# Possible Measurement of the Nucleon Axial-Vector Form Factor in Two-Pion Electroproduction Experiments

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Current-algebra techniques and the hypothesis of partially conserved axial-vector current are used to derive a low-energy theorem for the reaction $e+N\rightarrow e+N+\pi+\pi(\text{soft})$. Particular attention is paid to satisfying the requirements of gauge covariance. Except for recoil corrections, the resulting matrix element is proportional to the nucleon axial-vector form factors, and we suggest that this electromagnetic process may be used to measure $g_A(k^2)$.

## I. INTRODUCTION

MEASUREMENT of the momentum-transfer dependence of the nucleon axial-vector form factor $g_A(k^2)$ would clearly be of great interest, since it would give information about the spectrum of axial-vector mesons, just as our experimental knowledge of the nucleon electromagnetic form factors has provided much useful information about the vector mesons. Unfortunately, the elastic and inelastic weak-interaction experiments[1] to measure $g_A(k^2)$ are much more difficult than their electromagnetic counterparts, and as a result very little about $g_A(k^2)$ is known at present. Clearly, it would be useful to have alternative, even if very indirect, methods of measuring $g_A(k^2)$. We discuss in this paper the possibility of measuring $g_A(k^2)$ in the electroproduction reaction

$$e+N\rightarrow e+N+\pi+\pi(\text{soft}), \quad (1)$$

assuming the validity of the current algebra and of the partially conserved axial-vector current (PCAC) hypotheses. This possibility is suggested by the recent work of a number of authors,[2] showing that when current-algebra–PCAC methods are applied to the photoproduction reaction $\gamma+N\rightarrow N+\pi+\pi(\text{soft})$, which is the $k^2=0$ case of Eq. (1), the results of the old Cutkosky-Zachariasen static model[3] are obtained, with the dominant term coming from the matrix element of the axial-vector current $\langle N\pi|J_\lambda{}^A|N\rangle$.

In Sec. II we apply soft-pion methods to the reaction of Eq. (1), and get a relation between the matrix element for this process and the matrix elements for single-pion weak production and electroproduction, $\langle N\pi|J_\lambda{}^A|N\rangle$ and $\langle N\pi|J_\lambda{}^{\text{EM}}|N\rangle$. By carefully keeping all pion pole diagrams, we eliminate some discrepancies noted in the previous work on two-pion photoproduction. The matrix element $\langle N\pi|J_\lambda{}^A|N\rangle$ can be related, in turn, to the axial-vector form factor $g_A(k^2)$, using models analogous to the very successful CGLN[4] treatment of pion photoproduction.

In Sec. III we retain only the $I=J=\frac{3}{2}$ partial wave, treated in the CGLN approximation, and discuss the possibility of measuring $g_A(k^2)$ in the reaction $e+N\rightarrow N_{3,3}{}^*(1238)+\pi(\text{soft})$.

## II. DERIVATION

We will consider the electroproduction reaction

$$e(k_1)+N(p_1)\rightarrow e(k_2)+N(p_2)+\pi(q)+\pi^s(q_s), \quad (2)$$

with the superscript $s$ an isospin index. Letting $k=k_1-k_2$ be the four-momentum transfer between the electrons, the hadronic matrix element for Eq. (2) is

$$M_\lambda=\int d^4x d^4y\, e^{ik\cdot x}e^{-iq_s\cdot y}(-\Box_y{}^2+M_\pi{}^2)\times\langle N(p_2)\pi(q)|T(\phi_{\pi^s}(y)J_\lambda{}^{\text{EM}}(x))|N(p_1)\rangle. \quad (3)$$

We wish to find the limit of Eq. (3) when $\pi^s$ is soft, that is, as $q_s\rightarrow 0$. This can be done by the standard soft-pion methods[5]; the only delicate point is to insure that our soft-pion approximation for $M_\lambda$ satisfies gauge invariance.

Let us begin then by studying the gauge properties of $M_\lambda$. Multiplying Eq. (3) by $-ik_\lambda$, integrating by parts

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[1] For a discussion of the determination of $g_A(k^2)$ in neutrino experiments, see E. C. M. Young, CERN Report 67-12 (unpublished).

[2] T. Ebata, Phys. Rev. **154**, 1341 (1967); P. Carruthers and H. W. Huang, Phys. Letters **24B**, 464 (1967); P. Narayanaswamy and B. Renner, Nuovo Cimento **53A**, 107 (1968); S. M. Berman (unpublished) (Berman has also considered the extension to electroproduction); W. I. Weisberger (unpublished).

[3] R. E. Cutkosky and F. Zachariasen, Phys. Rev. **103**, 1108 (1956). [See also P. Carruthers and H. Wong, *ibid.* **128**, 2382 (1962).] The soft-pion result generalizes their model to a relativistic framework in the same way that Chew, Goldberger, Low, and Nambu extended the Chew-Low static model for $N_{3,3}{}^*$ photoproduction.

[4] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957). Hereafter referred to as CGLN.

