introduced in Sec. V. The result of this work is to replace the Druyvesteyn[3] expression, Eq. (2.2), by

$$V(z)=V_{1s}(z)+V_{2s}(z)+V_{3s}(z)-(2\pi/16\eta)R^4(\partial/\partial z)[n_a(z)kT_a(z)]. \quad (6.1)$$

Evaluating the quantities appearing in this expression requires Eqs. (5.8)–(5.10), (2.9), (2.11), (2.12), (3.43), (3.34), (3.23), (3.24), and (3.41). The gas flow and pressure difference between the ends of the discharge tube may then be obtained from Eqs. (2.15), (2.18), and (2.19). The following paper[30] compares the theory developed here with experimental results in gas discharges.

ACKNOWLEDGMENTS

The author thanks E. I. Gordon for many significant comments and R. C. Miller for a detailed critical reading of this paper.

[30] Arthur N. Chester, following paper, Phys. Rev. **169**, 184 (1968).

PHYSICAL REVIEW VOLUME 169, NUMBER 1 5 MAY 1968

Experimental Measurements of Gas Pumping in an Argon Discharge

ARTHUR N. CHESTER
Bell Telephone Laboratories, Murray Hill, New Jersey 07971
(Received 14 March 1967; revised manuscript received 9 November 1967)

Measurements have been made of the pressure difference between the cathode and anode bottles of a standard cw argon ion laser tube, with a positive column of 30-cm length and 1.25-mm radius. At several amperes current the pressure difference (anode minus cathode) is approximately constant at 0.25 Torr below 1.2 Torr, and is inversely proportional to pressure above that pressure. The results are compared with the predictions of various theories. The theory presented in the preceding paper accurately predicts both the magnitude of the pumping effect and its dependence upon pressure and current.

I. INTRODUCTION

IT is well known that a pressure difference is developed between the anode and cathode in a dc gaseous plasma, but at the current densities usually used in past work this has been a small effect and difficult to measure. However, in high current density discharges (1 A/mm^2) in small bore tubes (a few millimeters radius) gas pumping by the plasma is so strong that an external gas return[1] connected between cathode and anode must be used to keep the discharge from extinguishing itself. Even with such a return path for the gas, the pressure at the anode often builds up to several times the cathode pressure.

Recent discharge diagnostic work on the argon ion laser provides a unique opportunity to study the gas pumping effect in detail at high current densities in small-radius tubes. Various groups have recently measured electron, ion, and neutral-atom kinetic temperatures, number density and radial distribution of ions and neutral atoms, and axial electric field under a great variety of discharge conditions. Such data make it possible to compare experimental measurements in argon with the predictions of various theories in some detail.

In Sec. II we discuss the range of discharge parameters for which theoretical treatments of the gas pumping effect should apply; these considerations show that the theory presented in the preceding paper should be applicable to the argon discharge. In Sec. III we gather together certain experimental measurements of discharge parameters which are seen to be necessary in calculations using the theory, and in Sec. IV we give specific numerical predictions for argon discharges in various pressure regimes. In Sec. V we describe the experimental tube used in the present investigation and compare measured gas pumping with the predictions of various theoretical models. A long-standing discrepancy between theory and early experimental measurements in argon is examined in the Appendix.

II. VALIDITY OF THEORETICAL MODELS IN ARGON DISCHARGES

It is desirable to determine the ranges of pressure, current, and tube radius for which various theoretical models for the gas pumping effect may apply, since direct comparison of theory and experiment is to be undertaken. Relevant theoretical treatments of the problem have been given by Langmuir,[2] Druyvesteyn,[3]

[1] E. I. Gordon and E. F. Labuda, Bell System Tech. J. **43**, 1827 (1964).

[2] Irving Langmuir, J. Franklin Inst. **196**, 751 (1923).

[3] M. J. Druyvesteyn, Physica **2**, 255 (1935).

Halsted,[4,5] Leiby,[6,7] and the author,[8] and are summarized in the preceding paper.[8]

First determine the regime in which the viscous-flow assumption applies. At higher pressures the viscosity of argon is given by Kaye and Laby[9] as

$$\eta_{\mathrm{Ar}}=3.32\times10^{-5}(T/300)^{3/2}(0.473+T/300)^{-1}\quad \mathrm{kg/m\ sec} \quad (2.1)$$

for gas at kinetic temperature T.

To determine the pressure at which molecular flow becomes a better description of gas motion, one may define a pressure-dependent "effective viscosity" η' to be that value of viscosity which, when used in the Poiseuille equation[10] for viscous flow through a tube, gives a gas flow rate which agrees with that calculated from the Knudson formula[11] for molecular flow. The value of η' for argon turns out to be

$$\eta'_{\mathrm{Ar}}(\text{low pressures})=6.92\times10^{-5}(T/300°\mathrm{K})^{-1/2}pR\quad \mathrm{kg/m\ sec}, \quad (2.2)$$

if p is in Torr and R in mm. Comparing Eqs. (2.1) and (2.2) suggests that viscous flow is a better approximation than molecular flow in argon when the pressure-radius product is numerically greater than about $0.48(T/300°\mathrm{K})^2(0.473+T/300)^{-1}$ Torr mm. The quantity p will always be taken here to mean true gas pressure. Note that in the viscous-flow regime the true atom number density is proportional to the "reduced pressure" p_0 defined by

$$p_0\equiv(300°\mathrm{K}/T)p. \quad (2.3)$$

Another parameter relevant to the validity of the theories is the ion momentum-transfer mean free path λ_+ for low-energy (thermal) ions. This may be obtained by making use of the low-field values of ion mobility measured by Hornbeck, Varney, and Frost[12]:

$$\mu_+=0.124(T/300°\mathrm{K})^{1/2}/p \quad \mathrm{m^2/V\ sec} \quad (2.4)$$

(p in Torr). The low-field value of ion mobility applies for E/p_0, $E_r/p_0<20$ V/Torr cm, with E and E_r denoting the axial and radial electric field. Equation (2.4) assumes a temperature dependence for the mobility as given by the Langevin equation[13] and makes use of Eq. (2.3) as well. Although measurements by Chanin and Biondi[14] below room temperature suggest a somewhat slower variation with temperature, the temperature dependence of the measured electron density[15] in argon discharges is consistent with just the temperature dependence expressed by (2.4). Equation (2.4) is used with Holstein's formula[16] relating mobility to mean free path to obtain

$$\lambda_+=0.0195(T/300°\mathrm{K})/p \ \mathrm{mm} \quad (2.5)$$

(p in Torr), valid at least for thermal ions ($kT_i\sim0.025-0.15$ eV). The author has independently estimated the mean free path by comparing Webb's optical measurements of ion radial distribution[17] with radial profiles calculated by Self and Ewald[18] and finds values which are also in agreement with Eq. (2.5).

