PHYSICAL REVIEW VOLUME 169, NUMBER 5 25 MAY 1968

Backward Compton Scattering Sum Rule and the $\pi^0 \to 2\gamma$, $\eta^0 \to 2\gamma$ Decay Rates

S. RAI CHOUDHURY*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

AND

R. RAJARAMAN†
Institute for Advanced Study, Princeton, New Jersey

(Received 18 December 1967)

We use the superconvergence of certain Compton scattering (s-channel) helicity amplitudes for fixed s and large t, to derive a sum rule for t- and u-channel processes. The u-channel ($\gamma N \to \gamma N$) contribution contains the well-known nucleon pole terms and the continuum, which we replace by just the π-N intermediate states; we then feed in photoproduction data. The t-channel ($\gamma\gamma \to N\bar{N}$) contribution consists of the π, η poles and the continuum. We choose a suitable combination of superconvergent amplitudes such that the effects of 0^+, 2^+, and 1^+ resonances in the t channel are eliminated. Assuming that this takes care of most of the t-channel continuum, we get a sum rule for the $\pi^0 \to 2\gamma$ and $\eta^0 \to 2\gamma$ widths, or alternatively, by using the experimental widths, we can check the consistency of the superconvergence in question. A brief comparison is made with related work by Goldberger and Abarbanel, and by Pagels.

1. INTRODUCTION

THE exploitation of Regge asymptotic behavior, along with analyticity, has yielded in recent times some interesting sum rules connecting various observable quantities in photonic scattering processes.[1] These rules may be expected to remain as useful relations between static properties of the particles involved and their scattering amplitudes, inasmuch as no appeal is made to the detailed dynamics of the system in their derivation.

For the nucleon Compton-scattering process in the near forward direction, one of us has calculated a sum rule for the nucleon magnetic moment.[2] In this paper, we propose to study backward[3] Compton scattering and obtain, by using a combination of superconvergence and dispersion relations, the 2γ-decay rates of the π^0 and η mesons. The possibility of such a calculation based on superconvergence has also been suggested by Abarbanel and Goldberger.[4] However, we use a different combination of amplitudes from the one used by these authors, designed so as to eliminate contributions from the f_0 pole and possible 0^+, 1^+ poles in the $\gamma\gamma \to N\bar{N}$ channel. We have also included the pion photoproduction data to estimate the continuum contribution in the $\gamma N \to \gamma N$ channel.

It should be mentioned at this point that a calculation of the π^0 and η^0 photonic decay widths has also been performed by Pagels,[5] based on an analysis of forward Compton scattering. We feel that a certain aspect of Pagels's work needs reexamination and a brief discussion of this point is given in the Appendix. In any case, our sum rule is derived from quite a different point of view, using superconvergence of the amplitudes in the crossed channels, relative to the channel studied by Pagels.

2. KINEMATIC PRELIMINARIES AND NOTATIONS

We shall follow the notation and kinematics of Hearn and Leader,[6] of which we will only rewrite a minimal amount here. The s channel corresponds to Compton scattering, where s, t, and u are the standard Mandelstam variables. There are six invariant amplitudes in the problem, and we will use the choice $A_1 \cdots A_6$ introduced in H-L. These amplitudes satisfy the crossing relations

$$A_i(s,t,u) = A_i(u,t,s) \quad \text{for} \quad i=1, 2, 3, 6$$

and

$$A_i(s,t,u) = -A_i(u,t,s) \quad \text{for} \quad i=4, 5. \tag{1}$$

W, E, p, and θ refer, respectively, to the total energy, nucleon energy, nucleon momentum, and scattering angle in the c.m. frame of the s-channel process. $\Phi_1 \cdots \Phi_6$ refer to the six helicity amplitudes in the s channel described in H-L, while $M_1 \cdots M_6$ correspond to helicity amplitudes in the t channel ($\gamma\gamma \to N\bar{N}$). We will reproduce from H-L some of these amplitudes and their interrelationships in greater detail when we use them.

