trieste, Italy
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The spectroscopic factors of two-nucleon transfer reaction amplitudes are calculated with the microscopic theories known as the quasiparticle Tamm-Dancoff (QTD) and "second" Tamm-Dancoff (QSTD) methods. The latter theory describes vibrational states in terms of zero-, two, and four-quasiparticle (qp) excitations. The theory is applied to study (p,t) and (t,p) reactions in even tin isotopes. All the important lowest-lying states of these isotopes of both even and odd parity are included in the analysis. The QTD and QSTD eigenvectors correspond to several different residual interaction potentials. One of these potentials is the realistic potential of Tabakin renormalized for core polarization. The four-qp correlations cause a quite considerable reduction of the spectroscopic factors relative to those of the QTD theory. These factors are also appreciably sensitive to the effective nuclear force assumed.

1. INTRODUCTION

TWO-NUCLEON transfer reactions have attracted much attention in the past few years from both the theoretical and the experimental sides.

One of the characteristic advantages of these reactions is the natural, simple excitation of levels having two or more nucleons excited, particularly the excitation of some levels of collective character. Generally, states are favored having a large parentage based on the target nucleus in its ground state. The typical coherence of contributions to the reaction amplitudes coming from many different configurations of the two transferred nucleons can lead to very strong transitions to levels for which the interference is really constructive.

The assumption of the direct interaction mechanism and one of the usual formalisms based on perturbation theory, such as the distorted-wave Born approximation (DWBA), together with some simplifying assumptions such as the use of harmonic-oscillator radial wave functions, etc., lead to a factorization of the reaction amplitude into a factor G that depends upon details of the nuclear structure, and a "kinematic" factor B.

For example, Glendenning[1] writes the differential cross section as the coherent sum over L, S, J, and T:

$$\sum_M \left|\sum_N G_{NLSJT} B_{NL}{}^M(\mathbf{k}_1,\mathbf{k}_2)\right|^2, \qquad (1)$$

where L, S, and J are the orbital, spin, and total angular momenta of the transferred nucleon pair, and T is their isospin.

The branching ratios for different channels corresponding to different final nuclear states are essentially determined by the factors G involved, provided that the corresponding energy levels are sufficiently close to each other, and that the differences between the appropriate factors B are sufficiently small.

Recently, many theoretical studies have been performed, based on several simple microscopic models of nuclear structure. Of particular interest are quasiparticle (qp) microscopic models of the so-called vibrational nuclei. Recently, an extended ("second") quasiparticle Tamm-Dancoff theory (QSTD) of low-lying vibrational states has been developed.[2] It describes such states in terms of zero- two-, and four-qp excitations. It has been applied to Sn [3] and Cd [4] isotopes with success. The model evidently has a net superiority to the usual pure two-qp theories such as QTD (TD2) or QRPA (RPA2).

Our aim here is to apply the QSTD eigenvectors to calculate the spectroscopic factors of the Sn(t,p) and Sn(p,t) reactions, and to compare the results obtained with the corresponding predictions of QTD.

Several experimental groups have obtained and published data on (t,p) and (p,t) reactions on even tin isotopes. The early data by Bassani *et al.*[5] involve Sn(p,t) and Cd(p,t) reactions at the proton energy of 40 MeV. Several groups will publish their results in the near future. Bromley *et al.*[6] measured the cross sections of the $Sn^{120}(p,t)Sn^{118}$ reaction. Flynn[7] has studied the reactions $Sn^{122,124}(t,p)Sn^{124,126}$. Hansen[8] has supplied us with his yet unpublished data on the differential cross sections of the double stripping reactions $Sn^{118}(t,p)Sn^{120}$, with several final states of Sn^{120} and the incoming triton energy $E_t=11.28$ MeV.

In Sec. 2 we discuss in detail the spectroscopic factors

* A note on the same subject containing no numerical results of this paper will appear in *Nuclear Physics*.

† On leave of absence from Institute of Nuclear Research, Debrecen, Hungary.

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G appropriate to (p,t) and (t,p) reactions. In Sec. 3 we present some of our typical numerical results, based on the theory of Sec. 2.

2. TWO-NUCLEON TRANSFER SPECTROSCOPIC FACTORS IN THE QTD APPROXIMATIONS

Our analysis is based on the formula of Eq. (1) involving the factorization GB.

The nuclear structure factor G is defined by

$$G_{NLSJ}=\sum_{a\leqslant a'}(2-\delta_{aa'})^{1/2}\beta_{aa'LSJ}\Omega_n \times\langle n0,NL;\,L|n_al_a,n_{a'}l_{a'};\,L\rangle. \quad (2)$$

Here the pair of single-nucleon states (a,a') represents the transferred pair. The restriction $a\leqslant a'$ serves to avoid counting the same configuration twice. Ω_n is the overlap integral of the radial wave function $u_{n0}(\frac{1}{2}\nu r^2)$ of the relative motion of the nucleons of the transferred pair with the corresponding radial wave function of the same in the He^4, He^3, or H^3 nucleus; it is defined in Eq. (A10) of Ref. 1 as

$$\Omega_n=\int_0^\infty u_{n0}(\tfrac{1}{2}\nu r^2)u_{10}(a\eta^2r^2)r^2dr, \quad (3)$$

with $a=4$ or 3.

The symbol $\langle nl,NL;\lambda|n_al_a,n_{a'}l_{a'};\lambda\rangle$ is a Brody-Moshinsky "bracket" coefficient of the Talmi transformation. This implies, of course, that we limit ourselves to employing only harmonic-oscillator radial wave functions of single nucleons.