Webb's measurements of radial distribution[17] were also carried out for neutral atoms. His finding that the neutral-atom density is very accurately independent of radial position validates still another assumption of the gas pumping theories for argon ionic discharges.

Another quantity of concern is the size of the sheath thickness κ relative to the tube radius R. Calculation of the sheath thickness requires knowledge of the electron number density; this calculation will be deferred until the next section, it being sufficient to state at this point that $\kappa\ll R$ may be assumed for the pressure and current range which will be of interest here.

Finally, charge-exchange collisions between ions and neutral atoms must be considered. The momentum-transfer mean free path for ions against elastic collisions, λ_{el}, is[19]

$$\lambda_{\mathrm{el}}\approx0.05(T/300°\mathrm{K})/p \ \mathrm{mm} \quad (2.6)$$

(p in Torr) in argon, at thermal velocities. The mean free path for neutral atoms, λ_a, is almost the same[20]:

$$\lambda_a=0.0476(T/300°\mathrm{K})/p \ \mathrm{mm} \quad (2.7)$$

(p in Torr). The approximate equality of these quantities permits some simplifications in the theory, as presented in the preceding paper.

4 W. B. Bridges and A. S. Halsted, Hughes Research Laboratories Technical Report No. AFAL-TR-67-89, 1967, pp. 171–209 (unpublished).

5 W. B. Bridges, P. O. Clark, and A. S. Halsted, J. Quantum Electron. **QE-2**, xix, No. 3B-1 (April 1966).

6 C. C. Leiby, Jr. and H. J. Oskam, Phys. Fluids, **10**, 1992 (1967).

7 C. C. Leiby, Jr., and H. J. Oskam, in Proceedings of the Eighth International Ionization Conference, Vienna, 1967 (unpublished).

8 Arthur N. Chester, preceding paper, Phys. Rev. **169**, 172 (1968).

9 G. W. C. Kaye and T. H. Laby, *Tables of Physical and Chemical Constants* (John Wiley & Sons, Inc., New York, 1960), p. 38.

10 Saul Dushman, *Scientific Foundations of Vacuum Technique* (John Wiley & Sons, Inc., New York, 1962), p. 82.

11 John Strong, *Procedures in Experimental Physics* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1938), p. 99.

12 Earl W. McDaniel, *Collision Phenomena in Ionized Gases* (John Wiley & Sons, Inc., New York, 1964), pp. 466, 470.

13 Earl W. McDaniel, *Collision Phenomena in Ionized Gases* (John Wiley & Sons, Inc., New York, 1964), p. 429.

14 Earl W. McDaniel, *Collision Phenomenon in Ionized Gases* (John Wiley & Sons, Inc., New York, 1964), p. 471.

15 E. F. Labuda, C. E. Webb, R. C. Miller, and E. I. Gordon, Bull. Am. Phys. Soc. **11**, 497 (1966).

16 T. Holstein, J. Phys. Chem. **56**, 832 (1952).

17 C. E. Webb, in Proceedings of the Nineteenth Annual Gaseous Electronics Conference, Atlanta, Georgia, 1966 (to be published).

18 S. A. Self and H. N. Ewald, Phys. Fluids **9**, 2486 (1966).

19 Measurements by W. H. Cramer in *Collision Phenomena in Ionized Gases*, edited by Earl W. McDaniel (John Wiley & Sons, Inc., New York, 1964), p. 164.

20 G. W. C. Kaye and T. H. Laby, *Tables of Physical and Chemical Constants* (John Wiley & Sons, Inc., New York, 1960), p. 39. See also J. H. Jeans, *The Dynamical Theory of Gases* (Dover Publications, Inc., New York, 1954), p. 327.

Thus the requirements of the theoretical treatments, namely viscous flow, ion mean free path small compared with tube radius, and neutral-atom density constant as a function of radial position, are satisfied in argon for values of pR greater than $\sim$0.5 Torr mm at room temperature, and greater than $\sim$2.5 Torr mm at $T=1500°K$. Therefore the theoretical results obtained in the preceding paper may be applied to predict gas pumping effects in argon in this pressure range.

III. PLASMA MEASUREMENTS IN ARGON REQUIRED IN PREDICTING GAS PUMPING

To make specific theoretical predictions about the magnitude of the gas pumping effect certain fundamental plasma quantities must be known besides those summarized in Sec. II: the mean electron density, the axial electric field, and the kinetic temperatures of ions, neutral atoms, and electrons.

Unfortunately, measurements of many parameters in argon become easy to interpret only above what is generally referred to as the "discharge discontinuity." This is a particular value of discharge current, varying with tube radius and filling pressure, at which a discontinuity appears in the V-I characteristic of the tube. Simultaneously discontinuities appear in many other discharge properties, including the gas pumping effect and the intensity of light emission in spectral lines. The axial electric field and the electron temperature are strongly dependent on current below the discharge discontinuity. Similar effects also occur in other noble gas discharges.

Since some discharge parameters have been measured with confidence only above the discharge discontinuity, comparisons of gas pumping experiments with theory will be carried out only above the discontinuity, although the theory should be applicable to lower currents as well if sufficiently accurate measurements of electron temperature, electron density, and so forth become available. The discontinuity occurs at current densities of a few tenths of amperes per square millimeter in the pR range from 1 to 5 Torr mm, and its position will be quite apparent in the experimental V-I curves shown later.

Confining the theory to these higher discharge currents has the added advantage of allowing quite simple equations to be used to summarize the theoretical measurements of axial field and electron density. Experimental measurements of these quantities by Webb, Labuda, and Miller may be summarized as

$$E \approx 615 R^{-1} \text{ V/m} \tag{3.1}$$

(R in mm) for the axial electric field,[21,22] and

$$n_e = 1.0\times 10^{20} JRp(300°\text{K}/T_a)^{1/2} \text{ m}^{-3} \tag{3.2}$$

(J in A/mm^2, R in mm, p in Torr) for the mean electron number density.[15] The current density in the discharge is J and T_a is the atom kinetic temperature. Equation (3.1) neglects a small observed dependence of the axial field on current, which occurs even above the discharge discontinuity. This appears as a pressure-dependent resistance and its effects will be mentioned as a correction to the gas pumping theory in the section describing the experiments. The dependence of Eq. (3.2) on current density and gas temperature is consistent with the results that would be obtained theoretically, considering the energy balance in the discharge after the manner of Eqs. (50.10) and (50.11) in Francis,[23] and using Eq. (2.4) of this paper for the ion mobility.

