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Resistive States in High-Field Type-II Superconductors

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Restricting ourselves to high-field region, we calculate the longitudinal electrical current and the transverse heat current in the resistive states of the high-field type-II superconductors (i.e., type-II superconductors with a large Pauli paramagnetism). It is suggested that the measurements of the longitudinal resistivity together with magnetization data allows one to decompose the magnetization into components due to diamagnetic and paramagnetic currents.

I. INTRODUCTION

A NUMBER of theoretical and experimental works have been published recently on the resistive states of type-II superconductors.[1] In a previous paper,[2] referred to hereafter as CM, Caroli and Maki discussed the longitudinal resistivity and transverse heat transport in type-II superconductors for the high magnetic field region in the presence of an electric field. This was done within the framework of the current microscopic theory without referring to any phenomenological concepts such as the two-fluids model. In particular we established that the order parameter $\Delta(\mathbf{r}, t)$ moves, when it is small, [i.e., $|\Delta(\mathbf{r}, t)| \ll \pi T_{c0}$, where T_{c0} is the transition temperature in the absence of the magnetic field] with a uniform velocity $-u = -E/H$ in the direction perpendicular to both the external electric field E and the magnetic field H.

In contrast to the ordinary type-II superconductors exhibiting only negligible Pauli paramagnetism, there exists a large number of superconductors with a strong Pauli paramagnetism,[3] and consequently it is of great interest to examine the resistive states of these so-called "high-field superconductors." The purpose of the present paper is to extend the previous theory of the resistive states of the type-II superconductors to those with a large Pauli paramagnetic effect.

From recent theoretical work[4-6] we know that in the absence of electric field, for superconductors with a large Pauli paramagnetism, the order parameter in the mixed state in the high-field region is still given in terms of the Abrikosov solution;

$$\Delta(\mathbf{r}, t) = \sum_{n=-\infty}^{\infty} c_n e^{ikny} \exp\left[-eH\left(x - \frac{kn}{2eH}\right)^2\right], \quad (1)$$

where k and the c_n's are constant and n is an integer. Here we choose the static magnetic field H applied along the z axis. In Sec. II, we shall see that in the presence of an electric field E applied in the x direction the appropriate solution is

$$\Delta(\mathbf{r}, t) = \sum_{n=-\infty}^{\infty} c_n \exp[ikn(y+ut)] \times \exp\left[-eH\left(x - \frac{kn}{2eH} - \frac{iu}{4eHD}\right)^2\right], \quad (2)$$

where $u = E/H$ and $D = \frac{1}{3}lv$, l is the electronic mean free path, and v is the Fermi velocity. The order parameter moves, as in the ordinary type-II superconductors, in the directions of the y axis with a constant velocity $-u = -E/H$.

In Sec. III, following the prescription given in CM, we calculate the transport currents (i.e., electric and thermal currents) in the presence of the moving order parameter. It is worthwhile pointing out that the longitudinal resistivity as well as the transverse heat current have the contributions from the diamagnetic current terms only, since the paramagnetic current is always divergence-free. This effect results in significantly different expressions for the resistivity and the Ettingshausen coefficient in high-field type-II superconductors from those in the usual type-II superconductors. Therefore, making use of the resistive data in the mixed state, we can decompose the magnetization of the mixed state into two components: one due to the diamagnetic and one to the paramagnetic current, respectively.

II. TIME-DEPENDENT ORDER PARAMETER

We shall consider the following geometrical situation: A magnetic field H slightly smaller than H_{c2} is applied along the z axis, and an electric field E in the x direction. In order to describe the time-dependent order parameter we start with the following self-consistent equa-

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[1] For a comprehensive review of experiments and theories, see, e.g., Y. B. Kim and M. J. Stephen, in The Treatise on Superconductivity, edited by R. D. Parks (Marcel Dekker Inc., New York, to be published); more recent references can be found in Ref. 2.

[2] C. Caroli and K. Maki, Phys. Rev. **164**, 591 (1967).

[3] T. G. Berlincourt and R. R. Hake, Phys. Rev. **131**, 140 (1963); Y. B. Kim, C. F. Hemstead, and A. R. Strnad, *ibid.* **139**, A1163 (1965); Y. Shapira and L. J. Neuringer, *ibid.* **140**, A1638 (1965); **154**, 375 (1967); J. A. Cape, *ibid.* **148**, 257 (1966); R. R. Hake, *ibid.* **158**, 356 (1967).

[4] K. Maki, Physics **1**, 127 (1964).

[5] N. R. Werthamer, E. Helfand, and P. C. Hohenberg, Phys. Rev. **147**, 295 (1966).

