Nuclear Magnetic Moments of Cu^{60} and Cu^{62}: hfs Experiment and Configuration-Mixing Calculation*

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The hyperfine structures of 24-min Cu^{60} and 10-min Cu^{62} have been measured by the atomic-beam magnetic-resonance method. Our results are: Cu^{60} ($I=2$), $\Delta\nu=6033(4)$ Mc/sec, $\mu_I=+1.219(3)$ nm (diamagnetically corrected); Cu^{62} ($I=1$), $\Delta\nu=-2257.2(5)$ Mc/sec, $\mu_I=-0.380(4)$ nm. The spins were previously known. Configuration-mixing calculations in the $1f_{5/2}$-$2p$ shell reproduce fairly well the spectra and binding energies of Cu^{58}, Cu^{60}, and Cu^{62}, as well as these moments.

I. INTRODUCTION

NUCLEAR spins and magnetic moments are known for many copper isotopes (Table I). With one proton past a closed shell and the clear existence of regularities, they are a suitable place for the confrontation of theory and experiment.

The spins of Cu^{60} and Cu^{62} have been previously measured.[1,2] In Secs. II and III we report on measurements of their magnetic moments. The odd-odd Cu isotopes in Table I divide into two groups: $A=60$ and $A\geq 62$. According to the simple shell model, this is the result of filling the $2p_{3/2}$ neutron level at $A=61$; further neutrons must go into the $1f_{5/2}$ level. We report in Secs. IV and V a calculation which, although limited to $A=58$, 60, and 62 for reasons of time, reproduces their properties. In some details the calculated wave functions differ markedly from the simple shell-model description.

II. EXPERIMENTAL METHOD

The hyperfine structure (hfs) of an atom with $J=\frac{1}{2}$ is given by the Breit-Rabi formula[3]:

$$W(F=I\pm 1/2, M) = -\frac{\Delta\nu}{2(2I+1)} - \frac{g_I\mu_0 H}{h}M \pm \tfrac{1}{2}\Delta\nu\left(1+\frac{4Mx}{2I+1}+x^2\right)^{1/2}, \quad (1)$$

where $x=(-g_J+g_I)\mu_0 H/h\Delta\nu$. A plot of the term values $W(F,M)$ versus magnetic field H for Cu^{60}($I=2$, $\Delta\nu=+6033$ Mc/sec) and Cu^{62}($I=1$, $\Delta\nu=-2257$ Mc/sec) is shown in Fig. 1. The arrows indicate the transitions whose resonant frequencies were observed in this experiment.

The sign of $\Delta\nu$, the hfs separation at zero field, is not often measured. Inspection of (1) shows that although changing the sign of $\Delta\nu$ changes the values $W(F,M)$ it does not alter the ensemble of observed transition frequencies $|W_i-W_j|$. Most experiments seek instead to find the sign of g_I, which does affect the transition frequencies. The Fermi-Segrè formula[3] asserts that the ratio $\Delta\nu/(2I+1)g_I$ is constant for a given atomic configuration; for a single s electron this ratio can be calculated without serious uncertainty[3] and is positive. Thus, for copper, we may quote a $\Delta\nu$ with the same sign as the nuclear moment.

The atomic beam machine and detector arrangement have been described previously.[4] Atoms which have undergone a resonant transition in the C magnet are deflected by the B magnet to the "flop" collector, while those which have not are deflected to the "beam" collector. The clean iron collectors are removed from the machine and their β activity counted. The ratio of "flop" counts to "beam" counts (F/B) is used to determine the resonances. In this experiment, ordinary nickel powder was placed in an oven (Fig. 2) and heated to ~1400°C by electron bombardment while exposed

TABLE I. Measured ground-state hfs of copper isotopes.

Isotope	I	μ_{uncorr}	$\Delta\nu$	g_J
Cu^{60} (24 min)[a]	2[b]	+1.216(3)[c]	6033(4)	
Cu^{61} (3.3 h)[d]	$\frac{3}{2}$	+2.12(4)[c]	11225(200)	
Cu^{62} (9.7 min)[a]	1[d]	−0.379(4)[c]	−2257.2(5)	
Cu^{63}[e]	$\frac{3}{2}$	+2.2175(2)	11733.817	−2.00226(4)[f]
Cu^{64} (12.4 h)[d]	1[g]	−0.215(2)[c]	−1282.140(10)	−2.00228(2)[h]
Cu^{65}[e]	$\frac{3}{2}$	+2.3755(3)	12568.780	
Cu^{66} (5.2 min)[i]	1	−0.280(3)[c]	−1668.784(15)	

[a] Present work.
[b] Reference 1.
[c] Computed by us from the Fermi-Segrè formula. The errors are adjusted for possible hfs anomaly.
[d] Reference 2.
[e] The hfs of stable Cu is well known. These accurate values of μ and $\Delta\nu$ are taken from H. Figger, D. Schmitt, and S. Penselin, Colloq. Intern. Centre Natl. Rech. Sci. (Paris) **164**, 355 (1967); Z. Physik (to be published).
[f] L. Goodman (private communication).
[g] A. Lemonick and F. M. Pipkin, Phys. Rev. **95**, 1356 (1954).
[h] Value adopted in the present work.
[i] Ref. 19; C. S. Cussens, G. K. Rochester, and K. F. Smith (private communcation).