The last quantities which it is necessary to mention are the electron, ion, and neutral-atom kinetic temperatures T_e, T_i, and T_a.

Electron temperatures measured by Webb[15] in tubes of radii 1 and 1.5 mm range from 1.0 to 2.0 eV in the pressure range 2 to 20 Torr. They decrease slightly as p_0 increases, and are consistent with predicted values obtained by modifying Francis's treatment to include the important effects caused by ionization from neutral metastable levels. Space prohibits giving details of this comparison here. For most purposes the reader may estimate an electron temperature of 1.5 eV without being too far wrong, although more exact experimental values will be used in the theoretical predictions of Sec. V.

The ion and neutral-atom kinetic temperatures are complicated functions of pressure, current, and discharge tube radius. Measurements have been carried out by Webb, Labuda, and Miller[24] and by Ballik, Bennett, and Mercer,[25] by measuring the Gaussian component of spectral lines. For tubes of a few millimeters diameter and current densities up to about 1 A/mm^2 (but above the discharge discontinuity), the atom and ion temperatures are given by

$$T_a/300°\text{K} \approx T_i/300°\text{K} \approx 1+0.9I/R^{3/2} \tag{3.3}$$

(I in A, R in mm). This empirical formula is only approximately correct, and becomes a poor estimate at higher currents, where saturation appears to set in. However, at the current densities used in the present experimental work this equation is adequate. The temperature measurements of Webb, Labuda, and Miller, as well as the gas pumping measurements reported in this paper, were carried out in discharge tubes having water-cooled quartz bores of 1-mm thickness, and the parameters in Eq. (3.3) may not apply to tubes of other designs.

---

[21] E. I. Gordon, E. F. Labuda, Richard C. Miller, and C. E. Webb, in *Proceedings of the Physics of Quantum Electronics Conference, San Juan, Puerto Rico, 1965*, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill Publishing Co., Inc., New York, 1965).

[22] R. C. Miller (private communication).

[23] Gordon Francis, in *Handbook der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. XXII, p. 124.

[24] C. E. Webb, E. F. Labuda, and R. C. Miller (to be published).

[25] E. A. Ballik, W. R. Bennett, Jr., and G. N. Mercer, Appl. Phys. Letters 8, 214 (1966).

It is now possible to calculate the ion-sheath thickness by making use of Eqs. (2.4), (3.2), and (3.3). The required expressions are Child's equation[26] and the formula

$$i_w^+ = 18.2\mu_+ kT_e n_e \tag{3.4}$$

for the wall current per unit length, obtained from Francis.[23]

The result is finally that

$$\kappa/R \approx 0.00254 V_e^{1/4} I^{-1/2} \tag{3.5}$$

(V_e in V, I in A). Thus $\kappa \ll R$ for currents above the discharge discontinuity, as indicated in the previous section.

IV. PREDICTED GAS PUMPING IN ARGON

The theory of the preceding paper[8] may be used with the experimental results quoted in Secs. II and III to derive predictions for the gas pumping effect in argon. It is valid to neglect V_1 and the electron radial drift velocity and to assume that $\Delta p \ll p$ for that part of the pressure and current regime covered by the present experiments for which the theory is valid. The results are given as follows.

$$p_{\text{anode}} - p_{\text{cathode}} \equiv \Delta p\,, \tag{4.1}$$

and

$$\Delta p(\text{Torr}) = 8\eta_{\text{ret}} L_{\text{ret}} / \pi R_{\text{ret}}^4 n_{\text{ret}} kT_0 = 3.84\times 10^5 L_{\text{ret}} \Phi / p R_{\text{ret}}^4 \tag{4.2}$$

(L_{ret} in m, Φ in Torr m^3/sec, p in Torr, R_{ret} in mm). The quantities η_{ret}, n_{ret}, L_{ret}, and R_{ret} refer to gas viscosity, gas number density, length, and radius in a gas return path external to the discharge, if one is used. The pressure in the positive column is p, and the gas flux Φ is given by

$$\Phi = (T_0/T_a) p (1+c)^{-1} [V_{2s} + V_3 - (n_i/n_a) v_{di} \pi R^2]\,, \tag{4.3}$$

where T_a and T_0 are the temperatures of the gas in the discharge and in the gas return path, respectively. The mean ion and atom number densities are n_i and n_a, and v_{di} is the axial ion drift velocity ($v_{di} = \mu_+ E$). The quantity c is essentially the conductance of the tube bore for gas flow relative to that of the gas return tube:

$$c = (\eta_{\text{ret}}/\eta)(R/R_{\text{ret}})^4 (L_{\text{ret}}/L)(T_0/T_a)\,, \tag{4.4}$$

in which R and L are the discharge tube radius and length, and η and η_{ret} are the gas viscosities in the positive column and in the gas return. V_{2s} is given by

$$V_{2s} = (1 + z + 0.5z^2)\exp(-z) V_2\,, \tag{4.5}$$

with

$$z = 0.16\kappa/\lambda_{\text{el}} = 0.0081 V_e^{1/4} I^{-1/2} (300°\text{K}/T_a) pR \tag{4.6}$$

(V_e in V, I in A, p in Torr, R in mm) in argon. V_2 is given by

$$V_2 = (2\pi e E/\eta)(2.32) n_e R^4 f(\lambda_{\text{el}}/R) = 1.37\times 10^{-3} IR \times pR(0.473 + T_a/300°\text{K}) \times (300°\text{K}/T_a)^2 f(\lambda_{\text{el}}/R) \text{ m}^3/\text{sec} \tag{4.7}$$

(I in A, R in mm, p in Torr) in argon. The function f is

$$f(\lambda_{\text{el}}/R) = (\tfrac{1}{4}R^4) \int_0^R (R^2 - r^2) r J_0(2.4r/R) \times \exp[-(R-r)/\lambda_{\text{el}}] dr\,, \tag{4.8}$$

and is graphed in the preceding paper. V_3 is given by

$$V_3 = (2\pi e E/\eta)(2.32) n_e (0.054 R^3 \lambda_{\text{el}}) \times [\Delta(\bar{x}) - 0.866\Delta(0.193\bar{x})] = 0.371\times 10^{-5} IR(0.473 + T_a/300°\text{K})(300°\text{K}/T_a) \times [\Delta(\bar{x}) - 0.866\Delta(0.193\bar{x})] \text{ m}^3/\text{sec} \tag{4.9}$$

(I in A, R in mm). $\Delta(x)$ is defined by

$$\Delta(x) = (2/x\sqrt{\pi})\exp(-x^2) + (2 - x^{-2})\,\text{erf}x\,, \tag{4.10}$$

and is plotted in the preceding paper. In argon,

$$\bar{x} = 1.41 V_e / pR \tag{4.11}$$,

(V_e in V, p in Torr, R in mm), with $V_e = kT_e/e$. Finally, the ion transport term in argon is given by

$$(n_i/n_a) v_{di} \pi R^2 = 0.22\times 10^{-6} I (T_a/300°\text{K})/p \text{ m}^3/\text{sec} \tag{4.12}$$

(I in A, p in Torr), at pressures where Eq. (2.4) applies. For sufficiently low pressures ($p_0 R \lesssim 0.1$), the ion mobility is limited by wall collisions rather than by collisions with atoms, and Eq. (4.12) overestimates the gas pumping due to ion transport.

