between them. The other 15% of the wave function adds to give the calculated moment of −0.26 nm. It appears that a calculation on this isotope must be as complex as ours to be meaningful.

A much less detailed study of the Cu^{64} and Cu^{66} ground-state wave functions suggests that they may be well approximated by the addition of two and four $f_{5/2}$ neutrons, respectively, to the Cu^{62} wave function. Neutrons added in this way cause little change in the Cu^{62} magnetic moment. The moments calculated for Cu^{64} and Cu^{66} in this approximation will both be about −0.2 nm, in agreement with experiment.

ACKNOWLEDGMENTS

We wish to thank T. T. S. Kuo for providing us with his matrix elements and for discussions of the theory, E. H. Rogers, Jr., for help with the experiment, L. Goodman for sending us a value of g_J for copper, and K. F. Smith for permission to use unpublished measurements on Cu^{66}.

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Test of Time-Reversal Invariance through the Reactions $^{24}Mg+\alpha \rightleftharpoons {}^{27}Al+p$

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Time-reversal invariance has been tested via detailed balance in the compound nuclear reactions $^{24}Mg+\alpha \rightleftharpoons {}^{27}Al+p$. The relative differential cross sections agree within the experimental uncertainties. An upper limit for the ratio of the T-nonconserving to the T-conserving part of the reaction amplitude has been found to be $(2\text{–}3)\times10^{-3}$. A model-dependent upper limit between 4×10^{-4} and 3×10^{-3} has been derived for the relative strength of the T-odd part of the nuclear Hamiltonian.

I. INTRODUCTION

AFTER the discovery of parity nonconservation in weak interactions[1] in 1957 the question arose as to whether time-reversal (T) invariance might also be violated,[2] and by 1964 several experiments had been performed to test T invariance both in weak and in strong interactions. Various polarization experiments[3] and reciprocity tests[4] were the means by which upper limits of typically a few percent were found for the possible extent of a T-odd part of the amplitude in these experiments. Similar upper limits were derived by γ-γ angular-correlation experiments on oriented nuclei at low energies where the electromagnetic interaction was investigated.[5] An experimental test of T invariance in weak interactions has been reported by Burgy *et al.*[6] The interest in the question of T invariance was stimulated once more in 1964 when Christenson *et al.*[7] detected CP nonconservation of about 2×10^{-3} through the decay of the long-lived state of the K^0 meson into two pions, which, on the basis of the CPT theorem, implies that T invariance must also be violated. There is the possibility of a connection between CP nonconservation in the decay of the K_2^0 meson and T violations in nuclear interactions, since Bernstein, Feinberg, and Lee[8] and independently Prentki and Veltman[9] proposed that the forbidden mode of the K_2^0 decay might be due to an interference between a weak time-reversal-even interaction and a much stronger time-reversal-odd one. Since then, great effort has been made to reduce the experimental uncertainties in the experiments mentioned above. Recently, a polarization experiment in p-p scattering[10] yielded a T-nonconserving amplitude of less than 0.5% of the T-conserving one. Bodansky *et al.*[11] have been able to test relative cross sections in a detailed balance experiment with an accuracy of 0.3%, and for electromagnetic interaction studies using the Mössbauer

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effect Kistner[12] found a T-noninvariant impurity of $(1.0\pm 1.7)\times 10^{-3}$ in the amplitude of the nuclear γ decay for a mixed transition. Zech *et al.*[13] arrived in a similar experiment at an upper limit for the T-odd amplitude of about 5×10^{-3}.

In this paper a detailed balance experiment will be described which should provide a very accurate test of T invariance.[14]

The invariance of the nuclear Hamiltonian under the T operation leads to a symmetry of the scattering matrix[15] and from there directly to reciprocity relations for the cross section. One of those relations, the principle of detailed balance, states that the differential cross sections of two inverse reactions $a+A\rightleftharpoons b+B$, in the following denoted by $\sigma^{\rightarrow}(\theta)$ and $\sigma^{\leftarrow}(\theta)$, are connected, if they are considered at the same c.m. angles and energies, by the following formula:

$$\sigma^{\rightarrow}(\theta)=\sigma^{\leftarrow}(\theta)\left(\frac{p'}{p}\right)^2\frac{(2s_1'+1)(2s_2'+1)}{(2s_1+1)(2s_2+1)},$$

where p is the relative momentum and s_1 and s_2 are the spins of the particles in the entrance channel; the primes denote the corresponding quantities in the exit channel.

Henley and Jacobsohn[16] have pointed out that the principle of detailed balance should be most sensitive to T in reactions which precede via a complicated reaction mechanism and in which many competing channels are open; otherwise, detailed balance might hold irrespective of T invariance. Two examples of this are the case in which the scattering matrix breaks up into 2×2 matrices[16] and reactions involving well-localized interactions beyond the nuclear surface, as was recently shown by Robson[17] for direct reactions using distorted-wave Born-approximation theory.

A suitable experiment which involves the required complicated reaction mechanism seems to be a compound nuclear reaction[18] of the type $a+A\rightleftharpoons C^*\rightleftharpoons b+B$, where the compound nucleus C^* is excited in the continuum region where the mean level width Γ is much bigger than the mean level spacing D. The cross section as a function of energy then shows statistical fluctuations of the Ericson type due to the interference of many coherently excited resonances.[19] It can be assumed that in the presence of a T-odd force the T-conserving and the T-nonconserving amplitudes are uncorrelated in the sense of the statistical theory,[20] so that the interference of the two should change not only the absolute cross section but also the shape of the fluctuations in the two inverse reactions. Therefore, a measurement of detailed balance using relative cross sections should be capable of detecting such T-odd forces. Since the structure in the excitation functions of the two inverse reactions is mainly due to the behavior of the T-even contribution to the amplitude, it seems especially tempting to look for a violation of detailed balance in a deep minimum where the small T-odd part of the amplitude (if there is any) would be strongly enhanced relative to the T-even one.

The most favorable case, of course, will be furnished by a reaction where only one spin channel contributes to the cross section. In the case of several spin channels contributing, a T-violation effect which would show up clearly in each single channel would be obscured due to the incoherent mixture of the different uncorrelated spin channels.

