Strong Decays of Higher Baryons in a Quark Model

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An $SU(6)\times O(3)$ model of baryons, which had earlier been used by Mitra and Ross for the evaluation of the strong decay widths of negative parity $(\mathbf{70},1^-)$ baryons, is now extended to higher-lying baryonic states assumed to belong to the $(\mathbf{56},2^+)$ and $(\mathbf{70},3^-)$ representations, as Regge recurrences of the supermultiplets $(\mathbf{56},0^+)$ and $(\mathbf{70},1^-)$, respectively. Four different classes of supermultiplet transitions are examined. For the transitions $(\mathbf{56},2^+)\to(\mathbf{56},0^+)$, the results are in broad agreement with experiment, as well as those of $SU(6)_W$ and previous quark-model calculations. For the transitions $(\mathbf{56},2^+)\to(\mathbf{56},2^+)$ and $(\mathbf{56},2^+)\to(\mathbf{70},1^-)$, appreciable widths are predicted for several experimentally observable states, the first type being geometrically related to the $N^*N\pi$ coupling constant, and the second type being shown to involve a symmetry transition of only *one* unit (from symmetric to mixed symmetric wave functions). Results are also given for transitions from $(\mathbf{70},3^-)$ states.

1. INTRODUCTION

ONE of the areas of successful application of the quark model has been in the decays of hadrons. While weak and electromagnetic decays were some of the earlier specialties of the model,[1] strong decays of hadrons with pion or kaon emission have been receiving increasing attention in recent times. It was shown by Bechhi and Morpurgo[2] how the model could correlate the $NN\pi$ and $\rho\pi\pi$ coupling constants and also predict the decay of the 33 resonance, exactly as in an $SU(6)$ theory.[1] Mitra and Ross,[3] as well as others,[4,5] extended these ideas to all decays within the **56** of baryons, using an $SU(6)\times O(3)$ symmetry for the coupling constants, but breaking this symmetry in phase space. They obtained a number of interesting results for the (s- and d-wave) decays of the negative-parity baryons to the (familiar) positive-parity baryons in association with pseudoscalar mesons. More recently, Lipkin *et al.*[6] used the model to obtain the strong widths of the $L^P=2^+$ resonances. As for the decays of mesons, p-wave decays within the **35** states, taking account of mass-breaking effects, were computed by Cook,[7] while the cases of positive-parity mesons of all $L^P=1^-$ varieties corresponding to the $Q\bar{Q}$ structures $^1P_1$ and $^3P_{0,1,2}$ were treated by Mitra and Srivastava[8] and also by Uretsky.[9]

For all the processes, the basic mechanism is taken to be a Yukawa-type $\bar{Q}Q\Pi$ coupling connecting the emitted meson (Π) to the initial and final states of the quark (Q). While a formal dynamical explanation of such a simple mechanism is not available, dynamics seems otherwise to play a rather passive role in all these investigations, except for the assumptions of (I) the usual quark structures for the hadrons, and (II) the impulse approximation to the emission process in relation to these structures. However, if the various successes of the model that are listed above, provide any indication of its basic correctness, it appears worthwhile to pursue its consequences further, preferably with more specific assumptions. In this respect, the higher baryon resonances (a good number of which extend even to the third decade on the BeV scale, with accurate spin-parity determinations) seem to provide the natural choice. The purpose of this note is to consider the strong decays of the $L^P=2^+$ and $L^P=3^-$ baryons into those of lower L values (not necessarily $L^P=0^+$ alone),[10] which would eventually cascade down to $L^P=0^+$.

The basic structure of the baryons in this respect is taken to be given by an $SU(6)\times O(3)$ symmetry, which is broken in phase space, but not in the coupling constants, just as in MR.[3] Such a symmetry provides a very useful classification of the resonances, of which the $(\mathbf{56},0^+)$ and $(\mathbf{70},1^-)$ seem to agree quite well with

[1] For the earlier references on this subject, see R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High-Energy Laboratory, Chilton, Berkshire, England, 1966).
[2] C. Becchi and G. Morpurgo, Phys. Rev. **149**, 1284 (1966).
[3] A. N. Mitra and M. H. Ross, Phys. Rev. **158**, 1630 (1967); referred to as MR.
[4] R. J. Rivers, Phys. Letters **22**, 514 (1966).
[5] R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50A**, 617 (1967).
[6] H. J. Lipkin, H. R. Rubinstein, and H. Stern, Phys. Rev. **161**, 1502 (1967).
[7] P. A. Cook, Nuovo Cimento **48A**, 570 (1967).
[8] A. N. Mitra and P. P. Srivastava, Phys. Rev. **164**, 1803 (1967).
[9] J. Uretsky, Argonne National Laboratory Report, 1967 (unpublished).
[10] In this respect, the scope of this investigation is intended to extend appreciably beyond that of Ref. 6.

the existing data.[11] Indeed, the very assignments of the actual resonances are greatly facilitated[11] by the results of strong decays of the negative-parity baryons.[3] For the still higher resonances according to increasing L^P values, the most natural assignments would seem to be $(\mathbf{56},2^+)$, $(\mathbf{70},3^-)$, and so on, in accordance with the general idea of Regge recurrences.[11] These assignments can also be given a dynamical basis, provided one is prepared to give up Fermi statistics for quarks and use parastatistics[12] instead, in order to accommodate symmetric wave functions.[13] The group representations of the baryonic states are strongly dependent on the structure of the Q-Q forces. Thus while p-wave forces provide a large variety of baryonic states,[14] s-wave forces yield a surprisingly simple hierarchy of these,[15] such that the successive states are just $[\mathbf{56},(\text{even})^+]$ and $[\mathbf{70},(\text{odd})^-]$ in exact correspondence with the principle of angular momentum recurrences. To make specific predictions it is therefore convenient to take only these representations for the higher baryon resonances, so that a comparison with experiment will bear not only on the "elementary-meson hypothesis" in the strong-decay phenomena but, to a significant extent, also on the (dynamical?) question of the group representation themselves.