V. EXPERIMENTAL MEASUREMENTS IN ARGON

Experimental measurements of gas pumping in an argon discharge have been carried out to test the predictions of Sec. IV. The tube used was basically a water-cooled argon laser tube of the type illustrated in Ref. 1. The quartz discharge capillary was 30 cm long and 1.25-mm inner radius, with 1-mm-thick walls. The cathode and anode bottles were each roughly 20 cm long by 5 cm in diam. Gas had to flow through most of the volume of the electrode bottles before entering the gas return. The gas return consisted mainly of 61 cm of tubing of 2-mm inner radius. It was interrupted by 17.5 cm of 3-mm inner-radius tubing and by a glass stopcock whose passage added 2 cm of tube of 1-mm inner radius. In Eqs. (4.2) and (4.4), one therefore takes

$$L_{\text{ret}}/R_{\text{ret}}^4 = 0.61/16 + 0.175/81 + 0.02/1 = 0.060$$

(L_{ret} in m, R_{ret} in mm).

[26] James Dillon Cobine, *Gaseous Conductors* (Dover Publications, Inc., New York, 1958), pp. 125, 126, and 136. Quoted as Eq. (5.1) in Ref. 8.

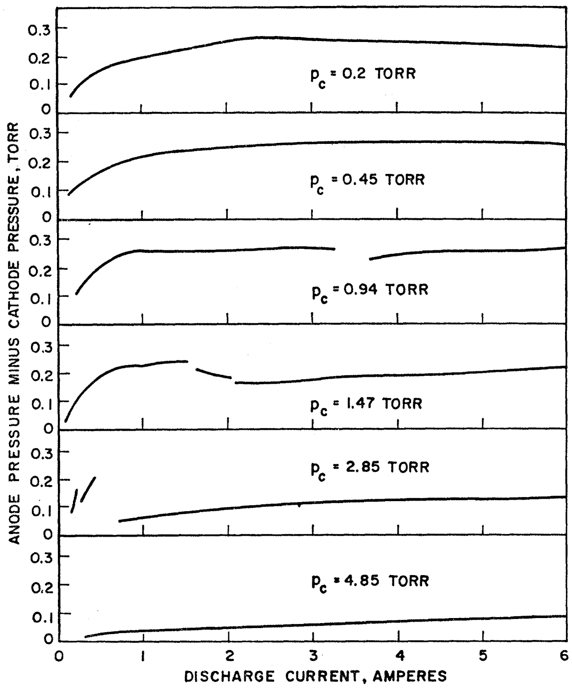

FIG. 1. Experimental measurements of anode pressure minus cathode pressure (Δp) as a function of discharge current I for various cathode pressures p_c.

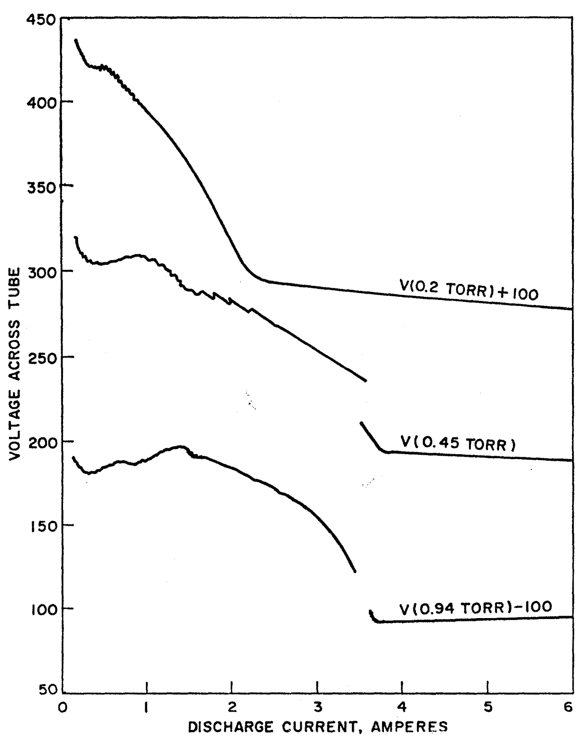

FIG. 2. Voltage drop across discharge tube as a function of current for various cathode pressures.

A capacitance manometer sensing head was connected between the electrode bottles, and separate thermocouple vacuum gauges were connected to each bottle. The tube was connected to the vacuum station at the cathode end, thus providing a large cold volume which served to hold the pressure at the cathode end constant. (If the valve leading to the vacuum station was closed when the tube was operating at lower pressures, the pressure in the entire tube could be observed to rise some 10 or 20% despite the quite large cold volumes remaining in the tubing leading to the vacuum measuring devices.) The pressure could also be monitored at the cathode by a 0.005–5-Torr McLeod gauge mounted on the vacuum station.

The thermocouple gauges were first calibrated against the McLeod gauge. The capacitance manometer was then calibrated against the thermocouple by

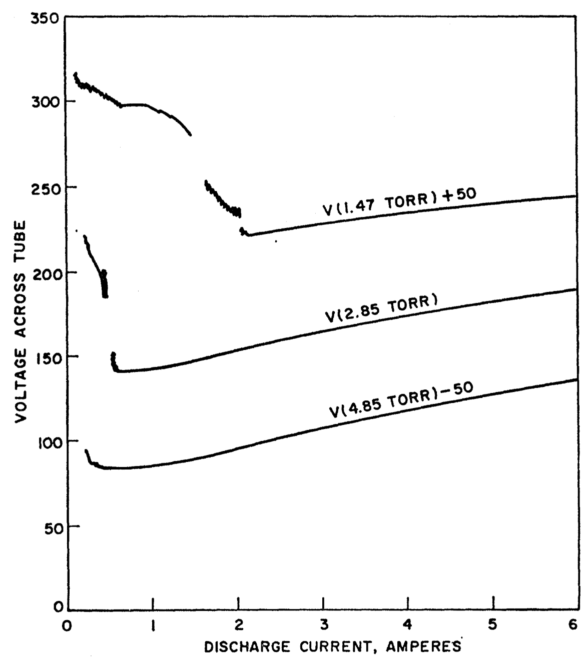

FIG. 3. Voltage drop across discharge tube as a function of current for various cathode pressures.

running a gas discharge to establish pressure differences. The standard deviation of the calibration factor obtained in this manner was 4.7%. Therefore, the absolute magnitude of the pressure differences measured may be in error by this amount, although their relative values are expected to be more accurate (1–2%).