[5] See, for example, S. L. Adler and F. J. Gilman, Phys. Rev. **152**, 1460 (1966).

with respect to $x$, and using $\partial_\lambda J_\lambda^{\rm EM}=0$ gives

$$-ik_\lambda M_\lambda=\int d^4x d^4y\ e^{ik\cdot x}e^{-iq_s\cdot y}(-\Box_y^2+M_\pi^2)$$
$$\times\langle N(p_2)\pi(q)\,|\,\delta(x_0-y_0)[J_0^{\rm EM}(x),\phi_{\pi^s}(y)]\,|\,N(p_1)\rangle. \quad (4)$$

In all simple canonical field theories involving pions one finds[6]

$$\delta(x_0-y_0)[J_0^{\rm EM}(x),\phi_{\pi^s}(y)]=i\epsilon_{3sc}\delta^4(x-y)\phi_{\pi^c}(y); \quad (5)$$

substituting this into Eq. (4) and finally integrating by parts with respect to $y$ gives

$$k_\lambda M_\lambda=-\epsilon_{3sc}(q_s^2+M_\pi^2)\int d^4y$$
$$\times e^{i(k-q_s)\cdot y}\langle N(p_2)\pi(q)\,|\,\phi_{\pi^c}(y)\,|\,N(p_1)\rangle$$
$$=-\frac{\epsilon_{3sc}(q_s^2+M_\pi^2)}{(k-q_s)^2+M_\pi^2}\int d^4y$$
$$\times e^{i(k-q_s)\cdot y}\langle N(p_2)\pi(q)\,|\,J_{\pi^c}(y)\,|\,N(p_1)\rangle. \quad (6)$$

As expected, when $\pi^s$ is on the mass shell, $k_\lambda M_\lambda=0$, but in the off-shell case the divergence of $M_\lambda$ is nonzero. Our soft-pion approximation for $M_\lambda$ will not actually satisfy Eq. (6) exactly, but will obey the approximate version

$$k_\lambda M_\lambda\approx-\frac{\epsilon_{3sc}(q_s^2+M_\pi^2)}{(k-q_s)^2+M_\pi^2}\int d^4y$$
$$\times e^{ik\cdot y}\langle N(p_2)\pi(q)\,|\,J_{\pi^c}(y)\,|\,N(p_1)\rangle, \quad (7)$$

obtained by neglecting $q_s$ in the matrix element of $J_\pi$ but keeping $q_s$ in the rapidly varying factor $(q_s^2+M_\pi^2)/[(k-q_s)^2+M_\pi^2]$. Clearly, Eqs. (6) and (7) are identical both in the soft-pion limit ($q_s=0$) and on the mass shell ($q_s^2=-M_\pi^2$).

In applying PCAC to Eq. (3), it is helpful to introduce the "proper part" $J_\lambda^{AP}$ of the axial-vector current, defined as follows: Let $a$ and $b$ be arbitrary hadron states, and let $q=p_a-p_b$. Then we define $J_\lambda^{AP}$ by

$$\langle a|J_\lambda^{AP}|b\rangle=\langle a|J_\lambda^{A}|b\rangle+\frac{q_\lambda}{M_\pi^2}\langle a|q_\sigma J_\sigma^{A}|b\rangle, \quad (8)$$

which implies that

$$\langle a|J_\lambda^{A}|b\rangle=\langle a|J_\lambda^{AP}|b\rangle-\frac{q_\lambda}{q^2+M_\pi^2}\langle a|q_\sigma J_\sigma^{AP}|b\rangle, \quad (9)$$

$$\langle a|q_\lambda J_\lambda^{AP}|b\rangle=\frac{q^2+M_\pi^2}{M_\pi^2}\langle a|q_\lambda J_\lambda^{A}|b\rangle. \quad (10)$$

Clearly, the proper current $J_\lambda^{AP}$ has no pion pole; Eq. (9) is thus a convenient decomposition of the axial-vector current into pion-pole and non-pion-pole pieces. [As an illustration, let us take $a$ and $b$ to be nucleons. Then $\langle N|J_\lambda^A|N\rangle\propto\bar{u}(g_A\gamma_\lambda\gamma_5+iq_\lambda h_A\gamma_5)u$. In the approximation in which the induced pseudoscalar form factor $h_A$ is given by $h_A=2M_Ng_A/(q^2+M_\pi^2)$, the proper part of $\langle N|J_\lambda^A|N\rangle$ is just the piece $\bar{u}g_A\gamma_\lambda\gamma_5u$.] Let us now introduce the PCAC hypothesis in the form

$$\partial_\sigma J_\sigma^{sA}=\frac{M_NM_\pi^2g_A}{g_r(0)}\phi_{\pi^s}. \quad (11)$$

Then using Eq. (10) we can write Eq. (11) as

$$\partial_\sigma J_\sigma^{sAP}=\frac{M_Ng_A}{g_r(0)}J_{\pi^s}, \quad (12)$$

which says that the divergence of the proper part of the axial-vector current is a smooth interpolating operator for the pion source. Thus, we can rewrite the gauge condition [Eq. (7)] in the alternative form

$$k_\lambda M_\lambda\approx\frac{i\epsilon_{3sc}(q_s^2+M_\pi^2)k_\sigma}{(k-q_s)^2+M_\pi^2}\frac{g_r(0)}{M_Ng_A}\int d^4y$$
$$\times e^{ik\cdot y}\langle N(p_2)\pi(q)\,|\,J_\sigma^{cAP}(y)\,|\,N(p_1)\rangle. \quad (13)$$