[6] K. Maki, Phys. Rev. **148**, 362 (1966).

tion[2]:

$$\Delta^\dagger(\mathbf{r},t)=-|g|\left\langle \exp\left[i\int_{-\infty}^{t}\mathcal{H}_I(t')dt'\right]\Psi^\dagger(\mathbf{r},t)\times\exp\left[-i\int_{-\infty}^{t}\mathcal{H}_I(t'')dt''\right]\right\rangle, \quad (3)$$

where $\langle\ \rangle$ means the average taken over the Gibbs ensemble of the normal state. The interaction Hamiltonian $\mathcal{H}_I(t)$ is given by

$$\mathcal{H}_I(t)=e\int n(\mathbf{r},t)\phi(\mathbf{r},t)d^3r+\int\{\Delta(\mathbf{r},t)\Psi^\dagger(\mathbf{r},t)+\Delta^\dagger(\mathbf{r},t)\Psi(\mathbf{r},t)\}d^3r, \quad (4)$$

where

$$n(\mathbf{r},t)=\sum_\sigma \psi_\sigma^\dagger(\mathbf{r},t)\psi_\sigma(\mathbf{r},t),$$

$$\Psi(\mathbf{r},t)=\psi_\uparrow(\mathbf{r},t)\psi_\downarrow(\mathbf{r},t),$$

and

$$\Psi^\dagger(\mathbf{r},t)=\psi_\downarrow^\dagger(\mathbf{r},t)\psi_\uparrow^\dagger(\mathbf{r},t). \quad (5)$$

The first term in Eq. (4) describes the perturbation due to a finite electric field, and $\phi(\mathbf{r})=-Ex$ is the scalar potential. The second term is simply the pairing interaction term in the generalized Hartree-Fock approximation, which is sufficient for the present purpose. Because we are now interested in the behavior of the order parameter in the high-field region, where $\phi(\mathbf{r},t)$ is small [i.e., $|\Delta(\mathbf{r},t)|\ll\pi T_{c0}$], we can reduce Eq. (3) to

$$\Delta^\dagger(\mathbf{r},t)=-i|g|\int_{-\infty}^{t}dt'\int d^3r'\langle[\Psi^\dagger(\mathbf{r},t),\Psi(\mathbf{r}',t')]\rangle\times\Delta(\mathbf{r}',t'). \quad (6)$$

Here the retarded product $i\langle[\Psi^\dagger(\mathbf{r},t),\Psi(\mathbf{r}',t')]\rangle\times\theta(t-t')$ has to be evaluated in the presence of both the magnetic field $\mathbf{H}$ and the electric field $\mathbf{E}$. In the following we restrict ourselves to the dirty limit (i.e., $l\ll\xi_0$, where l is the electronic mean free path and ξ_0 is the BCS coherence distance), since in the usual high-field superconductors this condition is amply satisfied. In this limit the effects of the magnetic and electric fields are taken into account by a simple transformation of differential operators [i.e.,

$$\partial/\partial t\rightarrow\partial/\partial t\pm 2ie\phi(\mathbf{r}),\qquad \nabla\rightarrow\nabla\mp 2ie\mathbf{A}(\mathbf{r}),$$

where $\mathbf{A}$ and ϕ are the vector and scalar potentials, respectively. Two signs refer to the operation on $\Delta(\mathbf{r},t)$ and $\Delta^\dagger(\mathbf{r},t)$, respectively]. In the absence of $\mathbf{H}$ and $\mathbf{E}$, the integral equation is converted into a differential equation as in CM, and we find

$$(1-|g|\langle[\Psi^\dagger,\Psi]\rangle_{\omega,\mathbf{q}})\Delta(\mathbf{q},\omega)=0. \quad (7)$$

Here $\langle[\Psi^\dagger,\Psi]\rangle_{\omega,\mathbf{q}}$ is the Fourier transform of the retarded product and is given by

$$\langle[\Psi^\dagger,\Psi]\rangle_{\omega,\mathbf{q}}=N(0)\left\{\ln\frac{\pi T}{\omega_D}+\frac{1}{2}\left[\left(1+\frac{b}{(b^2-I^2)^{1/2}}\right)\times\psi(\tfrac{1}{2}+\rho_-)+\left(1-\frac{b}{(b^2-I^2)^{1/2}}\right)\psi(\tfrac{1}{2}+\rho_+)-2\psi(\tfrac{1}{2})\right]\right\}. \quad (8)$$

$\psi(z)$ is the digamma function and

$$\rho_\pm=(1/2\pi T)[\tfrac{1}{2}(i-\omega)+(a\pm(b^2-I^2)^{1/2})],$$

$$a=(1/3\tau_{so})+\tfrac{1}{2}Dq^2,\qquad b=(1/3\tau_{so}),\qquad I=\mu H \quad (9)$$

and

$$D=\tfrac{1}{3}(lv).$$

Here $N(0)$ is the density of states of electrons and τ_{so} the spin-flip lifetime due to the spin-orbit scattering of impurities. In the derivation of the above equation we have made use of the renormalization procedure appropriate to the present situation as developed by Werthamer, Helfand, and Hohenberg[5] and by the present author.[6]

In the presence of $\mathbf{H}$ and $\mathbf{E}$ we understand ω and $\mathbf{q}$ in Eq. (8) to be

$$\omega=i\partial/\partial t-2e\phi(\mathbf{r})$$

and

$$\mathbf{q}=(1/i)\nabla-2e\mathbf{A}(\mathbf{r}). \quad (10)$$

It is then not difficult to show that $\Delta(\mathbf{r},t)$ given in (2) satisfies Eq. (7) [or Eq. (6)]. This follows because $\Delta(\mathbf{r},t)$ given by (2) is the solution of the differential equation

$$(-i\omega+Dq^2)\Delta(\mathbf{r},t)=\epsilon_0\Delta(\mathbf{r},t),$$

$$\epsilon_0=2DeH_{c2}. \quad (11)$$

Here we neglect a small shift of H_{c2} due to the electric field E.

