Threshold Electrodisintegration of the Deuteron

RONALD J. ADLER
Virginia Polytechnic Institute, Blacksburg, Virginia
(Received 11 January 1968)

We study threshold electrodisintegration of the deuteron caused by scattering of electrons at angles near 180°. The cross section for this reaction is calculated, using nonrelativistic deuteron wave functions. The deuteron D state is included in the calculation, as well as all of the relevant meson exchange currents. Disagreement of the calculated results with recent electron scattering experiments indicates that a basic difficulty exists in our present understanding of the deuteron wave function and the core radius of the nucleon. A crude Fourier transform is then performed on the experimental transition form factor to yield the configuration-space overlap charge density. The resultant overlap charge density indicates that a softer nucleon core is needed than is presently popular.

1. INTRODUCTION

THE present paper is motivated by the recent experiment of Rand *et al.*[1] on the scattering of electrons at 180° by deuterium. The cross section for the electrodisintegration of the deuteron found in that experiment is consistently greater than that predicted by the original Jankus theory[2] of e-D scattering, becoming over twice the theoretical value at a momentum transfer of $q^2=10$ F^{-2}. This is a very disturbing result since one expects that the standard calculational technique for e-D scattering[2–5] should be reasonably accurate near threshold and at low q^2 and begin to be untrustworthy only at higher q^2 when relativistic effects,[6–10] inaccuracies in deuteron models, etc., become crucial. The results of Rand *et al.* indicated that the discrepancies are much greater and occur at a somewhat lower value of momentum transfer than expected.

We have reformulated the theory of e-D scattering for large electron scattering angles. The original calculation of Jankus used a pure S-wave deuteron; we have included a deuteron D wave, meson exchange currents,[3,11] and some small kinematical relativistic corrections. In our opinion the resulting theory is basically complete within the context of a nonrelativistic wave-function model of the deuteron. Despite a drastically different theory, our numerical results for the e-D cross section are rather close to the older Jankus numbers.[1] Thus the discrepancy between theory and experiment persists and is even worsened since we believe the present theory is essentially complete.

[1] R. E. Rand, R. F. Frosch, C. E. Littig, and M. R. Yearian, Phys. Rev. Letters **18**, 469 (1967).
[2] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956); thesis, Stanford University, 1955 (unpublished).
[3] R. J. Adler, Phys. Rev. **141**, 1499 (1966); thesis, Stanford University, 1965 (unpublished).
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[11] R. J. Adler and S. D. Drell, Phys. Rev. Letters **13**, 349 (1964).

As we will discuss in Sec. 8, the only cure for this discrepancy seems to be a change in the deuteron wave function at small distances. We will make a crude estimate of the ground state and scattered wave-function overlap at small distances, $\sim$0.5 F, by Fourier transforming the experimentally measured form factor.

2. GENERAL CONSIDERATIONS

We first wish to demonstrate that when scattered backwards a Dirac electron emits a purely magnetic photon. This photon has the quantum numbers $J=1$ and $P=+$, so the transitions it can induce in a target system such as a ground-state deuteron are limited. For the electrodisintegration of the deuteron near threshold, the allowed final states of the (np) system are ^1S and the so-called α state,[12,13] which is predominantly ^3S with a very small admixture of ^3D due to the tensor force.

Consider an electron moving along the z axis with momentum k which is backscattered to momentum k' and emits a virtual photon. The momentum-space transition current for high-energy electrons with k and $k'\gg m$ is then

$$\begin{aligned} j_x&=u_f^\dagger\alpha_x u_i=-2i\left(\frac{kk'}{4m^2}\right)^{1/2}\chi_f^\dagger\sigma_y\chi_i\,,\\ j_y&=u_f^\dagger\alpha_y u_i=2i\left(\frac{kk'}{4m^2}\right)^{1/2}\chi_f^\dagger\sigma_x\chi_i\,,\\ j_z&=u_f^\dagger\alpha_z u_i\ll j_x,j_y\,,\\ j_0&=u_f^\dagger u_i\ll j_x,j_y\,. \end{aligned} \tag{2.1}$$

This current vanishes if the Pauli spinors χ_i and χ_f are the same and is thus pure spin flip. For a transition from spin up to spin down, the potential produced is

$$\begin{aligned}\mathbf{A}&=(-1/q^2)\mathbf{j}e^{-iqz}\\ &=(-1/q^2)(kk'/4m^2)^{1/2}(e^{-iqz},e^{-iqz+i\pi/2},0)\,,\end{aligned} \tag{2.2}$$

and the electromagnetic field associated with the virtual

[12] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 110.
[13] J. J. DeSwart, Physica **25**, 233 (1959).

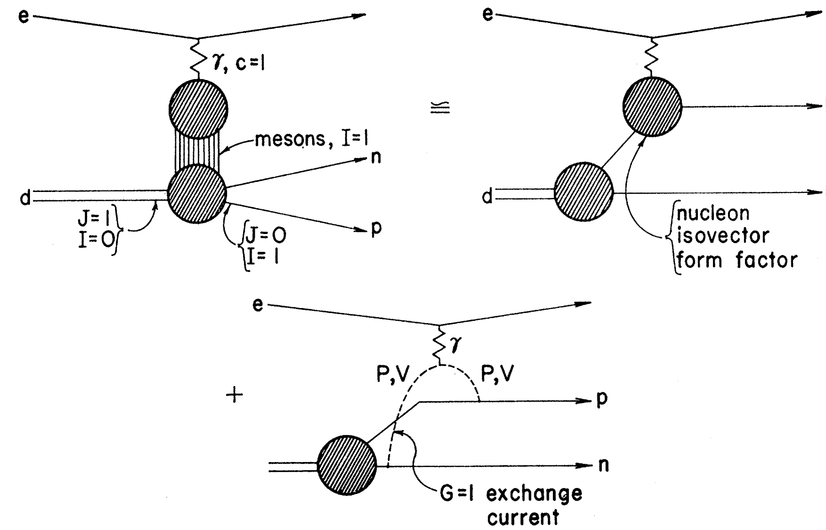

FIG. 1. Isotopics of scattering into singlet state.

photon is

$$\mathbf{E}=0, \qquad \mathbf{B}=(i/q)(kk'/4m^2)^{1/2}(e^{-iqz+i\pi/2},e^{-iqz},0). \qquad (2.3)$$

Thus such a photon can induce only magnetic transitions. Moreover, the potential due to a 3-vector current [Eq. (2.2)] is a pseudovector and has positive parity.

From the above comments, we can list the possible states of the (np) system. First, near threshold the centrifugal barrier suppresses all but S states. The magnetic character of the photon implies a positive parity. The ${}^1S$ state of the deuteron is clearly allowed by these criteria, and we will see that it dominates the final state. This is to be expected since a magnetic photon will easily flip one nucleon spin and produce a ${}^3S \rightarrow {}^1S$ transition.

In addition the ${}^3S$ state is possible. However, we will find that it is considerably less important than the ${}^1S$ state, although not negligible. It should be noted that the ${}^3S$ state that we speak of here actually should have a small admixture of ${}^3D$ and is more properly referred to as the α state.[12] In practice, this makes little difference for the small relative (np) momenta encountered near threshold.

