Meson-Meson Coupling Constants from Partial Conservation of Axial-Vector Current in Broken $SU(3)$

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Using the partial conservation of axial-vector current (PCAC) relation, we have obtained an expression for determining the couplings of vector mesons to two pseudoscalar mesons. The results are compared with the sum rules following from first-order symmetry. The decay width for $\phi \to K\bar{K}$ has been evaluated and compared with experiment.

RECENTLY, the hypothesis of partially conserved axial-vector currents (PCAC) has been exploited to yield relations[1] among the couplings of baryons and baryon resonances to baryons and pseudoscalar mesons in broken $SU(3)$. In deriving these relations it has been assumed that the axial-vector currents maintain their octet character to the first order in $SU(3)$ symmetry breaking. In view of the success obtained, we would like to extend these calculations to the case of VPP couplings.

The PCAC relation is

$$\partial_\nu A_\alpha{}^\nu(x) = C_\alpha \phi_\alpha(x)\,, \tag{1}$$

where $A_\alpha{}^\nu(x)$ is an axial-vector current density, $\phi_\alpha(x)$ is a pseudoscalar-meson field operator, and C is a constant which depends upon the $SU(3)$ octet index α in broken symmetry. We assume that we can write the matrix elements of both sides of Eq. (1) between a 0^- boson octet P and a 1^- vector boson octet V as follows:

$$\langle V(p_f)|\partial_\nu A_\alpha{}^\nu(0)|P(p_i)\rangle = \left(\frac{1}{E_iE_f}\right)^{1/2} C_{\alpha i f}\bar{\Phi}_\lambda(p_f)q_\nu \times[E_A(q^2)g^{\nu\lambda}+F_A(q^2)p^\nu q^\lambda]\phi(p_i)\,; \tag{2a}$$

$$\langle V(p_f)|j_\alpha(0)|P(p_i)\rangle = \left(\frac{1}{E_iE_f}\right)^{1/2} f_{if\alpha}K_{if\alpha}(q^2) \times\bar{\Phi}_\lambda(p_f)q^\lambda\phi(p_i)\,, \tag{2b}$$

where $p=(p_f+p_i)$, $q=(p_f-p_i)$, E_A and F_A are the axial-vector renormalization constants, $C_{\alpha if}$ denotes the antisymmetric $SU(3)$ coupling coefficients, $K_{if\alpha}(q^2)$ is the form factor for the VPP vertex normalized to $K_{if\alpha}(\mu_\alpha^2)=1$, and, finally, $j_\alpha(x)=(\Box^2+\mu_\alpha^2)\phi_\alpha(x)$.

Comparing the two Eqs. (2a) and (2b) at $q^2=0$, we get the relation

$$E_A(0)+(M^2-m^2)F_A(0)=\frac{C_\alpha}{\mu_\alpha^2}\frac{f_{if\alpha}K_{if\alpha}(0)}{C_{\alpha if}}\,, \tag{3}$$

where M (m) is the mass of the vector (pseudoscalar) meson of four-momentum p_f (p_i).

The evaluation of the right-hand side of Eq. (3) requires a knowledge of $K_{if\alpha}(0)$ and C_α/μ_α^2 for $\alpha=\pi$ and $\alpha=K$. We assume in our calculation that $K_{if\alpha}=K_{i'f'\alpha}$. This implies that the form factors do not depend sensitively on the mass of the external particles, and also that they do not change appreciably when the limit $\mu_\alpha^2\to 0$ is taken. The values of C_π/μ_π^2 and C_K/μ_K^2 can be determined directly from the decay rates of π and K, respectively.[2]

To determine $E_A(0)$ and $F_A(0)$ we first fix $\alpha=\pi$. Then for two different choices of (V,P)—say (ρ,π) and (K^*,K)—the right-hand side can be known completely in terms of a parameter d where we define $d=K_{if\pi}(0)/K_{ifK}(0)$ (in future we shall drop the suffix i, f in K). The $f_{\rho\pi\pi}$ and $f_{K^*K\pi}$ can be determined from the experimental values of the decay widths of ρ and K^*, respectively. Solving for these two sets of values we get

$$F_A'(0)=F_A(0)/K_K(0)=\sigma_{12}=(A_1-A_2)/[(M_1^2-m_1^2)-(M_2^2-m_2^2)]\,,$$

$$E_A'(0)=E_A(0)/K_K(0)=A_1-\sigma_{12}(M_1^2-m_1^2)\,,$$

where

$$A_i=(C_\pi/\mu_\pi^2)(f_{if\pi}/C_{\pi if})d$$

and the subscripts 1 and 2 denote the two choices of V and P. Fixing $d\approx1.25$, for best fit, taking $\Gamma(\rho\to\pi\pi)\approx120$ MeV and $\Gamma(K^*\to K\pi)\approx50$ MeV, we obtain

$$E_A'(0)\approx1.05\,,\quad F_A'(0)\approx0.06\,.$$

Knowing $E_A'(0)$ and $F_A'(0)$, we can now proceed to determine other VPP coupling constants by use of Eq. (3). Our results are given in Table I along with the $SU(3)$ predictions. The $\rho\pi\pi$ and $K^*K\pi$ coupling constants are those derived from the decay widths of ρ and K^*.