3. CHOICE OF AMPLITUDES AND DERIVATION OF THE SUM RULE

Consider the following three helicity amplitudes in the s channel:

---

* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. 68-1365

† On leave of absence from the University of Southern California, Los Angeles, Calif.

[1] S. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966).

[2] S. Rai Choudhury, Phys. Rev. Letters **19**, 96 (1967).

[3] By "backward" we shall mean $u=0$. The scattering is backward then only in the asymptotic region.

[4] H. D. I. Abarbanel and M. L. Goldberger, Phys. Rev. **165**, 1594 (1968).

[5] H. Pagels, Phys. Rev. **158**, 1566 (1967).

[6] A. C. Hearn and E. Leader, Phys. Rev. **126**, 789 (1962); referred to as H-L.

$$\Phi_2=\langle -\tfrac{1}{2}-1|T|\tfrac{1}{2}\,1\rangle=\frac{-\sin\frac{1}{2}\theta}{8\pi W}\times[E(A_1+A_2)-mp(A_4+A_5)-2pA_3],$$

$$\Phi_3=\langle \tfrac{1}{2}-1|T|\tfrac{1}{2}\,1\rangle=\frac{\cos\frac{1}{2}\theta}{8\pi W}\times[m(A_1+A_2)-Wp(A_4+A_5)],\tag{2a}$$

and

$$\Phi_6=\langle \tfrac{1}{2}-1|T|-\tfrac{1}{2}\,1\rangle=\frac{\sin\frac{1}{2}\theta}{8\pi W}\times[E(A_1+A_2)-mp(A_4+A_5)+2pA_3].$$

As is well known, these amplitudes have kinematic t singularities which have to be factored out. The reduced amplitudes, which are assumed to be analytic in the t plane except for dynamical singularities, are given by

$$\tilde{\Phi}_2=\Phi_2[\sin(\tfrac{1}{2}\theta)]^{-1},$$
$$\tilde{\Phi}_3=\Phi_3[\cos(\tfrac{1}{2}\theta)\sin^2(\tfrac{1}{2}\theta)]^{-1},\tag{2b}$$

and

$$\tilde{\Phi}_6=\Phi_6[\sin^3(\tfrac{1}{2}\theta)]^{-1}.$$

Further, by appealing to Regge asymptotic behavior of the helicity amplitudes, we note that for fixed s and large t, $\tilde{\Phi}_2$ behaves as $t^{\alpha(s)-1/2}$ while $\tilde{\Phi}_3$ and $\tilde{\Phi}_6$ behave as $t^{\alpha(s)-3/2}$, where $\alpha(s)$ is the position of the leading Regge trajectory in the s channel. We will be working at the point $s=0$—the choice of this point will be elaborated on below—and it may be reasonably expected that $\alpha(0)$ for the leading fermion Regge pole is less than $\frac{1}{2}$. Thus, $\tilde{\Phi}_2$ is convergent in t, whereas $\tilde{\Phi}_3$ and $\tilde{\Phi}_6$ are superconvergent, obeying

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{Im}\tilde{\Phi}_{3,6}(s,t',u')dt'=0.\tag{3}$$

Equation (3) is clearly a sum rule relating residues of poles of $\tilde{\Phi}_{3,6}$ to integrals over imaginary parts of the continuum amplitudes in the t and u channels. This is in principle the sum rule we use for evaluating the 2γ decay widths of the π^0 and the η, both of which occur as poles in the t channel.

Now, from the Hearn-Leader definition of the A_i, it can be seen that the π^0 and η poles occur only in A_3, which in turn is present only in $\tilde{\Phi}_6$. The reason for our introducing $\tilde{\Phi}_3$ and $\tilde{\Phi}_2$ will be clear from the following.

The sum rule (3) for $\tilde{\Phi}_6$ can be written as

$$-R_6{}^t+R_6{}^u-\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\,\mathrm{Im}\Phi_6(s,t',2m^2-t'-s)+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}du'\,\mathrm{Im}\tilde{\Phi}_6(s,2m^2-u'-s,u')=0,\tag{4}$$

where $R_6{}^u$ is the nucleon pole residue, $R_6{}^t$ is the sum of the π^0 and η pole residues, and m and μ are the nucleon and pion masses, respectively.