The spectroscopic factor β defined by Glendenning[1] for pickup reactions measures the fractional parentage of the ground state of the nucleus (A) plus two nucleons with the quantum numbers $\gamma(=n_1l_1j_1,\,n_2l_2j_2,\,\cdots)$, L, S, J, and T, in the final state of $(A+2)$. In the first-quantization notation it can be written as

$$\beta_{\gamma LSJ}(J_2J_1)=\binom{N+2}{2}^{1/2}\int\Psi_{J_1}{}^*(A,1,2) \times[\Psi_{J_2}(A)\phi_{\gamma LSJ}(1,2)]_{J_1}d(A)\;d(1)\;d(2). \quad (4)$$

The factor β, defined in the L-S coupling scheme, is related to the factor $B(Jj_1j_2)$ employed by Yoshida[9] by the usual recoupling in going over from the L-S to the j-j scheme:

$$\beta_{\gamma LSJ}=B(J,j_1j_2)\langle l_1\tfrac{1}{2}(j_1)l_2\tfrac{1}{2}(j_2)J\,|\,l_1l_2(L)\tfrac{1}{2}\tfrac{1}{2}(S)J\rangle. \quad (5)$$

Our microscopic spectroscopy and in particular our formulas given below are based on the second-quantization formalism.

After Yoshida[9] we define for the inverse (stripping) reactions $(A\rightarrow A+2)$

$$B(Jj_1j_2)=\sum_{MM_2}(J_2J;\,M_2M\,|\,J_1M_1) \times\langle\psi_{J_1M_1}|\mathfrak{A}^\dagger(j_1j_2JM)\psi_{J_2M_2}\rangle, \quad (6)$$

where

$$\mathfrak{A}^\dagger(j_1j_2JM)\equiv(1+\delta_{j_1j_2})^{-1/2} \times\sum_{m_1m_2}(j_1j_2;\,m_1m_2\,|\,JM)c_{j_1m_1}{}^\dagger c_{j_2m_2}{}^\dagger, \quad (7)$$

and $c_{jm}{}^\dagger$ is the creation operator of a nucleon with total angular momentum j, with z projection m.

In the following we limit our study to spherical nuclei in the vibrational region, where the Bardeen-Cooper-Schrieffer (BCS) pairing effect is most important. In order to simplify the formalism still further, we assume that the proton subshells are closed and form part of the "inert" core. Only a few valence neutron subshells are assumed to be active. Our initial state of (A) will be assumed to be the ground state of an even-even nucleus $(J_2=0)$.

All the low-lying nuclear states both of (A) and of $(A+2)$ can be reasonably described in terms of the Bogolyubov qp formalism. The most important defect of the formalism is that particle (neutron) number conservation is only approximate. In fact, the formalism is based on quantum statistical mechanics: The nucleus is treated as a grand canonical ensemble in the zero-temperature limit. We shall employ two model approximations for the description of both the initial ground state of (A) and any one of the final states of $(A+2)$. In the first, lowest approximation, $|\psi_{J_2}\rangle$ will be simply the qp vacuum itself, and $|\psi_{J_1M_1}\rangle$ will be eigenvectors of the usual QTD approximation made of qp pair creations. This description is an extension of the independent quasiparticle model (IQM) employed by Yoshida[9] in his calculations of the $B(Jj_1j_2)$ factors. The generalization consists in allowing for a configuration mixing due to a residual interaction potential. This approximate description of the nuclear structure has already been applied by Ching Liang Lin[10] to the two-nucleon transfer reactions with a considerable amount of success [(p,t) reactions in even Sn, Ni, Cd, Zr, and Fe isotopes]. The usual particle-hole (Tamm-Dancoff or RPA) model for the normal state (no "superconducting" gap Δ) has been applied by Broglia and Riedel[11] to their description of the $Pb^{206}(t,p)Pb^{208}$ reaction.

The next more sophisticated description is based on mixing zero-, two- and four-qp configurations (creations). Such a formalism has been recently described under the name of the quasiparticle second Tamm-Dancoff (QSTD) approximation,[2,3] and it has

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been applied with success to the spectra of the even tin isotopes. It is appropriate to describe not only the very lowest excited states, but also those of the so called two-phonon type. Actually, the latter states (e.g., 2_2^+, 4_1^+, 0_2^+) can hardly be explained in terms of double phonons. The QSTD theory contains the usual QTD theory as a special (limiting) case. It is enough to set all the four-qp components $b^E=0$ and the zero-qp constant $c^E=0$ in our QSTD equations in order to reproduce the QTD approximation.

The Bogolyubov qp operators are defined as

$$\alpha_\gamma^\dagger \equiv u_c c_\gamma^\dagger - S_\gamma v_c c_{-\gamma}, \quad \alpha_\gamma \equiv (\alpha_\gamma^\dagger)^\dagger, \tag{8}$$

where $\gamma\equiv(c, m_\gamma)$ and c is the set of quantum numbers characterizing a subshell; $-\gamma\equiv(c,-m_\gamma)$, and $S_\gamma\equiv(-)^{j_c-m_\gamma}$. In terms of qp's our two-nucleon transfer operator $\mathfrak{A}^\dagger$ can be written as

$$\begin{aligned}\mathfrak{A}^\dagger(j_1j_2JM)\equiv n_J(12)\{&-u_1u_2\mathbf{A}_{JM}^\dagger(12)\\&+(-)^{J-M}v_1v_2\mathbf{A}_{J-M}(12)+v_1u_2\bar{A}_{JM}(12)\\&+(-)^{j_1+j_2-J}u_1v_2\bar{A}_{JM}(21)\}\\&+(\hat{j}_1/\sqrt{2})v_1u_1\delta_{j_1j_2}\delta_{J0}\delta_{M0},\end{aligned} \tag{9}$$

where $n_J(12)\equiv[1+(-)^J\delta_{12}]^{-1/2}$, and

$$\mathbf{A}_{JM}^\dagger(aa')\equiv\sum_{m_\alpha m_{\alpha'}}(j_aj_{a'};m_\alpha m_{\alpha'}|JM)\alpha_{\alpha'}^\dagger\alpha_\alpha^\dagger, \tag{10}$$

$$\mathbf{A}_{JM}(aa')\equiv[\mathbf{A}_{JM}^\dagger(aa')]^\dagger,$$

$$\bar{A}_{JM}(aa')\equiv\sum_{m_\alpha m_{\alpha'}}S_\alpha(j_aj_{a'};-m_\alpha m_{\alpha'}|JM)\alpha_{\alpha'}^\dagger\alpha_\alpha. \tag{11}$$

Our phase conventions are all those of Ref. 2.