These conditions are fulfilled in the two inverse reactions $^{24}Mg+\alpha\rightleftharpoons{}^{27}Al+p$ at bombarding energies between 10 and 15 MeV.[21] The reactions are known to proceed mainly via the formation of a compound nucleus[22] and it can easily be shown that only one spin channel is present in these reactions at scattering angles $\theta=0°$ and $\theta=180°$. (See Sec. II and Appendix A.) Furthermore, protons and α particles of suitable energies are provided by the Heidelberg Tandem Van de Graaff, and targets of high purity are available. Thus, the idea of the experiment was to measure the differential cross sections in the above reactions at a backward angle close to 180° at a high maximum of the excitation functions where the contribution of a possible T-odd part of the amplitude is supposed to be negligibly small, as well as at places where the cross section is small, and to compare the relative cross sections.

It will be briefly outlined in Sec. II how upper limits for the relative strength of the T-odd part of the reaction amplitude and of the nuclear matrix elements are derived from the experimentally measured differential cross sections. The experimental apparatus will be discussed in detail in Sec. III. The experimental procedure and the results are presented in Sec. IV, whereas Sec. V contains a discussion of the results in terms of the

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[18] The effects of T violations in compound nuclear reactions have recently been discussed by T. J. Krieger, Bull. Am. Phys. Soc. **12**, 498 (1967); N. Rosenzweig, *ibid.* **12**, 894 (1967), P. A. Moldauer, Phys. Rev. **165**, 1136 (1968) gives a full discussion of T-violation effects in nuclear reactions and concludes that in fluctuating reactions any effect should be favored over direct reactions.

[19] T. Ericson, Ann. Phys. (N. Y.) **23**, 390 (1963).

[20] W. von Witsch, A. Richter, and P. von Brentano, in *Proceedings of the International Conference on Nuclear Physics, Gatlinburg, Tennessee, 1966* (Academic Press Inc., New York, 1967); discussion remark by H. A. Weidenmüller.

[21] A measurement of detailed balance in these reactions has already been performed by Kaufmann *et al.* in the region of single, isolated resonance [Phys. Rev. **88**, 673 (1952)].

[22] B. W. Allardyce, W. R. Graham, and I. Hall, Nucl. Phys. **52**, 239 (1964); G. M. Temmer, Phys. Rev. Letters **12**, 330 (1964).

relations stated in Sec. II and a discussion of the experimental uncertainties.

II. THEORETICAL CONSIDERATIONS

Using the notation of Bondorf and Leachman,[23] the differential cross section $\sigma(\theta)$ is written as a sum over "basic" partial cross sections

$$\sigma(\theta)=\sum_{\beta}\sigma_{\beta}(\theta)\,, \tag{1}$$

where $\beta=\{M,\mu;M',\mu'\}$ denotes the projections of the spins in the entrance and in the exit channels, respectively, and labels the different spin channels. In terms of amplitudes participating in the reaction, $\sigma(\theta)$ is expressed as

$$\sigma(\theta)=\sum_{\beta}|\hat{f}_{\beta}(\theta)|^2=\sum_{\beta}|f_{\beta}(\theta)+f_{\beta}'(\theta)|^2\,, \tag{2}$$

the reaction amplitudes $\hat{f}_\beta$ being decomposed into T-conserving ones f_β and into T-nonconserving ones f_β'.

In the case where only one spin channel contributes significantly, Eq. (2) becomes

$$\sigma(\theta)=|\hat{f}(\theta)|^2=|f(\theta)+f'(\theta)|^2\,,$$

where again f stands for the T-even part and f' stands for the T-odd part of the reaction amplitude $\hat{f}$. For compound nuclear reactions proceeding via the continuum region ($\Gamma/D>1$) but *not* in the region of single, isolated resonances, f and f' may be considered independent,[20] as will be shown in Appendix B. A measure of the average strength of the T-odd part of the reaction amplitude is then defined as[24]

$$\xi^2=\langle|f'|^2\rangle_{\rm av}/\langle|f|^2\rangle_{\rm av}\,, \tag{3}$$

where the symbol $\langle\ \rangle_{\rm av}$ denotes an average with respect to energy. If the cross section is measured in a minimum of the fluctuating excitation function, the minimum being by a factor ν smaller than the average cross section, any effect of T nonconservation will be enhanced since

$$\langle|f|^2\rangle_{\rm av}\approx\nu|f_{\rm min}|^2\,.$$

Therefore a new quantity ξ' is introduced which is related to ξ by

$$\xi'^2=\langle|f'|^2\rangle_{\rm av}/|f_{\rm min}|^2=\nu\xi^2\,. \tag{4}$$

In order to derive the value of ξ from the relative cross sections measured in the two reactions, the quantity

$$\Delta=\frac{\sigma^{\rightarrow}_{\rm min}-\sigma^{\leftarrow}_{\rm min}}{(\sigma^{\rightarrow}_{\rm min}+\sigma^{\leftarrow}_{\rm min})/2} \tag{5}$$

is considered. In the one-channel case, the two inverse cross sections can be expressed as

$$\begin{aligned}\sigma^{\rightarrow}_{\rm min}&=|f_{\rm min}|^2+2|f_{\rm min}||f'_{\rm min}|\cos(f,f')+|f'_{\rm min}|^2\,,\\ \sigma^{\leftarrow}_{\rm min}&=|f_{\rm min}|^2-2|f_{\rm min}||f'_{\rm min}|\cos(f,f')+|f'_{\rm min}|^2\,.\end{aligned} \tag{6}$$

Inserting relation (6) into Eq. (5) and neglecting the $|f_{\rm min}'|^2$ terms in Eq. (6), one finds

$$|\Delta|^2=\frac{16|f'_{\rm min}|^2}{|f_{\rm min}|^2}|\cos(f,f')|^2=\frac{16\nu|f'_{\rm min}|^2}{\langle|f|^2\rangle_{\rm av}}|\cos(f,f')|^2\,. \tag{7}$$

Since only the *average* strength of the T-nonconserving part of the reaction amplitude is of interest, $|f'_{\rm min}|^2$ has to be replaced by $\langle|f'|^2\rangle_{\rm av}$. This is done by means of the probability distribution of the differential cross section which, for a one-spin channel reaction, is[25]

$$P(\sigma/\bar{\sigma})=\exp(-\sigma/\bar{\sigma})\,. \tag{8}$$

Since $|f'|^2$ and $|f|^2$ can be assumed to fluctuate independently,[20] it follows from Eq. (8) that $P_a=\exp(-a)$ is the probability for finding[26]

$$|f'_{\rm min}|^2=|f'|^2\geq a\langle|f'|^2\rangle_{\rm av}\,.$$

Inserting this into Eq. (7) one gets

$$|\Delta|^2\geq\frac{16\nu a\langle|f'|^2\rangle_{\rm av}}{\langle|f|^2\rangle_{\rm av}}|\cos(f,f')|^2\,. \tag{9}$$

