Proton-Proton Final-State Interactions in the Reactions ^{3}He$(d,t)2p$, $d(^3$He$,t)2p$, $p(^3$He$,d)2p$, and ^{3}He$(^3$He$,^4$He$)2p$†

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The energy spectra of tritons, α particles, and deuterons from the reactions ^{3}He$(d,t)2p$, $d(^3$He$,t)2p$, ^{3}He$(^3$He$,^4$He$)2p$, and $p(^3$He$,d)2p$ were measured at forward angles 0° to 5°, using 36-MeV deuterons and 53-MeV ^{3}He from the Oak Ridge isochronous cyclotron. The reaction $d(^3$He$,t)2p$ at 0° was also studied at 74 MeV. All spectra had peaks in the high-energy region which are attributed to the enhancement of the cross section due to the final-state interaction of the two protons. The final-state interaction theory of Watson and Migdat gives an adequate representation of the data for the reactions ^{3}He$(d,t)2p$ and ^{3}He$(^3$He$,^4$He$)2p$ for the known proton-proton interaction. For the other two reactions, $d(^3$He$,t)2p$ and $p(^3$He$,d)2p$, the observed peaks are much narrower than the predictions of the Watson-Migdal theory. In the case of $d(^3$He$,t)2p$ the narrower peak could be explained by assuming the primary reaction mechanism to be a charge-exchange process. A search for a three-proton enhancement in the reaction ^{3}He$(^3$He$,t)3p$ was unsuccessful.

I. INTRODUCTION

Few-nucleon reactions are investigated for many reasons: to determine energy levels of very light nuclei, to elucidate the reaction mechanism, and to examine interactions between two particles in the final state. Our purpose in this experiment has been to investigate the two-proton final-state interaction from four different processes, to see in particular how unambiguously one can extract a scattering length from these reactions, and what effect the reaction mechanism has on the information one can obtain concerning final-state interactions. To anticipate our conclusions at this point, we find that indeed the reaction mechanism must be carefully considered in interpreting the data, and that scattering lengths deduced from final-state interactions must be extracted with great care, and that the applicability of any theory to particular experimental conditions must be examined critically. A method of treating three-body final states has been developed by Watson[1] and by Migdal.[2] The reaction is separated into two steps, a primary mechanism responsible for the three-body breakup followed by the final-state interaction. If the primary mechanism is known or if one assumes that it does not affect the final energy distributions, the final-state interaction provides an indirect means of investigating the scattering interaction of the reaction products. The theory as developed by Watson and by Migdal does not depend upon the details of the primary reaction mechanism except in stipulating that the interaction causing the transition must be short-range.

The Watson-Migdal theory has been applied, sometimes successfully, sometimes not, to reactions resulting in three-body final states involving either a neutron pair or a proton pair. Recently,[3] the ^{3}He spectra at lab angles of 6° and 8° from the reaction $t(d,^3$He$)2n$ for deuteron energies of 32.5 and 40 MeV were analyzed using the Watson-Migdal theory which gave the value for the neutron-neutron scattering length $a_n=-16.1\pm1.0$ F. The application of the Watson-Migdal theory was justified from the results of an analysis of the triton spectrum at a lab angle of 8° from the reaction ^{3}He$(d,t)2p$ for the same c.m. energy.[3] With this theory the latter reaction gave a proton-proton scattering length $a_p=-7.41_{-0.49}^{+0.39}$, which agrees with the results of low-energy proton-proton scattering.[4] The value of a_n from this experiment was in agreement with results from π^- capture by deuterium,[5] where a value of $a_n=-16.4\pm1.3$ F was found. Both of these values are in agreement with the calculations of Heller, Signell, and Yoder,[6] based on the charge symmetry of nuclear forces and low-energy p-p scattering results. In disagreement with the above results, similar analyses[7,8] of the forward proton spectra from the deuteron breakup reaction $d(n,p)2n$ for an incident neutron energy of 14 MeV gave $a_n=-21.2$ and $-23.6_{-1.6}^{+2.0}$ F. A disagreement with the predictions of the Watson-Migdal theory for a two-proton final state was also reported[9] for the reaction ^{3}He$(p,d)2p$ at a proton lab energy of 12 MeV. The width of the high-energy deuteron peak at 10° lab was found to be much narrower than predicted by the theory.

Other calculations have been made for three-body

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[9] T. A. Tombrello and A. D. Bacher, Phys. Letters **17**, 37 (1965).

nuclear reactions which include the effects of final-state interactions. An analysis[10] of the breakup of deuterons by protons and neutrons treated the final-state interaction between the neutron and proton with a Born approximation using zero-range potentials. A similar method was used[11] to analyze the experimental data on the breakup of deuterium by 8.9-MeV protons.[12] Calculations using more realistic potentials were made by Koehler and Mann[13] and by Koehler.[14] Recently, Henley, Richards, and Yu,[15] using various Born approximations, found it essential to include final-state interaction effects to explain the data of Jakobson, Manley, and Stokes[16] on the $d(^3\text{He},t)2p$ reaction at 21 MeV and the $^3\text{He}(d,t)2p$ reaction at 11.7 and 14.0 MeV. For their analysis, tritons were assumed to be a result of either a neutron pickup or of a charge exchange process. Also, it has been suggested by Phillips[17] that the final-state-enhancement peak can be drastically different from the Watson-Migdal prediction when the reaction takes place through a long-range charge-exchange process. The Phillips formulation of the charge-exchange process for the reaction[18] $d(n,p)2n$ yields a value of -14 ± 3 F for the n-n scattering length, which is in better agreement with the expectations of charge symmetry of nuclear forces than previously obtained[7,8] with the Watson-Migdal theory. Aaron and Amado[19] have made calculations for the process $d(n,p)2n$ at 14 MeV using an exact three-body theory with separable two-body potentials. They reproduce the major features of the data and conclude that Watson-Migdal theory is inadequate for this reaction.

