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Particle Motion and Heat-Exchange "Viscosity" in Superfluid Helium

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It is pointed out that a uniformly heated sphere experiences a nonzero drag force due to purely potential flow of superfluid. For small velocities, the resulting motion of heated trace particles in helium will arise from a competing drag due to normal and superfluid components. The motion of such particles will be strongly dependent upon experimental conditions, such as temperature, particle size, and lighting.

SEVERAL experiments[1-3] have been reported recently in which the motions of small tracer particles[4] (frozen H-D gas) have been interpreted as indicating the behavior of the superfluid component of He II. Although the superfluid is usually assumed to undergo pure potential flow, with resulting zero drag force (d'Alembert's paradox), no explanation of the tendency of such particles to follow the superfluid has yet been given.

We will show that it is possible to account for the observed tendency of small particles to follow the superfluid flow, but that the accuracy with which they can do so is strongly dependent upon the experimental conditions.

In particular, if the tracer particles are *heated*, the existence of heat-exchange[5] forces leads to a nonzero drag force, or effective "viscosity," for superfluid flow past the particles. If we imagine *potential* flow of superfluid around a uniformly heated sphere, with the normal fluid component at rest, the drag on the sphere may be readily calculated by the formalism of two-fluid hydrodynamics, as has been done previously for the torque.[5]

A simple calculation of the momentum given to the sphere by the superfluid as it converts to normal fluid to carry away the heat leads to the drag force

$$F_{\text{H.E.}}=\dot{Q}v_s/s_nT\,, \tag{1}$$

where $\dot{Q}$ is the heat exchanged between sphere and helium, v_s the flow velocity of the superfluid, and s_n the specific entropy of the normal fluid. The drag has the form of a "Stokes law" for the superfluid component.

In the simplest case, one might have tracer particles of such small size as to absorb radiant heat from external sources, equilibrate rapidly, and uniformly exchange heat with the helium. If we neglect nonlinear terms, then the steady-state velocity of such a particle will be determined by the condition of cancellation of the drag forces of normal and superfluid components. If u is the particle velocity, we expect for sufficiently small flow velocities that

$$O=(\dot{Q}/s_nT)(v_s-u)+6\pi\eta_nR(v_n-u)\,, \tag{2}$$

where we are using Stokes law for spheres of radius R.

As an example, reflecting experimental conditions, suppose $R=50\,\mu$, $T=1.5°\text{K}$, and let radiant heat per unit area supplied to the tracer particles be 10 mW cm^{-2}. Then, using the fact[6] that for the temperature in question $\eta_n\approx 10\,\mu P$, we readily find

$$u\cong(4/7)v_s+(3/7)v_n\,. \tag{3}$$

In this regard, it is well to recall that room-temperature blackbody radiation density corresponds to about 50 mW cm^{-2}, and that in several of the experiments[1,2] using tracer particles, external photographic lighting was employed.[7] Clearly, under such conditions, the heat-exchange is not negligible.

For a specified radiant power level, $\dot{Q}$ is proportional to the surface area of the trace particles, i.e., R^2, whereas the Stokes-law viscous drag of the normal fluid is proportional to R. Thus, for smaller particles, the normal fluid drag becomes relatively larger than the heat-exchange drag. The tracer particle velocity is, therefore, an inaccurate measure of *either* component velocity field unless account is taken of particle size. For this reason, measurements of circulation[1] in He II by the use of such tracer particles are probably unreliable.

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[1] W. A. Steyert, R. D. Taylor, and T. A. Kitchens, Phys. Rev. Letters **15**, 546 (1965).

[2] T. A. Kitchens, W. A. Steyert, R. D. Taylor, and P. P. Craig, Phys. Rev. Letters **14**, 942 (1965).

[3] D. Y. Chung and P. R. Critchlow, Phys. Rev. Letters **14**, 892 (1965).

[4] K. L. Chopra and J. B. Brown, Phys. Rev. **108**, 157 (1957).

[5] R. Penney, Phys. Fluids **10**, 2147 (1967).

[6] See, for example, F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1954), Vol. II, p. 68.

[7] The tracer particles employed in Refs. 1–4 were composed of frozen H-D gas and thus would not be expected to absorb significant energy in the visible. Pure solid H_2 and D_2 exhibit considerable absorption in the infrared from about 210 to 2.5 μ: see, for example, H. P. Gush *et al.*, Can J. Phys. **38**, 176 (1960). This fundamental absorption as well as any additional absorption due to trace impurities in the particles can readily account for heat input levels in the range of those discussed here.