In order to fix our attention let us limit ourselves to the reaction $(A+2)_{\text{g.s.}}(p,t)A$, or, by time reversal, to the reaction $A(t,p)(A+2)_{\text{g.s.}}$. The spin of the nucleus A is $J_2=J$, as the ground state of $(A+2)$ has spin $J_1=0$. In this case we have to calculate

$$\begin{aligned}B(Jj_1j_2)=\sum_M&(J_2J;M_2M|00)\\&\times\langle\Psi_{00}^{(A+2)}|\mathfrak{A}^\dagger(j_1j_2JM)|\psi_{JM}^{(A)}\rangle.\end{aligned} \tag{12}$$

In the notation of Refs. 2 and 3 any QSTD eigenvector is expressed in the form

$$\begin{aligned}|\Psi_{JM}^{(A)E}\rangle=\{&c_0^E\delta_{J0}+\sum_{a\leqslant a'}a_J^E(aa')A_{JM}^\dagger(aa')\\&+\sum_{(\alpha)}\sum_{b\leqslant b'\leqslant c\leqslant c'}b_{(\alpha)J}^E(bb'cc')\mathfrak{B}_{(\alpha)JM}^\dagger(bb'cc')\}|0\rangle,\end{aligned} \tag{13}$$

where $(c_0^N\delta_{J0},a_J^E,b_{(\alpha)J}^E)$ are the components of the eigenvector corresponding to the eigenvalue E (a real number in our case), and

$$A_{JM}^\dagger(aa')=n_J(aa')\mathbf{A}_{JM}^\dagger(aa'),$$

$$\begin{aligned}\mathfrak{B}_{(\alpha)JM}^\dagger(bb'cc')\equiv&\sum_{J'J''}c_{(\alpha)J}^{J'J''}(bb'cc')\\&\times[\mathbf{A}_{J'}^\dagger(bb')\otimes\mathbf{A}_{J''}^\dagger(cc')]_{JM},\end{aligned} \tag{15}$$

where $c_{(\alpha)J}^{J'J''}$ are defined in Ref. 2 as the orthonormalization coefficients computed from the Schmidt procedure; $|0\rangle$ is the qp vacuum.

The form of Eq. (13) also applies to the ground-state eigenvector ($|\psi_{00}^{(A+2)}\rangle$ in our case); for this we have $E=0$, but we use the notation $(c_0^G\delta_{J0},a_J^G,b_{(\alpha)J}^G)$. Instead, we reserve the notation $E=0$ for the ground state of (A). In fact, we consider also the particular case of the ground-to-ground transitions. In QTD this ground-state vector automatically degenerates to $(1\times\delta_{J0},0,0)$.

Because of the nature of the $\mathfrak{A}^\dagger$ operator and that of our QSTD vectors, the only nonvanishing contributions to $B(Jj_1j_2)$ are terms involving

(1) $c_0^Gc_0^E$ (0qp-0qp components),
(2) $c_0^Ga_J^E$(the 0qp-2qp),
(3) $a_0^Gc_0^E\delta_{J0}$(2qp-0qp),
(4) $a_0^Ga_J^E$(2qp-2qp),
(5) $a_0^Gb_{(\alpha)J}^E$(2qp-4qp),
(6) $b_{(\alpha')0}^Ga_J^E$(4qp-2qp),
(7) $b_{(\alpha)0}^Gb_{(\beta)J}^E$(4qp-4qp).

(1) The first term is

$$B_1(Jj_1j_2)=(\hat{j}_1/\sqrt{2})v_1u_1c_0^Gc_0^E\delta_{j_1j_2}\delta_{J0}. \tag{16}$$

It corresponds to the term given by Yoshida[9] for the ground-to-ground transition (if we set $c=1$ in both). For $E>0$, B_1 vanishes in QTD. It also vanishes together with the BCS energy gap Δ.

(2) The second term is

$$B_2(Jj_1j_2)=\hat{J}v_1v_2c_0^Ga_J^E(12). \tag{17}$$

If $c_0^G,a_J^E\to1$, this again corresponds to a term given by Yoshida.[9]

(3) The third term is

$$B_3(Jj_1j_2)=-u_1u_2a_0^G(12)\delta_{j_1j_2}c_0^E\delta_{J0}. \tag{18}$$

(4) The 2qp-2qp components of B are

$$\begin{aligned}B_4(Jj_1j_2)=&(\hat{j}_1/\sqrt{2})v_1u_1\sum_{p\leqslant p'}\delta_{j_pj_{p'}}a_0^G(pp')a_0^E(pp')\\&+\delta_{j_1j_2}\delta_{J0}\delta_{M0}-\sqrt{2}\hat{J}[\hat{j}_1^{-1}u_1v_2a_0^G(11)\\&+\hat{j}_2^{-1}v_1u_2a_0^G(22)]a_J^E(12).\end{aligned} \tag{19}$$

This term of B does not exist in QTD.