In this paper we report studies of four reactions involving two-proton interactions in the final state with the aim of exploring further the limitations of the Watson-Migdal theory. We have previously reported[20] results from an investigation of the reaction $d(^3\text{He},t)2p$ at 21 MeV c.m., at 0° and ~180°, which showed the importance of the primary reaction mechanism on the third-particle spectra. We include these results here both for completeness and because we have analyzed the data further. In addition, we report new triton spectra at 0° from the reaction $d(^3\text{He},t)2p$ at 74 MeV lab and the deuteron and α-particle spectra at 5° lab from the reactions $p(^3\text{He},d)2p$ and $^3\text{He}(^3\text{He},\alpha)2p$ using a 53-MeV ^{3}He beam.

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II. EXPERIMENTAL PROCEDURE AND RESULTS

Final-state interactions in three-body final states are most easily studied by limiting observation to that part of phase space which corresponds to a low relative velocity of two of the particles. This can be accomplished by detecting the third particle at a forward angle and near the maximum allowed energy. To make the measurements at small angles to the beam with the best possible resolution, we found the broad-range magnetic spectrograph at ORIC to be ideal. The arrangement of the experimental equipment is shown in Fig. 1. The spectrograph is of the type described by Borggreen, Elbek, and Nielsen,[21] but scaled up to ORIC energies. The extracted beam from the cyclotron was focused on the entrance slit to a 153° beam analyzing magnet. The energy-analyzed beam emerging from the exit slit was transmitted to the spectrograph scattering chamber by two quadrupoles to achieve a partial cancellation of energy dispersion. The energy spectrum of the observed particles was measured by placing nuclear emulsions along the focal plane of the spectrograph. For the reaction $d(^3\text{He},t)2p$ the target used was a deuterated polyethylene (CD_2) foil prepared by the Isotopes Division of the Oak Ridge National Laboratory. For the other reactions gas targets were used, and for this purpose windows of 0.0001-in. Havar foil were installed 11.75 in. upstream and 6 in. downstream from the center of the spectrograph scattering chamber.

The triton-energy spectrum from the reaction $^3\text{He}(d,t)2p$ at a lab angle of 5° for a beam energy of 36 MeV is shown in Fig. 2. The over-all resolution was about 100 keV FWHM, as deduced from the width of the triton line from the $^{14}\text{N}(d,t)^{13}\text{N}$ reaction to the ground state of ^{13}N on the nitrogen impurity in the gas target.

In Fig. 3 the triton spectra are shown for lab angles of 0° and 3° from the reaction $d(^3\text{He},t)2p$ with a 53-MeV ^{3}He beam. The beam energy was chosen such that the c.m. energy was the same as in the above $^3\text{He}(d,t)2p$ reaction. The 0° resolution was 40 keV FWHM, and for the 3° data the resolution was about 75 keV FWHM. Figure 4 shows the triton spectrum at 0° to a 74-MeV ^{3}He beam from the $d(^3\text{He},t)2p$ reaction with an over-all resolution of 84 keV FWHM.

The α-particle spectrum at 5° lab for the reaction $^3\text{He}(^3\text{He},^4\text{He})2p$ at 53 MeV lab is shown in Fig. 5. The resolution was approximately 200 keV FWHM. Because of the background of deuterons and tritons in the emulsion, there was some uncertainty in particle identification which led to a fluctuation of data points slightly larger than that expected from counting statistics. However, from track density the α particles could be distinguished with a fairly high degree of certainty, and the shape of the final-state-enhancement

[21] J. Borggreen, B. Elbek, and L. P. Nielsen, Nucl. Instr. Methods **24**, 1 (1963).

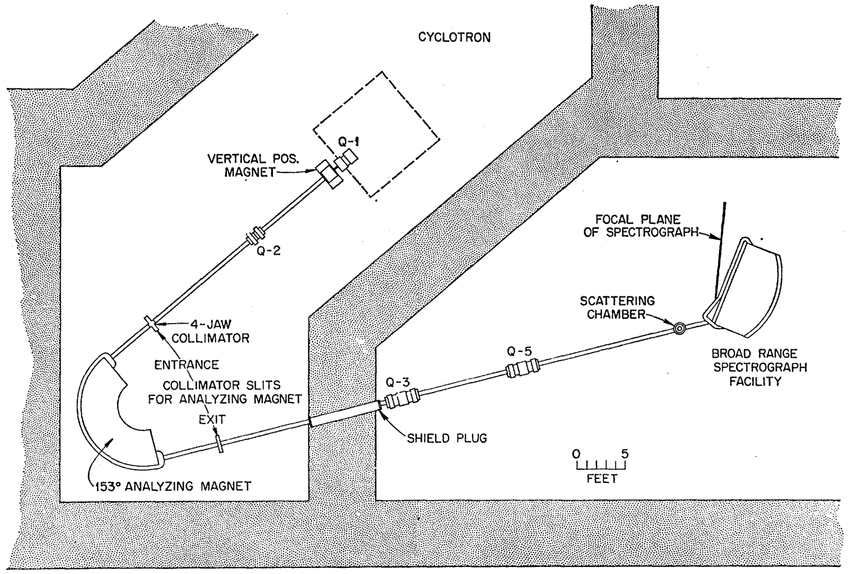

FIG. 1. Beam transport and analysis arrangement for the broad-range spectrograph facility at the Oak Ridge isochronous cyclotron.