In the following sections, we will calculate the transition amplitude for the ground-state deuteron going to the ${}^1S$ state. Since this dominates the magnetic disintegration near threshold, we will include the D wave of the ground-state deuteron and meson exchange currents. Since the ${}^3S$ state is less important, but not negligible, we will calculate only the impulse approximation for the ground state to ${}^3S$ transition. As noted above, we will not distinguish between the α and ${}^3S$ states but will treat the ${}^3S$ state as if it were a true eigenstate for the final (np) system. This is an excellent approximation since the mixing angle is only a few degrees.[12]

3. ISOTOPIC CONSIDERATIONS

We wish to limit ourselves in this section to the inelastic scattering of a ground-state deuteron into the singlet S state with isospin $I=1$. Near the threshold for inelastic scattering this transition dominates the cross section. Our problem thus reduces to the consideration of a photon induced isospin-flip transition from the ground state $(I=0)$ to the singlet state $(I=1)$, as shown in Fig. 1. It is apparent that if some system of mesons is to couple the deuteron to a photon it must have $I=1$ for the strong meson-deuteron vertex to be allowed, $C=-1$ for the electromagnetic meson-photon vertex to be allowed, and it must of course have zero charge. From these quantum numbers we can calculate the G parity of the meson system.[14]

The G parity is defined as the product of a rotation by π about the isospace y axis and charge conjugation

$$G=e^{i\pi I_y}C. \qquad (3.1)$$

Since the meson system must be the neutral member of an $I=1$ isotriplet, we see that under the rotation in isospace, the neutral member, or z component, changes sign; since the meson system also has $C=-1$, it clearly must have $G=1$. On the other hand, $G=-1$ for a single pion and is a multiplicative quantum number. Thus for a system of n pions

$$G=(-1)^n. \qquad (3.2)$$

We conclude that if the meson system can be considered as a bound or resonant state of pions then it is an eigenstate of G parity with $G=1$ and consists of an even number of pions.

For simplicity, we will limit ourselves to the nonstrange members of the pseudoscalar-meson octet P

[14] J. J. Sakurai, *Invariance Principles and Elementary Particles* (Princeton University Press, Princeton, N. J., 1964), p. 223.

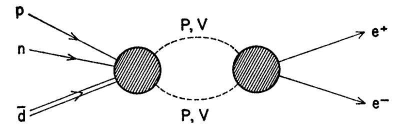

FIG. 2. Approximate saturation of unitarity in the crossed channel.

and the vector-meson octet V. Moreover, we will consider only pole-type diagrams of the form shown in Fig. 1. This leads us to meson exchange currents $(\pi\pi)$, $(\omega\pi)$, $(\rho\eta)$, and $(\rho\rho)$. We delete the neutral exchange currents $(\eta\eta)$ and $(\omega\omega)$. One should note that since only the $I=1$ part of the photon is involved our results must depend only on the isovector nucleon form factors and not on the isoscalar form factors. In the impulse approximation of the next section this will allow us to ignore the neutron-photon coupling during the calculation and merely replace the proton form factor with the isovector form factor at the end.

The exchange currents which we determined above have the virtue of containing only well established states of 2, 4, and 6 pions. We may therefore think of our choice of currents as an attempt to saturate unitarity with nonstrange vector and pseudoscalar-meson poles as shown in Fig. 2. We will not attempt to estimate the effects of strange-particle currents such as (KK).

4. IMPULSE APPROXIMATION

The simplest first approach to the problem of electron scattering from any nucleus is to assume that the electron interacts individually with the bound nucleons; this is the impulse approximation as pictured in Fig. 3. This approximation is limited by the following:

(1) The photon-nucleon interaction is usually described by a free nucleon form factor. The only way to avoid this difficulty is essentially to calculate, in some way, the nucleon form factor off the mass shell. We have no trustworthy way to do this. In the case of the deuteron the internal momenta are of the order of 50 MeV/c, so we do not expect off-mass-shell effects to be of great importance since $\mathbf{p}^2$ is then only a percent or so of M^2.

(2) One usually describes the $d\rightarrow n+p$ vertex in an ultimately noncovariant way, e.g., in terms of wave functions. Attempts are being made to treat the vertex by a covariant parametrization without ultimate recourse to nonrelativistic deuteron wave functions.[7–10] Such attempts have been at least partially successful and, among other things, serve to increase our trust in the validity of the wave-function approach for small momentum transfers.[7] Considerable progress in the relativistic two-body problem should be forthcoming. However, for low momentum transfers we believe the wave-function description should be accurate. The question remains as to what should be considered "low" momentum transfer.

(3) Nucleons are not the only particles in a nucleus and one must eventually consider the interaction of the photon with meson exchange currents,[3,11,9] which are not implicitly contained in the impulse approximation via nucleon form factors. We will deal with these currents in later sections of this paper.

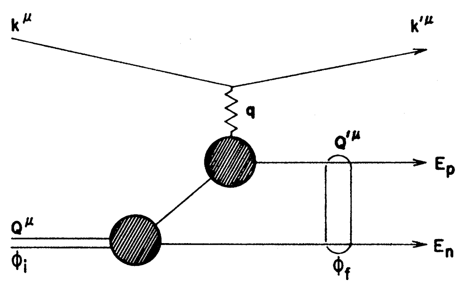

FIG. 3. The impulse approximation.

The S-matrix element for the interaction of two currents via one-photon exchange is given by the well-known general form (see Fig. 3)[15]

$$S=-ie^2(2\pi)^4\delta^4(Q'+k'-Q-k)\times(m^2M^2/kk'E_pE_n)^{1/2}j_\mu(1/q^2)J^\mu. \quad (4.1)$$

For the present problem, j_μ is the electron current in momentum space

$$j_\mu=\bar{u}(k')\gamma_\mu u(k)\,, \quad (4.2)$$

and J^μ is the deuteron transition current in momentum space, which we now wish to calculate in the impulse approximation. The transition current is simply the Fourier transform of the nucleon currents in configuration space. With respect to the deuteron center of mass, the proton is at a position $\frac{1}{2}\mathbf{y}$ and the neutron at $-\frac{1}{2}\mathbf{y}$. Thus, if we denote the nucleon current operators by $\Gamma_p{}^\mu$ and $\Gamma_n{}^\mu$, we can express the current in terms of the initial and final nonrelativistic wave functions as a three-dimension Fourier transform:

$$J_{ia}{}^\mu=\int\phi_f{}^\dagger(y)[\Gamma_p{}^\mu e^{i\mathbf{q}\cdot\mathbf{y}/2}+\Gamma_n{}^\mu e^{-i\mathbf{q}\cdot\mathbf{y}/2}]\phi_i(y)d^3y. \quad (4.3)$$

Although (4.3) is manifestly noncovariant, we wish to retain the lowest-order relativistic terms due to the Dirac nature of the nucleons. In particular, we wish to rewrite the operators $\Gamma_p{}^\mu$ and $\Gamma_n{}^\mu$ for use with Pauli spinors, but correct at least to second order in all three-momenta involved. To this purpose, we write the nucleon positive-energy Dirac spinors in terms of nucleon Pauli spinors χ as

$$u_\alpha(p)=\left(\frac{E+M}{2M}\right)^{1/2}\begin{pmatrix} I \\ \boldsymbol{\sigma}\cdot\mathbf{p}/(E+M)\end{pmatrix}\chi_\alpha \quad (4.4)$$

[15] S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, London, 1961), p. 2.

and use the following nucleon current operators:

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + (i\kappa/2M)\sigma^{\mu\nu}q_\nu F_2(q^2) \text{ for } \mu=0, \tag{4.5a}$$

$$\Gamma^\mu = F_1(q^2)(P^\mu/2M) + (i/2M)\sigma^{\mu\nu}q_\nu G_M(q^2) \quad \text{for } \mu=i, \tag{4.5b}$$

where q_ν is the 4-momentum transfer and P^μ is the sum of momenta before and after interaction. Then simple algebra results in the following expressions for the nucleon currents in terms of nucleon Pauli spinors χ:

$$\bar{u}_\alpha(p')\Gamma^0 u_\beta(p) = \chi_\alpha^\dagger\left[\left(1+\frac{\mathbf{p}^2}{2M^2}+\frac{\mathbf{q}^2}{8M^2}+\frac{\mathbf{p}\cdot\mathbf{q}}{2M^2}\right)G_E(q^2) + \frac{i}{4M^2}q^i p^k \sigma^l \epsilon^{ikl} G_R(q^2) + O(p^4)\right]\chi_\beta, \tag{4.6a}$$

$$\bar{u}_\alpha(p')\Gamma^i u_\beta(p) = \chi_\alpha^\dagger[(P^i/2M)G_E(q^2) - (i/2M)\epsilon^{ijk}\sigma^k q^j G_M(q^2) + O(p^3)]\chi_\beta. \tag{4.6b}$$