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[1] J. B. Reynolds, R. L. Christensen, D. R. Hamilton, W. M. Hooke, and H. H. Stroke, Phys. Rev. **109**, 465 (1958).
[2] B. M. Dodsworth and H. A. Shugart, Phys. Rev. **142**, 638 (1966).
[3] N. F. Ramsey, *Molecular Beams* (Oxford University Press, London, 1956). This book contains a more detailed discussion of hfs theory.
[4] E. A. Phillips, O. Ames, and S. S. Glickstein, Phys. Rev. **138**, B773 (1965). As mentioned here, the A and B magnets are now at 60% of full magnetization.

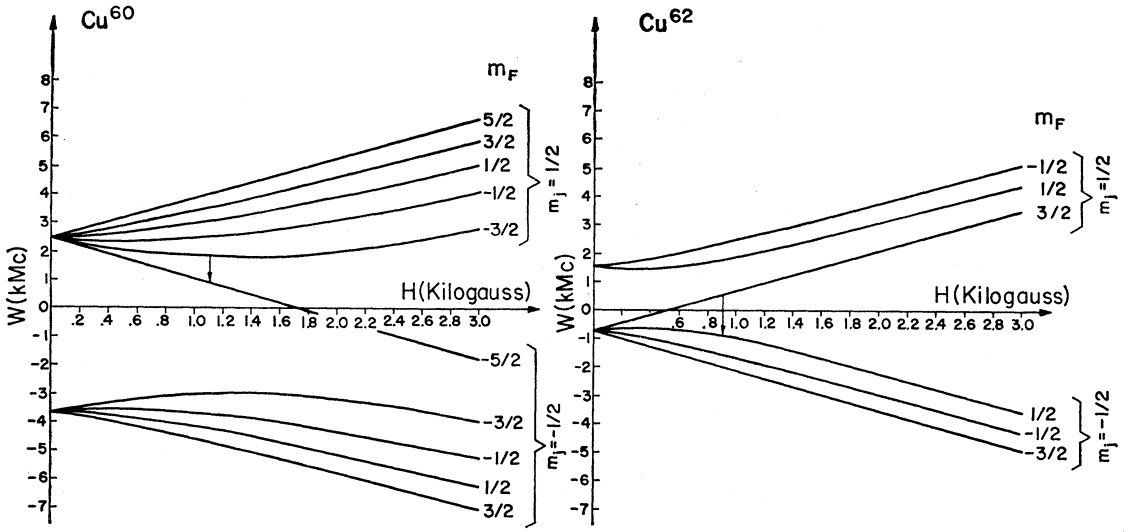

FIG. 1. The energy levels of Cu^{60} ($I=2$ and $J=\frac{1}{2}$) and Cu^{62} ($I=1$ and $J=\frac{1}{2}$) in a magnetic field.

to the external cyclotron beam, 0.3 μA of 18-MeV protons. The inner part of the oven was made of boron nitride, which we have found very useful for containing substances that alloy with metal ovens.

Of the five copper isotopes that can be produced by Ni(p,n)Cu, only Cu^{60} and Cu^{62} were observed in the beam. The others had less favorable half-lives, cross sections, and/or isotopic abundances. Although the natural abundance of Ni^{60} is seven times that of Ni^{62}, the observed resonance heights for Cu^{60} and Cu^{62} were nearly equal. (See Fig. 4). Investigation showed that Cu^{62} has a higher production cross section as well as a lower nuclear spin and a half-life more appropriate to the 8-min bombarding and counting times. The (F/B) ratio off-resonance was disturbingly high. We suggest that this is due to the state $(\frac{5}{2}, -\frac{3}{2})$ of Cu^{60}, which has $(dW/dH)\sim 0$ at 1300 G (Fig. 1). Such a state cannot be properly deflected by the B magnet, since the field on the beam side of the magnet is only 1000 G.[4] The height of the Cu^{60} resonance is then correspondingly reduced.

III. EXPERIMENTAL RESULTS

Decay curves of activated nickel showed a 25-min half-life with some curvature consistent with a 10-min component. Figure 3 plots the decay of "flop" and "beam" collectors taken at the spin-1 and spin-2 resonances. Since half the counting rate of the "flop" collector is present even off resonance, we interpret the curved decay of the spin-1 flop as a mixture of 10-min Cu^{62} from the resonance and 24-min Cu^{60} from the machine background (see above). The spin-2 flop contains only Cu^{60}. Thus we confirm the previous assignments[1,2] of spin 1 to Cu^{62} and spin 2 to Cu^{60}. A search showed no resonance at any other spin, integral or half-integral, from 0 to 4.

The one-quantum $\Delta F=0$ resonances (the arrows in Fig. 1) for both isotopes were observed at fields up to 252 G. The resonances at the highest field are shown in Fig. 4 and the data are summarized in Tables II and III. These tables also show the results of least-squares

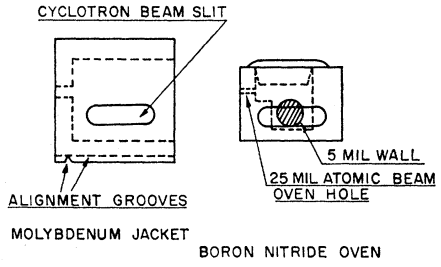

FIG. 2. Atomic-beam oven assembly.

