Experimental and Theoretical Investigation of the $Pb^{208}(p,t)Pb^{206}$ Reaction at 40 MeV*†

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The $Pb^{208}(p,t)Pb^{206}$ reaction induced by 40-MeV protons has been studied with 100-keV resolution. The shapes of the differential cross sections reproduce those predicted by distorted-wave Born-approximation calculations. The relative theoretical magnitudes of these cross sections for three different sets of Pb^{206} wave functions are in reasonable agreement with the experimental results. The similarity between the predictions of these three sets of wave functions for the observed levels indicates that the predicted strengths are primarily sensitive to the relative signs of the wave-function components and not their exact magnitudes. The over-all agreement between theory and experiment for each set of wave functions would be somewhat improved if the theoretically predicted ground-state transition were enhanced relative to that of the excited states by approximately 40%.

IT has been suggested that the two-nucleon transfer reaction can be useful in obtaining nuclear-structure information since the transition amplitude depends on correlations of the transferred pair.[1,2] Encouraged by recent distorted-wave Born-approximation (DWBA) calculations, which have adequately described the shapes of differential cross sections of such reactions,[3–6] we have examined the $Pb^{208}(p,t)Pb^{206}$ reaction at 40 MeV to investigate the spectroscopic information that can be extracted for levels of Pb^{206}. Since states of Pb^{206} have been depicted as two neutron holes in the doubly closed shell Pb^{208} nucleus, this reaction is well suited for such an investigation. To ascertain whether one can discriminate between theoretical wave functions by comparing their predictions with the experimental results, we have calculated differential cross sections for states in Pb^{206} described by three sets of wave functions. The present energy resolution of 100 keV is a significant improvement over that of a previous experiment at the same energy,[7] whose results were compared[3] with the predictions of a wave function set of True and Ford.[8] This set has also been employed in a comparison with data from a high-resolution $Pb^{208}(p,t)$ Pb^{206} experiment at 22 MeV at three scattering angles.[9]

An isotopically enriched 2 mg/cm² Pb^{208} target was bombarded by the 40-MeV proton beam of the Oak Ridge isochronous cyclotron and outgoing particles were detected in a ΔE-E telescope consisting of Si surface-barrier detectors of 500 and 4600 μ. Deuteron and triton spectra were identified by recording a particle's ($E\times\Delta E$) versus E spectrum in a 20×1000 channel two-dimensional mode of a multichannel analyzer. Triton energy spectra (e.g., see Fig. 1) were obtained by summing over the appropriate ($E\times\Delta E$) planes. Spectra were measured at 5° intervals from 15° to 50° (lab).

Scattered proton spectra were also measured by summing the energy signals from the two previously mentioned detectors with that from a 5-mm Li drifted Si detector mounted behind them. The energy resolution for the protons was 300 keV. The absolute cross sections were obtained by normalizing the elastic proton cross sections to those previously measured.[10] This relative determination of the absolute normalization agreed within 10% with that explicitly calculated from the measured solid angle and target thickness. The experimental error in the absolute cross sections is approximately 20%. The differential cross sections for the levels excited in Pb^{206} are shown in Fig. 2. The cross sections of levels resolved in the previous experiment[7] agree with those measured here within the experimental errors.

Angular-momentum assignments were made by comparing the experimental differential cross sections with theoretical DWBA angular distributions calculated in

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[10] L. N. Blumberg, E. E. Gross, A. van der Woude, A. Zucker, and R. H. Bassel, Phys. Rev. **147**, 812 (1966).