Similarly, the phase $|\cos(f,f')|$ may be replaced by any number b, where $0<b<1$, and it follows from the random distribution of the phases[19] that the probability for having

$$|\cos(f,f')|\geq b$$

is

$$P_b=|\arccos(b)|/\tfrac{1}{2}\pi\,.$$

Using this result and Eqs. (3) and (9), the quantity ξ is finally found to be

$$\xi\leq|\Delta|/4b(a\nu)^{1/2}\,. \tag{10}$$

From the two partial probabilities P_a and P_b, a total probability P is derived, which gives a confidence limit for all results obtained through (10). This confidence limit, which depends upon the choice of a and b, is a purely theoretical quantity and has no connection with the experimental uncertainties. Once a and b are chosen, the confidence limit can be increased only by measuring detailed balance at several independent points of the excitation function instead of at only one.

If the relative strength X of the T-odd part of the nuclear Hamiltonian is considered, according to Ericson[24]

[23] J. P. Bondorf and R. B. Leachman, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 34, 10 (1965).
[24] T. E. O. Ericson, Phys. Letters 23, 97 (1966).
[25] T. Ericson, Phys. Letters 4, 258 (1963).
[26] The symbol $f_{\rm min}'$ stands for the value which f' assumes at the minimum of f. Since f and f' are uncorrelated, one may put $f_{\rm min}'\equiv f'$.

and to Mahaux and Weidenmüller[27] an intrinsic enhancement factor $(W/\Gamma)^{1/2}$ occurs in nuclear reactions which proceed via a highly excited compound nucleus. The relative strength X is then defined as[24]

$$X \approx \xi (W/\Gamma)^{1/2}. \tag{11}$$

Here Γ is the mean width of the overlapping compound nucleus levels and W is the energy over which matrix elements are thoroughly mixed due to the complicated statistical nature of the reaction. Ericson identifies W with a single particle width which would be about 2 MeV in medium-heavy nuclei, whereas Mahaux and Weidenmüller conclude that W is connected with the width of a nuclear "doorway state" and therefore Ericson's estimate may be too large. In any case, however, one may assume $(W/\Gamma)^{1/2} \geq 1$, which permits at least the estimation of an upper limit for the relative strength of the T-odd part of the nuclear Hamiltonian. These considerations, however, are only meaningful if the T-odd part and the T-even part of the Hamiltonian are proportional to each other, as was pointed out by Moldauer.[18]

III. EXPERIMENTAL APPARATUS

Scattering chamber. The experiment was performed at the Heidelberg Tandem Van de Graaff accelerator in a scattering chamber of 50 cm diam. Figure 1 shows the experimental arrangement. The beam, coming from the left, was focused onto the target with a quadrupole magnet and passed through the target into a Faraday cup assembly. The beam current was integrated by means of a circuit calibrated to a relative accuracy of 0.1% over a period of several days. A magnet at the mouth of the cup and an electrode system biased at −400 V prevented secondary electrons from entering and escaping the Faraday cup. Along the beam path no collimating apertures were used in order to suppress the background in the spectra. An aperture 12 mm behind the target prevented particles being backscattered from the Faraday cup from entering the detectors.

Counters. Two counters placed symmetrically to the beam at $\theta_{lab} = 172°$ were used for the (α,p) and (p,α) measurements, whereas (α,α) and (p,p) standard cross sections for monitoring the beam energy were taken simultaneously at $\theta_{lab} = 160°$. The laboratory angle of 172° corresponds to a c.m. angle of 172.6° in the reaction ^{27}Al$(p,\alpha)^{24}$Mg. In order to attain the same c.m. angle in the two reactions, a lab angle of 171.9° must be chosen for the (α,p) reaction. This small difference of 0.1° in the lab system was achieved by putting the target 1.5 mm closer to the two counters in the (α,p) reaction, leaving the position of the counters unchanged. The 172° counters were provided with brass apertures which limited the solid angles viewed by the detectors to 4.28×10^{-3} sr and the angular resolution to ±2.12 deg. Antiscattering apertures mounted in front of the counters ensured that the counters viewed only the area surrounding the beam spot on the target. A pierced quartz disk which could be brought into the position of the target allowed the beam to be focused and the transmission to be optimized simultaneously. The α particles were detected with Si surface-barrier counters and the protons with Li-drifted Si counters, both cooled to −40°C. The energy resolution of the proton counters was about 30 keV, and the resolution of the α counters was about 40 keV. In the (α,p) experiment, 8 and 9 μ Al foils were placed in front of the proton counters in order to shift lines arising from (α,α) reactions to the low-energy end of the spectrum.

Electronics. The electronics used was conventional. The pulses from the detectors passed a charge-sensitive preamplifier and a main amplifier before being recorded in a multichannel pulse-height analyzer. Because of the low cross sections, no arrangement for the correction of dead time or pileup was necessary.

Targets. The targets consisted of isotopically enriched magnesium (99.96% ^{24}Mg) evaporated onto thin C foils, and of self-supporting Al foils, respectively. The thickness of the Mg targets has been carefully determined in several ways: by weighing before and after evaporation, by Rutherford scattering, and by energy loss. In the latter method, 6-MeV α particles have been scattered through 160 deg from the C backing, penetrating the Mg layer before and after scattering. When the target is turned through 180 deg, the particles are scattered directly from the carbon and the displacement of the peak in the spectrum is directly related to the thickness of the Mg layer. Similarly, the thickness of the self-supporting Al target was determined by weighing, by Rutherford scattering, and by the energy loss of low-energy α particles from a ^{241}Am source passing through the target. For the ^{24}Mg target used in the experiment, the different methods yielded a thickness of 33 ± 1.7 μg/cm^2, and 35 ± 2 μg/cm^2 was found for the ^{27}Al target. The methods utilizing the energy loss of the α particles yielded, furthermore, information concerning target inhomogeneity. It can be deduced from the shape of the lines in the (α,α) spectrum that the errors given above represent also an upper limit for the inhomogeneity.