(5) The 2qp-4qp cross terms can be written in the form

$$\begin{aligned}B_5(Jj_1j_2)=&\hat{J}v_1v_2n_J(12)\\&\times\sum_{a\leqslant a'}\sum_{(\alpha)}\sum_{r\leqslant r'\leqslant s\leqslant s'}\sum_{J'J''}(1/\sqrt{2})a_0^G(aa')\delta_{aja'}\\&\times b_{(\alpha)J}^E(rr'ss')c_{(\alpha)J}^{J'J''}(rr'ss')\langle0|\mathbf{A}_{00}(aa')\mathbf{A}_{J-M}(12)\\&\times[\mathbf{A}_{J'}^\dagger(rr')\otimes\mathbf{A}_{J''}^\dagger(ss')]_{J-M}|0\rangle,\end{aligned} \tag{20}$$

where the last vacuum matrix element is

$$\langle 0|\cdots|0\rangle \equiv P_{(J0)J(J'J'')}(12aa';rr'ss'), \quad (21)$$

in which P is defined explicitly in Eq. (A5) of Ref. 2. (6) Similarly, the 4qp-2qp term is

$$B_6(Jj_1j_2) = -n_J(12)u_1u_2 \times \sum_{a\leqslant a'}\sum_{(\alpha)}\sum_{r\leqslant r'\leqslant s\leqslant s'}\sum_{J'} b_{(\alpha)0}{}^G(rr'ss')n_J(aa')a_J{}^E(aa') \times c_{(\alpha)0}{}^{J'J'}(rr'ss')P_{(J'J')0(JJ)}(rr'ss';12aa'). \quad (22)$$

(7) Finally, the 4qp-4qp terms can be put in the form

$$B_7(Jj_1j_2) = \sum_{(\alpha),(\beta)}\sum_{b\leqslant b'\leqslant c\leqslant c'}\sum_{r\leqslant r'\leqslant s\leqslant s'} b_{(\beta)0}{}^G(bb'cc') \times b_{(\alpha)J}{}^E(rr'ss')\langle 0|\mathfrak{B}_{(\beta)00}(bb'cc')\{n_J(12)\bar{P}(12J) \times v_1u_2\bar{A}_{JM}(12)+\tfrac{1}{2}\hat{j}_1v_1u_1\delta_{j_1j_2}\delta_{J0}\delta_{M0}\} \times \mathfrak{B}_{(\alpha)JM}{}^\dagger(rr'ss')|0\rangle, \quad (23)$$

where $\bar{P}(abJ)\equiv 1-(-)^{j_a+j_b-J}(a\leftrightarrow b)$. The total of all contributions is in general

$$B(Jj_1j_2) = \sum_{i=1}^{7} B_i(Jj_1j_2). \quad (24)$$

In our Eqs. (16)–(23) we have used the approximation of neglecting the difference between the two qp vacuums, those of (A) and of $(A+2)$, in evaluating the matrix elements of the $\mathbf{A}$ and $\mathbf{A}^\dagger$ operators as simple vacuum expectation values. At the same time we have, somewhat inconsistently, used our QSTD eigenvectors always of the appropriate respective (A) and $(A+2)$ nuclei involved. The quantities u_a, v_a of the $\mathfrak{A}^\dagger$ operator refer to the BCS solution (qp vacuum) of (A).

In our quantum-statistical theory we deal essentially only with averages over a few adjacent isotopes; it is only the chemical potential λ which assures the correct number of nucleons, (A) or $(A+2)$ in our case; and it does so only in the uncorrelated ground state $|0\rangle$.

In order to correct for the difference between the qp vacuum of (A) and that of $(A+2)$ we apply the approximate method of an earlier paper by Yoshida.[12] One can, in fact, expand the BCS state of $(A+2)$ in terms of a series of qp pairs acting on the qp vacuum of (A). One can show that, for the lowest-order correction, the relation between the two states is

$$|0^{(A+2)}\rangle \cong \left[1+\tfrac{1}{2}\sum_j \frac{\Delta u_j}{v_j}\hat{j}\mathbf{A}_{00}{}^\dagger(jj)\right]|0^{(A)}\rangle, \quad (25)$$

where $\Delta u_j = u_j{}^{(A+2)} - u_j{}^{(A)}$.

In the following we shall discuss the effect of the correction $\Delta u_j \neq 0$, and compare some of our corresponding cumulative numerical results with those obtained with our previous formulas, i.e., with $\Delta u_j = 0$.

[12] S. Yoshida, Phys. Rev. **123**, 2122 (1961).

The above correction terms are evaluated after replacing the "final-state" bra $\langle 0|$ of $(A+2)$ in the original expressions for each one of our B_i by $\langle 0^{(A+2)}|$ of Eq. (25).

All our formulas given above are valid for $(A+2)_{J=0}\rightarrow(A)$ transitions. Actually, we could write the element

$$\langle \psi_{00}{}^{(A+2)}|\mathfrak{A}^\dagger(j_1j_2JM)\psi_{JM}{}^{(A)}\rangle$$

as

$$\langle \psi_{JM}{}^{(A)}|\mathfrak{A}(j_1j_2JM)\psi_{00}{}^{(A+2)}\rangle^*.$$

For the case of the general spins

$$B(Jj_1j_2)_{(A+2)J_1\rightarrow(A)J_2} = (-)^{J+J_1-J_2}(\hat{J}_1/\hat{J}_2) \times B(Jj_1j_2)_{(A)J_2\rightarrow(A+2)J_1}. \quad (26)$$

Equation (26) allows us to obtain immediately results for the time-reversed reaction, e.g., for the double stripping reaction (t,p), from our formulas (given explicitly only for $J_1=0$, however) for the two-neutron pickup reaction (p,t). The coefficients u_a, v_a here always correspond to the state on the right [e.g., to $|\psi_{JM}{}^{(A)}\rangle$ in Eq. (12) or to $|\psi_{00}{}^{(A+2)}\rangle$ in the case of the corresponding (p,t) reaction]. In this connection one has to use the appropriate Δu_a, the corrections referring always to the u_a as defined above; i.e., one has to change the signs of Δu_a in going from a given reaction to the corresponding time-reversed one.

3. NUMERICAL CALCULATIONS OF SPECTROSCOPIC FACTORS FOR (p,t) AND (t,p) REACTIONS ON EVEN TIN ISOTOPES

In the following we confine ourselves to reactions (t,p) and (p,t), i.e., where both the transferred nucleons are of the same nucleonic charge ($T=1$, $S=0$, $L=J$); generalizations to cases also involving $T=0$ components present no basic formal difficulties, although they would bear on important physical questions, such as the possibility of the neutron-proton pairing, etc., The formalism which we employ here does not involve the isotopic spin explicitly

References 2 and 3 have shown that a QSTD description of even-parity states of even tin isotopes seems to be rather successful both in predicting the observed level energies and in supplying reasonable microscopic wave functions.