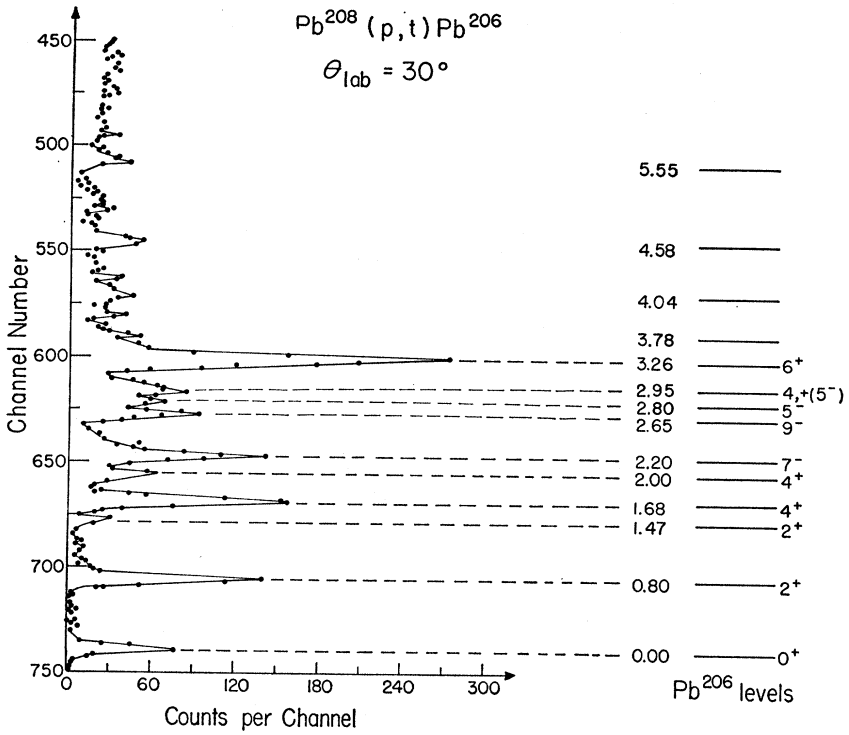

FIG. 1. $Pb^{208}(p,t)Pb^{206}$ spectra at $\theta_{lab}=30°$. The measured excitation energies and spin and parity assignments are indicated. Differential cross sections were obtained for those levels associated with dashed lines.

the zero-range approximation. Two-nucleon transfer form factors[4–6] were computed with neutron single-particle eigenfunctions of a Woods-Saxon potential (with a binding energy equal to $\frac{1}{2}$ the two-neutron separation energy) which were expanded into 10 harmonic-oscillator wave functions. This insured that the resulting form factor had the correct asymptotic form to 13 F. The magnitudes and shapes of theoretical