In Sec. 2 we describe the essential features of the wave functions of the $SU(6)\times O(3)$ supermultiplets $(\mathbf{56},2^+)$ and $(\mathbf{70},3^-)$, and indicate the J^P values of their various $SU(3)$ subgroups. We classify the decays into four broad categories and collect the necessary formulas for the decay widths in terms of (I) certain reduced widths or "coupling constants" (via the Wigner-Eckart theorem, applied to the spin and unitary spin matrix elements) and (II) phase-space factors parametrized in a convenient manner. Section 3 gives the numerical results for the different transitions of physical interest, together with a comparison of those from contemporary models like $SU(6)_W$, etc. For the conjectured particles, we give the results for alternative possibilities of spin and $SU(3)$ assignments corresponding to an assumed mass. Such a tabulation of the width variations for the conjectured particles is motivated by the possibility of (occasional) sensitive dependence of the decay widths to these assignments, so that a careful analysis of their decay widths might provide a useful probe into their spin-*cum*-$SU(3)$ structures. Section 4 gives a comparison with $SU(6)_W$ and other quark models. Section 5 is a summary of the results.

2. NECESSARY FORMALISM

As the calculational techniques to be used here are a straightforward extension of the pattern discussed in MR, we shall collect merely the essential features for convenience and adhere to the same notation, as far as possible.

For the $(\mathbf{56},2^+)$ states of an $[SU(6);L^P]$ classification the spatial wave function which is symmetric under parastatistics[12–15] may be taken as $\psi_{2M_L}{}^{(s)}$, so that the complete wave functions for the $\mathbf{8}$ and $\mathbf{10}$ $SU(3)$ states are[3]

$$\Psi(\mathbf{8})_J=[\psi_2{}^{(s)}\chi']_J\phi'+[\psi_2{}^{(s)}\chi'']_J\phi'' \tag{2.1}$$

and

$$\Psi(\mathbf{10})_J=[\psi_2{}^{(s)}\chi^s]_J\phi^s, \tag{2.2}$$

where we use the abbreviation[3]

$$[\psi_L{}^\alpha\psi_S{}^\beta]_J=\sum_{M_LM_S}C(LSJ;M_LM_S)\psi_{LM_L}{}^\alpha\psi_{SM_S}{}^\beta \tag{2.3}$$

for the spherical tensor product of the orbital $(\psi_{LM_L}{}^\alpha)$ and spin $(\chi_{SM_S}{}^\beta)$ functions of symmetry classifications α and β, respectively. (ϕ',ϕ'') and ϕ^s are the respective $SU(3)$ $\mathbf{8}$ and $\mathbf{10}$ functions; likewise (χ',χ'') and χ^s are the mixed symmetric (spin-$\frac{1}{2}$) and symmetric (spin-$\frac{3}{2}$) functions, respectively. The possible J values for the wave functions (2.1) and (2.2), corresponding to the representation $(\mathbf{56},2^+)$, are given by the following $[SU(3);J^P]$ components:

$$[8;\tfrac{3}{2}^+,\tfrac{5}{2}^+];\ [\mathbf{10},\tfrac{1}{2}^+,\tfrac{3}{2}^+,\tfrac{5}{2}^+,\tfrac{7}{2}^+]. \tag{2.4}$$

In a similar manner, for the $(\mathbf{70},3^-)$ states in the next hierarchy, the spatial functions are the mixed symmetric ones $\psi_{3M_L}{}'$ and $\psi_{3M_L}{}''$, so that the complete wave functions for the various allowed $SU(3)$ states are

$$\Psi(\mathbf{1})_J=[\psi_3{}''\chi'-\psi_3{}'\chi'']_J\phi^a, \tag{2.5}$$

$$\Psi(\mathbf{8}^d)_J=[\psi_3{}'\chi'-\psi_3{}''\chi'']_J\phi''+[\psi_3{}'\chi''+\psi_3{}''\chi']_J\phi', \tag{2.6}$$

$$\Psi(\mathbf{8}^q)_J=[\psi_3{}'\chi^s]_J\phi'+[\psi_3{}''\chi^s]_J\phi'', \tag{2.7}$$

$$\Psi(\mathbf{10})_J=[\psi_3{}'\chi'+\psi_3{}''\chi'']_J\phi^s, \tag{2.8}$$

where we have used the further abbreviation

$$[AB+CD]_J\equiv[AB]_J+[CD]_J \tag{2.9}$$

for the sum of two spherical tensor products of spin and orbital wave functions. In Eqs. (2.6) and (2.7), the octet states have been further classified in terms of spin-doublet $(\frac{1}{2})$ and spin-quartet $(\frac{3}{2})$ functions and distinguished by the respective superscripts d and q. The possible J values in (2.5)–(2.8) are indicated by the following $[SU(3);J^P]$ components of the represen-

[11] R. H. Dalitz, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 215.
[12] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).
[13] While the dynamical arguments for such an assumption are quite familiar (see Ref. 1), it was shown by one of us [A. N. Mitra, Phys. Rev. **151**, 1168 (1966)] how one could use this idea in a simple dynamical model to bring out the desired representations of $SU(6)\times O(3)$. In another paper [A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966)] it was shown how the structure of the baryon form factors stand strikingly in the way of antisymmetric quark wave functions for baryons.
[14] A. N. Mitra, Ann. Phys. (N. Y.) **43**, 126 (1967).
[15] A. N. Mitra and D. L. Katyal, National Physical Laboratory (Delhi) Report, 1967 (unpublished); also, Nucl. Phys. (to be published).

tation $(\mathbf{70},3^-)$ of $[SU(6);L^P]$:

$$[\mathbf{1};\tfrac{5}{2}^-,\tfrac{7}{2}^-],\quad [\mathbf{10},\tfrac{5}{2}^-,\tfrac{7}{2}^-],$$
$$[\mathbf{8};\tfrac{3}{2}^-,(\tfrac{5}{2}^-)^2,(\tfrac{7}{2}^-)^2,\tfrac{9}{2}^-]. \tag{2.10}$$