Photonic intermediate states have as usual been neglected in both channels, being suppressed by a factor e^2. Now, we can assume that the u-channel continuum is dominated by the N-π intermediate state, for which we will feed in photoproduction data. The nucleon pole residue is readily available in terms of the nucleon magnetic moments. This leaves the t-channel continuum, about which unfortunately not much experimental information is available. However, this contribution may be expected to be dominated by the $T=0$, 2π intermediate state. ($T=1$ is forbidden by charge conjugation and G parity.) The only established 2π resonances that qualify are the f^0(1250 MeV) with $J^P=2^+$ and a possible 0^+ contribution. The latter will occur only in the amplitudes A_1 and A_2 as can be readily seen from the definitions of the A_i in H-L. However, it is clear from Eq. (2) that A_1 and A_2 can be eliminated by using the superconvergent combination $\tilde{\Phi}_6-(E/m)\tilde{\Phi}_3$ instead of $\tilde{\Phi}_6$. For this combination then, the t-channel continuum is replaced in our approximation by the residue of the f^0 pole. This residue is of course not known, but we will presently show a way of eliminating it. It may also be noted that possible $J^P=1^+$ poles will not couple to A_4+A_5 or A_3 and will not contribute to our combination $\tilde{\Phi}_6-(E/m)\tilde{\Phi}_3$. Such a pole, although not allowed in the $T=0$ two-pion system, might have occurred via higher intermediate states in the t channel. Our superconvergent amplitude is then

$$\tilde{\Phi}_6-\frac{E}{m}\tilde{\Phi}_3=\frac{1}{\sin^2(\theta/2)}\left[\frac{\tilde{\Phi}_6}{\sin(\theta/2)}-\frac{E}{m}\frac{\tilde{\Phi}_3}{\cos(\theta/2)}\right]$$
$$=\frac{-(s-m^2)^2}{st}p\left[(A_4+A_5)\left(-m-\frac{EW}{m}\right)+2A_3\right],$$

using Eq. (2).

For fixed s, this may be replaced by

$$(1/t)[(A_4+A_5)(m+(EW/m))-2A_3]=(1/t)[(A_4+A_5)\tfrac{1}{2}m-2A_3]\quad\text{for}\quad s=0.\tag{5}$$

Our sum rule is just the superconvergence relation analogous to Eq. (4), where $\tilde{\Phi}_6$ is replaced by the amplitude in Eq. (5), with the integral evaluated at $s=0$. The π^0 and η poles occur in A_3, while the f^0 pole can occur only via (A_4+A_5) with some residue γ_f, since A_3 has negative parity. The nucleon pole occurs in all three. Collecting the residues of all these poles as given in H-L, we have

$$-g_{NN\pi}f_\pi\tau_3-g_{NN\eta}f_\eta I-\frac{m}{2\mu_f{}^2}\gamma_f I+\frac{e^2}{m}[(\tfrac{1}{4}\mu_p{}^2+\mu_p-\tfrac{1}{4})(I+\tau_3)+\tfrac{1}{4}\mu_n{}^2(I-\tau_3)]-\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}\mathrm{Im}[\tfrac{1}{2}mA_4(u,t,0)+\tfrac{1}{2}mA_5(u,t,0)-2A_3(u,t,0)]\frac{du}{2m^2-u}=0.\tag{6}$$

Here μ_p and μ_n refer to the total magnetic moments of the proton and the neutron, μ_f is the f^0 mass, I and τ_3 are 2×2 matrices acting on the nucleon isospinors, and the crossing property in Eq. (1) has been used in the last integral in going from $A_i(0,t,u)$ to $A_i(u,t,0)$. The $g_{NN\eta}$ and $g_{NN\pi}$ are strong coupling constants, while f_π and f_η are effective $\pi\gamma\gamma$ and $\eta\gamma\gamma$ coupling constants, respectively, and are related to the decay rates $1/\tau$ by the relations

$$f_\eta=-8\left(\frac{\pi}{\mu_\eta{}^3\tau_\eta}\right)^{1/2} \quad\text{and}\quad f_\pi=-8\left(\frac{\pi}{\mu^3\tau_\pi}\right)^{1/2}.$$

By using Eq. (6) for proton and neutron Compton scattering, the π^0 and η decay rates can be obtained, provided γ_f is known. This constant is determined as follows.