peak was well determined. Shown in Fig. 6 is the triton spectrum from the reaction ^{3}He(^{3}He,t)3p which was taken simultaneously with the above-mentioned α-particle spectrum. Only the high-energy region of the spectrum is shown and the cross section decreases smoothly to zero near the upper kinematic limit. There is thus no evidence for three-particle final states.

The deuteron spectrum taken at 5° lab for the reaction p(^{3}He,d)2p at 53 MeV lab is shown in Fig. 7. The resolution for this spectrum was about 260 keV FWHM. The energy spread is due mainly to the kinematic spread, which is large when the projectile is much heavier than the target particle.

III. THEORETICAL ANALYSIS

For completeness and ease in explaining our results, we briefly describe the theories used for our data analysis. We take the characteristic peak near the high-energy kinematic limit of the observed particle to be due to an enhancement of the cross section by the interaction of two protons in the final state.[22] The reaction proceeds in the following way:

$$A+B\rightarrow C+2p, \qquad (1a)$$

$$2p\rightarrow p+p, \qquad (1b)$$

where C is the observed particle and 2p is the unstable diproton. The cross section is given by

$$d^2\sigma/dEd\Omega=(2\pi/v\hbar)|T|^2\rho(E). \qquad (2)$$

Here v is the relative velocity of the two particles in the incident channel and the phase-space factor $\rho(E)$ is proportional to

$$E^{1/2}(E_{\max}-E)^{1/2},$$

where E is the c.m. energy of the observed particle and $E_{\max}$ is the maximum allowed E. The methods used for obtaining the transition amplitude T are discussed in the succeeding paragraphs.

The Watson-Migdal approach assumes that the final-state two-proton interaction is responsible for the energy distribution of the observed particle. In this case the transition amplitude may be written[2]

$$T=T_0e^{-i\delta}\sin\delta/C(\eta)k. \qquad (3)$$

Here T_0 contains all terms independent of the relative momentum k of the two protons. The s-wave phase

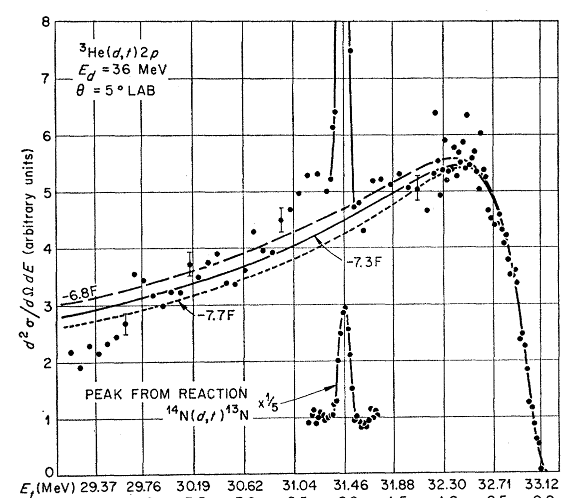

FIG. 2. Energy spectrum of tritons at 5° lab from the reaction ^{3}He(d,t)2p using 36-MeV deuterons. Typical statistical error bars are shown. The smooth curves are predictions of the Watson-Migdal theory for the indicated values of the two-proton scattering length a_p. Scales at the bottom indicate the triton lab energy and the relative energy of the two protons.

[22] W. T. H. van Oers and I. Šlaus, Phys. Rev. **160**, 853 (1967).

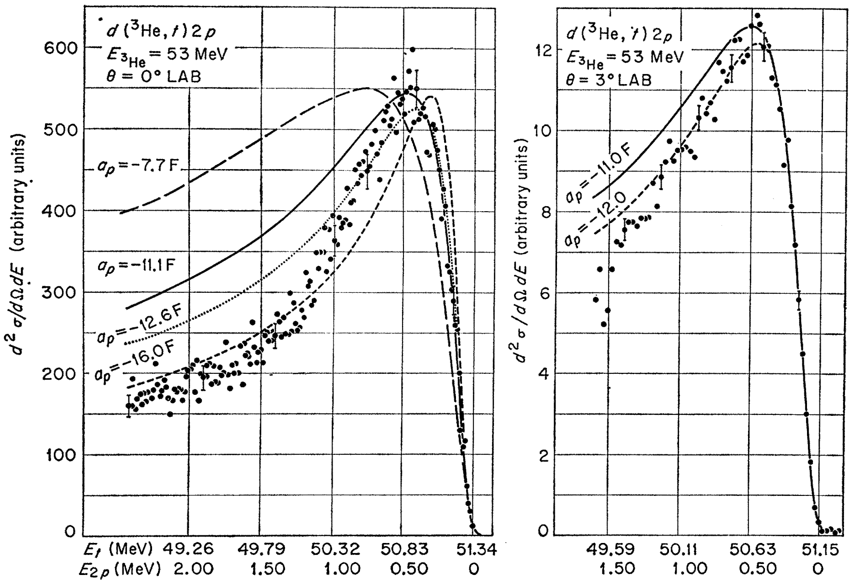

FIG. 3. Energy spectra of tritons at 0° and 3° lab from the reaction $d(^3\mathrm{He},t)2p$, using a 53-MeV ^{3}He beam. Typical statistical error bars are shown. The smooth curves are predictions of the Watson-Migdal theory for the indicated values of a_p.