Beam energy and beam spread. The absolute energy of the beam is defined only to about ±20 keV. If, however, a pronounced point of the excitation function (e.g. a high maximum) is used as a reference point, relative energy variations of several hundred keV are reproducible within a few keV. The energy spread of the beam (FWHM) was determined to be ≤3 keV.[28]

IV. EXPERIMENTAL PROCEDURE AND RESULTS

The experiment was performed in several steps. First, an excitation function of the reaction ^{27}Al$(p,\alpha)^{24}$Mg was

[27] C. Mahaux and H. A. Weidenmüller, Phys. Letters 23, 100 (1966).

[28] J. Ernst, thesis, Heidelberg, 1965 (unpublished).

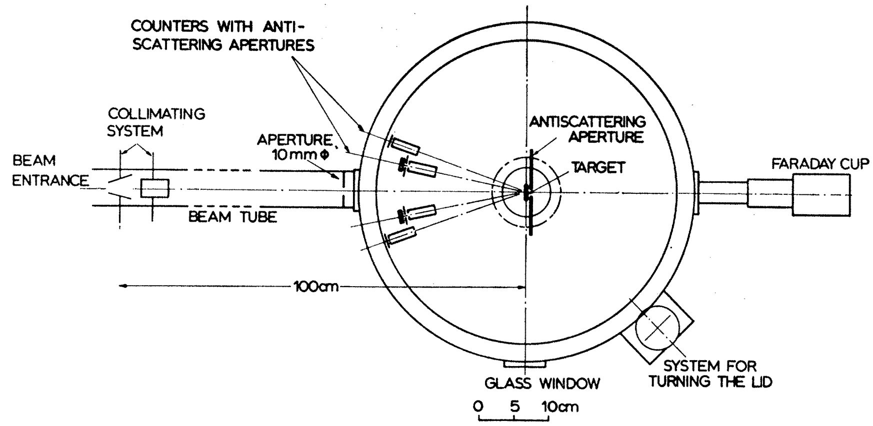

FIG. 1. Schematic sketch of the scattering chamber. The collimating system shown on the left side has not been used in the experiments in order to reduce the background in the spectra.

measured in 25-keV steps from 9 to 12 MeV proton energies at a laboratory angle of 170°. The cross section as a function of energy shows strong fluctuations with a typical coherence width Γ of about 45 keV (Fig. 2), as was already known from other experiments.[22] A very pronounced maximum was found in the excitation function at E_p around 10.30 MeV and a deep minimum nearby at about 10.55 MeV. This maximum proved to

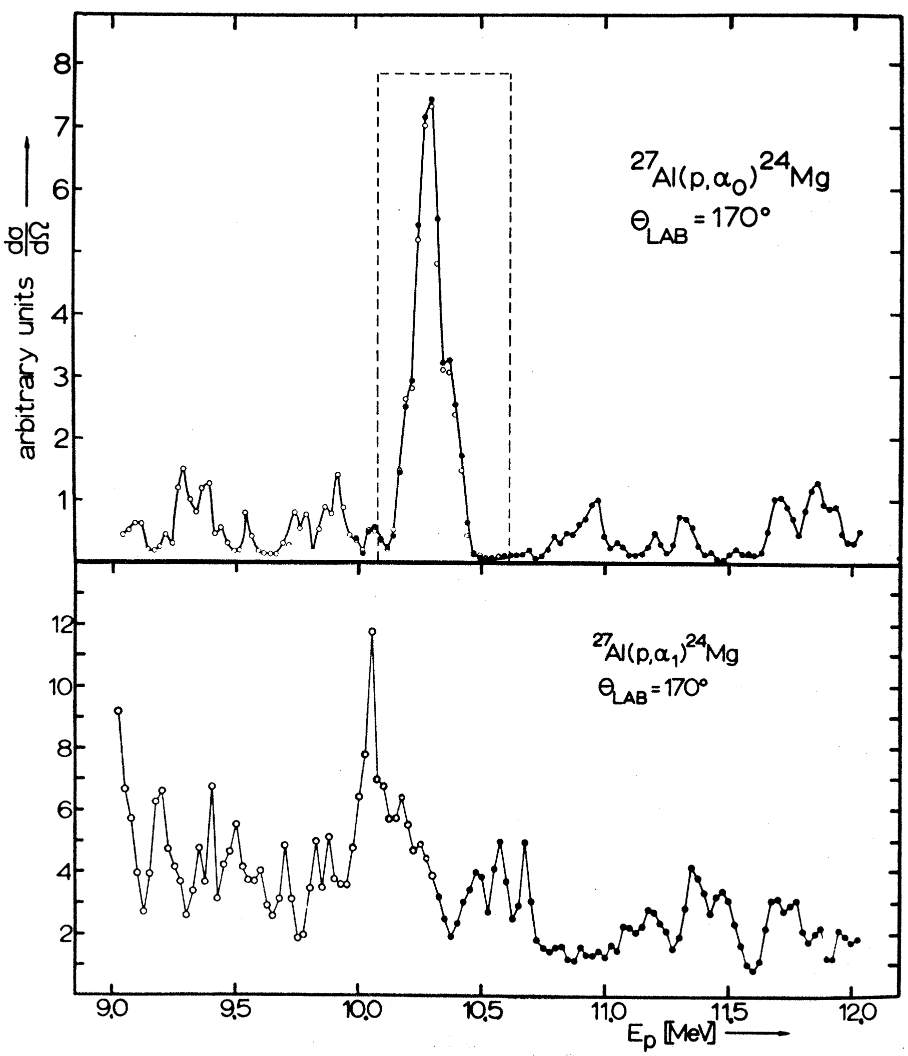

FIG. 2. Excitation functions of the reactions ^{27}Al$(p,\alpha_0)^{24}$Mg and ^{27}Al(p,α_1)-^{24}Mg at $\theta_{lab} = 170°$. The dashed line marks the peak which has been measured in detail as shown in Fig. 3. The structures of the α_0 and α_1 transitions are uncorrelated.