TABLE II. Summary of resonance data on Cu^{60} at magnetic fields between 24 and 252 G, as reduced by HYPERFINE-4A.

Result of least-squares fit:
$a=2413.1(13)$ Mc/sec, $\Delta\nu=6032.9(32)$ Mc/sec; $\mu=+1.216$; $\chi^2=0.3$ (moment positive); $\chi^2=105$ (moment negative).
Comparison isotope[a]:
Cu^{63}, $^2S_{1/2}$, $I=\frac{3}{2}$, $g_J=-2.00228(2)$, $\mu_I=+2.2175$, $\Delta\nu=11733.82$ Mc/sec.
Calibration isotope[a]:
K^{39}, $^2S_{1/2}$, $I=\frac{3}{2}$, $g_J=-2.002295$, $\mu_I=+0.3909$, $\Delta\nu=461.7197$ Mc/sec.

K^{39} Calibration frequency (Mc/sec)	Cu^{60} frequency (Mc/sec)	F_1, M_1, F_2, M_2	Residual and probable error (kc/sec)
18.965(2)	13.685(10)		+2(10)
28.180(2)	19.463(10)		−1(10)
56.620(2)	34.881(6)	$\frac{5}{2}$-$\frac{3}{2}$ $\frac{5}{2}$-$\frac{5}{2}$	−1(6)
56.450(10)	34.800(10)		+2(11)
147.904(3)	71.658(6)		+2(6)
432.104(7)	155.145(8)		−1(8)

[a] Reference 2.

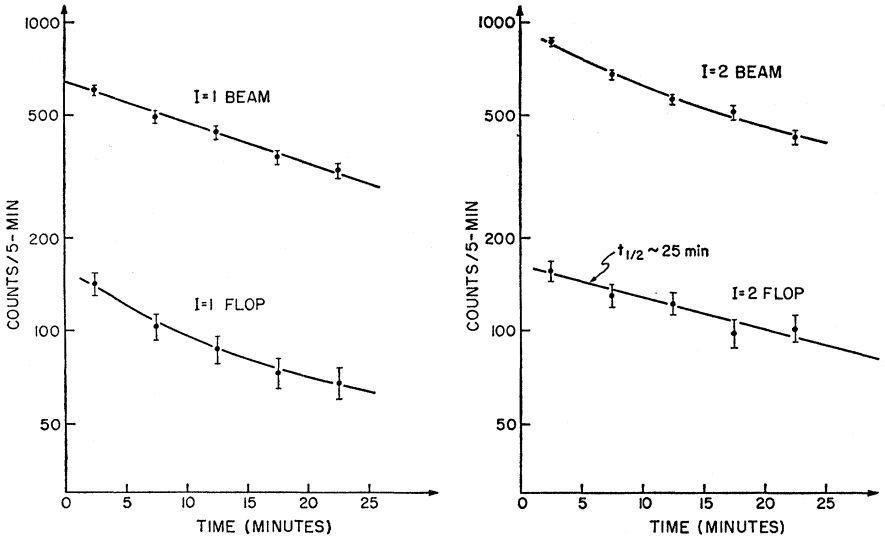

FIG. 3. Decay of flop and beam activity for Cu^{60} and Cu^{62} resonances.

fits to the data by the computer program HYPERFINE-4A. As a check on the consistency of the data, another fit was performed assuming g_J to be unknown. The values we obtain are $g_J(Cu^{60}) = -2.00236(25)$, $g_J(Cu^{62}) = -2.00230(19)$, in agreement with more accurate numbers (Table II).

Since 250 G is an unprecedented field for this machine, a check experiment was run on K^{41}. The result was $\Delta\nu = 254.008(3)$ Mc/sec, about two standard deviations away from the accepted value.[5] On the chance that this discrepancy is a real systematic error, we assign the following final numbers:

$$\begin{aligned} Cu^{60}: \quad & a = 2413.1(16) \text{ Mc/sec}; \\ & \Delta\nu = 6033(4) \text{ Mc/sec}, \\ Cu^{62}: \quad & a = -1504.8(3) \text{ Mc/sec}; \\ & \Delta\nu = -2257.2(5) \text{ Mc/sec}. \end{aligned}$$

TABLE III. Summary of resonance data on Cu^{62} at magnetic fields between 16 and 252 G, as reduced by HYPERFINE-4A.
Result of least-squares fit:
$a = -1504.78(24)$ Mc/sec, $\Delta\nu = -2257.17(36)$ Mc/sec; $\mu = -0.379$; $\chi^2 = 2.0$ (moment negative); $\chi^2 = 23$ (moment positive).
Comparison and calibration isotopes: See Table II.