To get a soft-pion approximation for $M_\lambda$, we substitute Eq. (11) into Eq. (3) and integrate by parts with respect to $y$. This gives

$$\frac{M_\pi^2}{q_s^2+M_\pi^2}M_\lambda=M_\lambda^{\rm ETC}+M_\lambda^{\rm SURF}, \quad (14)$$

with

$$M_\lambda^{\rm ETC}=-i\epsilon_{s3c}\frac{g_r(0)}{M_Ng_A}\int d^4x$$
$$\times e^{i(k-q_s)\cdot x}\langle N(p_2)\pi(q)\,|\,J_\lambda^{cA}(x)\,|\,N(p_1)\rangle \quad (15)$$

the equal-time commutator of $J_0^{sA}$ with $J_\lambda^{\rm EM}$,[7] and with

$$M_\lambda^{\rm SURF}=iq_{s\sigma}\frac{g_r(0)}{M_Ng_A}\int d^4xd^4y\ e^{ik\cdot x}e^{-iq_s\cdot y}$$
$$\times\langle N(p_2)\pi(q)\,|\,T(J_\sigma^{sA}(y)J_\lambda^{\rm EM}(x))\,|\,N(p_1)\rangle \quad (16)$$

the remainder. Separating Eq. (15) for $M_\lambda^{\rm ETC}$ into a

[6] When integrated over space with respect to $x$, Eq. (5) becomes $[I_3+\frac{1}{2}Y,\ \phi_{\pi^s}]=i\epsilon_{3sc}\phi_{\pi^c}$, which is just the statement that the pion is a particle with the quantum numbers $I=1$, $Y=0$. The local form, Eq. (5), follows from the integrated version in canonical field theories, since in such theories the charge density $J_0^{\rm EM}$ is a bilinear form in the canonical fields and momenta, and thus $[J_0^{\rm EM}(x),\phi_{\pi^s}(y)]|_{x_0=y_0}$ contains no gradient of $\delta$-function terms which vanish when integrated spatially.

[7] We have, of course, evaluated the equal-time commutator using the Gell-Mann algebra of currents [M. Gell-Mann, Physics 1, 63 (1964)]. The possible presence of Schwinger terms in the time-space commutators is irrelevant because of the cancellation of the Schwinger term and "seagull-diagram" contributions in soft-pion calculations. See, for example, S. L. Adler and R. F. Dashen, *Current Algebras* (W. A. Benjamin, Inc., New York, 1968), Chap. 3.

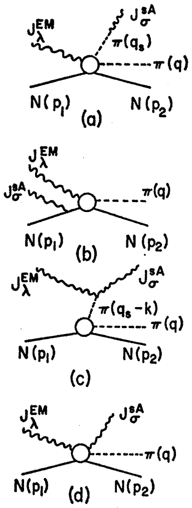


FIG. 1. Contributions to $M_\lambda^{\mathrm{SURF}}$ [Eq. (16)]. (a) Axial-vector current couples to a virtual pion. (b) Axial-vector current attaches to the initial external nucleon line in single-pion electroproduction. There is a similar diagram (not shown) in which the axial-vector current attaches to the final external nucleon line. (c) Axial-vector current and vector current attach to a virtual pion at the same space-time point [a "seagull" diagram]. (d) Axial-vector current couples to internal lines in the matrix element $\langle N(p_2)\pi(q)|\times T(J_\sigma^{sA}(y)J_\lambda^{\mathrm{EM}}(x))|N(p_1)\rangle$.

proper part and a remainder gives

$$M_\lambda^{\mathrm{ETC}}=-i\epsilon_{s3c}\frac{g_r(0)}{M_Ng_A}\bigg[\int d^4x\, e^{i(k-q_s)\cdot x}\langle N(p_2)\pi(q)|\times J_\lambda^{cAP}(x)|N(p_1)\rangle-\frac{(k-q_s)_\lambda(k-q_s)_\eta}{(k-q_s)^2+M_\pi^2}\int d^4x \times e^{i(k-q_s)\cdot x}\langle N(p_2)\pi(q)|J_\eta^{cAP}(x)|N(p_1)\rangle\bigg] \quad (17)$$

$$\approx -i\epsilon_{s3c}\frac{g_r(0)}{M_Ng_A}\bigg[\int d^4x\, e^{ik\cdot x}\langle N(p_2)\pi(q)|J_\lambda^{cAP}(x)\times|N(p_1)\rangle-\frac{(k-q_s)_\lambda k_\eta}{(k-q_s)^2+M_\pi^2}\int d^4x\, e^{ik\cdot x}\times\langle N(p_2)\pi(q)|J_\eta^{cAP}(x)|N(p_1)\rangle\bigg]. \quad (18)$$

In going from Eq. (17) to Eq. (18) we have neglected $q_s$ in matrix elements of the proper part $J_\lambda^{cAP}$ and its divergence $\partial_\eta J_\eta^{cAP}$, but have retained $q_s$ in the rapidly varying factor $(k-q_s)_\lambda/[(k-q_s)^2+M_\pi^2]$. The surface term $M_\lambda^{\mathrm{SURF}}$ contains four types of terms, shown in Figs. 1(a)–1(d). In Fig. 1(a), the axial-vector current couples to a virtual pion; it is easy to see that

$$M_\lambda^{\mathrm{SURF}(a)}=\frac{-q_s^2}{q_s^2+M_\pi^2}M_\lambda. \quad (19)$$