Among the VPP vertex coupling constants, we have

TABLE I. VPP coupling constants for $d\approx1.25$.

$f^2/4\pi$ \ VPP vertex	$\rho\pi\pi$	$K^*K\pi$	$\rho K\bar{K}$	$\omega^{(8)}K\bar{K}$	$K^*\eta K$
Present calculation	2.32	0.87	0.54	1.67	0.83
$SU(3)$	2.32	0.87	0.58	1.74	0.87

[1] K. Raman, Phys. Rev. **149**, 1122 (1966); **152**, 1517(E) (1966); R. H. Graham, S. Pakvasa, and K. Raman, *ibid.* **163**, 1774 (1967); Riazuddin, *ibid.* **136**, B268 (1964); P. G. O. Freund and Y. Nambu, Phys. Rev. Letters **13**, 221 (1964).

[2] With $\cos\theta_A=\cos\theta_V=0.978$ we obtain $C_K/\mu_K^2=0.17$ BeV; whereas from the pion decay, $C_\pi/\mu_\pi^2=0.13$ BeV.

the following sum rules[3]:

$$\frac{4}{\sqrt{2}}g_{\pi^-\bar{K}^0K^{*+}}=g_{\pi^+\pi^0\rho^-}-\frac{1}{\sqrt{2}}g_{\bar{K}^0K^+\rho^-}+\frac{3}{\sqrt{3}}g_{K^-K^+\omega^{(8)}}, \quad (4a)$$

$$g_{K^-K^+\omega^{(8)}}=g_{\eta K^-K^{*+}}-\frac{1}{\sqrt{6}}g_{\bar{K}^0K^+\rho^-}+\frac{1}{\sqrt{6}}g_{\pi^-\bar{K}^0K^{*+}}, \quad (4b)$$

$$(\sqrt{\tfrac{2}{3}})g_{K^-K^+\omega^{(8)}}=g_{\bar{K}^0K^+\rho^-}. \quad (4c)$$

Each of these sum rules is reasonably satisfied by our determined values of the coupling constants, as can be seen from Table II.

Unfortunately, $\rho K\bar{K}$ and $K^*\eta K$ vertices are not directly experimentally accessible. However, we can make a connection with the experiments if we calculate $\Gamma(\phi\to K\bar{K})$ by using $\omega^{(8)}K\bar{K}$ vertex and the idea of ω-ϕ mixing. Taking the usual values of $\cos\theta=0.77$ and $m_{\omega^{(8)}}=930$ MeV, we find $\Gamma(\phi\to K\bar{K})=3.6$ MeV. This may be compared favorably with the experimental ϕ width, 3.6 ± 0.8 MeV.[4] The agreement should be taken with some reservations because of the large uncertainty in the experimental value and also because our calculation involves off-mass-shell ($\mu_K^2\to 0$) coupling.

TABLE II. Comparison of VPP coupling constants with sum rules.

Sum rule	a	b	c
Right-hand side of sum rule	7.58	3.26	2.61
Left-hand side of sum rule	7.40	3.24	2.64

Thus we find that our method of calculation, even when involving large extrapolations, leads to results which are in agreement with those obtained from a different approach and also with experiment.

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Decay Widths of K Resonances from a Relativistic Quark Model

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The decay widths of K resonances are calculated in two relativistic quark models, using a unitary irreducible representation of the $SL(2,C)$ group. The first model that is used (essentially due to Gell-Mann) considers the K meson to consist of an active (nonstrange) quark and an inert (strange) quark. The $SL(2,C)$ group enters in an essential way to describe the orbital excitations of the K-meson tower. The second model, where the (strange) antiquark and the (nonstrange) quark are treated on an equal footing, involves the use of the two Dirac representations and an $SL(2,C)$ representation. In the second model we have one free parameter, and it is possible to fit satisfactorily the decay rates of $K^*\to K\pi$, $K_T\to K\pi$, $K_T\to K^*\pi$, and $K_A\to K^*\pi$. The possible generalization of this model to describe other hadrons is also discussed. The ratio of the s to d wave is predicted for the $K_A\to K^*\pi$ decay.

I. INTRODUCTION

QUARKS were originally introduced[1] to explain the occurrence of particular kinds of representations of $SU(3)$. This idea was pushed farther to imply that all the hadrons are bound states of real quarks.[2] Even though there has been no experimental evidence of a real (physical) quark, this model has many fascinating features.

Recent experiments have accumulated data on hadrons with high masses.[3] One of the main successes of the quark model has been in predicting the spin parities of these hadronic states. Whether the quarks are physically observable particles or purely mathematical objects is irrelevant to our discussion. The mesonic states are supposed to be bound states of the quark-antiquark system, and the excitation of the orbital angular momentum between them gives rise to the higher meson states.[4] Experimental data indicate no evidence for the excitation due to the creation of a quark-antiquark pair since, if any, it would lead to states belonging to **10**, $\overline{\mathbf{10}}$, and **27** representations of $SU(3)$. We therefore restrict

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