parameter solution therefore results with (3.5) and (3.7) satisfied. Now it is straightforward to show from our equations that neglecting the decuplet in only the $B\bar{B}$ channel directly gives (3.7) in agreement with this model. In fact, under CP and octet dominance (3.7) holds if, and only if, the decuplet contribution in the $B\bar{B}$ channel vanishes.

The Dashen-Frautschi-Sharp model will give (3.6) as well if their parameter R vanishes, so that the $B\bar{B}$ state is in an antisymmetric octet state. One may then deduce that the condition for this, in an $SU(3)$ model, is that the decuplet vanishes in the $B\pi$ channel.

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Nonlinear Sum Rules for High-Energy Hadron-Hadron Collisions in the Quark Model*

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By assuming that quark-quark double scattering contributes appreciably to high-energy hadron-hadron scattering, we obtain a quadratic and a cubic sum rule for hadron-hadron total cross sections. These sum rules are found to agree with experiment at an incident hadron momentum of 16 BeV/c.

THE idea of Lipkin[1] that high-energy hadron-hadron total cross sections can be calculated in the quark model by assuming simple addivity of the quark-quark cross sections has been widely used[2] to obtain cross-section sum rules under a variety of assumptions about the high-energy behavior of the quark-quark amplitudes. More recently, attention has been focused on the validity of the additivity assumption. It has been argued by Franco[3] that the existence of several quarks in the colliding hadrons implies that multiple-scattering effects cannot *a priori* be ruled out in such collisions. For this reason he has considered double-scattering corrections to the additive quark model, obtaining a subset of the additivity sum rules for which the disagreement with experiment is considerably reduced.[4] In the present work we will consider double-scattering corrections to the additive quark model, and will also make definite assumptions about the high-energy behavior of the quark-quark scattering amplitudes. We then obtain one sum rule which is quadratic, and another which is cubic, in the hadron-hadron cross sections, in addition to the linear sum rules found by Franco. We also obtain two inequalities among the cross sections. The new sum rules and inequalities are in good agreement with experiment.

We begin by assuming that the quark double-scattering corrections to the total hadron-hadron cross sections can be calculated using the Glauber approximation.[5] Then σ is given by

$$\sigma=\sum_{ij}\sigma(ij)+\left(\frac{1}{4\pi}\right)^2 \times\left(\frac{4\pi}{p}\right)^2\sum_{j\neq j'}\mathrm{Re}\Big\{\sum_i\int d^2q[f_{ij}(\mathbf{q})f_{ij'}(-\mathbf{q}) \times S_t(\mathbf{q})+f_{ji}(\mathbf{q})f_{j'i}(-\mathbf{q})S_p(\mathbf{q})] +\sum_{i\neq i'}\int d^2q\, f_{ij}(\mathbf{q})f_{i'j'}(-\mathbf{q})S_t(\mathbf{q})S_p(\mathbf{q})\Big\}, \quad (1)$$

where p is the momentum of the incident quark, $\sigma(ij)$ is the total cross section for the scattering of the ith quark in the projectile by the jth quark in the target, $f_{ij}(\mathbf{q})$ is the corresponding quark-quark scattering amplitude at three-momentum transfer $\mathbf{q}$, and all quantities are evaluated in the lab system. The quantities S_p and S_t are quark form factors in the projectile and target hadrons, respectively.

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[1] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966).
[2] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **2**, 105 (1965) [English transl.: JETP Letters **2**, 65 (1965)]; J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42**, 711 (1966); C. H. Chan, Phys. Rev. **152**, 1244 (1966); Y. T. Chiu, *ibid.* **155**, 1579 (1967).
[3] V. Franco, Phys. Rev. Letters **18**, 1159 (1967).
[4] Franco assumes that the relations of Eqs. (6) are satisfied by the corresponding quark amplitudes. With this assumption the additive quark model disagrees with experiment.
[5] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315; V. Franco and R. J. Glauber, Phys. Rev. **142**, 1195 (1966).

We now make the following additional assumptions:

(1) The quarks are so tightly bound in a hadron that the form factors can be set equal to unity.

(2) The quark-quark scattering amplitude $f_{ij}(\mathbf{q})$ has the following decomposition:

$$f_{ij}(\mathbf{q})=(p/4\pi)\sigma(ij)f(\mathbf{q}), \tag{2}$$

where

$$\text{Im}f(0)=1. \tag{3}$$

With these assumptions Eq. (1) becomes

$$\sigma=\sum_{ij}\sigma(ij)+g\sum_{j\neq j'}\left\{\sum_{i}\left[\sigma(ij)\sigma(ij')+\sigma(ji)\sigma(j'i)\right]+\sum_{i\neq i'}\sigma(ij)\sigma(i'j')\right\}, \tag{4}$$

where

$$g=\left(\frac{1}{4\pi}\right)^2\text{Re}\int d^2q\, f(\mathbf{q})f(-\mathbf{q}). \tag{5}$$

We now assume that at high energies $SU(3)$ invariance holds for the quark-quark cross sections, the charge-exchange cross section for quark-quark scattering is zero,[6] and a weak form of the Pomeranchuk theorem is valid.[7] The quark cross sections then satisfy the following relations:

$$\begin{aligned}\sigma(aa)=\sigma(ab)=\sigma(ac)&=\sigma(a\bar{c})\\=\sigma(a\bar{b})&\equiv\sigma_1,\\ \sigma(a\bar{a})&\equiv\sigma_2,\end{aligned} \tag{6}$$

where a, b, and c are the three quarks.