In Fig. 1(b), the axial current attaches to an external nucleon line in single-pion electroproduction; an expression for $M_\lambda^{\mathrm{SURF}(b)}$ can be obtained from the usual axial-current insertion rules and is given below. In Fig. 1(c), the axial current and vector current attach to a virtual pion at the same space-time point; this is a "seagull" diagram contributing to virtual radiative pion decay and may be calculated to be

$$M_\lambda^{\mathrm{SURF}(c)}=\epsilon_{s3c}q_{s\sigma}\frac{1}{(k-q_s)^2+M_\pi^2}\int d^4x \times e^{i(k-q_s)\cdot x}\langle N(p_2)\pi(q)|J_{\pi^c}(x)|N(p_1)\rangle\delta_{\lambda\sigma} \quad (20)$$

$$\approx -i\epsilon_{s3c}\frac{g_r(0)}{M_Ng_A}\frac{q_{s\lambda}k_\eta}{(k-q_s)^2+M_\pi^2}\int d^4x \times e^{ik\cdot x}\langle N(p_2)\pi(q)|J_\eta^{cAP}(x)|N(p_1)\rangle. \quad (21)$$

Finally, in Fig. 1(d) the axial current couples to internal lines in the matrix element

$$\langle N(p_2)\pi(q)|T(J_\sigma^{sA}(y)J_\lambda^{\mathrm{EM}}(x))|N(p_1)\rangle;$$

consequently, $M_\lambda^{\mathrm{SURF}(d)}$ is of order $q_s$ and may be neglected.

Comparing Eq. (21) with Eq. (18), we see that the effect of including the radiative pion decay diagram is to change the coefficient of the pion-pole term in $M_\lambda^{\mathrm{ETC}}$ from $(k-q_s)_\lambda$ to $(k-2q_s)_\lambda$. This eliminates the factor of two discrepancy noted by Carruthers and Huang,[2] who neglected $M_\lambda^{\mathrm{SURF}(c)}$, and leads to the satisfaction of the approximate gauge condition (13).

Combining all the terms, we may write our answer as follows:

$$M_\lambda=(2\pi)^4\delta^4(p_2+q-p_1-k)\left(\frac{M_N^2}{2p_{10}p_{20}q_0}\right)^{1/2}\times\bar{u}(p_2)N_\lambda u(p_1)+O(q_s), \quad (22)$$

$$N_\lambda=\epsilon_{s3c}\left[\frac{(2q_s-k)_\lambda k_\eta}{(k-q_s)^2+M_\pi^2}+\delta_{\lambda\eta}\right]\left(\frac{-ig_r(0)}{M_Ng_A}\right)O_\eta^{cAP}+\frac{g_r(0)}{2M_N}\tau^s\boldsymbol{q}_s\gamma_5\frac{p_2+iM_N}{2p_2\cdot q_s}O_\lambda^{\mathrm{EM}}+O_\lambda^{\mathrm{EM}}\frac{p_1+iM_N}{-2p_1\cdot q_s}\frac{g_r(0)}{2M_N}\tau^s\boldsymbol{q}_s\gamma_5,$$

where $O_\eta^{cAP}$ and $O_\lambda^{\mathrm{EM}}$ are defined by

$$\langle N(p_2)\pi(q)|J_\eta^{cAP}|N(p_1)\rangle=\left(\frac{M_N^2}{2p_{10}p_{20}q_0}\right)^{1/2}\times\bar{u}(p_2)O_\eta^{cAP}u(p_1), \quad (23a)$$

$$\langle N(p_2)\pi(q)|J_\lambda^{\mathrm{EM}}|N(p_1)\rangle=\left(\frac{M_N^2}{2p_{10}p_{20}q_0}\right)^{1/2}\times\bar{u}(p_2)O_\lambda^{\mathrm{EM}}u(p_1). \quad (23b)$$

The terms proportional to $O_\lambda^{\mathrm{EM}}$ are the single-pion electroproduction contribution $M_\lambda^{\mathrm{SURF}(b)}$ mentioned above.[8] Since the single-pion electroproduction matrix

[8] In writing the matrix element $O_\lambda^{\mathrm{EM}}$ we neglect the additional momentum $q_s$ carried by the intermediate nucleon. It is clear that the error is $O(q_s)$, consistent with our approximation.

element is gauge-invariant, we have $k_\lambda(p_2+iM_N)\times O_\lambda^{\mathrm{EM}}u(p_1)=k_\lambda\bar{u}(p_2)O_\lambda^{\mathrm{EM}}(p_1+iM_N)=0$, and thus the divergence of $N_\lambda$ is

$$k_\lambda N_\lambda=\epsilon_{s3c}\frac{q_s^2+M_\pi^2}{(k-q_s)^2+M_\pi^2}k_\eta\left(\frac{-ig_r(0)}{M_N g_A}\right)O_\eta^{cAP}. \quad (24)$$

Combining Eqs. (22)–(24), it is clear that the approximate gauge condition of Eq. (13) is satisfied. In particular, when $q_s^2=-M_\pi^2$, $k_\lambda N_\lambda=0$, so on-mass-shell Eq. (22) gives a gauge-invariant approximation to the matrix element for two-pion electroproduction.