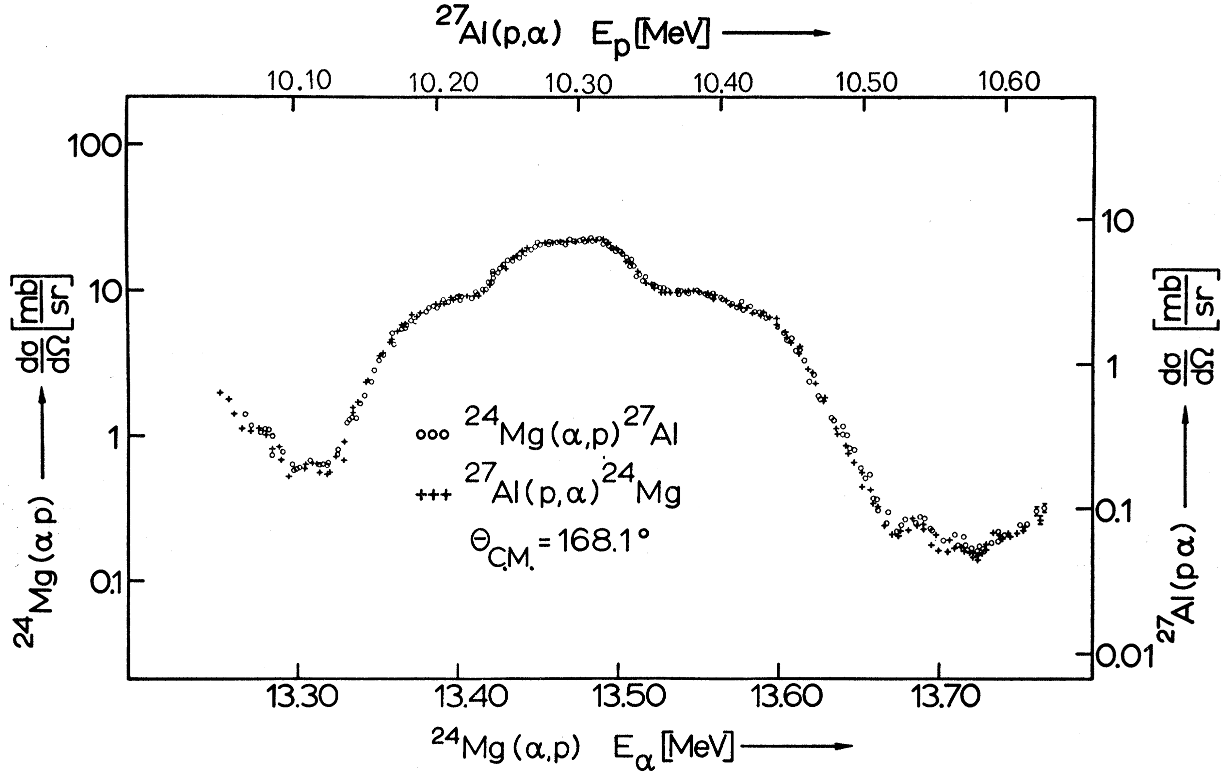

FIG. 3. Excitation functions of the reactions $^{27}Al(p,\alpha_0)^{24}Mg$ (crosses) and $^{24}Mg(\alpha,p_0)^{27}Al$ (circles) on a logarithmic scale. The statistical error is indicated for some points at the high-energy end of the curves. The different measurements have been normalized at the top of the maximum.

be very convenient for the normalization of both the cross sections and the energy scales of the two inverse reactions. This excitation function was also used to determine the average cross section.

Thereafter, both reactions were investigated in small energy steps in the region indicated in Fig. 2 by broken lines. In Fig. 3, the maximum and the adjoining minimum are shown on a logarithmic scale, the cross sections of the two reactions being normalized to each other at the maximum. The over-all agreement of the two curves is rather good and shows that detailed balance holds at least to a good approximation in the statistical reactions considered. The same result has been obtained for direct reactions by Bodansky *et al.*[11] Since it seems almost impossible to measure every point of the whole curve in Fig. 3 with sufficient accuracy, it was decided to measure only the top of the maximum, the minimum at E_p around 10.55 MeV, and a further small peak at E_p around 11.32 MeV with good counting statistics and to compare the relative cross sections of the two inverse reactions in the minimum and in the second maximum. Furthermore, the scattering angle of 172.6° in the c.m. system was chosen in the final measurement in order to approximate as closely as possible the case of only a single spin channel contributing to the reactions. It can be shown with the help of a Hauser-Feshbach calculation[29] that at this angle the contribution of additional spin channels is only about 10%.

The two reactions were investigated in the following manner: First, the high maximum was measured, then the minimum and the second maximum; finally, the measurements were repeated in reverse order to reproduce the data. Since the counting rates in the high maximum turned out to be the same at the beginning and at the end of the run, it was concluded that the targets were not dissipated. Beam currents of typically 500 nA were used in the (p,α) experiment and 150–250 nA in the (α,p) measurements.

Figures 4 and 5 show two typical spectra taken at the minimum where the cross section is about 25 μb/sr in the (p,α) and about 75 μb/sr in the (α,p) reaction. The background in both cases is very low and a well-defined peak is visible.

In Fig. 6, the top of the high maximum is shown where the cross sections of the two inverse reactions have been normalized to one another. The full points and the open circles represent the $^{24}Mg(\alpha,p)$ measurements, the full dots showing the reproduction of the open circles; the error bars indicate the statistical errors.

[29] W. von Witsch, P. von Brentano, T. Mayer-Kuckuk, and A. Richter, Nucl. Phys. **80**, 394 (1966); this reference uses Feshbach's formalism quoted in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 665.