It was found that the pressure at the cathode was not precisely constant at the value to which the system was filled. This was apparently due to a combination of gas pumping away from the cathode and over-all gas heating in the system, since the pressure would return to its initial value when the discharge was extinguished. However, the cathode pressure was monitored at all times by the cathode thermocouple gauge and its change never exceeded a few percent (plus or minus).

The data obtained are plotted in Fig. 1. The largest series ballast resistor usable with the available power supply, 400Ω, was used near the discharge discontinuity to observe as much structure as possible. The effect of the discharge discontinuity is quite apparent at the intermediate pressures, and so is the existence of other discontinuous changes at lower values of the current. Gas pumping was so pronounced that the long times required for the pressure to stabilize made it impossible to plot the pressure difference as the current was continuously varied. Instead, at each value of current the pressure difference was allowed to stabilize (which, near a discontinuity, might take several minutes) before taking a reading. With each fill a *V*-*I* curve was initially taken to establish the position of the discharge discontinuities. These data appear in Figs. 2 and 3.

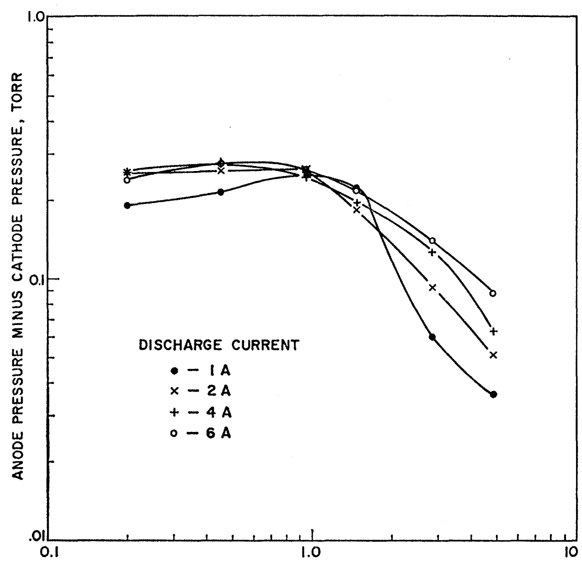

FIG. 4. Experimentally measured pressure difference as a function of cathode pressure.

It can be seen that discontinuities in the *V*-*I* curve often correspond to discontinuities in the pressure difference, which should not be surprising since the pressure difference depends so directly on the discharge parameters. The periodic structure in the *V*-*I* curves for 0.45 Torr around 2 A was repeatable, but its origin is not known. Similar series of jumps in the tube voltage have been observed by Webb, Labuda, and Miller.[24]

The data are replotted in Fig. 4 to exhibit the pressure dependence. The curves are quite regular except for the distortion in the 1-A curve caused by the presence of the discharge discontinuity (and to a small extent in the 2-A curve). If this distortion is neglected, the pressure dependence can be seen to be almost the

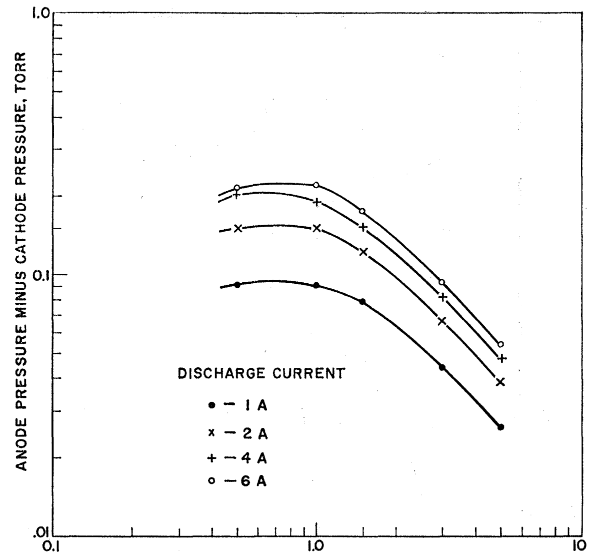

FIG. 5. Pressure difference predicted by the complete theory.

same at all currents, being roughly independent of p below $\sim$1.2 Torr and varying as p^{-1} above that pressure.

The predictions of the gas pumping theory as set forth in Eqs. (4.1–12) give the curves shown in Fig. 5 for the experimental tube used.

An interesting result is obtained if one uses instead of Eq. (4.3) the simplified form

$$\Phi = (T_0/T_a)p(1+c)^{-1}(2\pi eE/\eta) \times (2.32)n_e(0.054R^3\lambda_{e1})\Delta(\bar{x}). \quad (5.1)$$

This amounts to taking all the gas pumping as due to the new force term V_3, neglecting entirely the tradi-

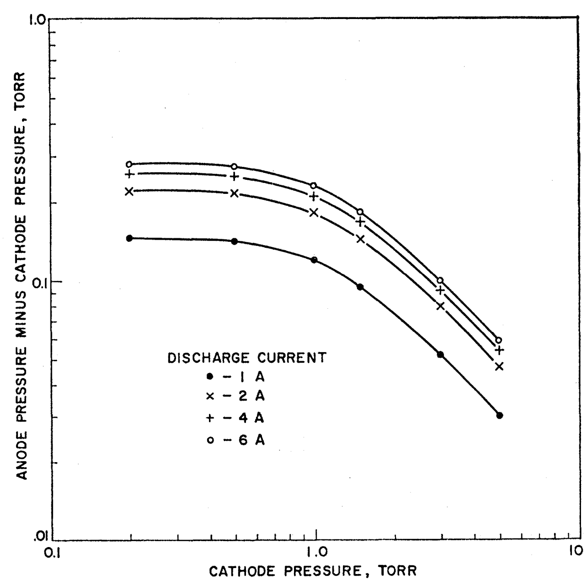

FIG. 6. Pressure difference predicted by a simplified theory including only the new force.