The electric, magnetic, and so-called "relativistic" form factors are defined as

$$\begin{aligned} G_E(q^2) &= F_1(q^2) + (q^2/4M^2)\kappa F_2(q^2), \\ G_M(q^2) &= F_1(q^2) + \kappa F_2(q^2), \\ G_R(q^2) &= F_1(q^2) + 2\kappa F_2(q^2). \end{aligned} \tag{4.7}$$

The remaining ingredients necessary for calculating the impulse-approximation current (4.3) are the wave functions for the ground-state deuteron and the scattered $l=0$ states. The nucleon current expressions (4.6) allow us to use nonrelativistic wave functions and Pauli spinors. For the ground-state deuteron, we use the well-known wave-function form[16]

$$\phi_i(y) = \frac{1}{(4\pi)^{1/2}}\left[\frac{u(y)}{y} + \frac{w(y)}{8y}S_{12}\right]\chi_M,$$

$$\chi_1 = \alpha\alpha, \quad \chi_{-1} = \beta\beta, \quad \chi_0 = (\alpha\beta+\beta\alpha)/\sqrt{2}. \tag{4.8}$$

For the scattered $l=0$ states, we use

$$\text{singlet; } \phi_f(y) = \frac{1}{(4\pi)^{1/2}}\left[\frac{Z_s(y,p)}{y}\right]\chi_s, \quad \chi_s = (\alpha\beta-\beta\alpha)/\sqrt{2},$$

$$\text{triplet; } \phi_f(y) = \frac{1}{(4\pi)^{1/2}}\left[\frac{Z_t(y,p)}{y}\right]\chi_{M'},$$

$$\chi_1 = \alpha\alpha, \quad \chi_{-1} = \beta\beta, \quad \chi_0 = (\alpha\beta+\beta\alpha)/\sqrt{2}, \tag{4.9}$$

where p is the relative (np) momentum.

The radial wave functions are normalized at large y to

$$Z(y,p)/y \to (4\pi)^{1/2}\sin(py+\delta)/py, \tag{4.10}$$

where δ is the phase shift of the singlet or triplet state.

[16] See Ref. 12, p. 99.

It now remains only to substitute the nucleon currents (4.6) and the wave functions (4.8) and (4.9) into the expression (4.3) for the impulse-approximation current. For the transition to the ^1S state this results in

$$J_{ia}{}^0 = 0,$$

$$J_{ia}{}^j = -i\epsilon^{jkl}q^k(\chi_s^\dagger\sigma_p{}^l\chi_M)[G_{MV}(q^2)/M][H_s(q^2)-J_s(q^2)] + q^l(\chi_s^\dagger\sigma_p{}^l\sigma_n{}^j\chi_M)[G_{MV}(q^2)/M]J_s(q^2), \tag{4.11}$$

where the deuteron structure is described by

$$\begin{aligned} H_s(q^2) &= \int_0^\infty Z_s(y,p)u(y)j_0(\tfrac{1}{2}qy)dy, \\ J_s(q^2) &= \frac{1}{\sqrt{8}}\int_0^\infty Z_s(y,p)w(y)j_2(\tfrac{1}{2}qy)dy. \end{aligned} \tag{4.12}$$

The j_0 and j_2 are spherical Bessel functions, and G_{MV} is the isovector nucleon magnetic form factor. The form of current obtained in (4.11) can be shown to be general; thus the modifications to be obtained when we consider meson exchange currents will simply change the functions $G_{MV}(H-J)/M$ and $G_{MV}J/M$.

For the ^3S state the transition current is

$$L_{ia}{}^0 = 0,$$

$$L_{ia}{}^j = -i\epsilon^{jkl}q^k(\chi_{M'}{}^\dagger\sigma_P\chi_M)[G_{MS}(q^2)/M][H_t(q^2)+J_t(q^2)] + q^l(\chi_{M'}{}^\dagger\sigma_P{}^j\sigma_n{}^l\chi_M)\{[3G_{MS}(q^2)/M]J_t(q^2) - [24G_{ES}(q^2)/M]K_t(q^2)\}, \tag{4.13}$$

where

$$\begin{aligned} H_t(q^2) &= \int_0^\infty Z_t(y,p)u(y)j_0(\tfrac{1}{2}qy)dy, \\ J_t(q^2) &= \frac{1}{\sqrt{8}}\int_0^\infty Z_t(y,p)w(y)j_2(\tfrac{1}{2}qy)dy, \\ K_t(q^2) &= \frac{1}{\sqrt{8}}\int_0^\infty \frac{Z_t'(y,p)y - Z_t(y,p)}{q^2y^2}w(y)j_2(\tfrac{1}{2}qy)dy. \end{aligned} \tag{4.14}$$

Here Z_t' is the derivative of Z_t with respect to y.

To get the cross section for electron scattering from the deuteron, we need merely square the S-matrix element (4.1), sum and average over spins, and integrate over proton and neutron momenta. This is completely straightforward and results in the following cross section for electrodisintegration into the ^1S state:

$$\left(\frac{d\sigma}{d\Omega dk'}\right)_{^1S} = \frac{\alpha^2 p[G_1{}^2+G_2{}^2]}{(2\pi)^2 3M\sin^2(\frac{1}{2}\theta)}\frac{k'}{k} \times[1+\sin^2(\tfrac{1}{2}\theta)+(k-k')^2/2kk'], \tag{4.15}$$

where

$$\begin{aligned} G_1(q^2) &= G_{MV}(q^2)[H_s(q^2)-J_s(q^2)], \\ G_2(q^2) &= G_{MV}(q^2)J_s(q^2). \end{aligned} \tag{4.16}$$

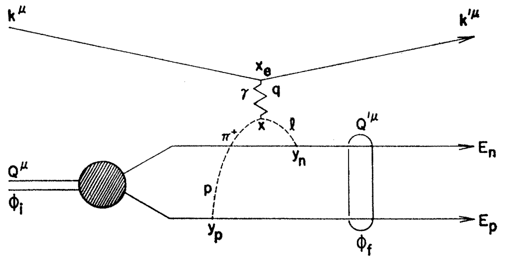

FIG. 4. $(\pi\pi)$ exchange diagram.

For the transition to the ^3S state, we obtain

$$\left(\frac{d\sigma}{d\Omega dk'}\right)_{^3S}=\frac{\alpha^2 p[G_3^2+G_4^2]}{(2\pi)^2 3M\sin^2(\frac{1}{2}\theta)}\frac{k'}{k}[1+\sin^2(\tfrac{1}{2}\theta)+(k-k')^2/2kk']+\frac{\alpha^2 pG_4^2}{(2\pi)^2 4M\sin^2(\frac{1}{2}\theta)}\frac{k'}{k}\times[((k-k')^2/2kk')\cot^2(\tfrac{1}{2}\theta)], \quad (4.17)$$

where

$$G_3(q^2)=\sqrt{2}G_{MS}(q^2)[H_t(q^2)+J_t(q^2)],$$
$$G_4(q^2)=3\sqrt{2}G_{MS}(q^2)J_t(q^2)-24\sqrt{2}G_{ES}(q^2)K_t(q^2). \quad (4.18)$$

We are particularly interested in angles near $\theta=\pi$, in which case $\cot(\frac{1}{2}\theta)=0$ and the ^1S and ^3S cross sections combine into a single convenient form

$$\frac{d\sigma}{d\Omega dk'}=\frac{\alpha^2 p[G_1^2+G_2^2+G_3^2+G_4^2]}{(2\pi)^2 3M\sin^2(\frac{1}{2}\theta)}\frac{k'}{k}\times[1+\sin^2(\tfrac{1}{2}\theta)+(k-k')^2/2kk']. \quad (4.19)$$

The cross section (4.19) is the net result of the impulse approximation. Before proceeding to consider meson exchange currents, we should note that the deuteron structure functions such as H_s are given in terms of the 3-vector $\mathbf{q}^2$ whereas the nucleon form factors are functions of the 4-vector $-q^2$. The fractional difference of these quantities is $\sim q^2/4M_D^2$. For $-q^2$ of 10 F^{-2} this amounts to only a few percent, but it should be kept in mind that an ambiguity does exist. We will discuss this further in Sec. 7.