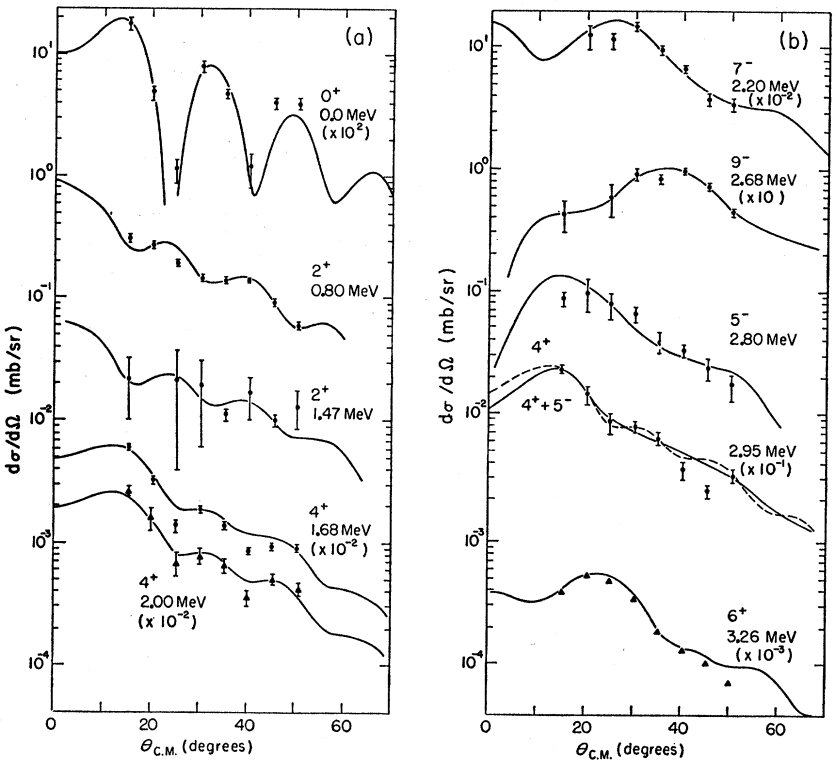

FIG. 2. Differential cross sections for the observed levels. The solid lines through the data points are zero-range DWBA calculations (described in the text). Where no error bars are indicated, the errors are less than the size of the data points.

TABLE I. Comparison of experimental results and theoretical predictions for observed levels in $Pb^{208}(p,t)Pb^{206}$ reaction. Columns 1–5 list the experimental information: (1) excitation energies of observed levels, (2) their spin and parity assignments given by (3) the references, and (4) the laboratory system angle of (5) the observed maximum cross sections. Columns 6–10 give the following theoretical quantities: (6) the eigenvalues of wave function set C (Ref. 10), (7) the spin and parity, and (8)–(10) the ratios of the predicted theoretical to the experimental differential cross sections for wave function sets A, B, and C (described in the text), respectively. The ratios have been normalized so that each ground-state ratio is unity.

Experiment					Theory				
				Max			$(d\sigma/d\Omega)^{th}/(d\sigma/d\Omega)^{expt}$		
E[a] (MeV)	J^π	Ref.	θ_{max} (deg)	$d\sigma/d\Omega$ (mb/sr)	E (MeV)	J^π	A	B	C
0.00	0^+	c	15	0.17	0.0	0^+_1	1.00	1.00	1.00
0.803	2^+	c	15	0.30	0.637	2^+_1	1.48	1.46	1.76
1.464	(2^+)[f]	c	15	(0.020)[f]	1.465	2^+_2	(1.40)[f]	(1.91)	(2.03)[f]
1.682	4^+	c	15	0.59	1.597	4^+_1	1.29	1.38	1.81
1.997	4^+	c, d, e	15	0.25	2.034	4^+_2	0.81	0.72	0.59
2.197	7^-	c, d, e	30	0.14	2.208	7^-_1	1.42	1.50	1.34
2.650	9^-	d	40	0.093	2.629	9^-_1	1.54	1.56	1.46
2.776	5^-	c, d	20	0.093	2.845	5^-_1	1.19	1.20	1.13
2.924	4^+	d, e	20	0.23	3.008	4^+_3	0.87	0.79	0.92
3.010	(5^-)[f]	e			3.096	5^-_2	(1.33)[g]	(1.31)[g]	(1.36)[g]
3.253	6^+	d	20	0.48	3.100	6^+_1	0.97	0.98	1.09
(3.78)[b]			20	0.076					
(4.04)[b]			20	0.059					
(4.58)[b]			20	0.085					
(5.55)[b]			15	0.034					

[a] Excitation energies are those of Ref. 9 except as noted.
[b] Excitation energies are from present experiment. The errors are ±50 keV. Angular distributions were unobtainable or had large errors.
[c] C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (John Wiley & Sons, Inc., New York, 1967).
[d] Assignment from present experiment.
[e] G. Vallois, J. Saudinos, and O. Beer, Phys. Letters **24B**, 512 (1967).
[f] Parentheses indicate tentative assignment or uncertain value.
[g] Ratios listed result from the sum of the $J^\pi=4^+_3$ and 5^-_2 predictions.

angular distributions are insensitive to the values of form factors beyond 12 F.