Therefore, we conclude that the order parameter moves with a constant velocity $-u=-E/H$ in the y direction as is the case in the ordinary type-II superconductors. Furthermore, the (imaginary) polarization in the direction of the electric field is controlled by $D=\frac{1}{3}(lv)$ (the diffusion constant) which is independent of the Pauli term.

III. TRANSPORT PROPERTIES

The transport currents in the resistive states of the mixed state are obtained following the procedure described in CM. A physical observable $\mathbf{Q}(\mathbf{r},t)$ in the resistive state is calculated by

$$\mathbf{Q}(\mathbf{r}t)=\left\langle \exp\left[i\int_{-\infty}^{t}\mathcal{H}_I(t')dt'\right]\mathbf{Q}(\mathbf{r},t)\times\exp\left[-i\int_{-\infty}^{t}\mathcal{H}_I(t'')dt''\right]\right\rangle. \quad (12)$$

In the high-field region where the order parameter $\Delta(\mathbf{r}, t)$ is small, we can simplify Eq. (12) as

$$\mathbf{Q}(\mathbf{r}, t) = Q_1(\mathbf{r}, t) + Q_2(\mathbf{r}, t), \tag{13}$$

$$Q_1(\mathbf{r}, t) = -ie\int_{-\infty}^{t} dt' \int d^3r' \times \langle [\mathbf{Q}(\mathbf{r}, t), n(\mathbf{r}', t')] \rangle \phi(\mathbf{r}', t), \tag{14}$$

and

$$Q_2(\mathbf{r}, t) = -\frac{1}{2}\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 \int d^3l \int d^3m \{\langle [[\mathbf{Q}(\mathbf{r}, t), \times \Psi(\mathbf{l}_1 t_1)]\Psi^\dagger(\mathbf{m}_1 t_2)] \rangle + \langle [[\mathbf{Q}(\mathbf{r}, t), \Psi^\dagger(\mathbf{m}_1 t_2)]\Psi(\mathbf{l}_1 t_1)] \rangle\} \times \Delta^\dagger(\mathbf{l}_1 t_1)\Delta(\mathbf{m}_1 t_2). \tag{15}$$

The first term in Eq. (13) is the expectation value of $\mathbf{Q}(\mathbf{r}, t)$ in the normal state, while the second term is the lowest-order correction to $\mathbf{Q}(\mathbf{r}, t)$ in $\Delta(\mathbf{r}, t)$ for the mixed state. In the following we shall calculate the electric and heat currents in the presence of a finite electric field $\mathbf{E}$, [i.e., we put $\phi(\mathbf{r}) = -\mathbf{E}\cdot\mathbf{r} = -Ex$].

A. Electric Current

The current operator is given by

$$\mathbf{j}(\mathbf{r}, t) = -\frac{ie}{2m}\sum_\sigma (\nabla - \nabla' - 2ie\mathbf{A}(\mathbf{r}, t)) \times \psi_\sigma^\dagger(\mathbf{r}', t)\psi_\sigma(\mathbf{r}, t)\,|_{\mathbf{r}=\mathbf{r}'}. \tag{16}$$

Substituting this in Eq. (14) we have

$$j_{1x}(\mathbf{r}, t) = E\sigma/(1+\eta^2),$$
$$j_{1y}(\mathbf{r}, t) = -E\sigma\eta/(1+\eta^2), \tag{17}$$

where $\sigma = e^2\tau N/m$, the conductivity of the metal in the normal state, $\eta = \tau\omega_c$ and $\omega_c = eH/2m$. From Eq. (15) $\mathbf{j}_2(\mathbf{r}, t)$ is evaluated (see Appendix) and we have

$$\mathbf{j}_2(\mathbf{r}, t) = \frac{e\tau N}{4\pi m T}(\mathbf{q}_1 - \mathbf{q}_2)\left\{\frac{1}{2}\left[1 + \frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_-) + \frac{1}{2}\left[1 - \frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_+)\right\}\Delta(1)\Delta^\dagger(2)\,|_{1=2=(\mathbf{r},t)}, \tag{18}$$

where $\rho_\pm$ is now given by

$$\rho_\pm = (1/2\pi T)[\epsilon_0 + b \pm (b^2 - I^2)^{1/2}]. \tag{19}$$

Here in the derivation of Eq. (18) we have made use of the relation

$$(-i\omega_1 + Dq_1^2)\Delta(1) = \epsilon_0\Delta(1),$$
$$(-i\omega_2 + Dq_2^2)\Delta^\dagger(2) = \epsilon_0\Delta^\dagger(2). \tag{20}$$