K^{39} Calibration frequency (Mc/sec)	Cu^{62} frequency (Mc/sec)	F_1, M_1, F_2, M_2[a]	Residual and probable error (kc/sec)
12.580(1)	15.761(5)		+3(5)
22.336(2)	26.788(5)		+6(5)
43.186(2)	47.729(8)		−6(8)
74.065(5)	74.275(20)	$\frac{3}{2}\ \frac{3}{2}\ \frac{3}{2}\ \frac{1}{2}$	+5(20)
74.420(2)	74.550(8)		−2(8)
154.060(2)	130.160(12)		−0(12)
432.101(5)	288.446(8)		+0(8)

[a] Our sign convention differs from that of Ref. 2.

[5] A. L. Bloom and J. B. Carr, Phys. Rev. 119, 1946 (1960).

The quoted errors in hfs reflect the computed least-squares values, and should be considered as standard deviations.

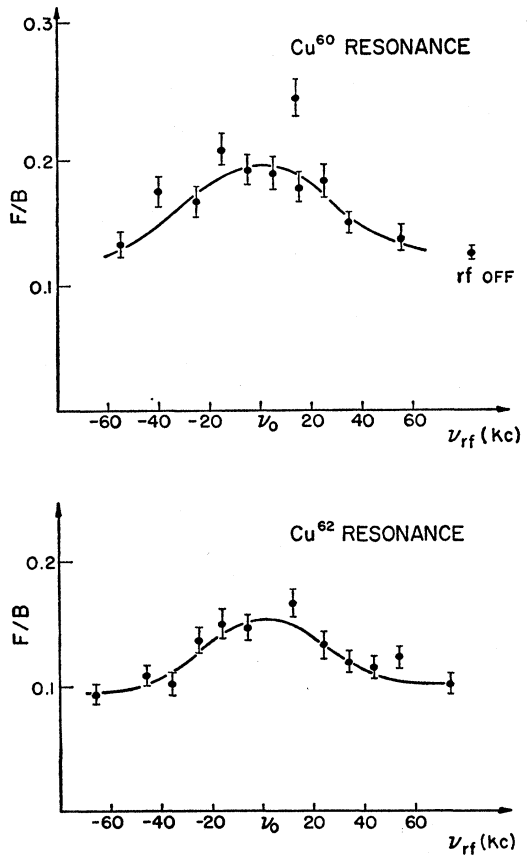

FIG. 4. Cu^{60} and Cu^{62} resonances at 252 G. The frequencies ν_0 are about 155 and 288 Mc/sec, respectively.

The nuclear moments of these isotopes may be computed from their hfs and the Fermi-Segrè formula.[3] Such a computation is incorrect to the extent of the hfs anomaly, which, however, can be computed from the nuclear wave function.[6] Calculations in the spirit of Stroke *et al.*,[6] using the wave functions in Sec. IV, give an anomaly of about 0.5% for Cu^{62} and one-tenth as much for Cu^{60}, referred to Cu^{63}. Adding double these numbers to our errors and correcting for diamagnetism, we obtain the values

$$Cu^{60}: \quad \mu_I = +1.219(3),$$
$$Cu^{62}: \quad \mu_I = -0.380(4).$$

IV. THEORY: ENERGY LEVELS OF $Cu^{58,60,62}$

The success of Auerbach[7] and of Lawson, MacFarlane, and Kuo[8] in describing the nickel isotopes with an effective interaction between configurations involving only the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ single-particle levels led us to undertake an extensive set of similar calculations for even-A copper isotopes. The method is to compute the Hamiltonian matrix between these configurations and diagonalize it to find energy levels and wave functions. We have followed the nickel calculations[7,8] in assuming the three single-particle levels to be at 0, 0.78, and 1.08 MeV, respectively. Since the interactions between many-particle configurations can be reduced by well-known techniques to appropriate sums of two-body interactions, we only need values for the two-body matrix elements within the f-p shell to be able to compute the Hamiltonian.

The values of the $T=1$ two-body matrix elements have been discussed in connection with the calculations on nickel isotopes.[8] The $T=0$ two-body matrix elements do not occur in the nickel calculations. We have used $T=0$ matrix elements of the Hamada-Johnston potential, calculated by Kuo.[9] They are sums of three components: The bare interaction uncorrected for polarizations of the Ni^{56} core, corrections due to the excitation of a single core nucleon to an empty level, and corrections due to the promotion of a pair of nucleons either from the $1f_{7/2}$ level to one of the three valence levels or from a valence level to the $1g_{9/2}$ level. Harmonic-oscillator radial wave functions for $\hbar\omega = 10.0$ MeV were used. A complete discussion of the calculation of these Hamada-Johnston matrix elements can be found in Ref. 10.

While the calculated bare interaction has been estimated to be within 15% of the real bare interaction,[10] it is very difficult to estimate the accuracy of the perturbative corrections or the importance of higher-order terms which have been neglected. In this region the perturbative corrections are more important for $T=1$ Hamada-Johnston matrix elements than for those with $T=0$, the average ratio of correction to bare interaction being about 100% for $T=1$ and only 15% for $T=0$.[9] Since much of the error in the final values is anticipated to come from these and other corrections, it is reasonable to expect that the $T=0$ calculated matrix elements will be closer to reality than their $T=1$ counterparts.

In fact, the renormalized Hamada-Johnston $T=1$ two-body matrix elements do not provide a satisfactory description of the energy levels of the nickel isotopes.[8] They tend to overestimate the level spacings and binding energies, the latter by as much as 5 MeV. Similar results were obtained when these matrix elements were used in calculating copper spectra. Although it is more appealing and fundamental to use a realistic force, we decided to use $T=1$ effective interactions instead.