In the angular region of 0°–90° (c.m.) the shapes of the theoretical differential cross sections for all angular-momentum transfers are found to be essentially independent of the theoretical nuclear wave functions employed, and thus L values can be assigned on the basis of calculations with form factors for pure configurations [e.g., $(3p_{1/2})^2$; $J=0$] only. Thus, as in single-particle transfer reactions, the nuclear-structure information resides in the magnitudes of the angular distributions.

The energies of the levels excited in the present experiment are shown in Fig. 1. Within the experimental error of ±50 keV they coincide with levels found by Flynn *et al.*,[9] which are listed in Table I with the J^π assignments from the present and previous experiments. Since the incident energy in the present experiment is significantly greater than the Coulomb and centrifugal barriers, the excitation of states with high angular momentum is enhanced compared to experiments at lower energies. Thus the 9^- and 6^+ assignments to the levels at 2.65 and 3.25 MeV, respectively, are based on the present experiment alone, although they were previously suggested by the calculations of True and Ford.[8] Judging from the shape of the 9^- differential cross section, we find that a 3^- state known to be located at 2.653 MeV[9] is negligibly excited in the present experiments. However, it is not possible to reason similarly that a $J^\pi=(5^-)$ level at 3.010 MeV[9] is not excited. Thus, Fig. 2 and Table I also contain results assuming the observed level at 2.95 MeV (Fig. 1) is a 4^+, (5^-) doublet.

The Pb^{206} wave functions of True and Ford[8] and of True[11] were employed to calculate the differential cross sections for all allowed (natural-parity) transitions below an excitation energy of 3.4 MeV. The ratios of the theoretical to experimental cross sections, which were obtained by fitting the differential cross sections with the theoretical distributions, are presented in Table II and are normalized so that the ground-state ratios are unity. Set A eigenfunctions are those calculated with a singlet-even force, while set B are those for singlet-even force plus collective coupling.[8] Set C eigenfunctions are similar to set B except that all possible neutron con-

TABLE II. Theoretical predictions of unobserved states. Listed are (1) the eigenvalues of wave function set C, (2) their spin and parity, (3) the known corresponding excitation energies with known spin and parity, and (4) the laboratory angle in the observed angular region (15°–50°) for (5)–(7) the maximum differential cross sections predicted with wave function sets A, B, and C, respectively.

E (MeV)	J^π	E[a] (MeV)	θ_{max} (deg)	Max $(d\sigma/d\Omega)^{th}$ (mb/sr) A	B	C
1.314	0^+_2	1.165	15	0.023	0.012	0.015
1.791	2^+_3	(1.785)	23	0.003	0.003	0.002
2.085	0^+_3		15	0.060	0.076	0.069
2.205	2^+_4		23	0.012	0.012	0.014
2.542	2^+_5		23	0.025	0.026	0.033
3.015	7^-_2		28	0.031	0.013	0.014
3.136	0^+_4		15	0.002	0.002	0.002
3.320	7^-_3		28	0.057	0.063	0.063

[a] Assignments are from C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (John Wiley & Sons, Inc., New York, 1967). Other levels in this excitation energy region of unknown spin and parity were found at 2.149, 2.314, 2.421, 3.116, 3.128, 3.191, and 3.383 MeV by Ref. 9 in the (p,t) reaction.

[11] W. W. True, Phys. Rev. **168**, 1388 (1968).

figurations between $N=82$ and $N=126$ were included.[10] The theoretical excitation energies listed are the eigenvalues of set C. The DWBA calculations employed a proton optical potential of Fricke *et al.*,[12] and the triton potential used by Glendenning.[3] The results presented in Table I were changed by less than 10% when different optical potentials were used or when nonlocal corrections in the local-energy approximation were made for the distorted waves.[13]

All three sets of predictions agree in qualitatively classifying strong and weak transitions. All of the experimentally observed transitions are predicted to be strong. The maximum cross sections in the observed angular region for the theoretical states which were not experimentally observed are listed in Table II with the same normalization as in Table I. These cross sections are generally small compared with the observed cross sections. If only pure configurations were assumed for the states of Pb^{206}, all states of the same spin would have roughly equal strengths.