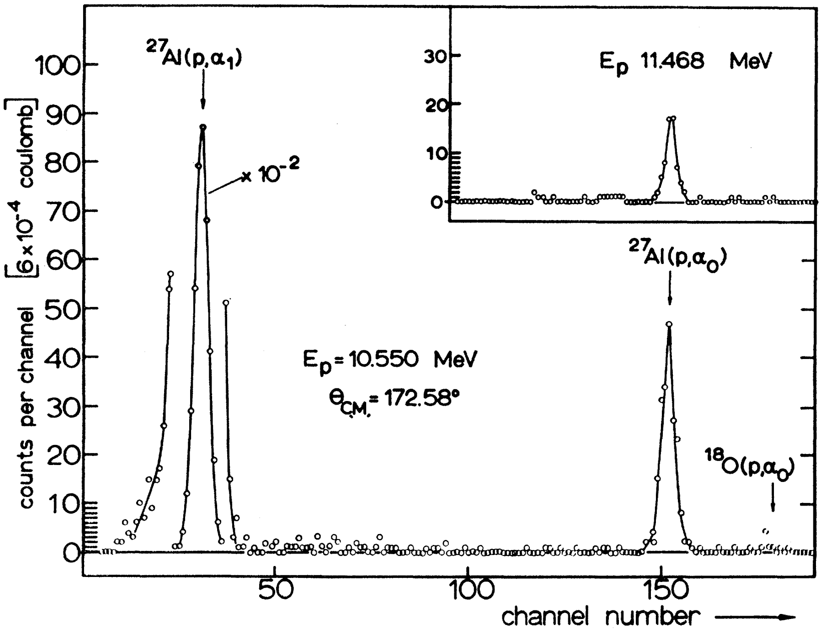

FIG. 4. Spectrum of the reaction $^{27}Al(p,\alpha)^{24}Mg$ at a c.m. angle of 172.6 deg, taken in the minimum of the excitation function at $E_p=10.55$ MeV. The insert in the upper right corner shows (in the same scale) part of a spectrum taken at a second minimum at $E_p=11.486$ MeV which, however, has not been used for the comparison of the relative differential cross sections.

Since the energy resolution in the entrance channel of the (α,p) reaction due to target thickness and beam spread is about 12 keV, but only 2 keV in the (p,α) reaction, a corresponding energy resolution of 12 keV was simulated in the (p,α) reaction by averaging afterwards over each six (p,α) points measured in 2-keV steps. The solid line in Fig. 6 gives the averaged (p,α) cross section.

Figures 7 and 8 show the relative cross section in the minimum and in the second maximum after normalization at the high maximum. The circles and dots again stand for the (α,p) and the solid lines represent the (p,α) reaction averaged over 12 keV. The relative cross sections of the two inverse reactions agree very well within the experimental uncertainties of 1.39 and 0.53%, respectively, which will be discussed in detail in Sec. V. The absolute cross sections agree within about 3%. (A very precise comparison of absolute cross sections was recently made by Thornton *et al.*[30])

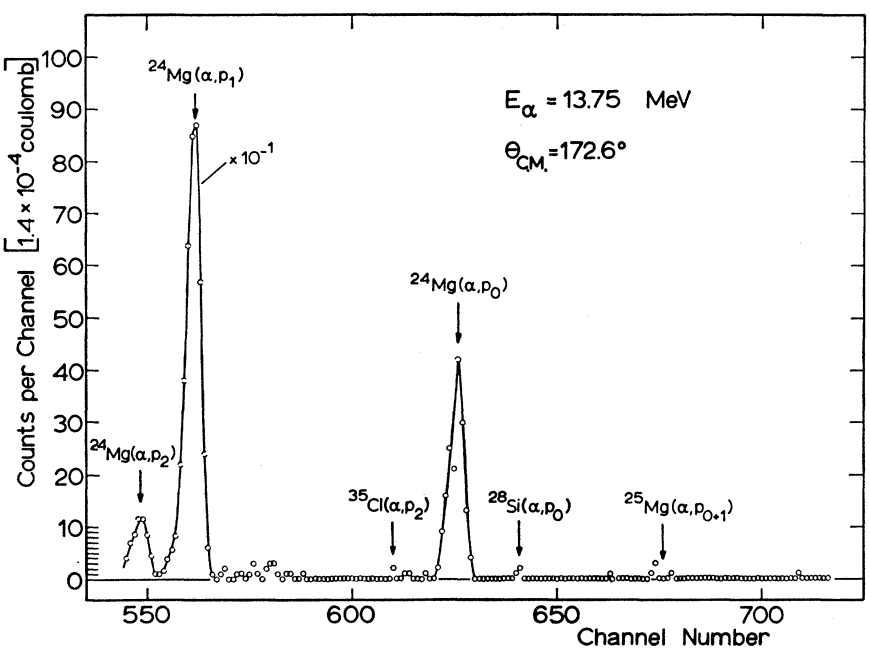

FIG. 5. Spectrum of the reaction $^{24}Mg(\alpha,p)^{27}Al$ at a c.m. angle of 172.6 deg, taken in the minimum of the excitation functions at $E=13.75$ MeV.

[30] S. T. Thornton, C. M. Jones, J. K. Bair, M. D. Mancusi, and H. B. Willard, Oak Ridge National Laboratory Report No. ORNL-4082, 1967, p. 2 (unpublished).

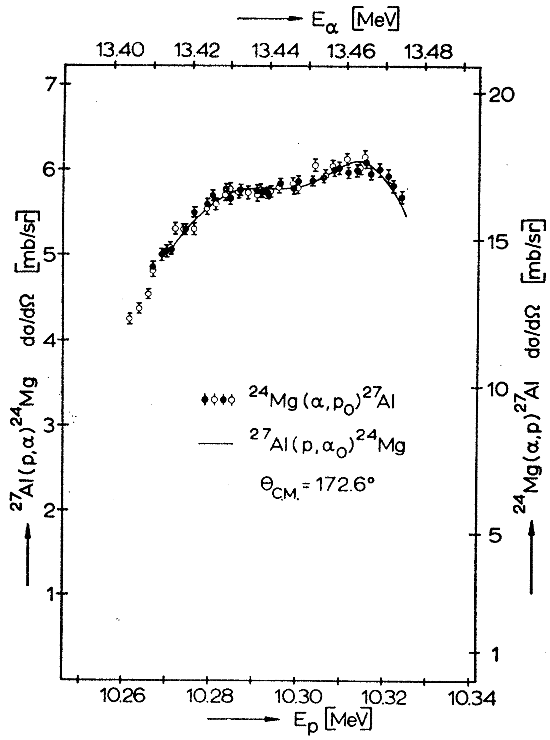

FIG. 6. Excitation functions of the reactions ^{24}Mg$(\alpha,p_0)^{27}$Al (circles and dots) and ^{27}Al$(p,\alpha_0)^{24}$Mg (full line) at $\theta_{c.m.}=172.6°$ on top of the maximum where the differential cross sections of the two inverse reactions have been normalized. The full dots show the reproduction of the open circles. The line represents the (p,α) cross section averaged over 12 keV. Not all available experimental points are shown. The error bars indicate the statistical errors.

V. DISCUSSION

Since only relative cross sections are compared in the two reactions, many experimental uncertainties become negligibly small, e.g. counter efficiency, exact target thickness, solid angle, and the absolute determination of the collected charge. The remaining errors will be discussed below.