shift for the two-proton interaction is given by δ, and $C(\eta)$ represents the Coulomb penetration factor:

$$C(\eta)=[2\pi\eta/(e^{2\pi\eta}-1)]^{1/2}, \tag{4}$$

where

$$\eta=e^2/\hbar v_{2p}.$$

By use of the effective-range expansion for the phase shift it can be shown that

$$C^2(\eta)k\cot\delta+H(\eta)/R=-1/a_p+\tfrac{1}{2}r_0k^2+\cdots. \tag{5}$$

The cross section may be written

$$\frac{d^2\sigma}{dEd\Omega}\propto\frac{C^2(\eta)\rho(E)}{C^4(\eta)E_{2p}+(\hbar^2/m_p)[-1/a_p-H(\eta)/R+\gamma_pE_{2p}]^2}, \tag{6}$$

where E_{2p} is the relative two-proton energy, m_p is the mass of the proton, a_p is the scattering length, r_0 is the effective range,

$$\gamma_p=r_0m_p/2\hbar^2,\quad R=28.82,$$

and

$$H(\eta)=\mathrm{Re}[\Gamma'(-i\eta)/\Gamma(-i\eta)]-\ln(\eta).$$

However, if one writes the complete Hamiltonian

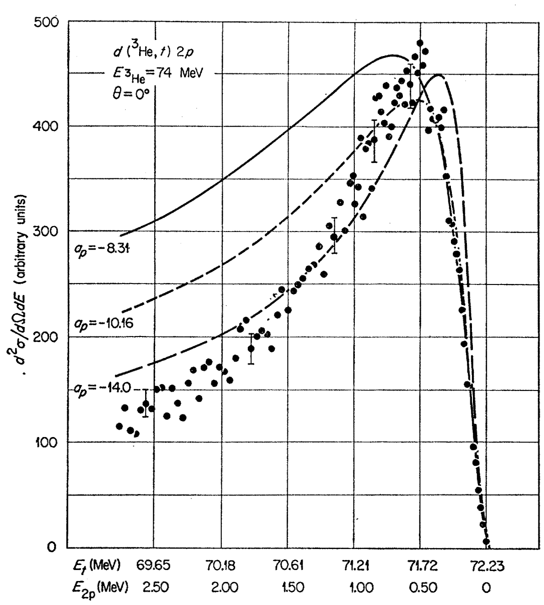

FIG. 4. Energy spectrum of tritons at 0° lab from the reaction $d(^3\mathrm{He},t)2p$, using a 74-MeV ^{3}He beam. Typical statistical error bars are shown. The smooth curves are predictions of the Watson-Migdal theory for the indicated values of a_p.

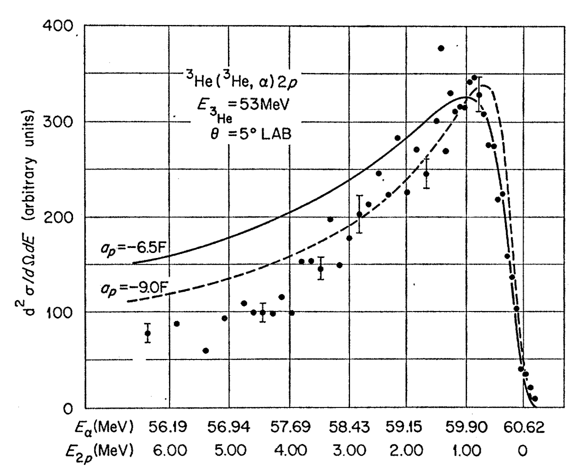

FIG. 5. Energy spectrum of α-particles at 5° lab from the reaction $^3\mathrm{He}(^3\mathrm{He},\alpha)2p$, using a 53-MeV ^{3}He beam. Typical statistical error bars are shown. Smooth curves are predictions of the Watson-Migdal theory for the indicated values of a_p.

describing the process (1) in the form[23]

$$H=H_i+V_i,$$

then the transition amplitude has the form

$$T=\langle\psi_f^{(-)}|V_i|\varphi_i^{+}\rangle. \tag{7}$$

Here H_i is the Hamiltonian describing the system in the initial state. The wave functions are defined by

$$H\psi_f=E\psi_f,\quad H_i\varphi_i=E\varphi_i,$$

with the appropriate boundary conditions. For the following calculations $\psi_f^{(-)}$ will be approximated by a product wave function for the particle C and the diproton system, i.e., $\psi_f^{(-)}=\psi_{2p}\psi_C$.