Substituting the explicit form of $\Delta(\mathbf{r}, t)$ given in Eq. (2), it is easy to see that the current has an oscillating part with harmonics of a basic frequency $\omega_0 = k\,|\,u\,|$, where $u = -E/H$ and k is of the order $\xi(T)^{-1} = (2eH_{c2}(t))^{1/2}$. This type of ac current is associated with the local variation of $\Delta(\mathbf{r}, t)$ due to the motion of the order parameter and has been previously pointed out by Kulik.[7] Making the space average of Eq. (18) we obtain

$$\langle \mathbf{j}_{2x} \rangle = \frac{e\tau_{\rm tr}N}{4\pi m T}\frac{u}{D}\left\{\frac{1}{2}\left[1 + \frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_-) + \frac{1}{2}\left[1 - \frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_+)\right\}\langle |\Delta|^2 \rangle = -M_d u/D, \tag{21}$$

$$\langle \mathbf{j}_{2y} \rangle = 0, \tag{22}$$

where we have defined M_d as

$$M_d = -\frac{e\tau_{\rm tr}N}{4\pi m T}\left\{\frac{1}{2}\left[1 + \frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_-) + \frac{1}{2}\left[1 - \frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_+)\right\}\langle |\Delta|^2 \rangle. \tag{23}$$

M_d is physically the diamagnetic contribution to the magnetization in the mixed state, which in the high-field superconductors is written as

$$M = (M_s - M_n) = M_d + M_p, \tag{24}$$

where M_p is the contribution due to the reduction of the Pauli paramagnetism in the mixed state and given[6] by

$$M_p = -\frac{\mu_s N}{4\pi m v^2 T}\frac{I}{(b^2-I^2)^{1/2}}\left\{\frac{1}{2}\left[1 + \frac{b}{(b^2-I^2)^{1/2}}\right] \times \psi^{(1)}(\tfrac{1}{2}+\rho_-) - \frac{1}{2}\left[1 - \frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_+) + \frac{2\pi T b}{2(b^2-I^2)}[\psi(\tfrac{1}{2}+\rho_-) - \psi(\tfrac{1}{2}+\rho_+)]\right\}\langle |\Delta|^2 \rangle. \tag{25}$$

Here μ is the Bohr magneton and v the Fermi velocity.

Making use of the relation[6]

$$4\pi M = -(H_{c2} - H)/[2\kappa_2^2(t) - 1]\beta_A, \qquad \beta_A = 1.16, \tag{26}$$

where $\kappa_2(t)$ is the second Ginzburg-Landau parameter, Eq. (21) can be rewritten as

$$\langle j_{2x} \rangle = -D^{-1}\frac{(1 - H/H_{c2})}{[2\kappa_2^2(t) - 1]\beta_A}A(t)E, \tag{27}$$

[7] I. O. Kulik, Zh. Eksperim. i Teor. Fiz. **50**, 1617 (1966) [English transl.: Soviet Phys.—JETP **23**, 1077 (1966)].

where

$$A(t)=M_d/(M_d+M_p)=\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_-)+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_+)\right\}$$

$$\times\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_-)+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_+)+\frac{2I^2}{\epsilon_0(b^2-I^2)^{1/2}}\left(\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_-)\right.\right.$$

$$\left.\left.-\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\psi^{(1)}(\tfrac{1}{2}+\rho_+)+\frac{2\pi Tb}{2(b^2-I^2)}\left[\psi(\tfrac{1}{2}+\rho_-)-\psi(\tfrac{1}{2}+\rho_+)\right]\right)\right\}^{-1}. \quad (28)$$

We also note here that Eq. (22) implies that, in the present approximation, there is no correction to the Hall current to the second order $\langle|\Delta|^2\rangle$. Since the Hall current is smaller than the longitudinal current by a factor at most τT_{c0}, and as our approximation consistently neglects terms of this order, a more delicate treatment of both the electric and magnetic fields in Eq. (7) is necessary for a calculation of the Hall current.

From Eqs. (17) and (21), we obtain the bulk resistivity in the mixed state;

$$\frac{R_s}{R_n}=\left\{1-\frac{4.95\kappa^2}{[2\kappa_2{}^2(t)-1]}\left(1-\frac{H}{H_{c2}}\right)A(t)\right\} \quad (29)$$

or

$$\frac{H}{R_n}\frac{\partial R_s}{\partial H}\bigg|_{H=H_{c2}}=\frac{4.95\kappa^2}{[2\kappa_2{}^2(t)-1]}A(t). \quad (30)$$

Here $\kappa=\kappa_2(1)$.

The above result is a generalization of the one due to Schmid[8] and CM.

In high-field type-II superconductors the temperature dependence of the resistivity in the mixed state is determined by two factors: the temperature dependence of $\kappa_2(t)$ and $A(t)$. Therefore measurement of the resistivity together with that of the magnetization M allow one to deduce $A(t)$ (i.e., the ratio of the diamagnetic contribution to M in high-field type-II superconductors). As we shall see, the temperature dependence of $A(t)$ is milder than that of $\kappa_2(t)$, and we expect that the slope given in Eq. (30) increases by a factor $\sim$10 at low temperatures, compared with the one close to $T=T_{c0}$. In fact, this kind of behavior has already been observed in the experiment by Kim *et al.*,[9] although they did not study the temperature variation of the slope in detail.

It may be of some interest to study the temperature dependence of $A(t)$. Since the general expressions are extremely complicated, we shall consider here only two limiting cases.