We have arbitrarily adopted the Auerbach effective interaction.[7] The average deviation between the Auerbach matrix elements and those of Lawson *et al.*[8] is small (0.18 MeV), and the results of this calculation are essentially unchanged when the Lawson numbers are used. Auerbach's method was to find 12 parameters: The matrix elements of the type $\langle j^2 J=0 | V | j'^2 J=0\rangle$, where j and j' are any of the three single-particle levels, and the "centers of gravity" of the possible states formed by coupling two particles in two valence levels, $\sum_J (2J+1)\langle jj'J|V|jj'J\rangle/(2j+1)(2j'+1)$. These parameters determine the energies of the 21 states in Ni^{58-66} which may be considered as seniority 0 or 1 (0_1^+, 0_2^+ for even A; $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{1}{2}^-$ for odd A.) A least-squares best fit to the data gave a rms deviation of 0.11 MeV and uncertainties of 10 to 20% in the parameters. Auerbach calculated the remaining off-diagonal matrix elements from the Kallio-Kolltveit potential and picked the remaining diagonal matrix elements to preserve the "centers of gravity" and provide a good fit to the seniority 0, 2, and 4 states in Ni^{58}, Ni^{60}, and Ni^{64}.

Our calculations were carried out in a basis set which was properly antisymmetrized with respect to the interchange of any two neutrons, but not of good isotopic spin. To simplify the problem, only configurations of seniority 0 and 2 were included. We justify this truncation by noting, with Auerbach,[7] that the lowest 0^+ state in the nickel calculations never contains more than 2% of seniority-4 components and the lowest 2^+ state never more than 0.5%. Thus we feel that restricting the seniority will not seriously upset the calculated low-energy copper spectrum.

Calculated matrices, ranging in size from 8×8 to 40×40, were diagonalized by computer to yield the spectra and eigenfunctions of Cu^{58}, Cu^{60}, and Cu^{62}. The

[6] H. H. Stroke, R. J. Blin-Stoyle, and V. Jaccarino, Phys. Rev. **123**, 1326 (1961).
[7] N. Auerbach, Phys. Letters **21**, 57 (1966); Nucl. Phys. **76**, 321 (1966).
[8] R. Lawson, N. MacFarlane, and T. T. S. Kuo, Phys. Letters **22**, 168 (1966).
[9] T. T. S. Kuo (private communication).
[10] T. T. S. Kuo and G. E. Brown, Nucl. Phys. **85**, 40 (1966); T. T. S. Kuo, Nucl. Phys. **A90**, 199 (1967).

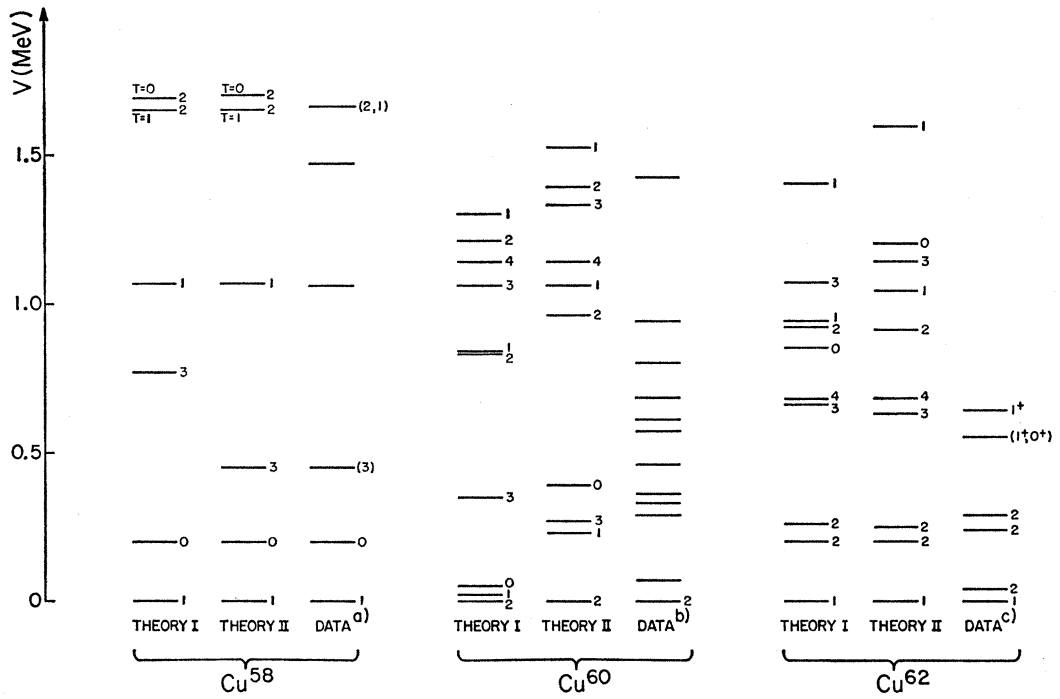

FIG. 5. Comparison of calculated and experimental spectra for Cu^{58}, Cu^{60}, and Cu^{62}; (a) Ref. 11, (b) Ref. 12, (c) Ref. 13.