Now suppose one approximates the diproton wave function contained in $\psi_f^{(-)}$ by its asymptotic form,

$$\psi_{2p}=e^{-i\delta}(F_0\cos\delta+G_0\sin\delta)/kr. \tag{8}$$

In Eq. (8), F_0 and G_0 are the s-wave regular and irregular Coulomb wave functions. If the integral in (7) over the spatial coordinates of the diproton wave function is such that F_0 and G_0 may be expanded in powers of r, then to first order

$$F_0\approx C(\eta)kr,\quad G_0\approx[1/C(\eta)][1+(r/R)\ln(r/R)].$$

Finally, using the effective range expansion (5) one may write

$$\psi_{2p}=e^{-i\delta}\frac{\sin\delta}{C(\eta)k}\left[\frac{1}{r}+\frac{1}{R}\ln(r/R)-\frac{1}{a}\right]. \tag{9}$$

Equation (9) placed into (7) factors directly into Eq. (3) for the Watson-Migdal transition amplitude. It can thus be concluded that the Watson-Migdal theory is valid only if the integral over the spatial coordinate of the diproton wave function has a short range.

Henley *et al.*[15] use the transition amplitude of Eq. (7) to obtain a plane-wave Born approximation (PWBA) to evaluate the cross section for the reactions ^{3}He$(d,t)2p$ and $d(^3$He$,t)2p$. They find for the pickup reaction ^{3}He$(d,t)2p$ that the transition amplitude is

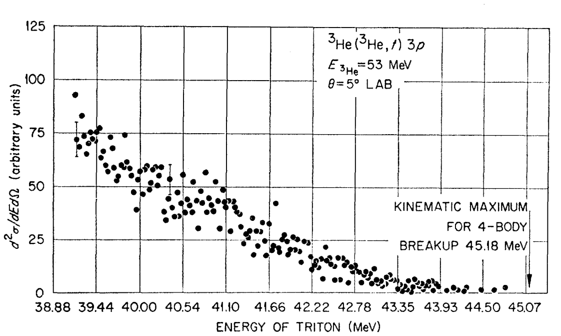

FIG. 6. Energy spectrum of tritons at 5° lab from the reaction ^{3}He(^{3}He,t)$3p$, using a 53-MeV ^{3}He beam.

[23] D. U. L. Yu and W. E. Meyerhof, Nucl. Phys. **80**, 481 (1965).

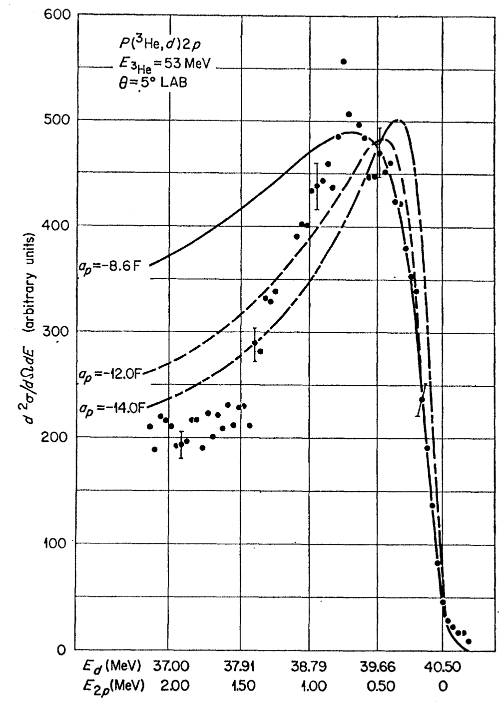

FIG. 7. Energy spectrum of deuterons at 5° lab from the reaction $p(^3$He$,d)2p$, using a 53-MeV ^{3}He beam. Typical statistical error bars are shown. The smooth curves are predictions of the Watson-Migdal theory for the indicated values of a_p.

given in essence by

$$T=A\int d\mathbf{r}\,\psi_{2p}^*(\mathbf{r})\psi_{^3\mathrm{He}}(\mathbf{r}), \tag{10}$$

where $\psi_{^3\mathrm{He}}(\mathbf{r})$ is that part of the ^{3}He wave function which carriers the information on the two protons relative to each other. This is schematically illustrated by diagram b of Fig. 8. The comparable transition amplitude for the charge-exchange reaction $d(^3$He$,t)2p$ is

$$T=B\int d\mathbf{r}\,\psi_{2p}^*(\mathbf{r})\varphi_d(\mathbf{r})e^{-i\frac{1}{2}\mathbf{Q}\cdot\mathbf{r}}, \tag{11}$$

where φ_d is the deuteron wave function, and $\mathbf{Q}$ is the momentum transfer. Diagram a of Fig. 8 illustrates this process. Other terms depending on the momentum transfer have not been explicitly shown in (10) and (11). For very forward angles and high c.m. energies, the momentum transfer is nearly zero and so the exponential in (11) may be replaced by unity.

Phillips[17] employs the impulse approximation to obtain an equation similar to (11) without the momentum transfer term, by considering a charge-exchange process for the reaction $d(p,n)2p$. His equation contains an overlap integral of the diproton wave function with the deuteron wave function.