## III. DISCUSSION

Let us now briefly consider the possibility of indirectly measuring $g_A(k^2)$ in the reaction $e+N\to e+N+\pi+\pi(\text{soft})$, by use of Eqs. (22)–(23). For simplicity, we will restrict ourselves to the case in which the soft pion is at rest (threshold) in the center-of-mass frame of the final baryons,[9] and in which the hard pion and nucleon emerge in the (3,3) resonance. At the soft-pion threshold, the kinematic structure of two-pion electroproduction becomes identical to the kinematic structure of the more familiar case of single-pion electroproduction; this makes it easy to compute the two-pion cross section from the matrix element in Eqs. (22)–(23). When the hard $\pi$ and $N$ form an $N_{3,3}^*$, the matrix elements in Eqs. (23a) and (23b) describe weak production of the (3,3) resonance from a nucleon target and have been extensively studied.[10] The vector matrix element [Eq. (23b)] is found to be dominated by the magnetic dipole[11] amplitude $M_{1+}^{(3/2)}$, while the axial-vector matrix element [Eq. (23a)] is dominated by the electric, longitudinal, and scalar amplitudes $\mathcal{E}_{1+}^{(3/2)}$, $\mathcal{L}_{1+}^{(3/2)}$, and $\mathcal{H}_{1+(g_A)}^{(3/2)}$. [The subscript $(g_A)$ indicates that the part of $\mathcal{H}_{1+}^{(3/2)}$ proportional to the induced pseudoscalar form factor $h_A$ is to be dropped and only the part proportional to the axial-vector form factor $g_A$ retained; this restriction arises because only the *proper* part of the axial-vector current appears in Eq. (23).] For momentum transfers $k^2$ less than 50 $\mathrm{F}^{-2}$, a model which should give a good approximation to $M_{1+}^{(3/2)}, \cdots$ is

$$\begin{aligned}M_{1+}^{(3/2)}&=M_{1+}^{(3/2)B}f_{1+}^{(3/2)}/f_{1+}^{(3/2)B},\\ \mathcal{E}_{1+}^{(3/2)}&=\mathcal{E}_{1+}^{(3/2)B}f_{1+}^{(3/2)}/f_{1+}^{(3/2)B},\\ \mathcal{L}_{1+}^{(3/2)}&=\mathcal{L}_{1+}^{(3/2)B}f_{1+}^{(3/2)}/f_{1+}^{(3/2)B},\\ \mathcal{H}_{1+(g_A)}^{(3/2)}&=\mathcal{H}_{1+(g_A)}^{(3/2)B}f_{1+}^{(3/2)}/f_{1+}^{(3/2)B},\end{aligned} \quad (25)$$

where $f_{1+}^{(3/2)}$ is the pion-nucleon scattering amplitude in the (3,3) channel and where the superscript $B$ denotes "Born approximation." Expressions for $f_{1+}^{(3/2)B}$, $M_{1+}^{(3/2)B}$, $\mathcal{E}_{1+}^{(3/2)B}$, $\cdots$ are given in the Appendix.[10]

TABLE I. Isospin coefficients.

| | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $e+p\to e+\pi^+(\text{soft})+N_{3,3}^{*0}$ | | | |
| $N_{3,3}^{*0}\to p+\pi^-$ | $-\frac{1}{6}$ | $-\frac{1}{12}$ | 0 |
| $\searrow n+\pi^0$ | $-\frac{1}{3}(\sqrt{2})^{-1}$ | $-\frac{1}{12}\sqrt{2}$ | $\frac{1}{6}\sqrt{2}$ |
| $e+p\to e+\pi^-(\text{soft})+N_{3,3}^{*++}$ | | | |
| $N_{33}^{*++}\to p+\pi^+$ | $\frac{1}{2}$ | 0 | $-\frac{1}{6}$ |

A straightforward calculation shows that, in terms of the weak (3,3) production multipoles, the cross section for $e+N\to e+N_{3,3}^*+\pi(\text{threshold})$ is given by

$$\sigma_1(k^2,W)\equiv\frac{1}{|\mathbf{q}_s|}\frac{d^3\sigma[e+N\to e+N_{3,3}^*+\pi(\text{soft})]}{dq_{s0}dk^2dW}\bigg|_{q_{s0}=M_\pi}$$
$$=\frac{\alpha^2}{\pi^3}\frac{g_r(0)^2}{M_N^2}\frac{(W+M_\pi)^2}{W^2+(W+M_\pi)^2-M_\pi^2}\frac{|\mathbf{q}|}{(k_{10}^L)^2}\times\left[\frac{1}{2k^2}\left(1+\frac{2k_{10}k_{20}-\frac{1}{2}k^2}{|\mathbf{k}|^2}\right)[|A|^2+3|B|^2+|C|^2]+\frac{4k_{10}k_{20}-k^2}{|\mathbf{k}|^2}|D|^2\right], \quad (26)$$