TABLE IV. Ground-state wave functions for Cu^{58}, Cu^{60}, and Cu^{62}.[a,b]

$$\psi(Cu^{58}\ I=1)=0.524[3_p3_n]-0.344[3_p5_n]-0.486[3_p1_n]+0.344[5_p3_n]+0.068[5_p5_n]+0.486[1_p3_n]+0.109[1_p1_n].$$

$$\psi(Cu^{60}\ I=2)=0.803[3_p3_n^3]+0.226[3_p3_n5_n^2(0)]+0.182[3_p3_n1_n^2(0)]+0.101[3_p5_n3_n^2(0)]+0.345[3_p1_n3_n^2(0)]+0.155[3_p1_n5_n^2(0)]$$
$$+0.069[5_p3_n^3]+0.069[1_p3_n^3]+0.103[1_p5_n3_n^2(0)]+0.212[\{5_p5_n(0)\}3_n^2(2)]+0.053[\{5_p5_n(0)\}3_n1_n]$$
$$+0.188[\{1_p1_n(0)\}3_n^2(2)].$$

$$\psi(Cu^{62}\ I=1)=0.244[3_p3_n^35_n^2(0)]+0.503[3_p3_n^31_n^2(0)]+0.132[3_p3_n5_n^4(0)]+0.203[3_p3_n5_n^2(0)1_n^2(0)]-0.193[3_p5_n3_n^4(0)]$$
$$-0.107[3_p5_n^3(5/2)3_n^2(0)]-0.121[3_p5_n3_n^2(0)1_n^2(0)]-0.050[3_p5_n^3(5/2)1_n^2(0)]-0.581[3_p1_n3_n^4(0)]$$
$$-0.356[3_p1_n3_n^2(0)5_n^2(0)]-0.135[3_p1_n5_n^4(0)]+0.053[5_p3_n^35_n^2(0)]+0.063[5_p3_n^31_n^2(0)]+0.104[1_p3_n^35_n^2(0)]$$
$$+0.085[1_p3_n5_n^4(0)]+0.055[1_p3_n5_n^2(0)1_n^2(0)]+0.189[1_p1_n3_n^4(0)]+0.095[1_p1_n3_n^2(0)5_n^2(0)].$$

[a] All components of the Cu^{58} wave function are reported. Cu^{60} and Cu^{62} wave functions as tabulated represent 99% of the calculated wave functions. Components with amplitude <0.05 have not been included.
[b] In this table we employ the abbreviations $2p_{3/2}\equiv 3$, $1f_{5/2}\equiv 5$, and $2p_{1/2}\equiv 1$.

spectra (Theory I) are compared with experiment[11–13] in Fig. 5, and the ground-state wave functions are given in Table IV.

The absolute energies of the copper ground states can be compared with experiment by extracting the interaction between the valence particles (V) from the measured binding energies.[14] If we assume that the binding energy of each valence particle to the nickel core is given by the Ni^{57}–Ni^{56} binding energy difference of 10.23 MeV,[14] we get

$$V(A)=-[\text{B.E.}(Cu^A)-\text{B.E.}(Ni^{56})-10.23(A-56)+\Delta E_c(A)],$$

where $\Delta E_c(A)$ is the Coulomb energy difference between Cu^A and Ni^A, as given by Sherr *et al.*[15] We compare calculated and observed values of $V(A)$ in Table V.

With only two nucleons outside the Ni^{56} closed shell, Cu^{58} provides a test of the matrix elements and the restriction to the f-p shell. The agreement with the experimental spectrum is quite satisfactory, as is the

TABLE V. Comparison of calculated valence nucleon interactions and magnetic moments with experiment.

	$V_{\text{Theory I}}$ (MeV)	$V_{\text{Theory II}}$ (MeV)	$V_{\text{Expt.}}$ (MeV)	$\mu_{\text{Theory I}}$ (nm)	$\mu_{\text{Theory II}}$ (nm)	$\mu_{\text{Expt.}}$ (nm)
Cu^{58}	−2.29	−2.29	−2.16	+0.78	+0.78	···
Cu^{60}	−3.64	−3.97	−4.44	+1.64	+1.56	+1.22
Cu^{62}	−3.74	−4.09	−4.50	−0.26	−0.25	−0.38

[11] R. Sherr and J. A. Nolen (private communication).
[12] R. G. Miller and R. W. Kavanagh, Nucl. Phys. **A94**, 261 (1967).
[13] S. Antman, H. Petterson, and A. Suarez, Nucl. Phys. **A94**, 289 (1967).
[14] L. Konig, J. Mattauch, and A. Wapstra, Nucl. Phys. **67**, 1 (1965).
[15] R. Sherr, A. G. Blair, and D. D. Armstrong, Phys. Letters **20**, 392 (1966).

fact that the calculated V is within 0.13 MeV (6%) of the measured value.

In the cases of Cu^{60} and Cu^{62} we have ignored possible configurations of seniority 4 and 6. The calculated spectra are in general agreement with experiment; in particular, the ground-state spins are reproduced. The valence nucleon interactions V are underestimated by about 0.8 MeV (20%). Inclusion of the high-seniority components which we have suppressed would necessarily increase the ground state V and improve the agreement between experiment and theory. It would also tend to increase the density of calculated levels.