5. MESON EXCHANGE CURRENTS: $(\pi\pi)$

In Sec. 4, we considered the impulse approximation for e-D scattering into the singlet deuteron state. In doing so, we implicitly included the mesonic interactions, in which only one nucleon is involved by using a phenomenological nucleon form factor. That is, we actually included intermediate states like ρ, ω, $(\pi\pi)$, and other meson systems which land on one nucleon and ignore the other. We now wish to widen our scope and consider a meson emitted by, say, the proton which interacts with the photon and lands finally on the neutron. This clearly is a three-body effect; both nucleons and the photon participate. One could also consider this as the photon interacting with the overlap of the nucleons' meson clouds; the photon hits a meson in one nucleon's cloud, which then transfers to the other nucleon's cloud. To reiterate our above comments, we can think of the present calculation as, equivalently, (1) an exchange current of mesons, (2) a nucleus-photon three-body effect, or (3) a proximity-induced distortion of the meson clouds of the two nucleons. The exchange currents should not be considered as a relativistic effect since they would be present also for arbitrarily heavy and slowly moving nucleons.

Since the transition to the ^1S state dominates the cross section near threshold, we will calculate exchange currents only for the ground state to ^1S transition and not for the transition to the ^3S state.

We first calculate the effect of the $(\pi\pi)$ exchange current pictured in Fig. 4. The transition in isospace is

$$(pn-np)/\sqrt{2}\rightarrow(pn+np)/\sqrt{2}. \quad (5.1)$$

The amplitude for $pn/\sqrt{2}\rightarrow np/\sqrt{2}$ involves the proton giving a π^+ to the neutron, as in Fig. 4, and has the amplitude $+|e|$. The amplitude for $-np/\sqrt{2}\rightarrow pn/\sqrt{2}$ is similarly $+|e|$. Thus we may consider the exchange of a π^+ only as pictured in Fig. 4, using $G^2/4\pi=14$, and double the resultant amplitude to account for isotopics.

The π-γ vertex is taken to be the usual form

$$e(p+l)^\mu A_\mu. \quad (5.2)$$

For the $\pi\pi$-d vertex, we introduce a convenient notation for spin operators between direct-product Pauli spin states

$$\sigma_p{}^j\sigma_n{}^l\equiv(\sigma^j:\sigma^l), \quad \sigma_p{}^t\equiv(\sigma^t:I), \quad \sigma_n{}^k\equiv(I:\sigma^k). \quad (5.3)$$

Then the $\pi\pi$-d vertex is the product of π-n and π-p vertices. In the nonrelativistic limit this is simply

$$\left[\frac{iG}{2M}(-p^j)\sigma_p{}^j\right]\left[\frac{iG}{2M}l^g\sigma_n{}^g\right]=\frac{G^2}{\frac{1}{4}M^2}p^jl^g(\sigma^j:\sigma^g). \quad (5.4)$$

Note that we have made yet another nonrelativistic approximation in (5.4) which is good only to $O(p^2)$.

With the γ-π and $\pi\pi$-d vertices given above, we can immediately write a Feynman amplitude for Fig. 4, with the dueteron temporarily described by functions $\psi(y_p,y_n)$.

$$S_{\pi\pi}=-2e^2\int d^4y_pd^4y_n\psi_f{}^\dagger(y_p,y_n)(\sigma^j:\sigma^g)\psi_i(y_p,y_n)\times\frac{G^2}{4M^2}l^gp^j\left[\frac{ie^{-ip\cdot(x-y_p)}}{p^2-m_\pi{}^2}\frac{d^4p}{(2\pi)^4}\right]\left[\frac{ie^{-il\cdot(y_n-x)}}{l^2-m_\pi{}^2}\frac{d^4l}{(2\pi)^4}\right]\times(p+l)^\mu\left[\frac{-ie^{-iq\cdot(x-x_e)}}{q^2}\frac{d^4q}{(2\pi)^4}\right]\bar{u}(k')\gamma_\mu u(k)\times e^{-i(k-k')\cdot x_e}(m^2/kk')^{1/2}(M^2/E_pE_n)^{1/2}d^4xd^4x_e. \quad (5.5)$$

This simplifies if we substitute center-of-mass (c.m.) and relative coordinates for the nucleons, and similar momentum variables

$$Y=\tfrac{1}{2}(y_p+y_n)\,,\quad \tau=p+l\,,$$
$$y=y_p-y_n\,,\qquad q=l-p\,. \tag{5.6}$$

The deuteron wave functions ψ are then written as internal $\phi(y)$, as in (4.8) and (4.9), and plane waves for the c.m. coordinate:

$$\psi_f(y_p,y_n)=e^{-iQ'\cdot Y}\phi_f(y)\,,$$
$$\psi_i(y_p,y_n)=e^{-iQ\cdot Y}\phi_i(y)\,. \tag{5.7}$$

With these coordinates, momenta, and wave functions, we can simplify the amplitude (5.5) to

$$S_{\pi\pi}=-ie^2(2\pi)^4\delta^4(Q'+k'-Q-k)(m^2/kk')^{1/2}$$
$$\times(M^2/E_pE_n)^{1/2}j_\mu(1/q^2)\frac{G^2}{64M^2}\int d^3y\,\frac{d^3\tau}{(2\pi)^3}\phi_f^\dagger(y)$$
$$\times(\sigma^j:\sigma^g)\phi_i(y)e^{i\tau\cdot y/2}\tau^\mu(q+\tau)^j(q-\tau)^g/$$
$$(\mathbf{p}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)\,, \tag{5.8}$$

with the restriction that $\tau^0=0$. This restriction implies that no energy can be transferred between the nucleons; it seems to be a standard feature of exchange-current calculations which use wave functions, and merely reflects the one-time nature of the nonrelativistic wave function.

If we compare the $(\pi\pi)$ exchange amplitude (5.8) to the general amplitude (4.1), we see that the following quantity can be identified as an exchange current:

$$J_{\pi\pi}{}^\mu=\frac{G^2}{64M^2}\int d^3y\,\frac{d^3\tau}{(2\pi)^3}\phi_f^\dagger(y)(\sigma^j:\sigma^g)\phi_i(y)e^{i\tau\cdot y/2}$$
$$\times\tau^\mu(q+\tau)^j(q-\tau)^g/(\mathbf{p}^2+m_\pi^2)(\mathbf{l}^2+m_\pi^2)\,. \tag{5.9}$$

Note that in simplifying the $(\pi\pi)$ amplitude from (5.5) to (5.8) we found $\tau^0=0$. This agrees with the general result of Secs. 2 and 4 that the deuteron transition current is magnetic and has no zeroth component.