As in MR, the basic interaction for the emission of a meson of momentum $\mathbf{q}$ and energy ω by "quark number one" of momentum $\mathbf{P}_1$, spin $\boldsymbol{\sigma}^{(1)}$ and $SU(3)$-spin $\lambda_\alpha^{(1)}$ is taken as

$$G\sum_{\alpha=1}^{8}\boldsymbol{\sigma}^{(1)}\cdot\left(\mathbf{q}-\frac{\omega}{M_Q}\mathbf{P}_1\right)\lambda_\alpha^{(1)}\pi_\alpha. \tag{2.11}$$

Here $\lambda_\alpha^{(1)}$ ($\alpha=1,\cdots,8$) are the usual Gell-Mann matrices and π_α are the corresponding meson operators.[16] We are interested in the following four types of transitions (A,B,C,D) between various supermultiplets, where the possible harmonics associated with the emitted mesons are indicated by appropriate letters within parentheses after each transition symbol:

$$\mathrm{A}(p):(\mathbf{56},2^+)\to(\mathbf{56},2^+), \tag{2.12}$$

$$\mathrm{B}(p,f):(\mathbf{56},2^+)\to(\mathbf{56},0^+), \tag{2.13}$$

$$\mathrm{C}(s,d,g):(\mathbf{56},2^+)\to(\mathbf{70},1^-), \tag{2.14}$$

$$\mathrm{D}(d,g):(\mathbf{70},3^-)\to(\mathbf{56},0^+). \tag{2.15}$$

These transitions are characterized by a set of universal overlap integrals involving the orbital functions of the initial and final supermultiplet states. The actual matrix elements for transitions between specific baryonic states are, of course, proportional to these integrals as well as geometrical factors, viz., the $SU(3)$ isoscalar factors and the reduced matrix elements between the initial and final spin states, via the Wigner-Eckart theorem. The latter factors may be lumped together into a single symbol g, as in MR. The overlap integrals must of course be parametrized in a suitable manner, in the absence of any knowledge of the orbital functions. Unfortunately the number of such integrals is now much larger than in MR, since for each possible harmonic of the emitted meson, there is an independent integral. Further, the "direct" and recoil terms in the $\bar{Q}Q\Pi$ operator (2.11) give rise to two independent overlap integrals for the same harmonic of the emitted meson. Thus the different overlap integrals associated with the supermultiplet transitions (2.12)–(2.15) are, respectively, (2,4,6,4). Their *relative* orders of magnitude which can be estimated as in MR, are shown in Table I, where R^{-1} is a typical expectation value of a quark momentum like $\mathbf{P}_1$. Under the assumption that[3,17] $(qR)^2\ll1$, the recoil terms seem to be dominant for the cases B(p), C(s), and D(d) (in spite of the factor ω/M_Q), while for the other cases it is safest to assume the direct term alone to govern the transitions, since, the q dependence being the same for both, it is not possible to discriminate between them at the present state of experimental knowledge. Such an assumption considerably reduces the number of independent parameters characterizing the various overlap integrals and gives rise as in MR to the forms for the various decay widths indicated by the last column of Table I. While the index n for the momentum damping factor was taken as unity in MR, it will be found in the present analysis (Sec. 3) that a value $n\approx\frac{2}{3}$ yields somewhat better results. As for the other parameters, the constant a_p, appearing in the widths for the $(\mathbf{56},2^+)\to(\mathbf{56},2^+)$ decays, is not an independent quantity, since the corresponding overlap integral is essentially a normalization integral, as was the case for the $(\mathbf{56},0^+)\to(\mathbf{56},0^+)$ and $(\mathbf{70},1^-)\to(\mathbf{70},1^-)$ transitions evaluated in MR. This constant is therefore *geometrically* related to the width parameter for the decay $\Delta(1236)\to N(938)+\pi$, apart from possible variations in the q values involved. The parameter α may be taken as $(600\ \mathrm{MeV})^{-2}$, as in MR. The determination of the other parameters is discussed in the next section.

TABLE I. Relative orders of magnitude of the different types of transition associated with the various meson harmonics. Direct and recoil terms are shown separately. The ω/M_Q factor in the "recoil" column is suppressed.

Transition	Direct	Recoil	Decay width
A(p)	q	q	$a_pg^2q^3(1+\alpha q^2)^{-n}$
B(p)	R^2q^3	q	$b_pg^2q^3\omega^2(1+\alpha q^2)^{-n}$
B(f)	R^2q^3	R^2q^3	$b_fg^2q^7(1+\alpha q^2)^{-3n}$
C(s)	Rq^2	R^{-1}	$c_sg^2q\omega^2$
C(d)	Rq^2	Rq^2	$c_dg^2q^5(1+\alpha q^2)^{-2n}$
C(g)	R^3q^4	R^3q^4	$c_gg^2q^9(1+\alpha q_2)^{-4n}$
D(d)	R^3q^4	Rq^2	$d_dg^2q^5\omega^2(1+\alpha q^2)^{-2n}$
D(g)	R^3q^4	R^3q^4	$d_gg^2q^9(1+\alpha q^2)^{-4n}$

3. NUMERICAL RESULTS

Among the large number of states (2.4) predicted by the $(\mathbf{56},2^+)$ representation of $SU(6)\times O(3)$, there are very few which seem to have been identified with a reasonable degree of certainty. The states $N(1688)$ of $J^P=\frac{5}{2}^+$, $\Delta(1920)$ of $J^P=\frac{7}{2}^+$, and $\Lambda(1820)$ of $J^P=\frac{5}{2}^+$ are most probably the respective Regge recurrences of $N(938)$, $\Delta(1236)$, and $\Lambda(1115)$, all of which lie in the

[16] M. Gell-Mann, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin Co., Inc., New York, 1964), p. 11.