$$\begin{aligned}A&=\frac{1}{g_A}a_1\mathcal{E}_{1+}^{(3/2)}+a_2\frac{|\mathbf{k}|}{p_{10}}M_{1+}^{(3/2)},\\ B&=a_2\frac{|\mathbf{k}|}{p_{10}}M_{1+}^{(3/2)},\\ C&=a_3\frac{|\mathbf{q}|}{p_{20}}M_{1+}^{(3/2)},\\ D&=\frac{1}{g_A}a_1\times\frac{(k_0-2M_\pi)\mathcal{L}_{1+}^{(3/2)}+2M_\pi(|\mathbf{k}|/k_0)\mathcal{H}_{1+(g_A)}^{(3/2)}}{k^2+2M_\pi k_0},\end{aligned} \quad (27)$$

where $k_{10}^L$ is the laboratory-frame initial electron energy, where $q_{s0}$ and all other noninvariant quantities refer to the center-of-mass frame of the final baryons, and where $W$ is the invariant mass of the resonating pion and nucleon. Values of the isospin coefficients $a_{1,2,3}$ are given in Table I. For comparison, the cross section for the ordinary (3,3) electroproduction reaction $e+p\to e+N_{3,3}^{*+}$ is

$$\sigma_2(k^2,W)\equiv\frac{d^2\sigma(e+p\to e+N_{3,3}^{*+})}{dk^2dW}=\frac{\alpha^2}{3\pi}\frac{|\mathbf{q}|}{(k_{10}^L)^2}\frac{1}{2k^2}\times\left(1+\frac{2k_{10}k_{20}-\frac{1}{2}k^2}{|\mathbf{k}|^2}\right)|M_{1+}^{(3/2)}|^2. \quad (28)$$

[9] That is, the frame defined by $\mathbf{p}_2+\mathbf{q}+\mathbf{q}_s=0$. In the case $\mathbf{q}_s=0$ which we consider, the center-of-mass frame of the final baryons is identical with the center-of-mass frame of the hard pion and nucleon (the $N_{3,3}^*$ rest frame).

[10] S. L. Adler (to be published).

[11] Our multipoles are a factor $(8\pi W/M_N e)$ times those of Ref. 4.

Looking at Table I, we see that the most promising reaction for the measurement of $g_A(k^2)$ is

$$e+p\to e+\pi^-(\text{soft})+N_{3,3}^{*++} \searrow p+\pi^+ \tag{29}$$

for the following three reasons: (1) The coefficient $a_1$ of the axial-vector multipoles is the largest in this case. (2) The coefficient $a_2$ vanishes and, consequently, the vector multipole $M_{1+}^{(3/2)}$ enters only through the very small recoil-correction term $|C|^2$. (3) In this case there is no soft-pion background coming from single-pion electroproduction, which can only lead to a soft $\pi^+$ or $\pi^0$.

Because the Born approximations $\mathcal{E}_{1+}^{(3/2)B}$ and $\mathcal{L}_{1+}^{(3/2)B}$ are known functions of $W$ and $k^2$, and are proportional to $g_A(k^2)$, Eq. (26) [apart from the small term $|C|^2$] is proportional to $g_A(k^2)^2$, and thus a measurement of $\sigma_1$ as a function of $k^2$ will determine the momentum transfer dependence of $g_A$.[12]

There is, however, a possible problem, which may be illustrated by comparing Eq. (26) with Eq. (28) for ordinary (3,3) resonance electroproduction. Just as $\sigma_1$ is proportional to $g_A(k^2)^2$, $\sigma_2$ is proportional to $F^V(k^2)^2$, where $F^V(k^2)$ is an isovector electromagnetic form factor. There seems to be some evidence that the axial-vector form factor $g_A(k^2)$ falls off considerably more slowly with $k^2$ than does $F^V(k^2)$. This in turn suggests that the soft pion$+N_{3,3}^*$ production cross section $\sigma_1$ falls off much more slowly with $k^2$ than does the $N_{3,3}^*$ cross section $\sigma_2$. Unfortunately, however, this conclusion is not correct. The reason is that the multipoles $M_{1+}^{(3/2)}$ and $\mathcal{E}_{1+}^{(3/2)}$ have different small-$|\mathbf{k}|$ threshold behavior,

$$\left.\begin{matrix} M_{1+}^{(3/2)}\sim|\mathbf{k}| \\ \mathcal{E}_{1+}^{(3/2)}\sim 1 \end{matrix}\right\}|\mathbf{k}|\to 0, \tag{30}$$

and this behavior, in the model of Eq. (25), persists into the physical region as well. As a result, the correct statement about the relative rates of decrease of $\sigma_1$ and $\sigma_2$ is that

$$\frac{\sigma_1(k^2)/\sigma_1(0)}{\sigma_2(k^2)/\sigma_2(0)}\approx\frac{[g_A(k^2)/g_A]^2}{[F^V(k^2)]^2}\frac{|\mathbf{k}|^2_{k^2=0}}{|\mathbf{k}|^2_{k^2}} \approx\frac{[g_A(k^2)/g_A]^2}{[F^V(k^2)]^2}\frac{(W-M_N)^2}{(W-M_N)^2+k^2}. \tag{31}$$

Even if $g_A(k^2)$ falls off appreciably more slowly than $F^V(k^2)$, the effect of the factor $(W-M_N)^2/[(W-M_N)^2+k^2]$ is to cause $\sigma_1$ to decrease more rapidly than $\sigma_2$.