1. Absence of Spin-Orbit Scattering

Substituting $b=(3\tau_{so})^{-1}=0$ in Eq. (28) we have

$$A(t)=\frac{\mathrm{Re}\psi^{(1)}(\frac{1}{2}+\rho(1+i\alpha))}{\mathrm{Re}[(1+i\alpha)\psi^{(1)}(\frac{1}{2}+\rho(1+i\alpha))]}, \quad (31)$$

where

$$\rho=\epsilon_0/4\pi T=\tau v^2eH_{c2}(t)/6\pi T, \qquad \alpha=\mu/eD.$$

The following asymptotics may be useful:

$$A(t)=1-\alpha^2\rho\{[28\zeta(3)/\pi^2]-\rho[2\pi^2-(784/\pi^4)\zeta^2(3)(1-\alpha^2)]\},\quad \text{for}\quad T\simeq T_{c0} \quad (32)$$

$$=[1/(1+\alpha^2)]\{1+\tfrac{1}{6}[\alpha^2/(1+\alpha^2)^2](4\pi T/\epsilon_0)^2\},\quad \text{for}\quad T\ll T_{c0}. \quad (33)$$

2. Strong Spin-Orbit Scattering Limit ($b\gg1$)

In this limit Eq. (28) reduces to

$$A(t)=\left(1+\frac{2I^2}{\epsilon_0 b}\right)^{-1}=\left[1+\frac{\tau_{so}\mu^2H_{c2}(t)}{e\tau v^2}\right]^{-1}. \quad (34)$$

We can see from these calculations that in either limit the temperature dependence of $\kappa_2(t)$ is much stronger[6] than that of $A(t)$, and generally gives rise to a steep slope of resistivity at low temperatures in fields close to $H_{c2}(t)$.

B. Heat Current

We shall be concerned here only with the additional heat current in the mixed state. The heat current is obtained by substituting $Q(\mathbf{r},t)$ in Eq. (15) by the heat-current operator

$$\mathbf{j}^h(\mathbf{r},t)=-(2m)^{-1}\sum_\alpha\{(\nabla-ie\mathbf{A})((\partial/\partial t')-ie\phi)+(\nabla'+ie\mathbf{A})((\partial/\partial t)+ie\phi)\}\psi_\sigma{}^\dagger(\mathbf{r}',t')\psi_\sigma(\mathbf{r},t)\,|_{\mathbf{r}'=\mathbf{r},t'=t}. \quad (35)$$

[8] A. Schmid, Physik Kondensierten Materie **5**, 302 (1966).
[9] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965).

From the evaluation of a triangular diagram, the following expression of $\mathbf{j}^h$ emerges (see Appendix A):

$$j^h(\mathbf{r},t)=(\mathbf{q}_1-\mathbf{q}_2)(\omega_2-\omega_1)(\tau N/8\pi mT)\{\tfrac{1}{2}[1+(b/(b^2-I^2)^{1/2})](\psi^{(1)}(\tfrac{1}{2}+\rho_-)+\tfrac{1}{2}\rho_-\psi^{(2)}(\tfrac{1}{2}+\rho_-)) + \tfrac{1}{2}[1-(b/(b^2-I^2)^{1/2})](\psi^{(1)}(\tfrac{1}{2}+\rho_+)+\tfrac{1}{2}\rho_+\psi^{(2)}(\tfrac{1}{2}+\rho_+))\}\Delta(1)\Delta^\dagger(2)\,|_{1=2=(\mathbf{r},t)}. \quad (36)$$

The space average of Eq. (36) results in

$$\langle j_{2x}^h\rangle=0,$$

$$\langle j_2y^h\rangle=-(\tau N/2\pi mT)\{\tfrac{1}{2}[1+(b/(b^2-I^2)^{1/2})](\psi^{(1)}(\tfrac{1}{2}+\rho_-)+\tfrac{1}{2}\rho_-\psi^{(2)}(\tfrac{1}{2}+\rho_-)) + \tfrac{1}{2}[1-(b/(b^2-I^2)^{1/2})](\psi^{(1)}(\tfrac{1}{2}+\rho_+)+\tfrac{1}{2}\rho_+\psi^{(2)}(\tfrac{1}{2}+\rho_+))\}\langle|\Delta|^2\rangle eE$$

$$=M_dL(t)E, \quad (37)$$

and

$$L(t)=\left\{2+\frac{\tfrac{1}{2}[1+b/(b^2-I^2)^{1/2}]\rho_-\psi^{(2)}(\tfrac{1}{2}+\rho_-)+\tfrac{1}{2}[1-b/(b^2-I^2)^{1/2}]\rho_+\psi^{(2)}(\tfrac{1}{2}+\rho_+)}{\tfrac{1}{2}[1+b/(b^2-I^2)^{1/2}]\psi^{(1)}(\tfrac{1}{2}+\rho_-)+\tfrac{1}{2}[1-b/(b^2-I^2)^{1/2}]\psi^{(1)}(\tfrac{1}{2}+\rho_+)}\right\}. \quad (38)$$

The above expression is a simple generalization of the one obtained by CM. It is interesting to note that both the electric and the heat currents have simple expressions in terms of M_d (rather than M, the total magnetization). This can be understood physically from the fact that the paramagnetic contribution to the current induced by the motion of the order parameter gives rise to rotational currents only, and neither contributes to the average electrical nor to the heat current.