[17] It may be seen from Table I that the q dependence in the recoil term is faithfully reproduced, in accordance with the order of the spherical harmonic of the emitted meson. However, the direct term often violates this principle by showing an "over-dependence" on this quantity, to a maximum extent of a factor q^2. This is understood by noting that the direct term is already proportional to vector $\mathbf{q}$ [see Eq. (2.11)], and that the integration over the quark variables in the overlap integral can only bring in additional q factors. In this way the recoil term can frequently overcome the disadvantage of the factor ω/M_Q (which some believe to be small) by showing a weaker q dependence than its direct counterpart, at least as long as the condition $(qR)^2\ll1$ is satisfied.

(**56**,0^+) representation.[18–20] It is probable that the $I=1$, $Y=0$ member of $J^P=\frac{5}{2}^+$ is $\Sigma(1910)$,[11] though its quantum numbers are not as certain as its existence.[21] The $I=\frac{1}{2}$, $Y=-1$ member of $J^P=\frac{5}{2}^+$ has been variously conjectured as $\Xi(1933)$ [6,21] and as $\Xi(2020)$,[19,20] the latter being unobserved so far. For the decuplet states of $J^P=\frac{7}{2}^+$ other than $\Delta(1920)$, the one which has been identified with some confidence is $\Sigma^*(2035)$ of $I=1$, $Y=0$,[11,21] but the other members are entirely speculative. So far no other members of the other J^P multiplets listed in (2.4) have been found. Presumably they lie much higher up for identification at the present stage,[11] or they are too elusive for present experimental sensitivity.[3]

The known members of the next supermultiplet of $L^P=3^-$ are even fewer. The ones which are known reasonably well[21,22] are the[18] $N(2190)$ and $\Lambda(2100)$, each of $J^P=\frac{7}{2}^-$, which are believed to be the respective Regge recurrences of $N(1518)$ and $Y_0^*(1520)$, with $J^P=\frac{3}{2}^-$.[23] No other members of the higher ($L^P=3^-$) negative-parity states, predicted by the list of (2.10), seem to have been established. For the resonance $\Sigma(2260)$, the only quantum numbers known are $I=1$, $Y=0$, so that its $SU(3)$ assignment (**8** or **10**), or J^P values are speculative. Similarly, another conjectured particle on the basis of rather poor evidence[22] is $\Xi(2460)$, whose spin or $SU(3)$ assignments are entirely speculative. As mentioned earlier (Sec. 1), we shall give some sample figures for the principal decay widths of these conjectured particles, on the basis of different J^P and $SU(3)$ assignments, so that any wide variations in these widths with the above assignments might provide possible probes into their quantum numbers. Table II gives a classification of these particles.

TABLE II. Classification of particles.

Repr.	member	J^P	$SU(3)$ assignment
(**56**,2^+)	$N(1688)$	$\frac{5}{2}^+$	Octet
	$\Lambda(1820)$	$\frac{5}{2}^+$	Octet
	$\Sigma(1910)$[a]	$\frac{5}{2}^+$	Octet
	$\Xi(2020)$[b]	$\frac{5}{2}^+$	Octet
	$\Delta(1920)$	$\frac{7}{2}^+$	Decuplet
	$\Sigma^*(2035)$	$\frac{7}{2}^+$	Decuplet
(**70**,3^-)	$N(2190)$	$\frac{7}{2}^-$	Octet
	$\Lambda(2100)$[c]	$\frac{7}{2}^-$	Singlet
	$\Sigma(2260)$[a]	$\frac{7}{2}^-$	Octet
	$\Xi(2460)$[b]	$\frac{7}{2}^-$	Octet

[a] The quantum numbers are still uncertain. See Refs. 19 and 20.
[b] The mass and the quantum numbers are purely tentative.
[c] See Ref. 23.

(A) (**56**,2^+) $\to$ (**56**,2^+) decays. As is the case for the numerical results for the decay widths, the simplest type of transition is A(p) defined by (2.12), which gives essentially a normalization integral (see preceding section), since only the "direct term" in the $\bar{Q}Q\Pi$ interaction (2.11) is effective. The best example of such a transition within the supermultiplet $L^P=2^+$ is provided by $\Delta(1920)\to N(1688)+\pi$, for which $q=185$ MeV/c, to be compared with $q=231$ MeV/c for the more familair $\Delta(1236)\to N(938)+\pi$. Since the geometrical factors are the same in both the cases, the only difference comes from the q dependence which was taken in MR to be of the form $q^3(1+\alpha q^2)^{-1}$ and which agrees with the corresponding expression in Table I, if $n=1$. This provides the estimate $\Gamma\approx 53.3$ MeV for the width $\Delta(1920)\to N(1688)+\pi$, when the interaction strength G in (2.11) is normalized to the width 120 MeV of the 33 resonance. It would be most interesting to look for such a transition experimentally, since the width is predicted to be appreciable. This will bear directly on the correctness or otherwise of the (**56**,2^+) classification of the higher-lying positive-parity states. In a similar way the decay width of $\Sigma^*(2035)\to \Lambda(1820)+\pi$ is predicted to be $\sim$19.0 MeV, which may be compared with the width $\sim$34 MeV for the more familiar transition $\Sigma^*(1385)\to\Lambda(1115)+\pi$ within the **56** of baryons.