The importance of the threshold behavior in Eq. (31) illustrates a problem which might invalidate Eq. (22), our soft-pion approximation for the two-pion production matrix element, and thus destroy the possibility of measuring $g_A(k^2)$ in the reaction Eq. (29). In deriving Eq. (22), we have neglected terms of first order or higher in the soft-pion four-momentum $q_s$. At $k^2=0$, we feel fairly justified in this approximation, since it leads to the Cutkosky-Zachariasen formulas, which seem to work. However, it is always possible that some of the terms of order $q_s$, which are negligible at $k^2=0$, increase rapidly relative to the terms of zeroth order in $q_s$ as $k^2$ increases, because of a different threshold behavior in $|\mathbf{k}|$. If this happened, the soft-pion approximation could become bad precisely in the large-$k^2$ region, where we must look to measure $g_A(k^2)$. Hopefully, this does not happen, but in using Eq. (22) to interpret two-pion electroproduction experiments, this danger must be kept in mind. A more detailed investigation of this problem is being undertaken.

## APPENDIX

We give here expressions for the Born approximations $f_{1+}^{(3/2)B}$, $M_{1+}^{(3/2)B}$, $\mathcal{E}_{1+}^{(3/2)B}$, $\mathcal{L}_{1+}^{(3/2)B}$, and $\mathcal{H}_{1+(g_A)}^{(3/2)B}$:

$$f_{1+}^{(3/2)B}=-\frac{g_r^2}{8\pi W|\mathbf{q}|^2}[W_-(p_{20}+M_N)A(\bar{a})+W_+(p_{20}-M_N)C(\bar{a})],$$

$$M_{1+}^{(3/2)B}=\frac{W^2|\mathbf{q}||\mathbf{k}|}{O_{2+}}\left(\frac{-g_r}{4M_N^2}\right)[F_1^V(k^2)+2M_NF_2^V(k^2)]\left[\frac{M_NW_-(p_{10}+M_N)}{W^2}\frac{A(a)}{|\mathbf{q}|^2|\mathbf{k}|^2}-\frac{W_+}{W^2}\frac{B(a)}{|\mathbf{q}||\mathbf{k}|}+\frac{M_NW_+}{W^2(p_{20}+M_N)}\frac{C(a)}{|\mathbf{q}||\mathbf{k}|}\right]+\text{nucleon and pion charge terms},$$

[12] A similar calculation would lead to a determination of $g_A(k^2)$ in electroproduction of a single soft pion. The relevant matrix elements are given in Ref. 5, which gives further references. Experimental data on single- and double-pion photoproduction reactions indicate that double-pion electroproduction may yield more reliable results for $g_A(k^2)$ than single-pion electroproduction. The reason is that the soft-pion matrix element seems to give an accurate description of the experimental results for two-pion photoproduction up to about 100 MeV above the $N_{3,3}^*+\pi$ threshold, while the single-pion photoproduction is dominated by $N_{3,3}^*$ production (which cannot be described by soft-pion methods) as soon as one goes away from threshold. In fact, it is interesting to note that the recent DESY results on $\gamma+p\to N_{3,3}^{*++}+\pi^-$ show a cross section rising less rapidly above threshold than indicated by earlier experiments and agree within experimental error with the prediction of the Cutkosky-Zachariasen model. The relevant experimental results and references are given in Fig. 9 of M. G. Hauser, Phys. Rev. **160**, 1215 (1967). If both methods of measuring $g_A(k^2)$ are feasible, one will be happy to have two independent determinations.

$$\mathcal{E}_{1+}{}^{(3/2)B}=W^2O_{1+}|\mathbf{q}|\left(\frac{-g_rg_A(k^2)}{2M_N}\right)\left[\frac{\frac{1}{2}W_-(p_{10}-M_N)}{W^2}\frac{A(a)}{|\mathbf{q}|^2|\mathbf{k}|^2}+\frac{2}{W^2}\frac{B(a)}{|\mathbf{q}||\mathbf{k}|}+\frac{\frac{1}{2}W_+}{W^2(p_{20}+M_N)}\frac{C(a)}{|\mathbf{q}||\mathbf{k}|}-\frac{3}{W^2}\frac{E(a)}{(p_{10}+M_N)(p_{20}+M_N)}\right],$$

$$\mathcal{L}_{1+}{}^{(3/2)B}=\frac{1}{k_0W}W^2O_{1+}|\mathbf{q}|\left(\frac{-g_rg_A(k^2)}{2M_N}\right)\left[\frac{M_N(p_{10}-M_N)W_++(\frac{1}{2}W_--p_{20})k^2}{W}\times\frac{A(a)}{|\mathbf{q}|^2|\mathbf{k}|^2}+\frac{M_N(p_{10}+M_N)W_--(\frac{1}{2}W_+-p_{20})k^2}{(p_{10}+M_N)(p_{20}+M_N)W}\frac{C(a)}{|\mathbf{q}||\mathbf{k}|}\right],$$