Our remaining task is to put $J_{\pi\pi}{}^\mu$ into the form of transition current given in (4.11). We may then pick off the contributions to the deuteron transition form factors, which we will call $G_1{}^{\pi\pi}$ and $G_2{}^{\pi\pi}$. Since we know the form of the wave functions $\phi(y)$ from (4.8) and (4.9), this is reasonably simple. We first perform the angular $\mathbf{y}$ integration; for this we utilize the following convenient properties of the tensor operator S_{12}:

$$S_{12}(\hat{y})\equiv3\hat{y}\cdot\sigma_p\hat{y}\cdot\sigma_n-\sigma_p\cdot\sigma_n=T^{lm}(\hat{y})(\sigma^l:\sigma^m)\,,$$
$$T^{lm}(\hat{y})=3\hat{y}^l\hat{y}^m-\delta^{lm}\,,$$
$$\int d\Omega\,T^{lm}(\hat{y})e^{i\tau\cdot y/2}=-4\pi j_2(\tfrac{1}{2}\tau y)T^{lm}(\hat{\tau})\,,$$
$$\int d\Omega\,e^{i\tau\cdot y/2}=4\pi j_0(\tfrac{1}{2}\tau y)\,. \tag{5.10}$$

Then the y integral becomes

$$\int d^3y\,\phi_f^\dagger(y)(\sigma^j:\sigma^g)\phi_i(y)e^{i\tau\cdot y/2}=\chi_s^\dagger(\sigma^j:\sigma^g)\chi_MH_S(\tau^2)$$
$$-\chi_s^\dagger(\sigma^j\sigma^t:\sigma^g\sigma^m)\chi_MT^{tm}(\hat{\tau})J_S(\tau^2)\,, \tag{5.11}$$

where the functions H_s and J_s are, as in (4.12),

$$H_s(\tau^2)=\int_0^\infty Z_s(y)u(y)j_0(\tfrac{1}{2}\tau y)dy\,,$$
$$J_s(\tau^2)=\frac{1}{\sqrt{8}}\int_0^\infty Z_s(y)w(y)j_2(\tfrac{1}{2}\tau y)dy\,. \tag{5.12}$$

The $\boldsymbol{\tau}$ integral can then be simplified to the form required, with the result

$$J_{\pi\pi}{}^j=-i\epsilon^{jkl}q^k\chi_s^\dagger(\sigma^l:I)\chi_M\frac{G^2}{32M^2}\zeta(q^2)$$
$$+q^l\chi_s^\dagger(\sigma^j:\sigma^l)\chi_M\frac{G^2}{32M^2}[\eta(q^2)-\zeta(q^2)]\,, \tag{5.13}$$

where

$$\eta(q^2)=\int\frac{d^3\tau}{(2\pi)^3}\frac{H_s(\tau^2)(\tau_1)^2}{[\frac{1}{4}(\mathbf{q}+\boldsymbol{\tau})^2+m_\pi^2][\frac{1}{4}(\mathbf{q}-\boldsymbol{\tau})^2+m_\pi^2]}\,,$$
$$\zeta(q^2)=\int\frac{d^3\tau}{(2\pi)^3}\frac{J_s(\tau^2)(\tau_1)^2}{[\frac{1}{4}(\mathbf{q}+\boldsymbol{\tau})^2+m_\pi^2][\frac{1}{4}(\mathbf{q}-\boldsymbol{\tau})^2+m_\pi^2]}\,. \tag{5.14}$$

The $(\pi\pi)$ current in (5.13) can be compared with the general current (4.11) and the definition (4.16) to obtain

$$G_1{}^{\pi\pi}(q^2)=\frac{G^2}{32M}\zeta(q^2),\ G_2{}^{\pi\pi}=\frac{G^2}{32M}[\eta(q^2)-\zeta(q^2)]\,. \tag{5.15}$$

These add coherently to the form factors G_1 and G_2 obtained using the impulse approximation (4.16).

The functions η and ζ defined in (5.14) may be considerably simplified by integrating over the angles of $\boldsymbol{\tau}$:

$$\eta(q^2)=\frac{1}{2(2\pi)^2}\int_0^\infty H_s(\tau^2)M(\tau,q)\tau^4d\tau\,,$$
$$\zeta(q^2)=\frac{1}{2(2\pi)^2}\int_0^\infty J_s(\tau^2)M(\tau,q)\tau^4d\tau\,,$$
$$M(\tau,q)=\frac{8}{q^2\tau^2}+\frac{2[(q^2+\tau^2+4m_\pi^2)^2-4q^2\tau^2]}{q^2\tau^3(q^2+\tau^2+4m_\pi^2)}$$
$$\times\ln\left[\frac{q^2+\tau^2+4m_\pi^2-2q\tau}{q^2+\tau^2+4m^2+2q\tau}\right]. \tag{5.16}$$

For $q=0$, one may also evaluate (5.14) as contour integrals, obtaining the particularly simple results

$$\zeta(0)=\frac{4}{3\pi}\int_0^\infty \frac{e^{-m_\pi y}}{y}(1+m_\pi y)\frac{Z_s(y)w(y)}{\sqrt{8}}dy,$$
$$\eta(0)=\frac{8}{3\pi}\int_0^\infty \frac{e^{-m_\pi y}}{y}(1-m_\pi y/2)Z_s(y)u(y)dy. \quad (5.17)$$

These are now in convenient form for numerical evaluation.

At this point, we should note that although we have so far used a bare pion-photon vertex, a pion form factor $G_\pi(q^2)$ may be easily included. This will result in $G_1^{\pi\pi}$ and $G_2^{\pi\pi}$ being multiplied by the function $G_\pi(q^2)$.

6. OTHER EXCHANGE CURRENTS: $(\omega\pi)$, $(\rho\eta)$, $(\rho\rho)$

In Sec. 5, we calculated the effect of the $(\pi\pi)$ exchange current; that is, the lowest mass and therefore the longest-range exchange current. We now wish to consider the heavier currents $(\omega\pi)$, $(\rho\eta)$, and $(\rho\rho)$.

We treat the $(\omega\pi)$ current first. Unlike the $(\pi\pi)$ exchange current, the $(\omega\pi)$ current involves parameters which are not very precisely known. Specifically, we can only estimate the $\omega\pi\gamma$ coupling from photoproduction data and $SU(3)$ symmetry. The ω-N coupling can be inferred by universality and $SU(3)$ symmetry.

In order to write an $(\omega\pi)$ exchange amplitude corresponding to Fig. 5, we must choose a form for the vertices for pion nucleon, ω nucleon, and $\omega\pi\gamma$. The pion-nucleon coupling is taken to be the standard limiting form for low momentum transfer that we used in Sec. 5, Eq. (5.5). We repeat it here:

$$(iG/2M)p^j(\chi_f^\dagger\sigma^j\chi_i). \quad (6.1)$$

For the ω-nucleon coupling we assume a form which is the same as photon-nucleon coupling. That is, the ω couples to a 4-vector current similar to (4.6):

$$\chi_f^\dagger\Gamma^0\chi_i=g_{\omega N}\chi_f^\dagger\chi_i,$$
$$\chi_f^\dagger\Gamma^i\chi_i=\chi_f^\dagger[g_{\omega N}(Pi/2M)-h_{\omega N}(i/2M)\epsilon^{ijk}l^j\sigma^k]\chi_i. \quad (6.2)$$

If one assumes that the exchange of an ω is responsible for the q^2 dependence of the isoscalar nucleon form factors, then

$$G_{ES}(0)\sim eg_{\omega N},\quad G_{MS}(0)\sim eh_{\omega N}. \quad (6.3)$$

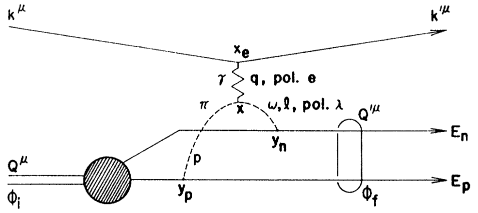

FIG. 5. $(\omega\pi)$ exchange diagram.

The remaining vertex, the $\omega\pi\gamma$ coupling, has been considered in Refs. 3 and 11. The only gauge-invariant and Lorentz-covariant form is (see Fig. 5)

$$(g_{\omega\pi\gamma}/m_\omega)\epsilon_{\alpha\beta\gamma\delta}e^\alpha q^\beta\lambda^\gamma p^\delta. \quad (6.4)$$

We shall assume that any q^2 dependence of $g_{\omega\pi\gamma}$ can be ignored for the values of q^2 we are dealing with; this is probably a reasonable assumption for $q^2\lesssim m_\omega^2=15.7$ F^{-2}. In summary, we know the vertices occurring in Fig. 5 to the extent that we can estimate $g_{\omega N}$ and $g_{\omega\pi\gamma}$.