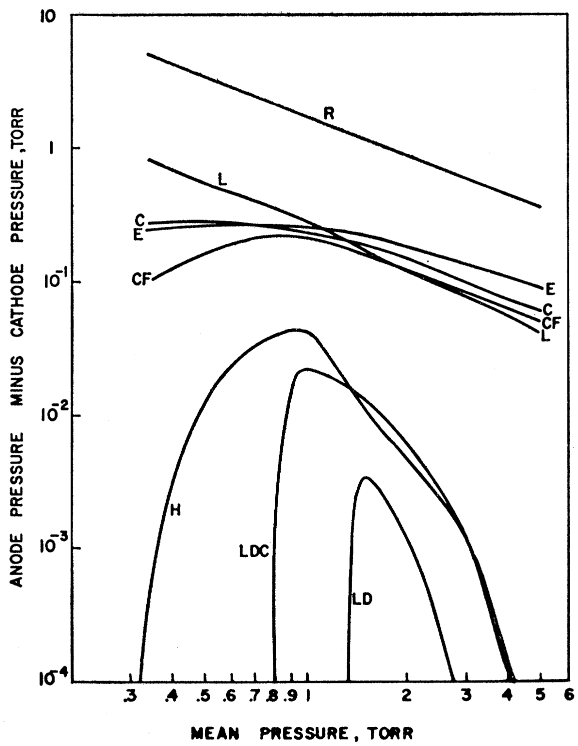

FIG. 7. Comparison of various theories with experimental results obtained at $I=6$ A. E, experiment; LD, original Langmuir-Druyvesteyn theory; LDC, Langmuir-Druyvesteyn theory with algebraic errors corrected; R, Rüttenauer's semiempirical formula, altered to allow for gas return path; H, Halsted theory; L, Leiby theory; CF, present theoretical predictions; C, simplified theory including only the new volume force (as in Fig. 6).

tionally used force terms and the "ionization correction" [which gives the expression in brackets in Eq. (4.9) instead of just $\Delta(\bar{x})$]. This drastically simplified form leads to the predictions shown in Fig. 6, not too different from those of Fig. 5 and even somewhat closer to the experimental results in Fig. 4. This similarity is deceptive, because at the lower pressures both V_3 and the "ionization correction" are quite significant; however, it happens that they largely compensate for one another, being of opposite algebraic sign, so that the net effect of including them both is not very large.

In comparing Fig. 5 with the experimental results in Fig. 4 it should be kept in mind that because of the viscous flow restriction the present theory may fail at values of pR below ~ 1 Torr mm, and should be fully applicable only at the highest pressures used in the experimental work. Despite this qualification, the agreement is quite good, regarding both functional form and absolute magnitude. The p^0 and p^{-1} forms below and above about 1.2 Torr agree precisely with the theory; they arise from the term V_3, as Fig. 6 shows. The current dependence predictions are also in a good agreement at the higher pressures. The theory predicts a dominant pressure-current dependence of the form

$$\Delta p \propto I(1.473+0.64I)/(1+0.64I)^2 p \qquad (5.2)$$

(p and Δp in Torr, I in A) for the experimental tube at high pressures. This approximates the shape of the experimental curves in Fig. 1 quite well at 4.85 and 2.85 Torr, and also fits those portions of the 1.47- and 0.94-Torr curves that lie above the discharge discontinuity. The departure at lower pressures is that the experimental curves are more closely clustered together at the higher currents than the theoretical ones.

The effect of allowing for the variation of axial electric field E with current may be seen by examining the V-I curves in Figs. 2 and 3. If Eq. (3.1) is used for the axial field just above the discharge discontinuity, it is seen that the electric field rises about 30% between 1 and 6 A at the highest pressure (4.85 Torr). The slope of the V-I curve decreases gradually with decreasing pressure, becoming negative at pressures below 0.5 Torr. (At the lowest pressure used, 0.2 Torr, the electric field falls by about 10% between currents of 2.5 and 6 A.) Since the pressure difference depends linearly upon the axial electric field, this would have the effect of spreading out the curves of Fig. 4 at high pressures, since at higher pressures the higher-current curves correspond to an increasing high electric field. The curves of Fig. 4 show just a hint of this spread from 2.85 to 4.85 Torr, which might represent the effects of this increasing axial field.

The theory should also be applicable at lower pressures if certain changes are made. One must use the low-pressure "viscosity" defined by Eq. (2.2); Eq. (2.3) is replaced by the relation appropriate to the molecular flow regime; Eq. (2.4) is replaced by an expression giving the high-field ion mobility, limited by wall collisions; and Eq. (3.2) must be replaced by an equation more suitable at low pressures.

Although no attempt has been made to calculate the *magnitude* of the predicted pumping at low pressures, these changes do predict the pressure-current dependence

$$\Delta p \propto I T_a^{-1/2} p^0, \qquad (5.3)$$

including only V_3 and the ion transport term, since V_2 is absent at low pressures.[8] This expression neglects some weaker variations with current and pressure due to changes in the electron temperature. The agreement in functional form with the low-pressure experimental data in Figs. 1 and 4 seems satisfactory, and it is therefore reasonable to hope that the theory of the preceding paper may be successfully extended to low pressures by making these changes. The principal difficulties with an accurate treatment of this regime are: (1) finding the ion mobility will involve a tedious calculation; and (2) experimental measurements of electron temperature and number density are not yet available at these pressures.

In Fig. 7 the predictions of all previous treatments known to the author are compared to the present experimental measurements at a representative value of discharge current, $I=6$ A. The theories compared are the Langmuir-Druyvesteyn treatment, in its original form[3] and with an algebraic error corrected[8]; the semi-empirical formula of Rüttenauer,[27] altered to allow for a gas return path; Halsted's[4] and Leiby's[6] treatments; and the predictions of the present theory,[8] as previously shown in Figs. 5 and 6. The changes required at lower pressures are not indicated but, as indicated in Eq. (5.3), such extension would probably improve agreement with experiment.

Finally, in the Appendix we discuss early experimental work in gas pumping and the origin of Rüttenauer's formula. A rather complete list of other experimental work is given by Leiby,[6] and Halsted has recently made a great number of measurements[4,28] in the noble gases. It is felt that comparison of these measurements with the present theory would be desirable for those gas systems in which the electron temperature and number density are known.

ACKNOWLEDGMENTS

The author is grateful to R. C. Miller and C. E. Webb for many discussions of their experimental work in argon discharges, to C. C. Leiby, Jr., for discussion of his work in advance of publication, and to E. W. Chapman for technical assistance.