It might be useful to extract from Eq. (37) the entropy carried by each flux (S);

$$S=\frac{1}{4eT}\frac{(H_{c2}-H)}{[2\kappa_2^2(t)-1]\beta_A}L(t)A(t). \quad (39)$$

We conclude this section with a brief examination of the temperature dependence of $L(t)$.

1. No Spin-Orbit Scattering ($b=0$)

In this case $L(t)$ in Eq. (38) reduces to

$$L(t)=\left\{2+\frac{\mathrm{Re}[\rho(1+i\alpha)\psi^{(2)}(\tfrac{1}{2}+\rho(1+i\alpha))]}{\mathrm{Re}\psi^{(1)}(\tfrac{1}{2}+\rho(1+i\alpha))}\right., \quad (40)$$

$$\cong 2-\frac{28\zeta(3)}{\pi^2}\rho+\left[2\pi^2(1-\alpha^2)+\frac{784\zeta^2(3)}{\pi^4}\right]\rho^2, \quad \text{for } T\simeq T_{c0} \quad (41)$$

$$=1+\frac{1}{6}\frac{1-3\alpha^2}{(1+\alpha^2)}\left(\frac{4\pi T}{\epsilon_0}\right)^2, \quad \text{for } T\ll T_{c0}. \quad (42)$$

2. Strong Spin-Orbit Scattering Limit ($b>1$)

$$L(t)=\{2+[\rho_-\psi^{(2)}(\tfrac{1}{2}+\rho_-)/\psi^{(1)}(\tfrac{1}{2}+\rho_-)]\}, \quad (43)$$

where

$$\rho_-\cong(2\pi T)^{-1}\{\tfrac{1}{2}\epsilon_0+\tfrac{3}{2}\tau_{so}(\mu H_{c2}(t))^2\}.$$

Asymptotic behaviors are

$$L(t)=2-[28\zeta(3)/\pi^2]\rho_-, \quad \text{for } T\simeq T_{c0} \quad (44)$$

$$=1+\tfrac{1}{6}(\rho_-)^{-2}, \quad \text{for } T\ll T_{c0}. \quad (45)$$

IV. CONCLUDING REMARKS

Extending the previous treatment by Caroli and Maki to the high-field type-II superconductors, we obtain the expressions of the electric and thermal currents in the resistive state for the high-field region ($H\simeq H_{c2}$). If M_d, the diamagnetic part of the total magnetization M, is substituted in place of M, we obtain an almost equivalent expression for the resistivity in the mixed state so that for normal type-II superconductors without any paramagnetic effect. The steep slope of the resistivity at $H\simeq H_{c2}$ at low temperatures in the high-field type-II superconductors results from $\kappa_2(t)$ being significantly smaller at low temperatures than κ. Therefore we can explain qualitatively in terms of the present theory, the experimental result of Kim *et al.*,[9] although more detailed analysis is needed to draw a definite conclusion.

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APPENDIX A: CALCULATION OF ELECTRIC AND HEAT CURRENTS

Since, in the dirty limit, the effect of both the electric and the magnetic fields is introduced into the theory by the transformation

$$\omega\to\omega\pm 2ie\phi(r), \qquad \mathbf{q}\to\mathbf{q}\mp 2ie\mathbf{A}, \quad (A1)$$

depending on whether ω and $\mathbf{q}$ act on Δ or Δ^+, it is sufficient to evaluate $\mathbf{j}(\mathbf{q},\omega)$ and $\mathbf{j}^{(h)}(q,\omega)$ in the absence of field. We then make use of the transformation and let $\mathbf{q}$ and ω tend to zero to get the averaged current.

Following exactly the same procedure as used in CM we first calculate the relevant thermal products, then the retarded product is obtained by an analytic continuation thereof.

The electric current $\mathbf{j}_2$ is given from (15) by

$$\mathbf{j}_2(\mathbf{r},\omega_1+\omega_2)=-(1/2m)[ie(\nabla'-\nabla)-2e^2\mathbf{A}(\mathbf{r})]$$
$$\times T\sum_\sigma\sum_{n=-\infty}^{\infty}\int d^3l d^3m$$
$$\times\langle G_{\omega_1-\omega_n}{}^{\sigma}(\mathbf{r},1)G_{\omega_n}{}^{-\sigma}(\mathbf{m},1)G_{-\omega_2-\omega_n}{}^{\sigma}(\mathbf{m},\mathbf{r}')\rangle_i$$
$$\times\Delta_{\sigma\omega_1}(1)\Delta_{-\sigma\omega_2}{}^{\dagger}(\mathbf{m})\,|_{\mathbf{r}=\mathbf{r}'}. \quad \text{(A2)}$$

$\langle\ \rangle_i$ stands for the average over random impurity configurations;

$$\omega_n=(2n+1)\pi T,\qquad \omega_1=2n_1\pi T,\qquad \omega_2=2n_2\pi T,$$

where n is any integer and n_1 and n_2 are positive integers. The above integral can be represented with a triangular diagram.