(B) (**56**,2^+) $\to$ (**56**,0^+) decays. The next simplest transitions are the B-type [Eq. (2.13)], involving f- or p-wave mesons for which a reasonable amount of data is available. Table III gives the results of calculations of such transitions for the various decay modes of experimental interest, to be compared with experiment as well as the $SU(6)_W$ results of Dobson.[20] The Table is so arranged that its earlier part gives pure f-wave decays with one adjustable parameter b_f (see Table I). For such decays, several experimental figures are fortunately available.[22] The later part of the Table involves cases in which the decays occur in mixtures of f and p waves. These are mostly cases for which little data are available, so that the emphasis in this part of the Table is mainly on a comparison with the $SU(6)_W$ predictions of Dobson.[20]

The f-wave decay widths in Table III are parametrized as

$$\Gamma_f=3.03g^2q^7(1+\alpha q^2)^{-2}, \tag{3.1}$$

where we have used the width of $N(1688)\to N+\pi$ as input and the index n of Table I as

$$n\approx\tfrac{2}{3}, \tag{3.2}$$

[18] We use the convention that, for the octet states of $J^P=\frac{5}{2}^+$, the particle symbols are (N,Λ,Σ,Ξ), exactly as for the ordinary baryons. For the decuplet states the corresponding symbols are, similarly, ($\Delta,\Sigma^*,\Xi^*,\Omega$).

[19] O. W. Greenberg and M. Resnikoff, Phys. Rev. **163**, 1844 (1967).

[20] P. N. Dobson, Jr., Phys. Rev. **160**, 1501 (1967).

[21] J. Meyer, Heidelberg International Conference on Elementary Particles, Heidelberg, 1967 (unpublished).

[22] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[23] If the $\Lambda(2100)$ of $\frac{7}{2}^-$ is a Regge recurrence of $Y_0^*(1520)$, it is more likely an $SU(3)$ singlet than the isoscalar member of an $SU(3)$ octet. For this particle, we should then use the notation $Y_0^*(2100)$ rather than $\Lambda(2100)$. However, there may be important mixing effects between these two states. We have, therefore, calculated the decay widths of this particle corresponding to both these assignments.

TABLE III. Numerical results for the principal decay widths (in MeV) in the $(\mathbf{56},2^+) \to (\mathbf{56},0^+)$ transitions. The decays based on pure f-wave transitions are shown in the first part of the Table, while the second part gives the net results for f- and p-wave transitions. For comparison, the $SU(6)_W$ predictions of Dobson are included.

$SU(3)$ transition $B' \to B$	Transition	Γ (MeV) (theory)	Γ (MeV) (experiment)	Γ (MeV) (Dobson)
$(8)_{5/2^+} \to (8)_{1/2^+}$	$N(1688) \to N\pi$	72.0 input	72.0	96.6
	ΛK	0.02	small	0.05
	$N\eta$	0.3	small	0.3
	$\Lambda(1820) \to N\bar{K}$	39.9	5.8±5.6	42.1
	$\Sigma\pi$	18.1	9.1±0.9	14.4
	$\Lambda\eta$	0.5	∼0.8	0.4
	$\Sigma(1910) \to N\bar{K}$	2.7	4.8	3.8
	$\Lambda\pi$	17.1	6.0	21.0
	$\Sigma\pi$	29.7	1.8	30.7
	$\Sigma\eta$	0.8	not observed	0.6
	$\Xi(2020) \to \Sigma\bar{K}$	45.0	not observed	33.6
	$\Lambda\bar{K}$	3.1	not observed	0.9
	$\Xi\pi$	2.7	not observed	3.0
	$\Xi\eta$	1.5	not observed	2.4
$(10)_{7/2^+} \to (8)_{1/2^+}$	$\Delta(1920) \to N\pi$	92.4	100.0	146.6
	ΣK	6.2	seen	4.1
	$\Sigma^*(2035) \to N\bar{K}$	27.3	25.6	38.0
	$\Lambda\pi$	40.9	40.0	65.8
	$\Sigma\pi$	19.3	seen	29.2
	$\Xi^*(2020) \to \Sigma\bar{K}$	9.3	not observed	...
	$\Lambda\bar{K}$	15.9	not observed	...
	$\Xi\pi$	13.9	not observed	...
$(10)_{7/2^+} \to (10)_{3/2^+}$	$\Delta(1920) \to \Delta(1236)\pi$	45.4	not observed	23.4
	$\Sigma^*(2035) \to \Sigma^*(1385)\pi$	20.6	seen	8.6
	$\Lambda(1820) - \Sigma^*(1385)\pi$	15.0 input	14.9±1.5	7.4
	$N(1688) \to \Delta(1236)\pi$	22.7	not observed	10.6
	$\Sigma(1910) - \Sigma^*(1385)\pi$	9.7	not observed	2.9
	$\Delta(1236)\bar{K}$	33.2	not observed	6.3
	$\Xi(2020) \to \Sigma^*(1385)\bar{K}$	9.4	not observed	1.7
	$\Xi^*(1533)\pi$	9.8	not observed	3.7

instead of $n=1$ used in MR. This happens to give a slightly better over-all fit to the experimental data.[22] One sees from Table III that the fits are rather satisfactory except for two large discrepancies in the decays of $\Sigma(1910)$ to $\Sigma\pi$ and $\Lambda\pi$ where the predicted values are much too large for experiment. This discrepancy which is also shared by the $SU(6)_W$ predictions for these cases (in close agreement with our results) probably indicates strong admixtures of $(I=1, Y=0)$ components in **8** and **10** states of the $(\mathbf{56},2^+)$ supermultiplet.

It is also possible to compare the width predictions of this model with those obtained from a Regge-recurrence model in $U(6)_W \times O(2)_W$ symmetry.[24] Thus for the ratio

$$R = \frac{\Gamma(\Delta(1920) \to N\pi)}{\Gamma(N(1688) \to N\pi)} = 1.4 \pm 0.4 \text{ (expt)}, \quad (3.3)$$

Freund *et al.*[24] obtained the value 1.84, to be compared with our prediction of 1.30. The $SU(6)_W$ prediction[20] for this ratio is 1.52. Sakita and Wali,[25] using a superconvergence dispersion relation saturated by the poles $N(938)$, $\Delta(1238)$, $N(1688)$, and $\Delta(1920)$, made an indirect evaluation of this ratio.[26]

For the f-wave decays of the decuplets in $(\mathbf{56},2^+)$, Barut and Tripathy[27] have recently obtained results using an $O(4,2)$ symmetry. Their figures do not seem to be as good as those of the quark model with $SU(6) \times O(3)$ symmetry.