APPENDIX: EARLY EXPERIMENTAL WORK IN GAS PUMPING

The early experimental work on gas pumping in discharge tubes was usually carried out in tubes of about 1-cm i.d., at pressures of the order of 1 Torr and at currents of about 1 A. These workers had no occasion to employ a gas return tube. Druyvesteyn[3] states that the most complete results were obtained by Rüttenauer,[27] who found that the pressure difference between the ends of a discharge tube containing He, Ne, or Ar could be approximately written

$$\Delta p/L = 0.24 IEM^{1/2}/pR^4 \tag{A1}$$

(p and Δp in Torr, L in m, I in A, E in V/m, R in mm), in which M is the molecular weight of the gas and p the average pressure. The other symbols have been previously defined. At the values of pR and I used by Rüttenauer, $\Delta p \ll p$. Druyvesteyn's theory neglects V_3 and therefore predicts

$$\Delta p = (8\eta L/\pi R^4) V_2, \tag{A2}$$

in which V_2 is given from Eqs. (4.7) and (4.8) by its high-pressure form:

$$V_2 = (2\pi/\eta) eE(2.32) n_e (1.25\lambda_+^3 R). \tag{A3}$$

Rewrite this, using

$$J = I/\pi R^2 = n_e eE\mu_- \tag{A4}$$

to get

$$\Delta p/L = 14.7 I\lambda_+^3/\mu_- R^5. \tag{A5}$$

In this range of pressures, λ_+ and μ_- are inversely proportional to the pressure p, so that Druyvesteyn's theory predicts

$$\Delta p/L \propto I/p^2 R^5, \tag{A6}$$

as he correctly points out.

Druyvesteyn then states that Rüttenauer's results, Eq. (A1), give $\Delta p/L \propto I/pR^4$ and therefore disagree with his predictions by a factor of pR. However, Druyvesteyn failed to take into account the variation of E with R. For example, suppose that E varies as $1/R$ [this is not only true for argon above the discharge discontinuity as shown by Eq. (3.1), but represents the principal R dependence in Schottky's formula for the axial field as quoted by Francis[23]]. Then in this case Rüttenauer's results would really give $\Delta p/L \propto I/pR^5$ and would only disagree with Druyvesteyn's formula by a factor of p.

On the other hand, the present theory predicts instead that the term V_3 dominates in this range of pR, giving approximately

$$\Delta p = (8\eta L/\pi R^4) V_3, \tag{A7}$$

in which the appropriate form for V_3 at these high values of pR is the small x form obtained from Eqs. (4.9) and (4.10):

$$V_3 = (5/6)(2\pi/\eta) eE(2.32) n_e (0.0896 R^3)(1.51\lambda_{el}) \times (\mu_+/ev_{thi}) kT_e (2.4048/R). \tag{A8}$$

Again use Eq. (A4) to obtain

$$\Delta p/L = 3.19 I\lambda_{el}\mu_+ V_e/\mu_- v_{thi} R^4. \tag{A9}$$

Consider now the degree of agreement between Eq. (A9) and the experimental work leading to Rüttenauer's result, Eq. (A1). Rüttenauer's work was carried out in four discharge tubes, three of inside radius 3.8 mm and one of radius 8 mm. He studied helium, neon, and argon in the pressure range 0.5–0.9 Torr.

The pressure dependence of Eq. (A9) is $1/p$, in agreement with Rüttenauer's results. He also quotes earlier work in argon extending to higher pressures (0.4 to 3 Torr) which also obtained this pressure dependence, so the earlier work seems to agree with the present theory in pressure dependence.

The linear variation of Δp with I (at sufficiently low current densities that the gas is not heated too far above room temperature) is also predicted by Eq. (A9). This behavior was observed by Rüttenauer, as

[27] A. Rüttenauer, Z. Physik 10, 269–274 (1922).

[28] Halsted finds fair agreement with a theory that neglects the term V_3. However, he works at current densities between 0.7 and 3.5 A/mm², and Eqs. (4.7), (4.8), (4.7)–(4.10) indicate that V_3 becomes less important at higher values of T_a, so that a theory neglecting this term is not too bad an approximation at high currents. A more accurate comparison of his experimental results with the theory presented here requires knowledge of the gas temperature as a function of current in their discharge tubes, and this is not available.

Eq. (A1) shows, and he quotes earlier work in nitrogen and argon which also showed linearity in discharge current.

To discuss the radius dependence of the pressure differences observed by Rüttenauer, it is preferable to examine his experimental data directly, rather than to rely on formula (A1). The reason for doing this is that his measurements comparing radius dependence were done at current densities around 0.01 A/mm^2, using only argon and only the two different radii just mentioned. In the smaller tube, he operated at pressures such that pR ranged from 2.45 to 2.95 Torr mm, while in the larger tube pR had values 4 and 6 Torr mm. The axial electric field he measured, however, was about three times as large in the smaller tube as in the larger one, rather than just twice as large, as one would expect from Eq. (3.1). But Eq. (3.1) is only valid above the discharge discontinuity, and Rüttenauer's work took place rather close to the discharge discontinuity, so it is not surprising that the electric field was seen to fall more strongly than $1/R$ as pR was increased at fixed current.

The radius dependence quoted by Rüttenauer was determined from four measurements in the 8-mm tube and three in the 3.8-mm tube. The pressure differences observed in the larger tube ranged from 0.01 to 0.25 Torr, with a claimed measurement accuracy of 0.004 Torr, but each datum point represents the average of a series of measurements. The pressure difference was obtained by reading McLeod gauges attached to cathode and anode bottles and subtracting their readings. To extract the radius dependence, compute the quantity $p\Delta p/IL$ from the data given by Rüttenauer. One finds

$$p\Delta p/IL=(2.86\pm0.3)\times10^{-5}, \tag{A10}$$

when $R=8$ mm, and

$$p\Delta p/IL=(143\pm3.7)\times10^{-5} \tag{A11}$$

when $R=3.8$ mm (p and Δp in Torr, I in A, L in cm). Thus this combination of measured quantities increased by a factor of 50 in going to the smaller tube.

Because the axial field increases so much (E/p, as measured by Rüttenauer, is about 1 V/Torr cm in the 8-mm tube and about 3 V/Torr cm in the 3.8-mm tube), at the lower values of pR the change of electron mobility due to the field must be included in Eq. (A9). From the mobility data presented by McDaniel,[29]

$$\mu_-=45/p \text{ m}^2/\text{V sec}, \tag{A12}$$

when $R=8$ mm and

$$\mu_-=31/p \text{ m}^2/\text{V sec}, \tag{A13}$$

when $R=3.8$ mm (p in Torr). The ion mobility remains just proportional to $1/p$ at these values of axial field.