As usual, the effect of impurity scattering is taken into account by means of the following renormalization:

1. In each Green's function ω has to be replaced by $\tilde{\omega}=\omega\eta_\omega=\omega(1+1/2\tau\,|\,\omega\,|\,)$, where τ is the total collision lifetime of the electron.

2. Each vertex corresponding to $\tilde{\Delta}_\pm(\mathbf{q},\Omega)$ or $\tilde{\Delta}_\pm{}^\dagger(\mathbf{q},\Omega)$ introduces a factor

$$\tilde{\Delta}_\pm{}^\dagger(\mathbf{q},\Omega)=\eta_{\pm,\mathbf{q},\Omega}\Delta^\dagger(\mathbf{q},\Omega),$$
$$\eta_{\pm,\mathbf{q},\Omega}=\frac{((\omega-\frac{1}{2}\Omega)\eta_{\omega-1/2\Omega}\pm iI)(\omega-\frac{1}{2}\Omega\mp iI+a+b)}{(\omega-\frac{1}{2}\Omega+a)^2-b^2+I^2},\quad \text{if}\quad \omega(\omega-\Omega)>0 \quad \text{(A3)}$$
$$=1,\qquad \text{if}\quad \omega(\omega-\Omega)<0,$$

where

$$I=\mu H,\qquad a=(3\tau_{\mathrm{so}})^{-1}+\tfrac{1}{6}(\tau\tau_{\mathrm{tr}})v^2q^2,\qquad b=(3\tau_{\mathrm{so}})^{-1}. \quad \text{(A4)}$$

Furthermore, a renormalization factor has to be multiplied on the vertex associated to the current operators. However, in the present calculation this modification gives rise to no effect on the final result and we shall neglect this. After these preliminaries it is easy to evaluate (A2) and we obtain

$$\mathbf{j}_2(\mathbf{q},\omega_1+\omega_2)=(e\tau N/4mT)(\mathbf{q}_1-\mathbf{q}_2)\Bigg\{(\omega_1+\omega_2+Dq_2^2-Dq_1^2)^{-1}$$
$$\times\Bigg\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2-}+\frac{\omega_2+2\omega_1}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{1-}+\frac{\omega_1}{4\pi T}\right)\right)$$
$$+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2+}+\frac{\omega_2+2\omega_1}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{1+}+\frac{\omega_1}{4\pi T}\right)\right)\Bigg\}$$
$$-(\omega_1+\omega_2+Dq_1^2-Dq_2^2)^{-1}\Bigg\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2-}+\frac{\omega_2}{4\pi T}\right)\right.$$
$$\left.-\psi\left(\frac{1}{2}+\rho_{1-}+\frac{2\omega_2+\omega_1}{4\pi T}\right)\right)$$
$$+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2+}+\frac{\omega_2}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{1+}+\frac{2\omega_2+\omega_1}{4\pi T}\right)\right)\Bigg\}\Bigg\}, \quad \text{(A5)}$$

and

$$\rho_{1,2\pm}=(1/2\pi T)\{\tfrac{1}{2}Dq_{1,2}^2+b\mp(b^2-I^2)^{1/2}\}.$$

Here $\mathbf{q}=\mathbf{q}_1+\mathbf{q}_2$. In the limit $\omega_1+\omega_2\ll\pi T_{c0}$. (A5) reduces to

$$\mathbf{j}_2(\mathbf{q},\omega_1+\omega_2)\cong\frac{e\tau N}{4\pi mT}(\mathbf{q}_1-\mathbf{q}_2)\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi^{(1)}(\tfrac{1}{2}+\rho_-)+\frac{(\omega_1+\omega_2)}{8\pi T}\psi^{(2)}(\tfrac{1}{2}+\rho_-)\right)\right.$$
$$\left.+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi^{(1)}(\tfrac{1}{2}+\rho_+)+\frac{(\omega_1+\omega_2)}{8\pi T}\psi^{(2)}(\tfrac{1}{2}+\rho_+)\right)\right\}\Delta(\mathbf{q}_1,\omega_1)\Delta^\dagger(q_2,\omega_2), \quad \text{(A6)}$$

where

$$\rho_\pm=1/2\pi T\{a\pm(b^2-I^2)^{1/2}\}\quad \text{and}\quad a=b+\tfrac{1}{2}\epsilon_0.$$

Here we have made use of the relations

$$(\omega_1+Dq_1^2)\Delta(1)=\epsilon_0\Delta(1),$$
$$(\omega_2+Dq_2^2)\Delta^\dagger(2)=\epsilon_0\Delta^\dagger(2). \quad \text{(A7)}$$