We note[6] that $A_4(s,t)+A_5(s,t)$ is given in terms of s-channel helicity amplitudes by

$$A_4+A_5=\frac{2\pi}{p^2}\left[\frac{-4E}{\cos\frac{1}{2}\theta}\Phi_3+\frac{2m}{\sin\frac{1}{2}\theta}(\Phi_6-\Phi_2)\right]. \tag{7}$$

From the asymptotic and analytic properties of Φ_2, Φ_3, and Φ_6 discussed before, we can see that A_4+A_5 obeys unsubtracted dispersion relations in t (although it is not superconvergent). Thus,

$$A_4(s,t,u)+A_5(s,t,u)$$
$$=\frac{\gamma_f I}{\mu_f{}^2-t}+\frac{e^2}{m^2-u}[\tfrac{1}{2}(\mu_p{}^2-1)(I+\tau_3)+\tfrac{1}{2}(\mu_n{}^2)(I-\tau_3)]$$
$$+\frac{1}{\pi}\int_{(m-\mu)^2}^{\infty}\mathrm{Im}\frac{[A_4(s,t',u')+A_5(s,t',u')]}{u'-u}du'. \tag{8}$$

However, since A_4 and A_5 are odd under $s\leftrightarrow u$ crossing,

$$0=A_4(0,2m^2,0)+A_5(0,2m^2,0)=\frac{\gamma_f}{\mu_f{}^2-2m^2}I+\frac{e^2}{m^2}[\tfrac{1}{2}(\mu_p{}^2-1)(I+\tau_3)+\tfrac{1}{2}\mu_n{}^2(I-\tau_3)]$$
$$+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}\mathrm{Im}\frac{[A_4(0,\,2m^2-u',\,u')+A_5(0,\,2m^2-u',\,u')]}{u'}. \tag{9}$$

The integral in Eq. (9) is once again obtained by introducing π-N intermediate states and using photoproduction data. The f_0 pole residue γ_f is then available from Eq. (9) and can be substituted into Eq. (6) for the π^0 and η decay widths.

We conclude this section with some explanation of how to deal with the point $s=0$, at which the sum rule is written. The amplitudes $A_i(0,t,u)$ are related by crossing to $A_i(u,t,0)$ which appear in the integral in Eq. (6). As $u\to\infty$, of course, this corresponds to backward scattering, but it is unphysical in the region of integration between $u=(m+\mu)^2$ and $u=2m^2$. This is clear from Fig. 1 and also from the expression

$$(\cos\theta_u)_{s=0}=1+\frac{2u(2m^2-u)}{(u-m^2)^2}.$$

However, it is also clear from the figure that $s=0$ gives a smaller unphysical region than any other value. $s<0$ will give a larger unphysical range in the low-energy end, while $s>0$ has an infinite unphysical range beyond some maximum u.

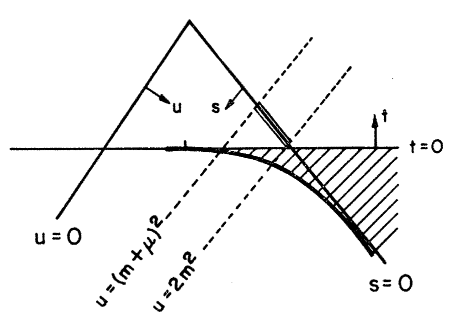

FIG. 1. The unphysical region of integration is shown as the thick line. The physical region is shaded, the integration being carried out on the line $s=0$.