[29] Earl W. McDaniel, *Collision Phenomena in Ionized Gases* (John Wiley & Sons, Inc., New York, 1964), p. 545.

Now make use of Eqs. (2.4) and (2.6), the room-temperature value $v_{\text{th}\,i}=354$ m/sec, and the expression

$$1 \text{ Torr}=147 \text{ kg/m sec}^2. \tag{A14}$$

One then finds that Eq. (A9) predicts

$$p\Delta p/IL=2.08V_e\times10^{-5}, \tag{A15}$$

when $R=8$ mm, and

$$p\Delta p/IL=59.2V_e\times10^{-5}, \tag{A16}$$

when $R=3.8$ mm (p and Δp in Torr, I in A, L in cm, V_e in V). These are to be compared with the expressions obtained from Rüttenauer's experimental measurements, Eqs. (A10) and (A11), respectively.

In the larger-radius tube one is more likely to be above the discharge discontinuity and therefore one expects electron temperatures of the order of 1 V. It is therefore encouraging that an electron temperature $V_e=1.375$ V makes Eqs. (A10) and (A15) agree exactly. Agreement between Eqs. (A11) and (A16) requires a higher electron temperature: $V_e=2.42$ V. However, in view of earlier remarks about electron temperatures at lower currents close to the discharge discontinuity, this value does not seem unreasonable. Thus the present theory gives a good prediction of the absolute magnitude of the pressure effects measured by Rüttenauer.

One may also compare the present theory [Eq. (A9)] with Rüttenauer's formula [Eq. (A1)]. The reason Rüttenauer tried to fit his data to a formula such as Eq. (A1) was that a then popular theory involving gas transport by the motion of negative argon ions predicted such a form for the pressure difference. Roughly speaking, as pointed out previously, Eq. (A1) gives a $1/R^5$ dependence on radius. However, formula (A9) gives $1/R^4$. The point to make here is that the electron temperature differed by a factor of $2.42/1.375=1.76$ in Rüttenauer's two tubes. His ratio of radii was $8/3.8=2.1$, not too different from 1.76, and moreover he was *looking* for a $1/R^5$ dependence instead of the $1/R^4$ of Eq. (A9). Thus it is proposed here that the change in electron temperature between his two tubes effectively added about a $1/R$ dependence, so that he was able to interpret the dependence V_e/R^4 as $1/R^5$.

Comparison with Rüttenauer's measurements in helium and neon is more difficult. Only the smaller-diameter tubes were used with these gases so that the dependence of the pressure diffierence upon radius was not obtained for these gases. However, Rüttenauer does find that $\Delta p/L$ is proportional to the discharge current and inversely proportional to the pressure, as in the case of argon. The $M^{1/2}$ dependence in Eq. (A1) was suggested by the older theory by which Rüttenauer was guided; however, since it was determined only by a comparison of two gases, argon and helium, the true dependence of $\Delta p/L$ on M may be more complicated than Eq. (A1) indicates.

Equation (A9) predicts $\Delta p/L \propto I/p$, just as Rüttenauer observed, but unfortunately the use of the high-pressure form for V_3 in Eq. (A8) [which leads to Eq. (A9)] cannot be justified in the case of helium and neon. The critical value of pR separating the high-pressure and low-pressure forms for V_3 may be written

$$pR \approx 1.06 V_e$$

(p in Torr, R in mm, V_e in V) in argon. However, according to Francis, V_e ranges from 5 to 15 V for helium and neon in Rüttenauer's experiments. (Such high electron temperatures have been observed by Labuda and Gordon[30] in helium-neon discharges.) Physically, this then means that the radial fields are much greater in helium and neon than in argon at similar values of pR, so that ion radial drift velocities are considerably greater in the lighter gases at a given value of pR. Thus V_3 will still have its "small x" form at some values of pR in argon but its "large x" form at the same values of pR in neon or helium.

Specifically, one finds that the crossover from the high-pressure form of V_3 to the low-pressure form should occur at

$$pR \approx 2.02 V_e \text{ in neon}$$

and

$$pR \approx 2.22 V_e \text{ in helium}$$

(p in Torr, R in mm, V_e in V). If the high values of electron temperature previously mentioned occurred in Rüttenauer's tubes, then it is clear that the appropriate form for V_3 in Eq. (A7) should be the *low*-pressure form (the large-x limit). As previously remarked in the case of argon, the large-x limit of V_3 is pressure-independent, so that the $1/p$ dependence found by Rüttenauer cannot be accounted for in this case.

The resolution of this remaining disagreement must await further work. Perhaps the electron temperature in Rüttenauer's range of current and pressures is lower than estimated here, in which case Eq. (A9) would still be applicable and would account for his measurements in helium and neon. Perhaps other pumping processes at work in these gases make V_3 an incomplete description of the pumping mechanism. Or perhaps, because of the small currents used by Rüttenauer, changes in electron temperature and axial field as a function of current obscured a true interpretation of the pressure dependence over the limited range of parameters he used in his work on helium and neon.

[30] E. F. Labuda and E. I. Gordon, J. Appl. Phys. 35, 1647 (1964).

PHYSICAL REVIEW VOLUME 169, NUMBER 1 5 MAY 1968

Asymptotic Electric Microfield Distributions in Low-Frequency Component Plasmas*

C. F. Hooper, Jr.
Department of Physics and Astronomy, University of Florida, Gainesville, Florida

(Received 21 December 1967)

A recently developed collective-coordinate technique is employed to calculate electric microfield distribution functions $P(\epsilon)$ for low-frequency-component plasmas. Values of ϵ considered range from 10 to 50, and both neutral-point and charged-point cases are treated. Comparison is made with appropriate asymptotic expressions.

I. INTRODUCTION

Recently, a collective-coordinate technique has been used to calculate electric microfield distributions in plasmas; both high-frequency- and low-frequency-component plasmas were considered.[1,2] The distribution functions graphed and tabulated in these papers were for values of field strength ϵ measured in units of the normal field strength, from zero to 10. However, for a number of practical applications such as spectral line broadening in plasmas, there is need for values of the microfield distribution function $P(\epsilon)$ for values of ϵ greater than 10.

It is the purpose of this paper to calculate and discuss the low-frequency-component microfield distribution functions for values of ϵ between 10 and 50. Both the neutral- and charged-point cases will be treated. Much of the work presented in this paper is based on the formalism developed in Refs. 1 and 2, hereafter referred to as I and II, respectively.

* This research was supported in part by the National Aeronautics and Space Administration.

[1] C. F. Hooper, Jr., Phys. Rev. 149, 77 (1966).

[2] C. F. Hooper, Jr., Phys. Rev. 165, 215 (1968).