We then obtain the following equalities among the hadron-hadron cross sections:

$$\sigma(pp)=\sigma(pn), \tag{7a}$$

$$\sigma(K^+p)=\sigma(K^+n), \tag{7b}$$

$$\sigma(K^-p)=\sigma(\pi^-p), \tag{7c}$$

$$\sigma(K^-n)=\sigma(\pi^+p), \tag{7d}$$

and

$$\sigma(pp)=\sigma(\Sigma^+p)=\sigma(\Sigma^-p)=\sigma(\Lambda p). \tag{8}$$

We also obtain

$$\sigma(pp)=9\sigma_1+72g\sigma_1^2, \tag{9a}$$

$$\sigma(\bar{p}p)=(4\sigma_1+5\sigma_2)+g(12\sigma_1^2+40\sigma_1\sigma_2+20\sigma_2^2), \tag{9b}$$

$$\sigma(\bar{p}n)=(5\sigma_1+4\sigma_2)+g(20\sigma_1^2+40\sigma_2\sigma_1+12\sigma_2^2), \tag{9c}$$

$$\sigma(K^+p)=6\sigma_1+30g\sigma_1^2, \tag{9d}$$

$$\sigma(K^-p)=(4\sigma_1+2\sigma_2)+g(12\sigma_1^2+16\sigma_1\sigma_2+2\sigma_2^2), \tag{9e}$$

$$\sigma(K^-n)=(5\sigma_1+\sigma_2)+g(20\sigma_1^2+10\sigma_1\sigma_2). \tag{9f}$$

[6] H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966).

[7] By the weak form of the Pomeranchuk theorem we mean that the theorem will be assumed to be valid except where an isosinglet annihilation channel contributes to the amplitude. See, for example, Ref. 6.

Eliminating σ_1, σ_2, and g from Eqs. (9), we obtain the following linear, quadratic, and cubic sum rules:

$$\begin{aligned}&[5\sigma(\bar{p}n)+10\sigma(K^-p)+10\sigma(K^+p)]\\&\qquad=[20\sigma(K^-n)+4\sigma(\bar{p}p)+\sigma(pp)],\end{aligned} \tag{10}$$

$$\begin{aligned}&[5\sigma(pp)\sigma(\bar{p}p)+30\sigma(K^+p)\sigma(K^-p)+8\sigma(pp)\sigma(K^+p)\\&\quad+45\sigma(pp)\sigma(K^-n)]=[50\sigma(pp)\sigma(K^-p)\\&\quad+3\sigma(\bar{p}p)\sigma(K^+p)+5\sigma(pp)^2+30\sigma(K^+p)^2],\end{aligned} \tag{11}$$

$$\begin{aligned}&[25\sigma(pp)^2\sigma(K^-p)+9\sigma(K^+p)^2\sigma(K^-p)+25\sigma(pp)^2\sigma(K^+p)\\&\quad+90\sigma(K^+p)^3+168\sigma(pp)\sigma(K^+p)\sigma(K^-n)\\&\quad+81\sigma(K^+p)\sigma(K^-n)^2]=[30\sigma(pp)\sigma(K^+p)\sigma(K^-p)\\&\quad+84\sigma(pp)\sigma(K^+p)^2+50\sigma(pp)^2\sigma(K^-n)\\&\quad+180\sigma(K^+p)^2\sigma(K^-n)+54\sigma(pp)\sigma(K^-n)^2].\end{aligned} \tag{12}$$

From the conditions $\sigma_1\geqq 0$, $\sigma_2\geqq 0$ we also obtain the following inequalities:

$$12\sigma(K^+p)\geqq 5\sigma(pp) \tag{13a}$$

and

$$\begin{aligned}&5[35\sigma(pp)+45\sigma(\bar{p}p)+144\sigma(K^-p)]\geqq 3[165\sigma(\bar{p}n)\\&\qquad+140\sigma(K^+p)+24\sigma(K^-n)].\end{aligned} \tag{13b}$$

The cross-section equalities of Eqs. (7) and the linear sum rule Eq. (10) have been previously obtained by Franco.