Counting statistics. In order to normalize the two excitation functions to each other, only the points lying on the flat plateau (at E_α around 13.44 MeV and E_p around 10.29 MeV) were used, so that the result is insensitive to a small energy shift of each point. The uncertainty in the normalization factor arising from counting statistics of all the points within the plateau is 0.27%. In the minimum, only the points in the flat region 10.54 MeV$\leq E_p \leq$10.56 MeV and 13.715 MeV$\leq E_\alpha \leq$13.74 MeV, respectively, were used for the comparison of the two inverse cross sections. The statistical error of these points is 0.80% in the (α,p) and 0.74% in the (p,α) reaction.

In the second maximum, which lies more than 1 MeV away from the high maximum, the energy scales of the two reactions have been renormalized using the steep low-energy slopes of the peaks, while the points on top and on the high-energy sides of the peaks were used for the comparison of the relative cross sections. The statistical errors are 0.35% in the (α,p) and 0.19% in the (p,α) case.

Background corrections. The high (α,p_1) and (p,α_1) peaks, respectively, were used to determine the proper boundaries for the small (α,p_0) and (p,α_0) peaks in the minimum. Extensive examination of reaction kinematics ensured that no lines from target impurities were lying under the peaks of the reactions being studied. In the (α,p) case, the background was also estimated by bombarding a carbon foil without magnesium evaporated onto it. The uncertainty due to background corrections could finally be reduced to 0.5% both in the (α,p) and in the (p,α) reaction. For the maxima, background corrections were negligibly small.

Scattering angle. Since two counters placed symmetrically to 180° were used, no very exact alignment of the symmetry axis of the scattering chamber with respect to the beam was necessary. Similarly, the effect of a wandering of the beam spot on the target is automatically corrected for. It was estimated from the angular distributions, which are shown for the minimum and for the high maximum in Figs. 9 and 10, that the uncertainty in the relative cross sections caused by these effects is less than 0.3% in the minimum and negligibly small in the second maximum. A small deviation in the relative position of the two counters does not influence the result since their relative positions remained un-

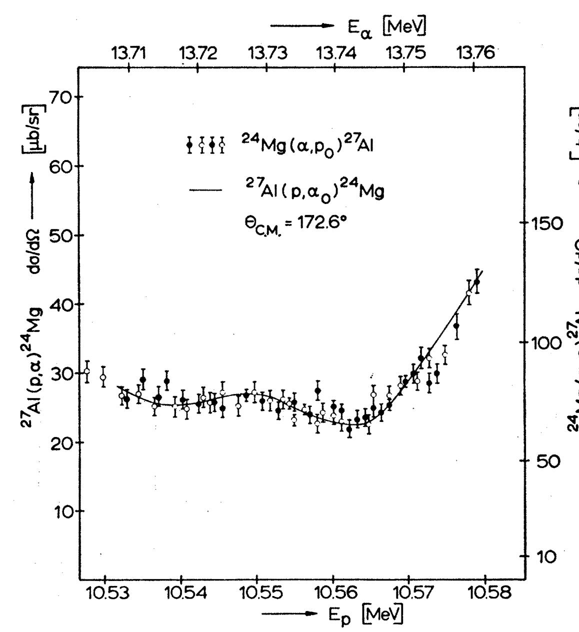

FIG. 7. Part of the excitation functions of the reactions ^{24}Mg$(\alpha,p_0)^{27}$Al (circles and dots) and ^{27}Al$(p,\alpha_0)^{24}$Mg (line) taken at $\theta_{c.m.}=172.6°$ in the minimum. The full dots show the reproduction of the curve given by the circles. The line represents the averaged (p,α) cross section as explained in the text. The two excitation functions have been normalized at the maximum (Fig. 6). The error bars indicate the statistical errors.

changed in both experiments. The uncertainty of ± 0.25 mm connected with the position of the target relative to the counters induces an error of 0.2% in the minimum only.

Target thickness. As mentioned in Sec. III, the thickness of the Mn target could be determined within an uncertainty of $\pm 5\%$. Through the averaging procedure mentioned above, this error causes an uncertainty in the averaged (p,α) cross section which is 0.18% in the minimum and 0.2% in the second maximum.

Bremsstrahlung corrections. It has been shown[31] that bremsstrahlung, which is produced inside the nucleus when either the α particle or the proton strikes the target, and which gives rise to a downward shift of the bombarding particle energy, leads to a correction in the cross sections of the order of 10^{-3}. This correction has therefore been neglected.

All these errors are treated as standard deviations and allow a maximum deviation of the relative cross sections of the two inverse reactions of 1.39% in the minimum and 0.53% in the second maximum. These results are an improvement of the ones given in Ref. 14.

With the help of formula (10), an upper limit for the T-nonconserving part of the reaction amplitude can be derived from these experimentally found deviations. Since the cross section in the minimum is a factor of 31 below the average cross section, an enhancement factor $\sqrt{\nu}=5.57$ arises. In the second maximum, $\sqrt{\nu}$ is only 1.13, but this disadvantage is partially cancelled by better counting statistics and by negligible background problems. Putting $a=0.5$ and $b=0.5$ and using formula

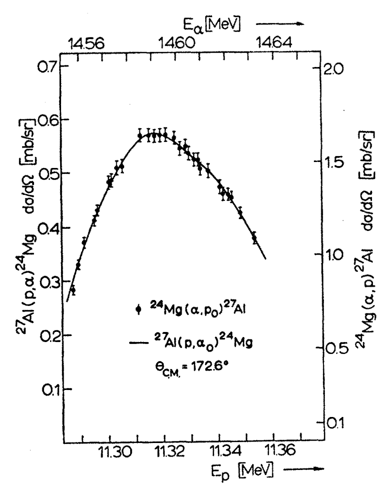

FIG. 8. Excitation functions of the reactions $^{24}Mg(\alpha,p_0)^{27}Al$ (dots) and $^{27}Al(p,\alpha_0)^{24}Mg$ (line) taken at $\theta_{c.m.}=172.6°$ in the second maximum. The line again represents the (p,α) cross section averaged over 12 keV. The two excitation functions have been normalized at the high maximum and the energy scales of the two reactions have been renormalized using the steep low-energy slopes of the peaks. The error bars indicate the statistical uncertainties. Not all the experimental points are shown.