It is interesting to see how sensitive the results are to variations in the two-body matrix elements. To this end the matrix element $\langle p_{3/2}^2\ T=0\ J=3\ |V|\ p_{3/2}^2\ T=0\ J=3\rangle$ was altered by -0.32 MeV (25%), which gives a better fit to the energy of the 3+ state in Cu^{58}. The results of this variation of one matrix element are reported as "Theory II" in Fig. 5 and Table V. Small changes occur in the spectra of Cu^{60} and Cu^{62}, and their calculated binding energies were improved by about 0.35 MeV. We do not claim that this adjustment has any meaning beyond an indication of the effect of reasonable variations in the parameters.

V. THEORY: MAGNETIC MOMENTS OF Cu^{58}, Cu^{60}, AND Cu^{62}

The nuclear magnetic moments of the ground states of these copper isotopes can be calculated from the above wave functions. To do this we use empirical values for the moments of single particles in the f-p shell. This approach is well justified when the moment of the odd-odd nucleus (Z,N) is computed from odd-group moments of the same configurations observed in $(Z, N-1)$ and $(Z-1, N)$.[16] Then essentially all of the corrections to the moment of the odd-odd nucleus appear also in the odd-group moments.[17] Such observations are not all available for the copper isotopes, and we have been forced to rely on other nearby moments. It should be noted that the $1f_{5/2}$ moments do not affect the final answers very much, and that by experiment and theory $p_{1/2}$ moments are relatively independent of A. The empirical values we have chosen are in Table VI.

There are also nondiagonal contributions to the magnetic moment of a mixed wave function, which occur between components differing only by the promotion of one particle from one member of a spin-orbit doublet to the other.[18] These are proportional to the quantity (g_l-g_s) for the particle involved, which we assume to have its free-space value.

TABLE VI. Empirical odd-group magnetic moments and their sources.[a]

	μ_p (nm)	μ_n (nm)
$2p_{3/2}$	+2.16 [Cu^{61}]	−0.75 [Ni^{61}]
$1f_{5/2}$	+1.38 [$Rb^{83,85}$]	+0.35 [Ni^{61}]
$2p_{1/2}$	−0.15 [$Y^{89,91}$]	+0.55 [Ge^{71}]

[a] *Nuclear Data Sheets*, compiled by K. Way *et al.* (U. S. Government Printing Office, National Academy of Sciences—National Research Council, Washington, D. C., 1965), Appendix 1.

In any such semiempirical calculation, it is important to avoid counting the same effect twice, once empirically and once theoretically. It has been shown by Caine[16] that the nondiagonal corrections to the odd-group moments in an odd-odd nucleus (which we have elected to describe empirically) must be of seniority 4 if the nuclei $(Z, N-1)$ and $(Z-1, N)$ are of seniority 1. Auerbach has indicated that odd-A nickel ground states contain less than 4% of seniority-3 components. Thus the nondiagonal contributions from our mixed wave functions of seniority 2 do not represent double counting. Since Cu^{58} is supposed to be a two-particle nucleus, its odd groups are unique and there can be no question of double counting in its moment.

The magnetic moment of 3.3-sec Cu^{58} has not been measured. Our predicted value, +0.78 nm, is somewhat different from the lowest-order odd-group estimate $(p_{3/2}p_{3/2}J=1$, +0.47 nm).

For Cu^{60} the calculated and measured moments differ by 0.42 nm. This is not unreasonable in the absence of the empirical moments of Cu^{59} and Ni^{59}. We note that the lowest-order odd-group estimate $(p_{3/2}p_{3/2}J=2$, +0.94 nm) has been altered in the right direction, mostly by an off-diagonal term of +0.64 nm.

In the case of Cu^{62} we have used the empirical moments of Cu^{61} and Ni^{61} and so anticipate better results. The agreement to 0.13 nm is quite satisfying. For the similar isotopes Cu^{64} and Cu^{66} various odd-group configurations have been suggested[2,19–21]: $p_{3/2}p_{3/2}$, +0.47 nm; $p_{3/2}f_{5/2}$, −0.83 nm; $p_{3/2}p_{1/2}$, +1.52 nm. The negative moment has been assumed to favor the $p_{3/2}f_{5/2}$ assignment, although Kisslinger and Sorenson[21] reject this possibility because it implies a hindrance to the β decay, which is not seen. Eighty-five percent of our wave function is $p_{3/2}p_{3/2}$ or $p_{3/2}p_{1/2}$; these configurations contribute a diagonal magnetic moment of +0.95 nm, which is cancelled by the −0.92 nm off-diagonal term

[16] C. A. Caine, Proc. Phys. Soc. (London) **A69**, 635 (1956).

[17] The use of empirical single-particle moments involves a certain error in these corrections, since they depend on the occupation numbers of all shells (Ref. 18). Our case is unusual in that we can estimate this error. Comparison of our wave functions for $Cu^{60,62}$ with those of Auerbach (Ref. 7) for $Ni^{59,61}$ reveals a systematic shift of neutron population from the $f_{5/2}$ shell to the p shells. We have estimated the effect of this on the corrections, using the techniques of Ref. 18, and it changes by about 30% the differences between calculated and measured moments given in the text.