It is also interesting to compare the theoretical predictions for our reactions based on spectroscopic factors computed with our four-qp microscopic theory with those of a pure two-qp theory.

In our numerical results reported below, we have employed the eigenvectors of the low-lying states of even tin isotopes of Ref. 3 for both the QSTD and the QTD.

The low-lying states of Sn nuclei are described in Ref. 2, and in Ref. 3 in terms of five subshells: $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$. The double-magic 50-50 core is assumed to be "inert." Reference 3 employs the

TABLE I. Values of $B(0j_1=j_2)$ for the reaction $Sn^{118}(p,t)Sn^{116}$ $(0_1^+,0_2^+,0_3^+)$ calculated with the QTD eigenvectors of Ref. 3 for the potentials P_2 and Gauss (a), (b), and (c), with the approximation $\Delta u_a=0$.

State of Sn^{116}	j_1	j_2	P_2	Gauss (a) $t=1$	Gauss (b) $t=0$	Gauss (c) $t=-0.555$
0_1^+	$\frac{1}{2}$	$\frac{1}{2}$		0.4688	0.4999	0.4999
	$\frac{3}{2}$	$\frac{3}{2}$		0.5501	0.5516	0.5515
	$\frac{5}{2}$	$\frac{5}{2}$		0.3293	0.3363	0.3402
	$\frac{7}{2}$	$\frac{7}{2}$		0.3863	0.3906	0.3907
	$\frac{11}{2}$	$\frac{11}{2}$		−0.6305	−0.6185	−0.6118
0_2^+	$\frac{1}{2}$	$\frac{1}{2}$	0.2414	0.2053	0.0360	0.0242
	$\frac{3}{2}$	$\frac{3}{2}$	0.0352	0.0636	−0.0028	−0.0113
	$\frac{5}{2}$	$\frac{5}{2}$	0.0750	0.1271	0.0035	−0.0058
	$\frac{7}{2}$	$\frac{7}{2}$	0.1256	0.0082	−0.0077	−0.0069
	$\frac{11}{2}$	$\frac{11}{2}$	0.0543	0.0555	0.0045	0.0042
0_3^+	$\frac{1}{2}$	$\frac{1}{2}$	−0.2250	−0.0897	0.0148	−0.0047
	$\frac{3}{2}$	$\frac{3}{2}$	0.1454	−0.0456	−0.0150	0.0127
	$\frac{5}{2}$	$\frac{5}{2}$	−0.0887	0.1101	−0.0077	0.0092
	$\frac{7}{2}$	$\frac{7}{2}$	0.0808	0.8735	0.0045	−0.0071
	$\frac{11}{2}$	$\frac{11}{2}$	0.0173	0.0161	−0.0035	0.0048

unperturbed single-neutron wave functions of a harmonic oscillator, as in Kuo *et al.*[13] The range parameter of the harmonic-oscillator functions assumed is $\sqrt{\nu}=\sqrt{M\omega_0/\hbar}=0.454$ F^{-1}, and the unperturbed single-neutron energies are taken for all our cases from Table 5(a) of Ref. 13. The residual interaction potentials $V(1,2)$ are of two types: (1) the Gaussian form of Ref. 3,

$$V(1,2)=-V_0\exp(-r_{12}^2/r_0^2)(P^s+tP^t), \quad (27)$$

and (2) the standard P_2 force,

$$V(1,2)=-\chi(5/4\pi)r_1^2r_2^2P_2(\cos\theta_{12}). \quad (28)$$

In Eq. (27), P^s and P^t are the singlet-even and triplet-odd projection operators, respectively. V_0 is fixed at 31.0 MeV and $r_0=2.0$ F. Three values of the parameter t are considered: (a) $t=1$ (Wigner force), (b) $t=0$ (singlet states only), (c) $t=-0.555$ (a Rosenfeld-type mixture). For the P_2 force of Eq. (28) we assume the value of $\chi=(0.275$ MeV$)\,\nu^2$, which fits the observed first excited 0^+ level 0_2^+ of $A=116$ (at 1.76 MeV). The BCS solutions for the quantities u_a, v_a, and E_a (involving the chemical potential λ and the energy gaps Δ_a) are all determined with the same two-body force, which is our residual interaction in the case of the Gaussians (a), (b) and (c); for the residual P_2 force we use the BCS solution appropriate to the Gaussian (a) as the pairing force.

The most important seem to be the ground-to-ground $(0_1^+\rightarrow 0_1^+)$ transitions, for which a specific enhance-

TABLE II. Values of $B(0\,j_1=j_2)$ for the reaction $Sn^{118}(p,t)Sn^{116}$ $(0_1^+,0_2^+)$ calculated with the QSTD eigenvectors of Ref. 3, both with $\Delta u_a=0$ and $\Delta u_a\neq 0$ for the potentials P_2 and Gauss (a), (b), and (c).

State of Sn^{116}	j_1	j_2	$\Delta u_a\neq 0$ P_2	$\Delta u_a\neq 0$ Gauss $t=1$	$\Delta u_a=0$ P_2	$\Delta u_a=0$ Gauss (a) $t=1$	Gauss (b) $t=0$	Gauss (c) $t=-0.555$
0_1^+	$\frac{1}{2}$	$\frac{1}{2}$	0.3603	0.3600	0.3039	0.2889	0.2647	0.2394
	$\frac{3}{2}$	$\frac{3}{2}$	0.3167	0.3349	0.2208	0.2316	0.2162	0.2049
	$\frac{5}{2}$	$\frac{5}{2}$	0.2477	0.2283	0.2441	0.2237	0.2311	0.2242
	$\frac{7}{2}$	$\frac{7}{2}$	0.2520	0.2764	0.2582	0.2717	0.2803	0.2731
	$\frac{11}{2}$	$\frac{11}{2}$	−0.5301	−0.6886	−0.4824	−0.4701	−0.4725	−0.4567
0_2^+	$\frac{1}{2}$	$\frac{1}{2}$	0.2191	−0.2049	0.1347	−0.1538	0.2737	0.2422
	$\frac{3}{2}$	$\frac{3}{2}$	−0.0305	−0.0967	0.0028	−0.0488	0.0558	0.0154
	$\frac{5}{2}$	$\frac{5}{2}$	0.0371	−0.0302	−0.0365	−0.0169	−0.0745	−0.1355
	$\frac{7}{2}$	$\frac{7}{2}$	−0.2288	0.1981	−0.2269	0.2112	−0.2346	−0.2202
	$\frac{11}{2}$	$\frac{11}{2}$	−0.0598	−0.1649	0.0659	−0.1173	0.0812	0.0720