[31] D. A. Robertson (private communication).

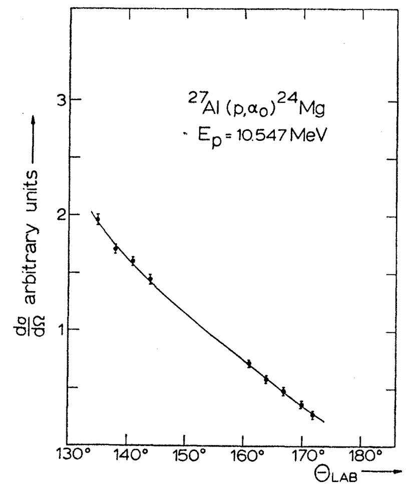

FIG. 9. Angular distribution of the reaction $^{27}Al(p,\alpha_0)^{24}Mg$ taken at $E_p=10.547$ MeV in the minimum. The error bars show the statistical errors.

(10), one finds

$$\xi \leq 3\times 10^{-3} \quad (85\% \text{ confidence}),$$
$$\xi \leq 2\times 10^{-3} \quad (60\% \text{ confidence}),$$

the latter result being obtained if only the minimum is used for the comparison. As mentioned in Sec. II, these confidence limits stem only from the fact that the above results are interpreted as upper limits for the *average* strength of the (fluctuating) T-nonconserving part of the reaction amplitude.

If the nuclear matrix elements are considered instead of the reaction amplitude, the intrinsic enhancement factor $(W/\Gamma)^{1/2}$ must be taken into account.[24] According to Mahaux and Weidenmüller,[27] this factor is equal to or just slightly greater than unity, so that the

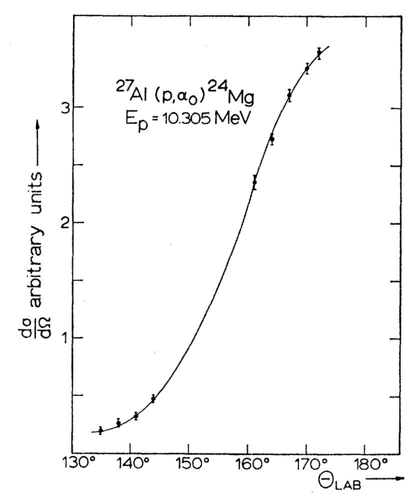

FIG. 10. Angular distribution of the reaction $^{27}Al(p,\alpha_0)^{24}Mg$ taken at $E_p=10.305$ MeV on top of the high maximum. The errors represent the statistical uncertainties.

limits given above for the amplitudes would also represent upper limits for the relative strength X of the matrix elements of the nuclear Hamiltonian H' which is odd with respect to T. Taking, however, Ericson's estimate in Eq. (11), $(W/\Gamma)^{1/2}$ is about 7 and the upper limit for the relative strength of the T-odd part of the nuclear Hamiltonian becomes

$$X \leq 4\times10^{-4}.$$

The actual value of X might lie somewhere between 4×10^{-4} and 3×10^{-3}.

VI. CONCLUSION

In the preceding sections a test of detailed balance using the reactions ^{24}Mg$(\alpha,p)^{27}$Al and ^{27}Al$(p,\alpha)^{24}$Mg was described. The experimental results are compatible with no violation of T invariance and an upper limit of 3×10^{-3} with 85% confidence has been derived for the possible amount of the T nonconserving fraction of the reaction amplitude. A tentative upper limit for the T-odd part of the nuclear Hamiltonian is also given.

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APPENDIX A

If the spins of the incoming particle and of the target nucleus are denoted by i and I, the angular momentum by l, and the corresponding values in the exit channel by i', I' and l', and if one chooses the z axis in the direction of the incoming particles, then one gets for the reaction ^{27}Al$(p,\alpha)^{24}$Mg

$$i=\tfrac{1}{2},\quad I=\tfrac{5}{2},\quad \mu=\pm\tfrac{1}{2},\quad M=\pm\tfrac{1}{2},\pm\tfrac{3}{2},\pm\tfrac{5}{2},\quad l_z=0,$$
$$i'=I'=\mu'=M'=l_z'=0,$$

where μ, μ', M, and M' are defined in Sec. II. Conservation of angular momentum demands for the z components the following relation:

$$\mu+M=\mu'+M'=0.$$

This means that either $\mu=+\frac{1}{2}$ and $M=-\frac{1}{2}$ or $\mu=-\frac{1}{2}$ and $M=+\frac{1}{2}$. Therefore, only $\beta=(+\frac{1}{2},-\frac{1}{2};0,0)$ or $\beta=(-\frac{1}{2},+\frac{1}{2};0,0)$ are possible combinations. From invariance under parity transformation it follows that $\sigma_\beta=\sigma_{-\beta}$, and one gets immediately $\sigma=2\sigma_\beta$. The same holds, of course, for the inverse reaction.

APPENDIX B

If the nuclear Hamiltonian is written $H=H_0+H'$, where H_0 denotes the T-conserving and H' the T-nonconserving part of $H(H_0\gg H')$, then the S matrix for a compound reaction in the continuum region may be written as[27,32]

$$S_{l,m}=-i\sum_{\lambda}\frac{\Gamma_{l\lambda}{}^{1/2}\Gamma_{\lambda m}{}^{1/2}}{E-\mathcal{E}_\lambda}-i\sum_{\mu,\nu}\frac{\Gamma_{l\mu}{}^{1/2}H_{\mu\nu}{}'\Gamma_{\nu m}{}^{1/2}}{(E-\mathcal{E}_\mu)(E-\mathcal{E}_\nu)},\quad(12)$$

where the first term stands for the T-conserving and the second one for the T-nonconserving part of S. Since, according to the statistical theory,[19] the $\Gamma^{1/2}$'s are uncorrelated (this means that the expectation value $\langle\Gamma_\lambda{}^{1/2*}\Gamma_\mu{}^{1/2}\rangle=0$ for $\lambda\neq\mu$), the correlation between the two terms in Eq. (12) vanishes also because $H_{\mu\nu}{}'=-H_{\nu\mu}{}'$. The above-considered S matrix is easily transformed into the amplitudes f and f', which then are also independent.

[32] H. A. Weidenmüller and K. Dietrich, Nucl. Phys. **83**, 332 (1966).