Equations (10) and (11) are similar in form. Because the deuteron wave function has a much greater spatial

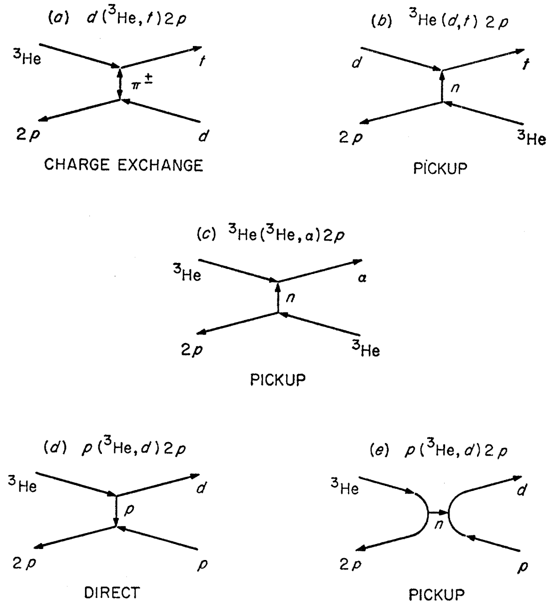

FIG. 8. Schematic illustrations of some simple reaction processes discussed in the text. (*a*) is the one-pion charge-exchange process characterized by the $d(^3\text{He},t)2p$ reaction. (*b*) and (*c*) illustrate the low-momentum transfer process for neutron pickup as characterized by the $^3\text{He}(d,t)2p$ and $^3\text{He}(^3\text{He},\alpha)2p$ reactions, respectively. The direct proton stripping process in the $p(^3\text{He},d)2p$ reaction is shown in (*d*), while the high-momentum transfer process for neutron pickup in the same reaction is illustrated in (*e*).

extent than the ^{3}He wave function, the integral of Eq. (11) has a larger spatial extent than Eq. (10). For this reason the Watson-Migdal calculation may not be expected to give valid results.

The transition amplitude for the reaction $p(^3\text{He},d)2p$ may also be evaluated in the PWBA. Two reaction amplitudes contribute to this process as schematically illustrated in Fig. 8. For high c.m. momenta and forward angles the direct stripping process (diagram *d* of Fig. 8) should dominate. Calculations of the cross section were made for the direct process above and for the direct and pickup process (diagram *e* of Fig. 8) combined. The relative weight of the amplitudes was essentially obtained by an absolute calculation of each transition amplitude in the PWBA. Gaussian wave functions and a Gaussian two-nucleon potential were used in the calculations.[15]

Computer programs were written to calculate the cross section using the various approximations for the transition amplitudes. Each cross section was then converted to the laboratory system and folded into a Gaussian distribution corresponding to the over-all experimental resolution for comparison with the experimental results. Also included for most calculations was a least-squares search procedure to find the value of a_p and r_0 which gives the best fit to the experimental data. In general, the shape of the theoretical curve was not sensitive to the effective range parameter, and throughout the calculations it was held fixed at $r_0=2.65$ F, the value determined from low-energy proton-proton scattering.[4]

IV. DISCUSSION

A. Reactions $^3\text{He}(d,t)2p$ and $d(^3\text{He},t)2p$

The triton spectra from the reaction $^3\text{He}(d,t)2p$ at 5° lab and for a bombarding energy of 36 MeV, Fig. 2, are adequately described by the final-state interaction theory of Watson and Migdal. A least-squares fit to the region $0.0\leq E_{2p}\leq 1.5$ MeV gives a scattering length $a_p=-7.3\pm0.6$ F. [The errors quoted are standard deviations, or the value of the parameter change required to cause a change in χ^2 of $(2\chi^2)^{1/2}$.] This is in agreement with the experimental results of Baumgartner *et al.*[3] for the same reaction at a slightly lower energy.

In contrast with the above results, the triton spectra shown in Fig. 3 for the reaction $d(^3\text{He},t)2p$ at 53 MeV lab were not reproduced by the Watson-Migdal theory and the known proton-proton interaction. For the 0° data a least-squares fit to the region $0.0\leq E_{2p}\leq 0.5$ MeV gives an $a_p=-11.1\pm0.7$ F, by fitting the region $0.0\leq E_{2p}\leq 1.0$ MeV, $a_p=-12.6\pm0.6$ F. Using a similar procedure for the 3° data fitting the region $0.0\leq E_{2p}\leq 0.5$ MeV gives $a_p=-11.0\pm0.7$ F and for the region $0.0\leq E_{2p}\leq 1.0$, $a_p=-12.0\pm0.6$ F. Figure 4 shows the results of similar calculations for the $d(^3\text{He},t)2p$ reaction for a ^{3}He energy of 74 MeV. Fitting the region $0.0\leq E_{2p}\leq 0.5$ MeV gives $a_p=-8.3\pm0.8$ F and for the region $0.0\leq E_{2p}\leq 1.0$ MeV, $a_p=-10.2\pm0.7$ F. These values are closer to the free *p*-*p* value. However, the over-all character of the fits is very similar to that at the lower energy.

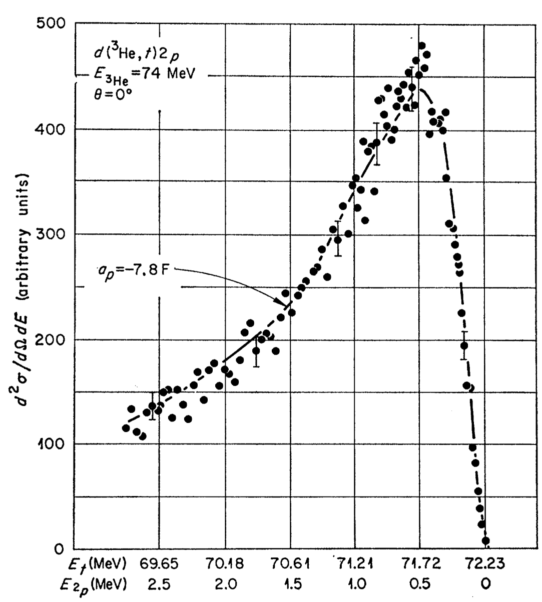

FIG. 9. Energy spectrum of tritons at 0° from the reaction $d(^3\text{He},t)2p$, using a 74-MeV ^{3}He beam. The data are the same as shown in Fig. 4. The smooth curve is the prediction of the PWBA assuming the reaction is dominated by a charge-exchange process.