The contribution from the unphysical part for $s=0$ is obtained by writing the $A_i(u,t,0)$ in terms of helicity amplitudes, which expand in partial waves as

$$\mathrm{Im}\Phi_{\lambda\mu}(u,t,0)=\frac{1}{2p}\sum_J(2J+1)\,\mathrm{Im}\Phi^J(u)d_{\lambda\mu}{}^J(\theta_u).$$

The $d_{\lambda\mu}{}^J(\theta)$, being polynomial in $\cos\theta$, can be used to extrapolate for the unphysical $|\cos\theta|>1$ region, with the partial-wave amplitudes $\mathrm{Im}\Phi^J(u)$ being given by unitarity expansion with πN intermediate states. The calculation is presented in the next section.

4. CALCULATION AND RESULTS

The integrands in Eqs. (6) and (9) involve the absorptive parts of the amplitudes (A_4+A_5) and A_3 in the s channel. In the approximation of retaining only the π-N intermediate state, these can be expressed as bilinear products of photoproduction amplitudes, of which we retain the s, $p_{3/2}$, and $D_{3/2}$ waves only.[6] The procedure is well known[7] and the result is

$$\mathrm{Im}[A_4(s,t,0)+A_5(s,t,0)]$$
$$=2(\pi/p^2)[-3Eq(1-\cos\theta)\alpha_1+\tfrac{3}{2}mq(1-\cos\theta)\alpha_2$$
$$-\tfrac{1}{2}mq(1+3\cos\theta)\alpha_3-\tfrac{1}{2}mq\alpha_4], \tag{10a}$$

[7] We assume that the resonant $D_{3/2}$ state is excited by isovector photons only. See Ref. 2.

$$\mathrm{Im}A_3(s,t,0)=(2\pi W/p)[\tfrac{3}{4}q(1-\cos\theta)\alpha_2 +\tfrac{1}{4}q(1+3\cos\theta)\alpha_3+\tfrac{1}{4}q\alpha_4]; \quad (10b)$$

q is the c.m. momentum of the pion in the reaction $\gamma+N\rightarrow\pi+N$ corresponding to a total c.m. energy $\sqrt{s}$; $\cos\theta$ is the scattering angle in Compton scattering corresponding to $u=0$:

$$\cos\theta=1+[2s(2m^2-s)/(s-m^2)^2]; \quad (10c)$$

and the α's are related to the CGLN (Chew-Goldberger-Low-Nambu) photoproduction multipole amplitudes $M_{l\pm}$, $E_{(l+1)\pm}$ by

$$\alpha_1=(M_{1+}-E_{2-}+3E_{1+}+3M_{2-})\times(E_{1+}-M_{1+}-E_{2-}-M_{2-})^* -(3E_{1+}-3M_{2-}+M_{1+}+E_{2-})\times(-E_{1+}+M_{1+}-E_{2-}-M_{2-})^*, \quad (10d)$$

$$\alpha_2=(-E_{1+}+M_{1+}-E_{2-}-M_{2-})\times(E_{1+}-M_{1+}-E_{2-}-M_{2-})^* +(E_{1+}-M_{1+}-E_{2-}-M_{2-})\times(-E_{1+}+M_{1+}-E_{2-}-M_{2-})^*, \quad (10e)$$

$$\alpha_3=-(M_{1+}-E_{2-}+3E_{1+}+3M_{2-})\times(3E_{1+}-3M_{2-}+M_{1+}+E_{2-})^* -(3E_{1+}-3M_{2-}+M_{1+}+E_{2-})\times(M_{1+}-E_{2-}+3E_{1+}+3M_{2-})^*, \quad (10f)$$

$$\alpha_4=-8E_{0+}E_{0+}^*. \quad (10g)$$

The multipole amplitudes are taken from the energy-dependent parametrization given recently by Walker.[8] The s-wave contribution is retained in the integral up to $s=100\mu^2$, as was done earlier,[2,9] whereas the integration of the other waves is carried out well past the second resonance region. The photoproduction integral has to be evaluated for both the proton and the neutron case, in order that the π^0 and η^0 widths may be independently obtained by taking the difference and sum of the two cases, respectively. In our approximation,[7] all multipoles except E_{0+} are the same for proton and neutron. The final numerical results, upon computing the integral and substituting for the nucleon magnetic moment, are