$$\mathcal{K}_{1+(g_A)}{}^{(3/2)B}=O_{1+}|\mathbf{q}|\frac{g_rg_A(k^2)}{2M_N}\left[(\tfrac{1}{2}W_--q_0)\frac{A(a)}{|\mathbf{q}|^2|\mathbf{k}|}-\frac{(\frac{1}{2}W_+-q_0)}{(p_{10}+M_N)(p_{20}+M_N)}\frac{C(a)}{|\mathbf{q}|}\right], \tag{A1}$$

with

$$W_\pm=W\pm M_N,\quad O_{1+}=[(p_{10}+M_N)(p_{20}+M_N)]^{1/2},\quad O_{2+}=[(p_{10}+M_N)/(p_{20}+M_N)]^{1/2},$$
$$a=(2p_{20}k_0+k^2)/(2|\mathbf{q}||\mathbf{k}|),\quad \bar{a}=(2p_{20}q_0-M_\pi{}^2)/(2|\mathbf{q}|^2). \tag{A2}$$

The functions $A$ through $E$ are defined by

$$A(a)=1-\tfrac{1}{2}a\ln\left(\frac{a+1}{a-1}\right),\quad B(a)=\tfrac{1}{2}\left[a+\tfrac{1}{2}(1-a^2)\ln\left(\frac{a+1}{a-1}\right)\right],$$
$$C(a)=-\tfrac{1}{2}\left[3a+\tfrac{1}{2}(1-3a^2)\ln\left(\frac{a+1}{a-1}\right)\right],\quad E(a)=\tfrac{1}{2}\left[\tfrac{2}{3}-a^2+\tfrac{1}{2}a(a^2-1)\ln\left(\frac{a+1}{a-1}\right)\right], \tag{A3}$$

and $F_1{}^V(k^2)$ and $F_2{}^V(k^2)$ are, respectively, the isovector nucleon charge and magnetic form factors, normalized so that $F_1{}^V(0)+2M_NF_2{}^V(0)=4.7$. For reasons explained in Ref. 10, only the part of $M_{1+}{}^{(3/2)B}$ proportional to the total nucleon isovector magnetic moment (given explicitly in the equation above) is used in Eq. (25); the part proportional to the nucleon and pion charges should be dropped.

# Errata

**Unified Formulation of Effective Nonlinear Pion-Nucleon Lagrangians,** P. Chang and F. Gürsey [Phys. Rev. 164, 1752 (1967)]. Equation (4.1a) should read

$$\mathbf{J}_{5\mu}=-\bar{\xi}\gamma_\mu\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\xi+\frac{\partial_\mu\boldsymbol{\pi}}{f}\left(\frac{\sin2f\sqrt{\pi^2}\cos2f\sqrt{\pi^2}}{2f\sqrt{\pi^2}}\right)+f\left\{\frac{\boldsymbol{\pi}(\partial_\mu\pi^2)}{2f^2\pi^2}\left(1-\frac{\sin2f\sqrt{\pi^2}\cos2f\sqrt{\pi^2}}{2f\sqrt{\pi^2}}\right)+\bar{\xi}\gamma_\mu\boldsymbol{\tau}\times\boldsymbol{\pi}\xi\frac{\sin2f\sqrt{\pi^2}}{2f\sqrt{\pi^2}}\right\}+2f^2\bar{\xi}\gamma_\mu\gamma_5\left(\frac{\pi^2\boldsymbol{\tau}-(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\boldsymbol{\pi}}{1+\cos2f\sqrt{\pi^2}}\right)\left(\frac{\sin2f\sqrt{\pi^2}}{2f\sqrt{\pi^2}}\right)^2\xi.$$

Equation (4.3a) should read

$$\mathbf{J}_{5\mu}=-\bar{\xi}\gamma_\mu\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\xi+\frac{\partial_\mu\boldsymbol{\pi}}{f}\frac{1-f^2\pi^2}{(1+f^2\pi^2)^2}+f\left\{\frac{\boldsymbol{\pi}}{2}\frac{(\partial_\mu\pi^2)}{(1+f^2\pi^2)^2}+\bar{\xi}\gamma_\mu\boldsymbol{\tau}\times\boldsymbol{\pi}\xi\frac{1}{1+f^2\pi^2}\right\}+2f^2\bar{\xi}\gamma_\mu\gamma_5\left(\frac{\pi^2\boldsymbol{\tau}-(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\boldsymbol{\pi}}{2}\right)(1+f^2\pi^2)^3\xi,$$

and Eq. (4.5a) should read

$$\mathbf{J}_{5\mu}=-\bar{\xi}\gamma_\mu\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\xi+\frac{\partial_\mu\boldsymbol{\pi}}{f}(1-4f^2\pi^2)^{1/2}+f\left\{\frac{\boldsymbol{\pi}(\partial_\mu\pi^2)}{(1-4f^2\pi^2)^{1/2}}+\bar{\xi}\gamma_\mu\boldsymbol{\tau}\times\boldsymbol{\pi}\xi\right\}+2f^2\bar{\xi}\gamma_\mu\gamma_5\left(\frac{\pi^2\boldsymbol{\tau}-(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\boldsymbol{\pi}}{1+(1-4f^2\pi^2)^{1/2}}\right)\xi.$$