TABLE III. Values of $B(Jj_1j_1)$ for the reaction $Sn^{118}(p,t)Sn^{116}$ $(0_{1,2}^+,2_{1,2}^+,4_{1,2}^+)$ calculated with the QTD eigenvectors of Ref. 3 for the potentials P_2, and Gauss (a) with $\Delta u\neq 0$.

		0_2^+					2_1^+		2_2^+				4_1^+		4_2^+	
j_1	j_2	0_1^+	Gauss (a)	P_2	j_1	j_2	Gauss (a)	P_2	Gauss (a)	P_2	j_1	j_2	Gauss (a)	P_2	Gauss (a)	P_2
$\frac{1}{2}$	$\frac{1}{2}$	0.5684	0.3787	0.4286	$\frac{1}{2}$	$\frac{3}{2}$	0.3954	0.5464	0.4504	0.4282	$\frac{1}{2}$	7/2	−0.1952	0.0167	0.0204	0.0573
$\frac{3}{2}$	$\frac{3}{2}$	0.7475	0.2048	0.1598	$\frac{1}{2}$	$\frac{5}{2}$	0.3273	0.3959	0.1262	0.2870	$\frac{3}{2}$	$\frac{5}{2}$	0.1274	0.0828	−0.2323	0.0782
$\frac{5}{2}$	$\frac{5}{2}$	0.3297	0.1845	0.1347	$\frac{3}{2}$	$\frac{5}{2}$	−0.1248	−0.2609	−0.2755	0.0156	$\frac{3}{2}$	7/2	−0.1253	0.3871	0.1795	0.0166
7/2	7/2	0.3885	0.0754	0.1968	$\frac{3}{2}$	7/2	0.2572	0.7328	0.0642	−0.1060	$\frac{5}{2}$	7/2	−0.0984	−0.0067	0.6570	−0.2170
11/2	11/2	−1.0520	0.0198	0.0127	$\frac{5}{2}$	7/2	0.4451	0.2025	0.1110	−0.1834	$\frac{5}{2}$	$\frac{5}{2}$	−0.0603	−0.0112	0.2008	0.0137
					$\frac{3}{2}$	$\frac{3}{2}$	0.2235	0.2314	0.4557	0.1031	7/2	7/2	−0.1768	0.0883	0.8778	0.3952
					$\frac{5}{2}$	$\frac{5}{2}$	0.1937	0.2666	0.1087	0.0987	11/2	11/2	0.4932	0.2084	0.0617	−0.1136
					7/2	7/2	0.4416	0.2560	0.2894	−0.1765						
					11/2	11/2	−0.2667	−0.1254	0.2867	0.3190						

[13] T. T. S. Kuo, E. Baranger, and M. Baranger, Nucl. Phys. **79**, 513 (1966).

ment due to the superconductivity effect (the energy gap) has been pointed out by Yoshida.[9] The specificity of QSTD for the 0_1^+ eigenvectors is a high (40–50%) percentage of the four-qp components, while the 0_1^+ energy shifts relative to the uncorrelated ground state $|0\rangle$ are appreciable only for the force (1c) if $A=116$, and for the force (2) if $A=120$.[14]

All our numerical results presented below were obtained on the computers of the Centro di Calcolo of the University of Trieste.

Some of our results on the spectroscopic factors $B(Jj_1j_2)$ are presented in Tables I–III. We give results both for $\Delta u_a=0$ and $\Delta u_a\neq 0$ in order to study the effect of the first-order correction for the difference between the BCS solutions appropriate to the ground states of $A+2$ and A.

In Table I we give $B(J=0\ j_1j_2)$ for the $Sn^{118}(p,t)$-Sn^{116} reaction in the crudest approximation of $\Delta u_a=0$ and QTD. In the cases 0^+ we have $j_1=j_2$. The main interest of Table I is the dependence of our spectroscopic factors B on the residual nuclear force. In the case of the $0_1^+\rightarrow 0_1^+$ transition with $\Delta u_a=0$ only the differences between the corresponding BCS solutions can be responsible for any difference in $B(0\ j_1=j_2)$. The same BCS solution for the potential (1a) is also used for the construction of the eigenvectors of P_2 and Gauss (a) —hence we have no difference in the corresponding $B(0_1^+\ j_1=j_2)$ for these cases.

The negative sign of $B(0\frac{11}{2}\frac{11}{2})$ for the $0_1^+\rightarrow 0_1^+$ transition stems from the negative sign of $v_{11/2}$ (cf. Ref. 2). The absolute values of $B(0\frac{11}{2}\frac{11}{2})$ are large.

We note rather impressive differences (except for the transition $0_1^+\rightarrow 0_1^+$) between the cases of different t (a rather strong dependence on spin-exchange forces). These differences are even larger than the corresponding differences between the cases of P_2 and Gauss (a).