The calculation of the $(\omega\pi)$ exchange current proceeds precisely as the $(\pi\pi)$ exchange. A current may be extracted from the S matrix, and additional contributions to the form factors G_1 and G_2 result:

$$G_1^{\omega\pi}(q^2)=(g_{\omega\pi\gamma}Gg_{\omega N}/64m_\omega e)[\Omega(q^2)+\beta(q^2)+\sigma(q^2)],$$
$$G_2^{\omega\pi}(q^2)=-(g_{\omega\pi\gamma}Gg_{\omega N}/64m_\omega e)\sigma(q^2). \quad (6.5)$$

The functions Ω, β, and σ are integrals over the deuteron and (np) wave functions, similar to the η and ζ of Sec. 5:

$$\Omega(q^2)=\int\frac{d^3\tau}{(2\pi)^3}\frac{H_s(\tau^2)(\tau_1)^2}{[\frac14(\mathbf{q}+\boldsymbol{\tau})^2+m_\pi^2][\frac14(\mathbf{q}-\boldsymbol{\tau})^2+m_\pi^2]},$$
$$\beta(q^2)=\int\frac{d^3\tau}{(2\pi)^3}\frac{J_s(\tau^2)(\tau_1)^2}{[\frac14(\mathbf{q}+\boldsymbol{\tau})^2+m_\pi^2][\frac14(\mathbf{q}-\boldsymbol{\tau})^2-m_\pi^2]}, \quad (6.6)$$
$$\sigma(q^2)=\int\frac{d^3\tau}{(2\pi)^3}\frac{J_s(\tau^2)(\tau_1)^2[2+3(\tau_3)^2q/\tau^3]}{[\frac14(\mathbf{q}+\boldsymbol{\tau})^2+m_\pi^2][\frac14(\mathbf{q}-\boldsymbol{\tau})^2+m_\pi^2]}.$$

The functions Ω, β, and σ are evidently slowly varying for $q^2\ll m_\omega^2$, as may be inferred from (6.6). Since, moreover, the $(\omega\pi)$ contributions to G_1 and G_2 are quite small and the parameters $g_{\omega N}$ and $g_{\omega\pi\gamma}$ not well known, we will content ourselves with estimating Ω, β, and σ by their values at $q^2=0$. Thus we will use the following expressions obtained by contour integration of the τ integral in (6.6) with $q^2=0$:

$$\Omega(q^2)\simeq\Omega(0)=\frac{8}{3\pi(m_\omega^2-m_\pi^2)}\times\int_0^\infty\left(m_\omega^2\frac{e^{-m_\omega y}}{y}-m_\pi\frac{e^{-m_\pi y}}{y}\right)Z_s(y)u(y)dy,$$

$$\beta(q^2)\simeq\beta(0)=\frac{-8}{5\pi(m_\omega^2-m_\pi^2)}\times\int_0^\infty\left[m_\omega^2\frac{e^{-m_\omega y}}{y}\left(1+\frac{3}{m_\omega y}+\frac{3}{m_\omega^2y^2}\right)-m_\pi^2\frac{e^{-m_\pi y}}{y}\left(1+\frac{3}{m_\pi y}+\frac{3}{m_\pi^2y^2}\right)\right]\frac{Z_s(y)w(y)}{\sqrt{8}}dy, \quad (6.7)$$

$$\sigma(q^2)\simeq\sigma(0)=10\beta(0)/3.$$

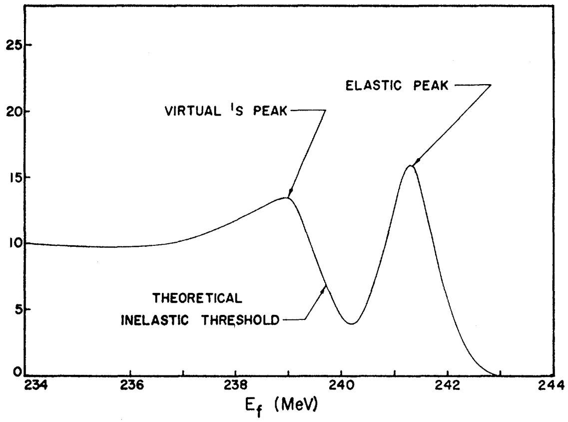

FIG. 6. Differential cross section for 325-MeV electrons scattered at 180° from the deuteron (from Ref. 1) in 10^{-36} cm²/sr (MeV/c).

Only a brief comment is necessary for the $(\rho\eta)$ exchange current. Since this is a vector-meson plus pseudoscalar-meson current, like the $(\omega\pi)$ current, we need merely replace ω by ρ and π by η in the preceding calculation. A sign change also occurs because of the different isotopics of the two systems. Thus we have additional contributions to the transition form factors

$$G_1^{\rho\eta}(q^2)=-(g_{\rho\eta\gamma}g_{\eta N}g_{\rho N}/64m_\rho e)[\bar{\Omega}(0)+\bar{\beta}(0)+\bar{\sigma}(0)],$$
$$G_2^{\rho\eta}(q^2)=(g_{\rho\eta\gamma}g_{\eta N}g_{\rho N}/64m_\rho e)\bar{\sigma}(0). \qquad (6.8)$$

The $\bar{\Omega}$, $\bar{\beta}$, and $\bar{\sigma}$ are defined as Ω, β, and σ with the masses of ω and π replaced by ρ and η.

An analysis of the $(\rho\rho)$ exchange current yields the following results. The ρ couples to a nucleon via one spin-independent term and one spin-dependent term due to the nucleon magnetic moment. The spin-independent term has no effect on the exchange current. The spin-dependent magnetic term is down by a momentum factor, and hence has a very small effect, about 10^{-3} of the $(\pi\pi)$ exchange contribution. The $(\rho\rho)$ can therefore be safely ignored.

7. COMPARISON WITH EXPERIMENT

The e-D cross section may now be calculated. As previously stated, we will limit ourselves to low (np) kinetic energies and an electron scattering angle of 180°. For a deuteron model, we will use wave functions obtained by Partovi from a Hamada-Johnston potential.[17–19] These wave functions have a hard core of 0.48 F and approach one-pion-exchange potential (OPEP) functions asymptotically. The scattered state wave functions are for (np) c.m. energies of 1.5 and 3.0 MeV which correspond to relative (np) momenta of 0.190 and 0.269 F^{-1}.

$$E_{\mathrm{c.m.}}=\mathbf{p}^2/M. \qquad (7.1)$$

We first note briefly the general qualitative features of 180° electron scattering from the deuteron.[1,2] For a fixed q^2, or fixed incident electron energy E_i, a narrow elastic peak will be observed centered about some final electron energy. (See Fig. 6.) For E_f several MeV lower, the threshold for inelastic scattering occurs. The inelastic cross section rises rapidly to a peak due to the virtual ^1S (np) state, then remains relatively flat forming a shoulder to the ^1S peak. Finally, it rises again to the so-called quasi-elastic peak[9,10] far outside the region shown in Fig. 6. We are interested in the relatively flat shoulder portion to the left of the ^1S peak. Rand and co-workers obtained a cross section in this region that is considerably higher than predicted by previous theory.