The slight modification in the representation of the momentum damping factor from $n=1$ in MR to $n=\frac{2}{3}$ in the present case, which was made in order to "tone down" the damping to some extent, has been found to produce very small changes in the results of MR. Thus for some typical cases in MR, the changes in width (in MeV) are as follows[28]:

$$\begin{aligned} &N^*(1690) \to N\pi && (17.0 \to 18.0), \\ &Y_1^*(1765) \to Y_1^*(1385)\pi && (\ 5.0 \to \ 4.1), \\ &N^*(1518) \to N\pi && (52.0 \to 48.9), \\ &Y_1^*(1765) \to Y_0^*(1520)\pi && (16.0 \to 16.2). \end{aligned}$$

In the present case, however, the generally larger phase space available makes the predictions more sensitive to this parameter, and a somewhat lower value $(\approx\frac{2}{3})$ of the index seems to be indicated by the data.

The decays of the $(\mathbf{56},2^+)$ states into decuplets and

[24] P. G. O. Freund, A. N. Maheshwari, and E. Schonberg, Phys. Rev. **159**, 1232 (1967).
[25] B. Sakita and K. C. Wali, Phys. Rev. Letters **18**, 29 (1967).
[26] This figure of Ref. 25 is discussed in detail in Ref. 24.
[27] A. O. Barut and K. C. Tripathy, Phys. Rev. Letters **19**, 1081 (1967).
[28] For the negative-parity $(\mathbf{70},1^-)$ baryons, we are using the same notation as MR.

TABLE IV. Predicted decay widths of some typical $(\mathbf{56},2^+)\rightarrow(\mathbf{70},1^-)$ transitions. For purposes of calculation, the resonances $N^*(1518)$ of $J^P=\frac{3}{2}^-$, and $N^*(1540)$ of $J^P=\frac{1}{2}^-$, are taken as pure $\mathbf{8}^d$ and $\mathbf{8}^q$ states, respectively, as in MR. The d- and g-wave contributions to these transitions (in MeV) are shown as proportional to two arbitrary dimensionless parameters γ_d and γ_g, respectively.

$SU(3)$ transition	Individual transition	Γ (MeV) (theory)
$(8)_{5/2^+}\rightarrow(1)_{3/2^-}$	$\Sigma(1910)\rightarrow Y_0^*(1520)\pi$	$154.5\gamma_g+10.0\gamma_d$
$(10)_{7/2^+}\rightarrow(8)_{5/2^-}$	$\Delta(1920)\rightarrow N^*(1690)\pi$	$0.7\gamma_g+1.7\gamma_d$
	$\Sigma^*(2035)\rightarrow Y_1^*(1765)\pi$	$1.5\gamma_g+4.1\gamma_d$
$(8)_{5/2^+}\rightarrow(8)_{3/2^-}$	$N(1688)\rightarrow N^*(1518)\pi$	$5.6\times10^{-4}\gamma_g+4.6\times10^{-3}\gamma_d$
$(10)_{7/2^+}\rightarrow(10)_{1/2^-}$	$\Delta(1920)\rightarrow N^*(1518)\pi$	$194.8\gamma_g+2.9\gamma_d$
	$\Sigma^*(2035)\rightarrow N^*(1518)\bar{K}$	$11.8\times10^{-3}\gamma_g+1.3\times10^{-2}\gamma_d$
	$\Delta(1920)\rightarrow\Delta(1680)\pi$	$0.9\gamma_g+0.$
	$\Delta(1920)\rightarrow N^*(1540)\pi$	$10.5\gamma_g+0.$
	$\Sigma(1910)\rightarrow Y_0^*(1405)\pi$	$0\quad+5.1\gamma_d$

mesons, which are mixtures of p- and f-wave mesons, have been parametrized in Table II as

$$F\Gamma_f+P\Gamma_p, \tag{3.4}$$

where $\sqrt{F}$ and $\sqrt{P}$ are certain geometrical factors (Clebsch-Gordan coefficients) governing the f and p admixtures in decay amplitudes, Γ_f is given by (3.1), and Γ_p (see Table I) by

$$\Gamma_p=0.39g^2q^3\omega^2(1+\alpha q^2)^{-2/3}. \tag{3.5}$$

The input width for these cases is normalized to 15 MeV for $\Lambda(1820)\rightarrow\Sigma^*(1385)+\pi$ whose experimental width is 14.9 ± 1.5 MeV. In the absence of experimental data for such decays, the only comparison that can be made of the calculated figures is with the $SU(6)_W$ predictions which, however, are not in good agreement with our results.

(C) $(\mathbf{56},2^+)\rightarrow(\mathbf{70},1^-)$ decays. Another interesting class of decays is represented by the transitions $C(s,d,g)$ of Eq. (2.14) between the $(\mathbf{56},2^+)$ and $(\mathbf{70},1^-)$ supermultiplets. For the pure g-wave decays, we make the parametrization (see Table I)

$$\Gamma_g=C_gg^2q^9(1+\alpha q^2)^{-8/3} \tag{3.6}$$

characteristic of a direct transition. For the d-wave decays, in general, *both* the direct and recoil terms could be appreciable, but in order to economize on the parameters we arbitrarily take the widths of the form

$$\Gamma_d=C_dg^2q^5(1+\alpha q^2)^{-4/3}. \tag{3.7}$$

Since the cases of physical interest seem to be mainly those for which a mixture of d- and f-wave decays are operative, we shall not consider s-wave decays in such transitions. Certain typical widths for d- and g-wave decays are shown in Table IV in terms of two dimensionless parameters γ_d and γ_g. While at the present stage of experiment it is not possible to compare these results with observation, we wish to assert that the quark model *predicts* the absolute widths to be *appreciable* for the following reason: The overlap integral for these transitions is one involving a symmetrical (S) initial wave function and a mixed symmetrical (M) final wave function. Since the basic interaction (2.11) causes a change in the state of *one* quark at a time, one expects the "symmetry-selection rule" $\Delta(\text{symm})=1$ to be valid.[29] Such a symmetry selection rule implies a large overlap between S and M functions, M and A functions, but *not* S and A functions. This was also the basis of calculation of the $(\mathbf{70},1^-)\rightarrow(\mathbf{56},0^+)$ widths in MR. Experimental detection of $(\mathbf{56},2^+)\rightarrow(\mathbf{70},1^-)$ would therefore be of great interest.