The two reactions were studied at the same c.m. energy, so that the difference in the shape of the data is due to differences in reaction mechanisms for the processes. For the reaction ^{3}He$(d,t)2p$, one would expect a pickup process to dominate. When the triton comes out parallel to the projectile, in the reaction $d(^3$He$,t)2p$, one might expect a charge-exchange process to dominate. In the charge-exchange process the two protons are created out of the two nucleons originally in the deuteron and it is tempting to assume that the cross section for producing two protons with a given relative momentum depends on the relative momentum of the two nucleons in the original deuteron. Then a measure of the yield would be the degree of overlap of the two-proton wave function and the deuteron wave function. This is just what appears in the charge-exchange calculation in the impulse approximation [Eq. (11)]. The excellent agreement of this calculation with the experimental results for the 74-MeV ^{3}He beam is shown in Fig. 9 and might be taken as an indication that this is the dominant mechanism. The somewhat poorer agreement with the 53-MeV data (as shown in Fig. 10) might be an indication that the impulse approximation is not as well justified at the lower energy. In addition, distortion effects and other reaction mechanisms may become increasingly important as the energy is lowered. However, even for the 53-MeV data the impulse approximation gives much better agreement than the Watson-Migdal calculation.

If one assumes a pickup mechanism, then the two protons come from the original ^{3}He after a neutron is removed, and the appropriate integral is the overlap of the two-proton part of the ^{3}He wave function with the two-proton final-state wave function. Since the ^{3}He nucleus is much more tightly bound than the deuteron, the two protons from the ^{3}He have a smaller spatial distribution, or a larger momentum distribution, than the two nucleons in the deuteron. This would cause the overlap with the two protons in the final state to be better over a wider range of two-proton relative momenta, producing a wider experimental peak. This also gives an indication of why the Watson-Migdal theory fits the experimental data better where the pickup mechanism is expected to dominate. An approximation made in the Watson-Migdal theory is that $kr \ll 1$. This requires that the reaction volume be small and that one only consider the region where the particles which are to interact in the final state have a small relative momentum. When the two protons come from the original deuteron, which has a mean radius of about 5 F, for two-proton relative energies as large as 1 MeV, $k=0.16$ F^{-1} and $kr \simeq 0.8$. When the two protons come from the ^{3}He particle, which has a mean radius of about 2 F, $kr \simeq 0.3$. Clearly, for the latter case the assumption of the Watson-Migdal theory is better satisfied.

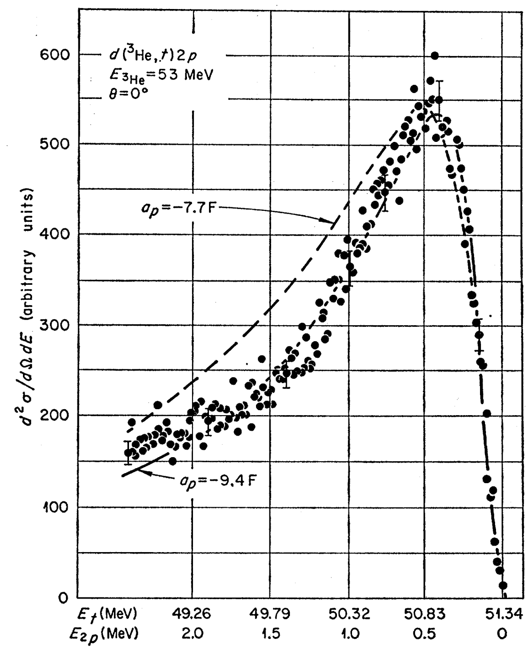

FIG. 10. Energy spectrum of tritons at 0° from the reaction $d(^3$He$,t)2p$, using a 53-MeV ^{3}He beam. The data are the same as shown on the left side of Fig. 3. The smooth curves are predictions of the PWBA assuming the reaction is dominated by a charge-exchange process. The calculations differ by the indicated values assumed for the two-proton scattering length.

B. Reaction ^{3}He$(^3$He$,^4$He$)2p$

The ^{3}He$(^3$He$,^4$He$)2p$ reaction presumably takes place through a pickup process similar to the reaction ^{3}He$(d,t)2p$ and is illustrated by diagram c in Fig. 8. Perhaps for this reason our experimental results (see Fig. 5) are fairly well reproduced by the Watson-Migdal theory. Agreement with the Watson-Migdal theory for the reaction ^{3}He$(^3$He$,^4$He$)2p$ has also been reported[24] for bombarding energies of 43.7 and 53.0 MeV and has prompted the suggestion[24,25] that the reaction $t(t,\alpha)2n$ may therefore be an attractive reaction for studying the n-n scattering length. Unfortunately, ~50-MeV triton beams are not available and experiments with ~20-MeV tritons may be plagued by the sequential decay process[26]

$$t+t \to {}^5\mathrm{He}^*(16.7\ \mathrm{MeV})+n \to {}^4\mathrm{He}+n+n\,,$$

which can contribute α particles near the final-state interaction peak.