$$f_\pi g_{\pi NN}=e^2(0.53+0.09), \quad (11a)$$

$$f_\eta g_{\eta NN}=e^2(0.79-0.92), \quad (11b)$$

where we have exhibited the pole and continuum contributions separately. The result for the π meson corresponds to a mean lifetime of 0.53×10^{-16}. The result for the η meson of course depends on the η-N coupling constant and if we use the value obtained by Ball[10] in his fit to the reaction $\pi+N\rightarrow\eta+N$, we get for the η^0 a width of 65 eV.

[8] R. L. Walker, quoted in Ref. 9.
[9] S. L. Adler and F. J. Gilman, Phys. Rev. **152**, 1460 (1966).
[10] J. S. Ball, Phys. Rev. **149**, 1191 (1966).

5. COMMENTS

The π^0 lifetime agrees reasonably with experiment.[11] For the η^- meson, experimental information is much less definite than with the π^0 meson. Our result for the η meson differs from that of Pagels[5] roughly by a factor of 2, and both are in considerable disagreement with a recent value of 1.21 ± 0.26 keV obtained by Bemporad *et al.*[12] This last result is, however, far from being a confirmed one and we must await more precise experimental values of the η width for an assessment of the result for the η width. We would like to emphasize, however, that there is almost a total cancellation between the pole and the continuum contributions for the η^0 sum rule and hence the η^0 width is extremely sensitive to slight variations in the latter. The continuum contribution cannot be calculated very precisely because of lack of experimental information. The agreement otherwise of the η^0 width with experimental values is thus not a reasonable measure of the validity of our sum rules. However, when the contribution of the meson-pole term to the sum rule is small compared to other terms, as is true with the η pole in our case, the sum rule should be more properly regarded as a consistency condition to be satisfied by the scattering data rather than a way of calculating the meson decay widths. From this point of view, the η sum rule seems to be satisfied quite reasonably by experimental data.

APPENDIX

A t-channel Regge-pole term in $A_3(s,t)$ has the form

$$R_\pi(s,t)=\frac{\beta(t)}{\sin\pi\alpha}\left[\frac{t^{1/2}(t-4m^2)^{1/2}}{\Lambda}\right]^{\alpha(t)} \times\left[P_{\alpha(t)}\left(\frac{2s-2m^2+t}{[t(t-4_m{}^2)]^{1/2}}\right)+P_{\alpha(t)}\left(\frac{2m^2-2s-t}{[t(t-4m^2)]^{1/2}}\right)\right]. \quad (A1)$$

$\beta(t)$ is determined by the residue of the Regge pole, whereas Λ is a scaling factor. At $t=\mu^2$, this expression reduces to an usual Feynman pole. Two assumptions now enter Pagels's derivation. The function $\beta(t)$ is taken to remain unchanged as we go from the pole to the point $t=0$. This may be reasonable but has to be examined. The second assumption amounts to saying that the integral

$$\frac{1}{\pi}\left[\int_{-\infty}^{-s_0}+\int_{s_0}^{\infty}\right]\frac{\mathrm{Im}R_\pi(s',0)}{s'-m^2}$$

equals the Feynman pole contribution at $t=0$, where s_0 is a point somewhat above the resonating regions. On

[11] For references on the experimental value of the π^0 lifetime, see Ref. 5.
[12] C. Bemporad *et al.*, Phys. Letters **25B**, 380 (1967).

evaluating, one easily sees that it differs from the pole term by a factor

$$2^{2\alpha(0)}[(s_0-m^2)/\Lambda]^{\alpha(0)},$$

assuming $\alpha(t)$ to be linear in t. We cannot determine this factor, since we do not know Λ, but for the η meson this can be quite different from unity. Both the behavior of $\beta(t)$ near $t=0$ and an estimate of Λ can be obtained from high-energy Compton-scattering data (Regge region), for which we have no information so far.