[18] H. Noya, A. Arima, and H. Horie, Progr. Theoret. Phys. (Kyoto) Suppl. 8, 33 (1958).

[19] G. K. Rochester and K. F. Smith, Phys. Letters 8, 266 (1964).

[20] M. H. Brennan and A. M. Bernstein, Phys. Rev. **120**, 927 (1960).

[21] L. S. Kisslinger and R. A. Sorenson, Rev. Mod. Phys. **35**, 853 (1963).

between them. The other 15% of the wave function adds to give the calculated moment of −0.26 nm. It appears that a calculation on this isotope must be as complex as ours to be meaningful.

A much less detailed study of the Cu^{64} and Cu^{66} ground-state wave functions suggests that they may be well approximated by the addition of two and four $f_{5/2}$ neutrons, respectively, to the Cu^{62} wave function. Neutrons added in this way cause little change in the Cu^{62} magnetic moment. The moments calculated for Cu^{64} and Cu^{66} in this approximation will both be about −0.2 nm, in agreement with experiment.

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Test of Time-Reversal Invariance through the Reactions $^{24}Mg+\alpha \rightleftharpoons {}^{27}Al+p$

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Time-reversal invariance has been tested via detailed balance in the compound nuclear reactions $^{24}Mg+\alpha \rightleftharpoons {}^{27}Al+p$. The relative differential cross sections agree within the experimental uncertainties. An upper limit for the ratio of the T-nonconserving to the T-conserving part of the reaction amplitude has been found to be $(2\text{–}3)\times10^{-3}$. A model-dependent upper limit between 4×10^{-4} and 3×10^{-3} has been derived for the relative strength of the T-odd part of the nuclear Hamiltonian.

I. INTRODUCTION

AFTER the discovery of parity nonconservation in weak interactions[1] in 1957 the question arose as to whether time-reversal (T) invariance might also be violated,[2] and by 1964 several experiments had been performed to test T invariance both in weak and in strong interactions. Various polarization experiments[3] and reciprocity tests[4] were the means by which upper limits of typically a few percent were found for the possible extent of a T-odd part of the amplitude in these experiments. Similar upper limits were derived by γ-γ angular-correlation experiments on oriented nuclei at low energies where the electromagnetic interaction was investigated.[5] An experimental test of T invariance in weak interactions has been reported by Burgy *et al.*[6] The interest in the question of T invariance was stimulated once more in 1964 when Christenson *et al.*[7] detected CP nonconservation of about 2×10^{-3} through the decay of the long-lived state of the K^0 meson into two pions, which, on the basis of the CPT theorem, implies that T invariance must also be violated. There is the possibility of a connection between CP nonconservation in the decay of the K_2^0 meson and T violations in nuclear interactions, since Bernstein, Feinberg, and Lee[8] and independently Prentki and Veltman[9] proposed that the forbidden mode of the K_2^0 decay might be due to an interference between a weak time-reversal-even interaction and a much stronger time-reversal-odd one. Since then, great effort has been made to reduce the experimental uncertainties in the experiments mentioned above. Recently, a polarization experiment in p-p scattering[10] yielded a T-nonconserving amplitude of less than 0.5% of the T-conserving one. Bodansky *et al.*[11] have been able to test relative cross sections in a detailed balance experiment with an accuracy of 0.3%, and for electromagnetic interaction studies using the Mössbauer

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[1] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, Phys. Rev. **105**, 1413 (1957).
[2] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).
[3] P. Hillman, A. Johansson, and G. Tibell, Phys. Rev. **110**, 1218 (1958); A. Abashian and E. M. Hafner, Phys. Rev. Letters **1**, 255 (1958); C. F. Hwang, T. R. Ophel, E. H. Thorndike, and R. Wilson, Phys. Rev. **119**, 352 (1960); D. G. McDonald, W. Haeberli, and L. W. Morrow, *ibid.* **133**, B1178 (1964).
[4] L. Rosen and J. E. Brolley, Jr., Phys. Rev. Letters **2**, 98 (1959); D. Bodansky, S. F. Eccles, G. W. Farwell, M. E. Rickey, and P. C. Robinson, *ibid.* **2**, 101 (1959).
[5] E. Fuschini, V. Gadjokov, C. Maroni, and P. Veronesi, Nuovo Cimento **33**, 709 (1964); **33**, 1309 (1964).
[6] M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Rings, and V. L. Telegdi, Phys. Rev. Letters **1**, 324 (1958).
[7] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).
[8] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).
[9] J. Prentki and M. Veltman, Phys. Letters **15**, 88 (1965).
[10] R. Handler, S. C. Wright, L. Pondrom, P. Limon, S. Olsen, and P. Kloeppel, Phys. Rev. Letters **19**, 933 (1967).
[11] D. Bodansky, W. J. Braithwaite, D. C. Shreve, D. W. Storm, and W. G. Weitkamp, Phys. Rev. Letters **17**, 589 (1966); W. G. Weitkamp, D. W. Storm, D. C. Shreve, W. J. Braithwaite, and D. Bodansky, Phys. Rev. **165**, 1233 (1968).