(D) $(\mathbf{70},3^-)\rightarrow(\mathbf{56},0^+)$ decays. Finally we give the results for the mesonic decays of certain $(\mathbf{70},3^-)$ states to the particles of $(\mathbf{56},0^+)$. While the decaying mesons could be either or both of g or d waves, the presently available experimental figures are mainly for g-wave mesons. According to Table I, we parametrize such decay widths as

$$\Gamma_g'=d_gq^2q^9(1+\alpha q^2)^{-8/3}. \tag{3.8}$$

An extra source of uncertainty is in the appearance of octet states in pairs, which we classify as spin-doublet and spin-quartet, respectively, as in MR. Table V gives the results for some of the transitions of experimental interest, listing under separate columns the transitions from $\mathbf{8}^d$ and $\mathbf{8}^q$ states. Most of the table is speculative, since we have considered several alternative possibilities of $SU(3)$ and J^P assignments for the particles not yet established. The only detailed comparison that can be made at this stage is that with the corresponding $SU(6)_W$ predictions[20] which are also listed. It appears that the $SU(6)_W$ results are generally in conformity with $\mathbf{8}^d$ for certain types of transitions and with the $\mathbf{8}^q$ assignments for certain other types.

4. COMPARISON WITH RELATED MODELS

It may be of interest to compare in some detail the present $SU(6)\times O(3)$ model with the $SU(6)_W$ approach of Dobson[20] and the quark model of Lipkin *et al.*[6] The relation between $SU(6)_W$ and the quark model has been shown to be very close by Lipkin *et al.*, since the basic $\bar{Q}QP$ interaction operator $\boldsymbol{\sigma}\cdot\mathbf{q}$ transforms like the

[29] S. Das Gupta and A. N. Mitra, Phys. Rev. **159**, 1285 (1967).

TABLE V. Decay widths for (70,3⁻) — (56,0⁺) transitions in terms of pure *g*-wave mesons. For the conjectured particles like Σ(2260) and Ξ(2466), results are shown for variations in J^P and $SU(3)$ assignments. The $SU(6)_W$ predictions of Dobson are also included.

$SU(3)$ transition $B' \to B$	Transition	Γ (MeV) (theory) $^2(8)$	$^4(8)$	Γ (MeV) (Dobson)
$(8)_{7/2^-} \to (8)_{1/2^+}$	$N(2190) \to N\pi$	60.0 input	60.0 input	62.6
	ΛK	9.6	4.3	2.8
	$N\eta$	8.8	0.0	0.9
	ΣK	0.8	2.8	0.2
	$\Lambda(2110) \to N\bar{K}$	29.8	13.3	11.4
	$\Sigma\pi$	2.9	11.6	3.6
	$\Lambda\eta$	0.5	1.8	...
	$\Sigma(2260)$ — $N\bar{K}$	2.0	8.0	1.2
	$\Lambda\pi$	3.3	13.2	5.7
	$\Sigma\pi$	43.7	7.5	9.5
	$\Sigma\eta$	0.8	3.4	...
	$\Xi(2460) \to \Lambda\bar{K}$	16.2	0.0	2.2
	$\Sigma\bar{K}$	38.5	38.5	34.3
	$\Xi\pi$	3.6	13.6	1.5
	$\Xi\eta$	14.7	6.3	4.1
$(8)_{9/2^-} \to (8)_{1/2^+}$	$\Sigma(2260)$ — $N\bar{K}$		30.2	...
	$\Lambda\pi$		52.5	...
	$\Sigma\pi$		30.1	...
	$\Sigma\eta$		12.6	...
	$\Xi(2460)$ — $\Lambda\bar{K}$		0.0	...
	$\Sigma\bar{K}$		154.0	...
	$\Xi\pi$		52.6	...
	$\Xi\eta$		25.2	...
$(1)_{7/2^-} \to (8)_{1/2^+}$	$Y_0^*(2110) \to N\bar{K}$	29.8		...
	$\Sigma\pi$	33.3		...
	$\Lambda\eta$	4.5		...

component of a vector in the direction **q** of the emitted meson, exactly as expected for this operator under W-spin rotations. Further, for transitions between two like baryon states belonging to the same **56** but different L excitations (e.g., $L=2$ and 0, respectively), the transformation operator involves only $L_z=0$, so that even a nonzero L excitation of the initial baryon state does not play any role in an $SU(6)_W$-invariant interaction. One would therefore expect a strong similarity between the results of the quark model, as well as $SU(6)_W$, for transitions involving like baryon states, e.g., $N_{5/2}(1688) \to N_{1/2}(938)+\pi$, etc., except possibly for a difference in input figures and the explicit forms of parametrization of the decay widths. However, the situation is somewhat different for transitions between unlike baryon states. Such situations which do not seem to have been covered by the analysis of Ref. 6, may well demand nonzero values of L_z for the initial state. For example, consider the transition $N_{5/2}(1688) \to \Delta_{3/2}(1236)+\pi$, where the intrinsic spin functions in the initial state have only $S=\frac{1}{2}$, while those for the product state have $S=\frac{3}{2}$, under an $SU(6)\times O(3)$ quark model. Therefore a transition from an initial $L=2$ state of $J_z=\frac{3}{2}$ to a final $L=0$ state of $J_z=S_z=\frac{3}{2}$ necessarily requires $L_z=+1$ for the initial state. Such a requirement, which is specific to our $SU(6)\times O(3)$ quark model, makes the L excitations play a more active role in the decay transitions than envisaged in an $SU(6)_W$ theory. One should therefore expect differences in the decay predictions of an $SU(6)\times O(3)$ quark model and those of an $SU(6)_W$ theory, especially when unlike baryons are involved in the transitions; and these differences should increase with the L excitation of the initial state.