In Table II we have again $B(0\ j_1=j_2)$ for the reaction $Sn^{118}(p,t)Sn^{116}(0_1^+,0_2^+)$ calculated in the four-qp QSTD theory and both with $\Delta u_a=0$ and $\Delta u_a\neq 0$. Here even for the transition $0_1^+\rightarrow 0_1^+$ one can distinguish between the different residual interaction potentials. The differences due to the $\Delta u_a\neq 0$ effects are not very large, however. The differences between the $B(0\ j_1=j_2)$ of the various different potentials are even more pronounced here than in the case of Table I (QTD). The main interest of Table II, however, lies of course in the comparison between QSTD and QTD (we compare here the appropriate columns of the two tables with $\Delta u_a=0$). The four-qp components of the QSTD vector of 0_1^+ appear to reduce considerably in absolute value

[14] However, the high percentage of the b components in $|0_1^+\rangle$ may be exaggerated by the lack of Hartree-Fock-Bogolyubov self-consistency; the percentage of the possible six-qp components should be small, for the reasons explained in Refs. 2–4. We may point out that our most important 4qp modes in 0_1^+ are of seniority zero; the total weight of the seniority $v=4$ components is generally small, as should be expected; the depletion of the qp-vacuum component in $|0_1^+\rangle$ is due to a rather high spuriousness of $|0\rangle$ which, in QSTD, is projected out (fluctuations of $\hat{N}^2$, etc.).

TABLE IV. Values of the spectroscopic factors G_{NJ} (table entries are $10^3\times G$) for the reactions $Sn^{118}(p,t)Sn^{116}(J_n^\pi)$ calculated with the QSTD eigenvectors of Ref. 3 for the Gaussian potentials (a) and (c); here $\Delta u=0$.

J_n^π / N	0_1^+		0_2^+		2_1^+		2_2^+		4_1^+		4_2^+		3_1^-		5_1^-		7_1^-	
	(a)	(c)	(a)	(c)	(a)	(c)	(a)	(c)	(a)	(c)	(a)	(c)	(a)	(c)	(a)	(c)	(a)	(c)
0	1.75	0.90	−0.37	−0.13	1.23	1.78	1.24	0.34	0.14	1.82	0.82	0.082	0.003	−0.65	0.05	0.13	7.23	16.51
1	−4.04	−6.97	−1.80	−3.08	−5.00	−6.38	−4.66	−0.40	−4.67	−29.31	−5.66	−50.12	−0.08	−7.07	−15.93	−28.54	−39.88	−71.6
2	18.61	7.41	5.17	2.67	9.30	15.87	12.22	1.28	10.95	158.58	24.60	293.90	0.32	40.03	44.03	73.99	...	...
3	−56.99	−94.02	−23.88	−38.62	−59.81	−169.95	−71.68	−26.33	−1.48	−2.28	−1.62	1.44	−0.40	−56.50	...	...	...	...
4	263.29	150.26	−45.25	−14.89	4.60	1.01	−1.44	2.33	...	...	...	...	...	...	...	...	...	...
5	−20.74	−24.57	−5.22	−3.18	...	...	...	...	...	...	...	...	...	...	...	...	...	...

all the respective $B(0\ j_1=j_2)$ for the transition $0_1^+ \rightarrow 0_1^+$. This is the effect of our QSTD correlations in the ground state. An opposite effect of the 4qp terms on the absolute values of B is to be noted for the $0_1^+ \rightarrow 0_2^+$ transition. The QSTD predictions here represent a complete qualitative change relative to the QTD ones.

In Table III QTD results are presented for the same reaction $Sn^{118}(p,t)Sn^{116}$ with $\Delta u_a \neq 0$ for the final states 0_1^+, 0_2^+, 2_1^+, 2_2^+, 4_1^+, and 4_2^+.

Generally, the dependence of the respective $B(J j_1 j_2)$ on the residual interaction potential is, except for the cases with 4^+ in the final state, much less dramatic here than in the case of Table I. For the transitions $0_1^+ \rightarrow 4_1^+$ and $0_1^+ \rightarrow 4_2^+$ our B are indeed rather sensitive to the residual potential assumed. For the effect of $\Delta u_a \neq 0$ alone, one should compare the first three columns of Table III with the corresponding columns of Table I for the transitions $0_1^+ \rightarrow 0_1^+$ and $0_1^+ \rightarrow 0_2^+$ with the P_2 force and the Gaussian (a). One can note a general tendency to increase B as one includes the $\Delta u_a \neq 0$ corrections.

A theoretical model intermediate between the theories QTD and QSTD would obtain if we assumed the uncorrelated ground state, i.e., assumed the qp vacuum itself to be the ground state, and used the unmodified QSTD eigenvectors for the excited states. This variant is not unjustified because of all the uncertainties in the QSTD theory of the ground state.

This theory predicts numerical values of the parentage factors $B(J j_1 j_2)$ (and, similarly, of the spectroscopic factors G_{NJ}) typically in between the corresponding results of QTD and QSTD. In fact, the components B_i, $i=3$–7, of Eqs. (18)–(23) are zeros in this variant, but in the surviving terms B_2 the $a_J{}^E(12)$-components are those of QSTD eigenvectors [smaller than the QTD components $a_J{}^E(12)$].

Finally, we compute from Eq. (2) the spectroscopic factors G_{NLSJ} of the transferred pairs. They are superpositions of the $B(J j_1 j_2)$ appropriate to all the components of each one of our QTD or QSTD vectors. The $\Omega_n(n=0;\ 1,\ 2,\ \cdots,)$ integrals are computed numerically with $\sqrt{\nu}=0.454$ F^{-1}, $a=3$, and η^2, given by Glendenning.[1]

In Table IV we give G_{NJ} computed for the reactions $Sn^{118}(p,t)$ with the final states of Sn^{116} of $J_n{}^\pi = 0_{1,2}^+$, $2_{1,2}^+$, $4_{1,2}^+$, 3_1^-, 5_1^-, and 7_1^-. The residual nuclear forces are the Gaussian potentials (a) (Wigner) and (c) (Rosenfeld) of Ref. 3. The number of nonzero N factors G_{NJ} is in each $J_n{}^\pi$ case determined from the energy conservation selection rule of the Talmi transformation for the neutron pairs involved.[15]

We observe that typical G_{NJ} are negative for odd N and positive for even N; this situation is characteristic of states of the collective type (capable of being strongly excited); in fact, the coherence of the sum $\sum_N (-)^N G_{NJ}$ is a well-known rough measure of the relative strength of excitation of a given final state $J_n{}^\pi$. We observe that the state 0_2^+ is not collective in this sense, and, in fact, it is well known to be only weakly excited in (p,t) and (t,p) reactions[8] in contrast to, e.g., the 2_1^+ state.