C. Reaction $p(^3$He$,d)2p$

The results from the reaction $p(^3$He$,d)2p$ are shown in Fig. 7. The Watson-Migdal theory does not repro-

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[25] E. E. Gross, J. J. Malanify, B. J. Morton, and A. Zucker, Bull. Am. Phys. Soc. **12**, 465 (1967).

[26] J. J. Malanify, E. E. Gross, and R. Woods, Bull. Am. Phys. Soc. **12**, 1175 (1967).

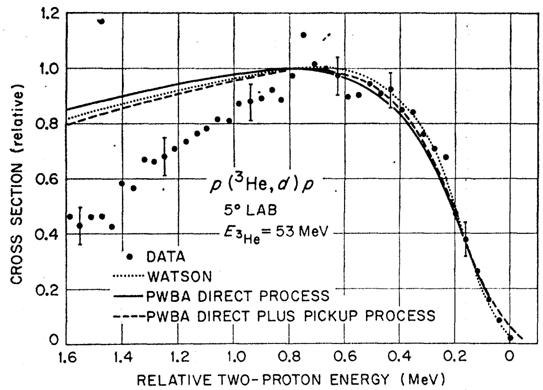

FIG. 11. A comparison of the PWBA and Watson-Migdal calculations for the $p(^3\mathrm{He},d)2p$ reaction is shown. The PWBA calculation uses the accepted scattering length of -7.69 F, while the Watson-Migdal calculation was fitted to the data by allowing the scattering length to vary. The best value was -9.1 F. The PWBA uses Gaussian wave functions and a Gaussian potential.

duce the shape of the enhancement peak for this reaction. As in the case of $d(^3\mathrm{He},t)2p$, the peak is much narrower than the prediction. This reaction may proceed through the two reaction mechanisms discussed in Sec. III. Figure 11 shows the theoretical predictions for a PWBA of the direct process and of the direct and pickup processes combined. In these calculations Gaussian forms were used for the internal wave functions of the ^{3}He and deuteron. In addition, the diproton wave function of Eq. (8) and a Gaussian potential of the form $V=V_0\exp(-\beta^2r^2)$ were used.[15] The results do not differ greatly from the Watson-Migdal curve. It should be noted that the PWBA calculations used the accepted value of the two-proton scattering length (-7.63 F), while the Watson calculation used a scattering length of -9.1 F to fit the data. In the PWBA case one may vary the range of the two-nucleon interaction (β) instead of the two-proton scattering length.

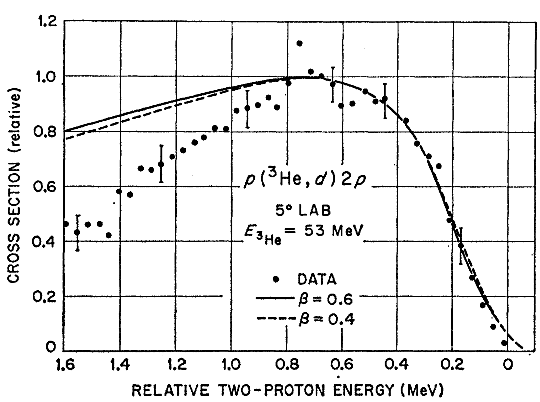

FIG. 12. The effect of the range of the nuclear potential on the shape of the energy spectrum for the reaction $p(^3\mathrm{He},d)2p$. A Gaussian potential of the form $V=V_0\exp(-\beta^2r^2)$ for the effective nucleon-nucleon interaction and a scattering length of -7.69 F was used.

Figure 12 shows that the calculations are reasonably insensitive to β. Also, as the range of the interaction is increased (β is decreased), the calculated peak becomes narrower, but for reasonable values of β ($\sim$0.6 F^{-1}) the curve does not reproduce the data for relative two-proton energies above 0.7 MeV.

Calculations in a distorted-wave formalism are planned. It would also be of value to do this reaction [and the reaction $^3\mathrm{He}(p,d)2p$] at higher c.m. energies.

V. SUMMARY AND CONCLUSIONS

In Table I we summarize the reactions which we have investigated in order to study the two-proton final-state interaction. We list the laboratory beam energy, laboratory scattering angle, and the c.m. energy in the final-state system. Comparison of reactions 1 and 2 has revealed that the width of the final-state interaction peaks is strongly dependent upon the reaction mechanism as predicted by Phillips.[17] Furthermore, the rather good agreement between the simple overlap integral of Eq. (11) with the data of reactions 2 and 3 in Table I is strong evidence for the charge exchange process assumed to be operative here. Reactions 1 and 4 seem to be reasonably well understood in terms of the Watson theory (i.e., a zero-range formulation of a pickup process). The last reaction in Table I cannot be so easily explained and the comparatively low c.m. energy of these data may be noteworthy in this respect since distortion effects are likely to be important.[22]

TABLE I. Summary of the two-proton final-state reactions studied. Column 4 refers to the energy in the c.m. system between the observed particle and the two protons.

Reaction	Beam energy (MeV)	Laboratory scattering angle (deg)	Final-state c.m. energy (MeV)
$^3\mathrm{He}(d,t)2p$	36	5	20.1
$d(^3\mathrm{He},t)2p$	53	0, 3	19.7
$d(^3\mathrm{He},t)2p$	74	0	28.1
$^3\mathrm{He}(^3\mathrm{He},{}^4\mathrm{He})2p$	53	5	39.4
$p(^3\mathrm{He},d)2p$	53	5	7.8

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