PHYSICAL REVIEW VOLUME 169, NUMBER 5 25 MAY 1968

Use of Born Approximations in N/D Calculations

P. D. B. Collins and R. C. Johnson
Physics Department, University of Durham, Durham City, England
(Received 27 December 1967)

The N/D equations have been solved with the first, second, and third Born approximations to the left-hand cut, for nonrelativistic, single-channel potential scattering, with potentials involving combinations of attraction and repulsion of different ranges, and the results are compared with the exact solution of the Schrödinger equation. It is found that for the sort of potential strengths which occur in strong-interaction dynamics, the third Born approximation is satisfactory. It is known that the first Born approximation, which is commonly used, suffers from several defects in that long-range repulsions can produce attractive effects, and "ghosts" appear on the physical sheet, and we explore the way in which the approximation breaks down. It is concluded that in dynamical calculations, such as those involving the strip approximation, much more satisfactory results are likely to be obtained if the left-hand cut is calculated from a few iterations of the potential.

I. INTRODUCTION

It has been known for some time that the forces which generate strongly interacting particles are likely to contain both attractive and repulsive components. In particular, it is known that the exchange of a Pomeranchon (P) trajectory gives rise to a long-range repulsion.[1]

This has created severe difficulties for the usual sort of N/D calculations which are used to solve dynamical problems.[2] In such calculations it is usual to impose unitarity on an amplitude whose left-hand cut is given by just the first Born approximation; that is, the left-hand cut of the amplitude is assumed to contain just the cut of the potential. This is not, of course, the same as taking the first Born approximation to the amplitude, in that exact unitarity (or at least exact within the framework of the possibility multichannel calculation which we wish to perform) is imposed on the right-hand cut, but the effects of the reaction to the potential on the left-hand cut are ignored. It has been found that if a repulsive force is combined with an attraction (the two having different ranges), the effect of the repulsion if often to give stronger binding, i.e., to act as an attraction.

This fact was commented on by Kayser,[3] and has been noted since by many authors,[4–7] particularly in the context of the Dashen-Frautschi type of perturbation calculation.[8,9]

What is worse, if the repulsion is really strong it is possible for "ghosts," by which is meant in this context resonances with negative residues, to appear.[2] These violate causality and so must be due to the inadequacy of our approximations.

In calculations involving the "new form of the strip approximation" it was found necessary to remove the P repulsion by normalizing the potential,[2] though it was realized at the time that the validity of this procedure was doubtful, and that the P repulsion probably represents the physically important effects of the presence of infinitely many channels with thresholds above the resonance region.[1]

It is to be expected that these defects of the strip approximation would be removed if we were able to use an exact expression for the left-hand cut, but in general, this is prohibitively difficult to calculate. The question thus arises as to the order of the Born approximation to the left-hand cut which is needed to give satisfactory accuracy in this sort of problem. The best way of trying to assess this is to examine the situation in single-channel potential scattering, where we can compare the solution of the N/D equations, for various types of potentials treated in various Born approximations, with the exact solution to the corresponding Schrödinger equation. We know, of course, that if we

[1] G. F. Chew, Phys. Rev. **140**, B1427 (1965).
[2] P. D. B. Collins, Phys. Rev. **142**, 1163 (1966).
[3] B. Kayser, Berkeley Report, 1965 (unpublished).
[4] G. Auberson and G. Wanders, Nuovo Cimento **46**, 78 (1966).
[5] R. F. Sawyer, Phys. Rev. **142**, 991 (1966).
[6] B. Kayser, Phys. Rev. **165**, 1760 (1968).
[7] H. Banerjee, Nuovo Cimento **50**, 993 (1967).
[8] R. F. Dashen and S. C. Frautschi, Phys. Rev. **135**, B1190 (1964); **137**, 1318 (1965).
[9] R. F. Dashen, Phys. Rev. **135**, B1196 (1964).