The above qualitative considerations are probably enough to explain the differences of our decay results from those of Dobson,[20] the differences being more marked in Table V ($L^P=3^-$) than in Table III ($L^P=2^+$). In this connection, one must also keep in mind the detailed forms of parametrization of the decay rates, which differ significantly in the two cases. Thus, while the centrifugal effects in both cases are similarly considered, the emphasis on the damping factors for high-wave mesons differs widely. Further, the "elementary" nature of the baryon in Dobson's approach, introduces certain relativistic baryon kinematics in the decay rates which find no counterpart in our $SU(6)\times O(3)$ model where the baryon masses or energies do not appear at all in the formulas.

As for a comparison of our results with those of Lipkin *et al.*,[6] the main difference probably lies in the scope of the investigation. According to their own claims, the authors of Ref. 6 are mainly interested in examining the $L^P=2^+$ quark structures of the baryons, irrespective of whether or not these structures are a part of a (bigger) group, such as $SU(6)\times O(3)$. Indeed, they do not assume anything more than isospin invariance, much less $SU(3)$. In our approach we are more interested in examining the effect of a full $SU(6)\times O(3)$ symmetry in the structure of the decay-coupling constants, as a direct extension of MR for $L^P=1^-$ baryon decays. We break this group structure only in the phase space, which we believe to be the main symmetry-breaking effect. Consideration of the $SU(6)\times O(3)$ group clearly has much greater predictive power than the (limited) model of Ref. 6 which is necessarily confined to like baryons among **56** members. The bigger group also facilitates the evaluation of transition rates between other supermultiplet states involving unlike baryons as well as nonzero L values in the final baryons. This is manifested through our evaluation of the additional types of transitions $2^+\to 2^+$, $2^+\to 1^-$, and $3^-\to 0^+$, none of which were considered in Ref. 6.

Another question of interest, which was pointed out by Lipkin *et al.*,[6] concerns certain polarization experiments on the final baryon to check on the interference between two partial waves for the emitted meson, where applicable. The argument used in Ref. 6 goes as follows: In a transition of the form $N_{5/2}(1688)\to\Delta_{3/2}(1236)+\pi$, the pion will come out with p or f waves, both of which, however, involve the same radial integral. Thus, the amplitudes for the p and f waves should bear geometrical relations to each other, and this competition between different partial waves can be checked by polarization measurements. Before commenting on such a possibility, we record a similar situation in negative-parity baryon decays[3] relating to the interference between s and d mesons, in the process $Y_1^*(1660)\to Y_1^*(1385)+\pi$. A similar argument also leads to a geometrical relationship between the two amplitudes and hence to a definite prediction for the final baryon polarization. It seems to us, however, that the argument is probably oversimplified. A look at Table I, for the structure of the decay rates, shows that the similarity of the decay amplitudes for p and f waves (and hence their geometrical relationship) applies only to the direct term in the transition, but not to the recoil term which seems to be a "more faithful index" of the centrifugal barrier (see also Sec. 2), insofar as it shows the f wave more heavily suppressed than the p wave. A similar result was discussed in MR in detail for s versus d waves. Therefore, if we consider the recoil term to be more important for the p wave, and the direct term to be dominant for the f wave, and parametrize accordingly (as we have actually done in Sec. 3), then the two amplitudes become totally incoherent and are no longer geometrically related. Such an effect would mar the possibility of any specific prediction on the baryon polarization, since no phase relation exists between the direct (f) and the recoil (p) amplitudes; unlike the (strictly geometrical) relationship between both the direct amplitudes. Since we believe this parametrization to be a more realistic representation of the decay widths (see also MR), the possibility of making specific quark-model predictions on the baryon polarizations is much more remote, in our estimation, than conjectured in Ref. 6.

5. CONCLUSION

We have tried to present a straightforward extension of the $SU(6)\times O(3)$ model of MR for the evaluation of decay widths from the several higher-lying baryonic states, by classifying the transitions under four distinct heads. The cases of greatest experimental interest at the moment are represented by the $(\mathbf{56},2^+)\to(\mathbf{56},0^+)$ transitions, for which good agreement is achieved with the experimental data, with a minimum of parametrization (especially for the pure f-wave decays). In particular, the parameter $\alpha=(600\text{ MeV})^{-2}$ is the same as used earlier (in MR) for the $(\mathbf{70},1^-)$ decays. For certain other types of decays, the model makes several interesting predictions whose experimental status should be more carefully explored. Thus, the $(\mathbf{56},2^+)\to(\mathbf{56},2^+)$ decays, which are governed essentially by the $NN\pi$ coupling constant according to this model,[30] have some particularly interesting predictions, such as an appreciable width ($\sim$50 MeV) for the $\Delta(1920)\to N(1688)\pi$ transition. Similarly, the decays of $(\mathbf{56},2^+)\to(\mathbf{70},1^-)$ are predicted to be appreciable in terms of a "symmetry selection rule," viz., *one unit* of symmetry charge (which was also the main basis of calculations in MR for the negative-parity decays). Several experimentally detectable ratios for such decays are presented. It is hoped that at least some measurements for such cases will soon be available.

For the higher-lying negative-parity baryons, the richness of predictions is, at this stage, too far ahead of experiment.

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[30] As other octets such as $J^P=\frac{3}{2}^+$ or $\frac{1}{2}^+$ are not known at present. we have not considered (h,f)-wave decays within $(\mathbf{56},2^+)\to(\mathbf{56},2^+)$ transitions.