In comparing the predictions of the different sets of wave functions, we find that the differences between the predictions of sets A and B are small for the states of Table I and are somewhat larger for the states of Table II. The predictions of sets B and C are quite similar except for the highly correlated 2^+_1, 4^+_1, and 6^+_1 states (lowest 2^+, 4^+, and 6^+ states), which are enhanced for the latter set. The predictions for the ground-state cross sections of sets A, B, and C are in the ratio of 1:1:1.2. Figure 3 shows the maximum values of the predicted cross sections for the states of Table I. In addition, the prediction for the ground-state cross section of the wave function of Mukherjee and Cohen[14] is 1.29 that of set A.

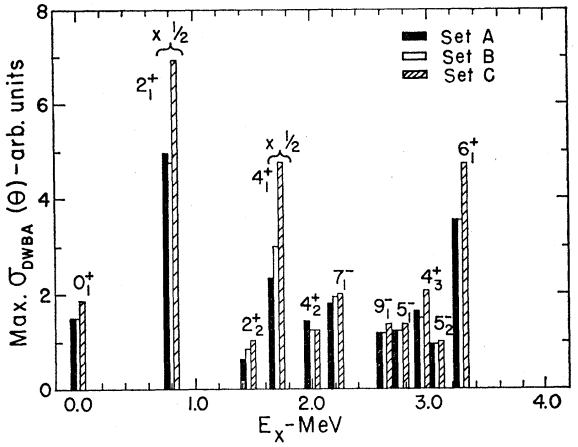

FIG. 3. Maximum values of the DWBA differential cross sections for the $Pb^{208}(p,t)Pb^{206}$ reaction predicted by wave function sets A, B, and C (described in text).

The theoretical predictions and experimental results for a given level are in agreement relative to the ground state if the ratios presented in Table I are unity. Within the experimental uncertainty of approximately 15%, only the predictions for the 5^-_1, 4^+_3, and 6^+_1 states are in agreement with the experiment relative to the ground state. With one exception (second 4^+ state) the remaining states are theoretically too large, generally by 30–50%. The average deviation from experiment would be reduced if the normalization was taken so that the ground-state ratio was less than unity, or if other effects which were not considered increased the predictions of the ground state relative to the excited-state strengths. For example, using the wave function of Broglia and Riedel,[5] which includes pairing correlations, we find an increase in the predicted ground-state transition of 1.83 times that of set A.

Some interesting nuclear information can be obtained from a comparison of the relative strengths of states of the same spin. For example, for the first two 4^+ states the experimental strength is divided more nearly equally than the theoretical strength as predicted by all three sets of wave functions. Similarly, the experimental γ transition rates of the lowest 7^- state to the first and second 4^+ states are also closer to each other than theoretically predicted.[8] The disagreement between the experimental results and theoretical predictions is smallest for wave function set A. Similarly, set A also best reproduces the experimental ratio of the second 2^+ to the first 2^+ strength, although there is a large uncertainty in the experimental second 2^+ strength.

As has been generally recognized, two-nucleon transfer reactions are sensitive to the relative signs of the components of the wave functions involved. The present over-all agreement of the experimental results with the theoretical predictions for all three sets of wave functions indicates that these wave functions give an adequate description of the relevant signs. The observed states are generally the lowest state of each spin and the contributions from each of their components are positive for the wave function sets employed. The (p,t) reaction strengths for states of this type are comparatively insensitive to the detailed differences of these three sets, i.e., to the exact magnitudes of the various wave function components. This is similarly true for theoretical electromagnetic transition rates.[8] It appears that the magnitudes of wave-function components are more easily and uniquely determined by other methods, such as one-nucleon transfer reactions, and the relative signs of these components can be determined by two-nucleon transfer reactions.

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