Using a 7040 computer and the Partovi wave functions, we have calculated the deuteron structure functions H_s, J_s, etc. These are shown in Fig. 7 for an (np) c.m. energy of 3.0 MeV. The numbers $\Omega(0)$, $\beta(0)$, etc., are

$$\Omega(0)=-0.146,\quad \beta(0)=0.042,\quad \sigma(0)=0.140,$$
$$\bar{\Omega}(0)=-0.471,\quad \bar{\beta}(0)=0.019,\quad \bar{\sigma}(0)=0.063. \qquad (7.2)$$

The coupling constants that occur in the $(\omega\pi)$ and $(\rho\eta)$ currents may be crudely estimated. These currents have a very small effect so crude estimates will suffice. The ρ-nucleon coupling, according to Sakurai's universality assumption, is

$$g_{\rho N}\simeq 2.6. \qquad (7.3)$$

$SU(3)$ with no ω-ϕ mixing then gives an ω-nucleon

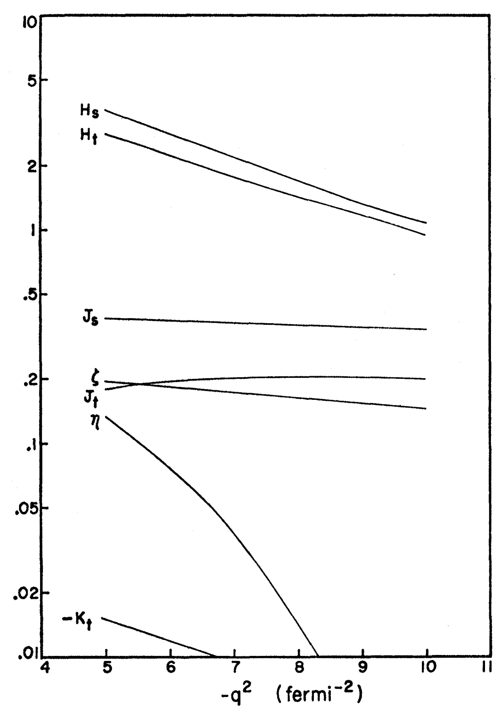

FIG. 7. Deuteron structure functions for $E_{\mathrm{c.m.}}=3.0$ MeV. Functions ζ and η are in F$^{1/2}$; others are in F$^{3/2}$.

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[17] F. Partovi, Ann. Phys. (N. Y.) **27**, 79 (1964).
[18] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).
[19] E. F. Erickson kindly supplied us with numerical values of the wave functions used.

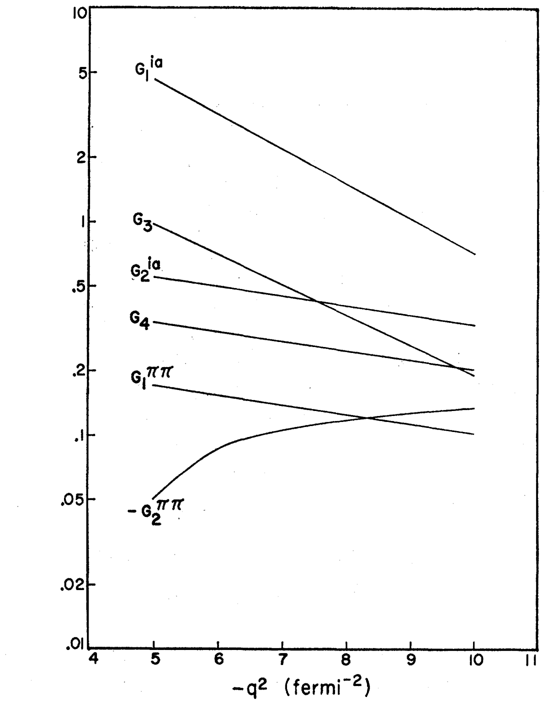

FIG. 8. Deuteron form factors for $E_{\text{c.m.}}=3.0$ MeV. Units are $F^{3/2}$.

coupling of

$$g_{\omega N}\simeq\sqrt{3}g_{\rho N}\simeq 4.5. \tag{7.4}$$

The η-nucleon coupling also follows from $SU(3)$ if an f/d ratio of 0.6 is assumed;

$$g_{\eta N}\simeq 3.8. \tag{7.5}$$

The $\omega\pi\gamma$ and $\rho\eta\gamma$ couplings are on rather shaky ground. A previous analysis of the $\rho\pi$ exchange current in

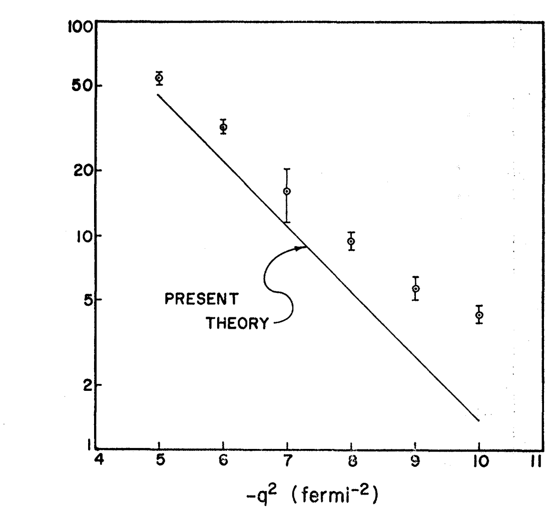

FIG. 9. Inelastic e-D scattering cross section for $E_{\text{c.m.}}=3.0$ MeV. Units are 10^{-36} cm^2/sr (MeV/c).

elastic e-D scattering[11] indicated that

$$g_{\rho N}g_{\rho\pi\gamma}\simeq 0.48, \tag{7.6}$$

so

$$g_{\rho\pi\gamma}\simeq 0.18. \tag{7.7}$$

A recent analysis of photoproduction by Donnachie and Shaw[20] indicates that $g_{\rho\pi\gamma}$ may be smaller yet, so we will use

$$g_{\rho\pi\gamma}\lesssim 0.18. \tag{7.8}$$

$SU(3)$ then yields

$$g_{\rho\eta\gamma}\simeq g_{\omega\pi\gamma}\simeq\sqrt{3}g_{\rho\pi\gamma}\simeq 0.31. \tag{7.9}$$

One may note that for this value we have, numerically,

$$g_{\rho\eta\gamma}^2/4\pi\simeq g_{\omega\pi\gamma}^2/4\pi\simeq e^2/4\pi=1/137.03. \tag{7.10}$$

The form factors G_1 and G_2 that appear in the cross section may now be calculated. We used a universal nucleon form-factor dependence[21]

$$G(q^2)\propto(1-q^2/m^2)^{-2};\quad m^2=0.71\text{ BeV} \tag{7.11}$$

and a ρ pole for the pion form-factor dependence, and obtained the functions G_1 and G_2 shown in Fig. 8. The contributions of $(\omega\pi)$ and $(\rho\eta)$ exchange to G_1 and G_2 are quite small;

$$G_1^{\omega\pi}\simeq 0.008,\quad G_2^{\omega\pi}\simeq -0.032,$$
$$G_1^{\rho\eta}\simeq 0.015,\quad G_2^{\rho\eta}\simeq 0.003. \tag{7.12}$$

The final cross section is given by (4.19), which we repeat here;

$$\frac{d\sigma}{d\Omega dk'}=\frac{\alpha^2 p[G_1^2+G_2^2+G_3^2+G_4^2]}{(2\pi)^2 3M\sin^2(\frac{1}{2}\theta)}\frac{k'}{k}\times\left[1+\sin^2(\tfrac{1}{2}\theta)+\frac{(k-k')^2}{2kk'}\right], \tag{7.13}$$

$$G_1=G_1^{ia}+G_1^{\pi\pi}+G_1^{\omega\pi}+G_1^{\rho\eta},$$
$$G_2=G_2^{ia}+G_2^{\pi\pi}+G_2^{\omega\pi}+G_2^{\rho\eta}.$$

The theoretical cross section must be multiplied by a radiative correction factor[2] before comparison with a specific experiment. According to Yearian,[22] this factor is nearly constant at 0.92 for the experiment under consideration. As noted in Sec. 4, we also have an ambiguity in the interpretation of q^2; according to Rand *et al.*,[1] the use of q^2 equal to the negative of the 4-momentum transfer squared results in correct threshold behavior of the inelastic cross section. In any case, the difference between using $-q^\mu q_\mu$ and $\mathbf{q}^2$ in the deuteron structure functions is only a few percent in the cross section. Our theoretical cross section is plotted as a function of q^2 in Fig. 9 and compared to the experi-

[20] A. Donnachie and G. Shaw, Ann. Phys. (N. Y.) **37**, 333 (1966).

[21] M. Goitein, J. R. Dunning, Jr., and R. Wilson, Phys. Rev. Letters **18**, 1018 (1967).