In Table IV we note rather remarkable differences between the G_{NJ} corresponding to the Wigner force [Gaussian (a)] and those calculated with the Rosenfeld mixture [Gaussian (c)]. These differences are particularly dramatic for the 3_1^- state.

The corresponding factors G_{NJ} computed with the QTD eigenvectors are typically much larger (by up to 45% for large G_{NJ}). This again shows the importance of even the sometimes small admixtures of the four-qp terms in the QSTD eigenvectors and of the coupling of $|0\rangle$ with the four-qp terms for $J^\pi = 0^+$.

In Table V we give our G_{NJ} for the QSTD eigenvectors obtained with the P_2 force. Although these G_{NJ} are generally similar to those of the force Gaussian (a) of Table IV, some of them are considerably different.

In the "intermediate" theory QSTD with the qp vacuum $|0\rangle$ as the ground state we find numerical values of G_{NJ} typically in between those of QTD and QSTD proper. For example, for the reaction $Sn^{118}(p,t)Sn^{116}(0_2^+)$ we find with the Gaussian (a) force the values of $10^3 \times G_{N0}$, $N=0,\ 1,\ \cdots 5$: 0.04, −3.39, 12.02, −43.43, 11.56, −5.88. With the force P_2 the same $10^3 \times G_{N0}$ are: −0.46, −3.26, 2.28, −39.33, −64.34, 0.81. For the reaction $Sn^{118}(p,t)Sn^{116}(2_1^+)$ we find with the P_2 force: $10^3 \times G_{N2}$, $N=0,\ 1,\ \cdots,\ 4$, 3.19, −12.01, 37.47, −208.98, and 3.08.

In addition to the phenomenological forces of the Gaussian—and of the P_2—type, we have also considered one case of an effective nuclear force derived from a realistic nucleon-nucleon potential. Gmitro *et al.*[16] have obtained QSTD eigenvectors for the realistic potential of Tabakin renormalized for the Kuo-Brown core polarization corrections. These eigenvectors give a rather good over-all agreement with the data for Sn^{116} and Sn^{120}. We have computed the corresponding spectro-

TABLE V. Values of the spectroscopic factors G_{NJ} (table entries are $10^3 \times G$) for the reactions $Sn^{118}(p,t)Sn^{116}$ $(0_{1,2}^+, 2_{1,2}^+, 4_{1,2}^+)$ calculated with the QSTD eigenvectors of Ref. 3 for the P_2 force; here $\Delta u = 0$.

N \ $J_n{}^\pi$	0_1^+	0_2^+	2_1^+	2_2^+	4_1^+	4_2^+
0	1.79	−0.16	0.79	0.41	0.55	0.82
1	−4.00	−2.36	−4.40	0.09	−6.44	−20.15
2	18.41	4.89	19.46	−5.23	8.68	8.47
3	−56.64	−29.81	−92.29	−52.24	1.39	−1.26
4	269.08	−18.74	−7.48	1.12	···	···
5	−21.27	−2.90	···	···	···	···

[15] We may mention that, e.g., for the states 6^- all the G_{NJ} vanish identically (forbidden transitions) just because of the above selection rule, while the corresponding $B(J j_1 j_2)$ are nonzero. This result is then a specific property of the harmonic-oscillator wave functions in our particular subspace of the single-particle spectrum. This shows one of the limitations of the harmonic-oscillator model.

[16] M. Gmitro, J. Hendekovic, and J. Sawicki, Phys. Rev. (to be published); and Phys. Letters **26B**, 252 (1968).

TABLE VI. Values of the spectroscopic factors G_{NJ} (tables entries are $10^3 \times G$) for the reactions $Sn^{118}(p,t)Sn^{116}$ (J_n^π) calculated with the QSTD eigenvectors of Ref. 16 for the Tabakin potential renormalized for core polarization; here $\Delta u = 0$.

N \ J_n^π	0_1^+	0_2^+	2_1^+	2_2^+	4_1^+	4_2^+	3_1^-	5_1^-	7_1^-
0	3.266	0.584	3.064	−1.273	9.392	1.376	0.136	2.812	68.001
1	−10.611	1.341	−13.802	1.523	−48.190	−49.849	−16.716	−69.466	−261.773
2	38.600	1.649	53.187	12.460	249.761	308.700	75.794	161.774	···
3	−143.854	17.161	−243.746	172.609	−11.959	6.217	−97.886	···	···
4	491.441	77.581	6.055	6.172	···	···	···	···	···
5	−39.434	2.290	···	···	···	···	···	···	···

scopic parentage factors $B(Jj_1j_2)$ and G_{NJ} in the $\Delta u = 0$ approximation for the reaction $Sn^{118}(p,t)Sn^{116}$-(J_n^π). In Table VI we give G_{NJ} for this case with $J_n^\pi = 0_{1,2}^+$, $2_{1,2}^+$, $4_{1,2}^+$, 3_1^-, 5_1^-, and 7_1^-. These results are only qualitatively similar to those of Table IV [more similar to the Gaussian (c) (Rosenfeld) case].

Note added in proof. The corresponding QTD values of G_{NJ} are considerably larger. For example, with $\Delta u = 0$ we find for the $O_1^+ \rightarrow O_1^+$ transition $G_{N0} \times 10^3$: 4.03, −13.11, 48.99, −177.95, 607.10; and −49.41 for $N = 0, 1, \cdots, 5$, respectively.

The rather impressive differences between the G_{NJ} computed for the same final state with QSTD eigenvectors corresponding to different effective nuclear forces show that these spectroscopic factors are sensitive to the details of such forces.

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