Calculation of the heat current can be carried out in a parallel fashion. Starting from

$$\mathbf{j}_2^{(h)}(\mathbf{r}_1\omega_1+\omega_2)=(2m)^{-1}\left[\frac{\partial}{\partial t'}\nabla+\frac{\partial}{\partial t}\nabla'\right]T\sum_\sigma\sum_{m=-\infty}^{+\infty}\int d^3l d^3m$$
$$\times\langle G_{\omega_1-\omega_n}{}^\sigma(\mathbf{r},1)G_{\omega_n}{}^{-\sigma}(\mathbf{m},1)G_{-\omega_2-\omega_n}{}^\sigma(\mathbf{m},\mathbf{r}')\rangle\Delta_{\sigma\omega_1}(1)\Delta_{-\sigma\omega_2}{}^\dagger(\mathbf{m}),\qquad r=r',\ t=t' \quad \text{(A8)}$$

we arrive at

$$\mathbf{j}_2^{(h)}(\mathbf{q},\omega_1+\omega_2)=\frac{4\tau}{4m}(\mathbf{q}_1-\mathbf{q}_2)\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2-}+\frac{\omega_2}{4\pi T}\right)\right.\right.$$
$$-\psi\left(\frac{1}{2}+\rho_{2-}+\frac{2\omega_1+\omega_2}{4\pi T}\right)\Bigg)+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2+}+\frac{\omega_2}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{2+}+\frac{2\omega_1+\omega_2}{4\pi T}\right)\right)$$
$$+\frac{\omega_2-Dq_1^2}{\omega_1+\omega_2-D(q_1^2-q_1^2)}\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2-}+\frac{2\omega_1+\omega_2}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{1-}+\frac{\omega_1}{4\pi T}\right)\right)\right.$$
$$\left.+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{2+}+\frac{2\omega_1+\omega_2}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{1+}+\frac{\omega_1}{4\pi T}\right)\right)\right\}$$
$$+\frac{\omega_2+Dq_1^2}{\omega_1+\omega_2+D(q_1^2-q_2^2)}\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{1-}+\frac{2\omega_2+\omega_1}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{2-}+\frac{\omega_2}{4\pi T}\right)\right)\right.$$
$$\left.\left.+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]\left(\psi\left(\frac{1}{2}+\rho_{1+}+\frac{2\omega_2+\omega_1}{4\pi T}\right)-\psi\left(\frac{1}{2}+\rho_{2+}+\frac{\omega_2}{4\pi T}\right)\right)\right\}\right\}. \quad \text{(A9)}$$

In the limit $\omega_1+\omega_2\rightarrow 0$, the above expression reduces to

$$\mathbf{j}_2^{(h)}(\mathbf{q},\omega_1+\omega_2)=\frac{N\tau}{8\pi mT}(\mathbf{q}_1-\mathbf{q}_2)(\omega_2-\omega_1)$$
$$\times\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right](\psi^{(1)}(\tfrac{1}{2}+\rho_-)+\tfrac{1}{2}\rho_-\psi^{(2)}(\tfrac{1}{2}+\rho_-))\right.$$
$$\left.+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right](\psi^{(1)}(\tfrac{1}{2}+\rho_+)+\tfrac{1}{2}\rho_+\psi^{(2)}(\tfrac{1}{2}+\rho_+))\right\}$$
$$\times\Delta(\mathbf{q},\omega_1)\Delta^\dagger(\mathbf{q}_2,\omega_2). \quad \text{(A10)}$$

As can be seen from the above expressions, the relevant currents can always be expressed as a sum of two terms, each having an equivalent expression to one in the type-II superconductor without the Pauli term, except that now $\rho=\epsilon_0/4\pi T$ is replaced by $\rho_\pm$ and a weighting factor $\frac{1}{2}[1+b/(b^2-I^2)^{1/2}]$ [or $\frac{1}{2}[1-b/(b^2-I^2)^{1/2}]$] is applied.

APPENDIX B: THERMAL CONDUCTIVITY IN HIGH-FIELD SUPERCONDUCTORS

The method used in the calculation of the electronic contribution to the thermal conductivity in the ordinary type-II superconductor can be easily extended to the one in the high-field type-II superconductors. We shall not go into details here, since it is a trivial repetition of the calculation done by Caroli and Cyrot.[10] The final expression can be written as

$$\frac{K_s}{K_n}=1-\frac{3\langle|\Delta|^2\rangle_{\text{av}}}{2(\pi T)^2}\left\{\frac{1}{2}\left[1+\frac{b}{(b^2-I^2)^{1/2}}\right]\right.$$
$$\times(\rho_-^2\psi^{(2)}(\tfrac{1}{2}+\rho_-)+\rho_-\psi^{(1)}(\tfrac{1}{2}+\rho_-))+\frac{1}{2}\left[1-\frac{b}{(b^2-I^2)^{1/2}}\right]$$
$$\left.\times(\rho_+^2\psi^{(2)}(\tfrac{1}{2}+\rho_+)+\rho_+\psi^{(1)}(\tfrac{1}{2}+\rho_+))\right\}, \quad \text{(B1)}$$

which can be compared with

$$\frac{K_s}{K_n}=1-\frac{3\langle|\Delta|^2\rangle_{\text{av}}}{2(\pi T)^2}\{\rho^2\psi^{(2)}(\tfrac{1}{2}+\rho)+\rho\psi^{(1)}(\tfrac{1}{2}+\rho)\} \quad \text{(B2)}$$

in the type-II superconductors without Pauli paramagnetism.

[10] C. Caroli and M. Cyrot, Physik Kondensierten Materie **4**, 285 (1965).