[22] M. Yearian (private communication).

ment of Rand *et al.* The theoretical curve is for an (np) center of mass energy of 3.0 MeV, and the experimental points are for $p=0.98p$ threshold; both are on the flat portion of the cross section curve (see Fig. 6).

As is obvious from Fig. 9, the theoretical cross section is far too small; it is off by 20% at $q^2=5$ F^{-2} and 100% at $q^2=9$ F^{-2}. In Sec. 8, we will discuss possible remedies.

8. CONCLUSIONS

From the comparison of the present theory with experiment, we may draw the following conclusions:

(1) The present theory disagrees with experiment. The ratio of experimental to theoretical cross sections rises from about 1 at $q^2=5$ F^{-2} to 2 at $q^2=9$ F^{-2}.

The transition to the ^1S state dominates the cross section. Since the form factor G_3 is about 25% of G_1 and G_4 is less yet, the ^3S state contributes only about 12% to the cross section.

(3) The $(\omega\rho)$, $(\rho\eta)$, and $(\rho\rho)$ exchange currents contribute a negligible amount to the cross section. The $(\pi\pi)$ is not negligible but contributes about 10% to G_1 and 50% to G_2. Since G_1 dominates the cross section, the over-all effect of $\pi\pi$ is of the order of 20%.

(4) The impulse approximation to G_1 dominates the cross section. Thus the most believable reason for the diagreement with experiment is that G_1, which depends mainly on the structure function H_s, is incorrect.

(5) Relativistic corrections to the impulse approximation[6,7,9] should be expected to be small when the difference between 3-momentum transfer and 4-momentum transfer squared is small. This fractional difference is $q^2/4M_D{}^2$, which is only 2% at $q^2=10$ F^{-2}. As a result, we feel the most likely source of error is in the deuteron wave function.

Motivated by the above conclusions, we have attempted to find a qualitative answer to what is wrong

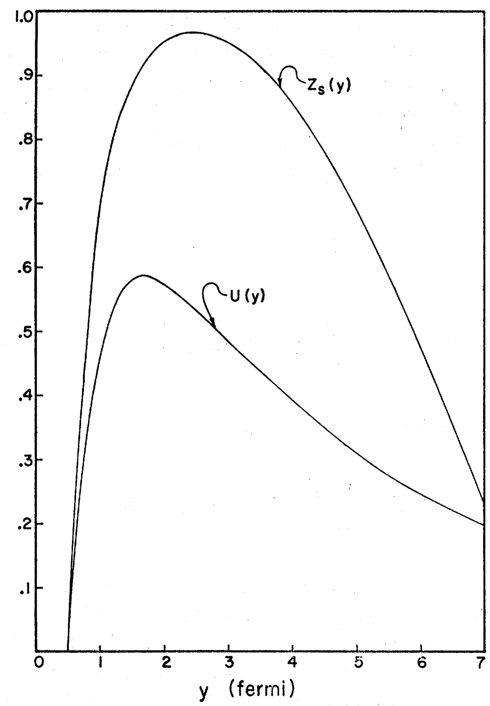

FIG. 10. Partovi wave functions for $E_{\text{c.m.}}=3.0$ MeV. $u(y)$ is in F$^{-1/2}$, and $Z_s(y)$ is in fermi.

with the deuteron wave function. The Partovi wave functions we used[17–19] are shown in Fig. 10.

We first varied the core radius, keeping the shape of the wave functions the same. For core radii around 0.35 F instead of the original 0.48, the theory looked much better, but the theoretical cross section still fell off too rapidly.

We next made a crude inversion of the experimental results to get the product $u(y)Z_s(y)$. To do this, we assumed the error in the cross section was entirely due

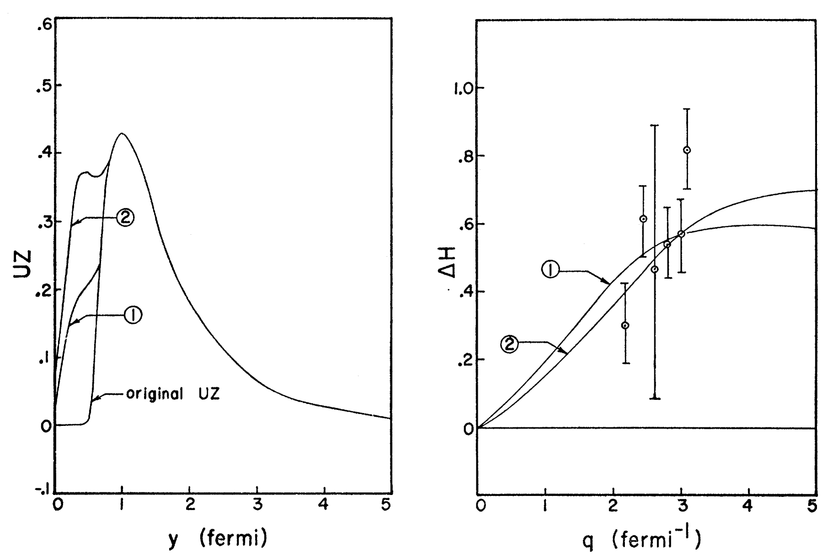

FIG. 11. Crude estimate of ΔH and corresponding change in wave function. ΔH is in F$^{3/2}$, uZ_s in F$^{1/2}$.

to an error in the theoretical H, which we may call ΔH. Comparison of the theoretical and experimental cross sections indicates that this function ΔH may rise slightly between $q^2=5$ F^{-2} and $q^2=10$ F^{-2}, but is roughly constant at 0.6 ± 0.3. If this ΔH arises from an addition $\delta(y)$ to $u(y)Z_s(y)$, then by the definition of H

$$H(q^2)=\int_0^\infty u(y)Z_s(y)j_0(\tfrac{1}{2}qy)dy\,,$$

$$\Delta H(q^2)=\int_0^\infty \delta(y)j_0(\tfrac{1}{2}qy)dy\,, \tag{8.1}$$

which may be inverted to yield

$$\delta(y)=\frac{1}{2\pi}\int_0^\infty \sin(\tfrac{1}{2}qy)\Delta H(q^2)dy\,. \tag{8.2}$$

Several forms for ΔH were chosen so as to give a correct H in the region of $q^2=5$ to 10; these functions and the resultant function $\delta(y)$ are shown in Fig. 11. This is clearly very crude but seems to indicate that the wave function should go to zero more slowly than the Partovi model, i.e., the nucleon core is smaller and/or softer. Specifically, a hard core of more than 0.3 F seems to be ruled out.

The question naturally arises as to how well $H(q^2)$ must be known to obtain a reasonably accurate function uZ_s and whether one might hope to invert direct experimental results. To answer this, we first obtained H from uZ_s, then inverted to obtain uZ_s again but used a cutoff on the integral over H. We found that a cutoff value of $q=9$ F^{-1} was necessary to give reasonably good results down to $y=0.3$ F. This is three times the present momentum transfer measured and is in a region where relativistic effects should be considerable. Clearly, a quantitative inversion is not justified at present.

The next step in this problem is to consider a number of different existing deuteron models and see if any of them produce better results than the one used here. In conjunction with this, we are attempting to relate the np scattering to deuteron form factors in a very simple way so that the errors may be easily propagated. This would allow us to see precisely how trustworthy the theoretical deuteron form factors are. Specifically, we would like to parametrize np scattering with a small number of parameters, say 4 or 6, and obtain the deuteron form factors as analytic functions of these parameters.

Finally, although we feel that deuteron model uncertainty is the most obvious uncertainty in the theory, relativistic effects and other contributions to e-D scattering could certainly be unexpectedly large and produce the present disagreement.

ACKNOWLEDGMENTS

The author would like to thank Professor M. Yearian for informing him of his experimental results and providing the original motivation for this work. C. E. Littig was helpful in the earlier stages, and Professor R. A. Arndt provided invaluable assistance with the computer work.