We will test the sum rules at the same hadron lab momentum for all processes.[8] We shall use the 16-BeV/c data of Galbraith *et al.*,[9] since at higher energies some of the cross sections are not as well known. Like Franco, we find that there are discrepancies of up to 4 mb in the equalities (7c) and (7d); for example, $\sigma(K^-p)=21.3\pm0.4$ mb while $\sigma(\pi^-p)=25.1\pm0.3$ mb. For the linear sum rule Eq. (10), the left-hand side (lhs) is 647 ± 19 mb and the right-hand side (rhs) is 642 ± 12 mb. The quadratic and cubic sum rules are satisfied within the errors. We find for the quadratic sum rule Eq. (11) l.h.s.$=(6.10\pm0.13)\times10^4$ mb^2 and r.h.s.$=(5.99\pm0.12)\times10^4$ mb^2. For the cubic sum rule Eq. (12), we have l.h.s.$=(4.74\pm0.13)\times10^6$ mb^3 and r.h.s.$=(4.80\pm0.15)\times10^6$ mb^3. We have also examined the two inequalities Eqs. (13) and have found that they are also satisfied within the errors. The three equalities Eqs. (8) involving hyperon-nucleon cross sections cannot be compared with experiment at the present time.

There exists a certain freedom in testing the sum rules (10)–(12) because of the discrepancy in Eqs. (7c) and (7d); we find, however, that if we replace either or both of $\sigma(K^-p)$ and $\sigma(K^-n)$ by $\sigma(\pi^-p)$ and $\sigma(\pi^+p)$, respec-

[8] In this model the sum rules should be tested at the same quark lab momentum. There is some uncertainty as to what this means in comparing meson-baryon and baryon-baryon cross sections. If we assume that the quark momentum is given by the hadron momentum divided by the number of quarks composing the hadron, then this would imply that we should take $K_M=\frac{2}{3}K_B$, where K_M and K_B are the meson and baryon momenta, respectively; however, the meson-nucleon cross sections vary very little in the region 10–16 BeV/c, so that such an assumption seems to be unnecessary.

[9] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965).

tively, the sum rules are still satisfied quite well. In order to obtain agreement with experiment in our model, quark double scattering must give a large positive contribution ($\gtrsim 85\%$) to the total cross sections. The question of whether higher terms in the Glauber expansion might not also be important therefore arises. In order to test this hypothesis we have performed a least-squares analysis of the data at this energy, including successively double- and triple-scattering corrections. This analysis will be presented elsewhere; however, our results indicate that there is little statistical improvement of the fit obtained with double scattering when triple-scattering terms are included. If one assumes $SU(2)$ rather than $SU(3)$ symmetry of the quark cross sections, then it becomes very difficult to eliminate the parameters of the model to obtain sum rules. However, one result of such an assumption is that the equalities (7c) and (7d), which are not well satisfied, are no longer obtained from this model. Furthermore, our least-squares analysis then indicates that the double-scattering corrections are considerably reduced. This latter result is similar to that of Barnhill,[10] who used this parametrization of the quark amplitudes in a somewhat different model.

We obtain the results of the additive quark model by putting $g=0$ in Eq. (9). In addition to the relations which we have already obtained, this yields the result

$$2\sigma(pp)=3\sigma(K^+p).$$

The left-hand side is 77.4 ± 1.2 mb while the right-hand side is 51.0 ± 0.3 mb, in very poor agreement with experiment. We therefore conclude that it may not be valid to use the additive quark model to rule out $SU(3)$ invariance of the quark cross sections, as double-scattering corrections are undoubtedly important.

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New Criterion for Nonoscillating High-Energy Behavior of Scattering Amplitudes. II. Applications*

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The condition for nonoscillation of scattering amplitudes at high energy, proposed in a previous paper by the authors, is applied to show that (i) fast oscillations can be incorporated into the picture by taking into account the finite resolution of physical apparatus, (ii) finite-energy sum rules are equivalent to the fulfillment of the criterion, (iii) Regge cuts lead to nonoscillating behavior in our sense, (iv) the criterion may still be satisfied when the leading singularities in the complex l plane are more general than the usual cuts and poles.

I. INTRODUCTION

In a previous paper,[1] a new criterion defining nonoscillating behavior of scattering amplitudes at high energy was proposed and discussed. The criterion amounts to a requirement that the limit

$$\lim_{E\to\infty}\frac{a(uE)}{a(E)}=\psi(u)\,,\quad a(E)=\operatorname{Im}T(E,t) \tag{1}$$

exists. Here E is the lab energy of the incoming particle. As shown in Ref. 1, this existence implies that $\psi(u)$ is of the form u^α. The function T is the scattering amplitude, and we drop the dependence on the momentum transfer t since we consider t to be fixed.

As shown in Ref. 1, the standard high-energy results follow from this assumption. Thus, e.g., we could prove that (1) implies Regge-type behavior, including signature, asymptotic equality of differential cross sections, and the Pomeranchuk theorem. On the other hand, it is amusing that, after writing the former article,[1] we discovered that the criterion (1) has been previously studied in the mathematical literature,[2] in a quite different context, and that the functions satisfying it are referred to as "regularly varying functions."

In the present paper we extend the analysis of Ref. 1, giving new results, partly our own and partly obtained from Ref. 2. In Ref. 1, it was shown that sufficiently slow oscillations (with constant relative amplitude) could be incorporated into the high-energy picture without changing it, since they satisfy criterion (1). In

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