

very satisfactory results including the interesting mass formula Eq. (20) are obtained independent of the choice of the cutoff. However, it is not clear, and still remains to be investigated, whether we could obtain self-consistent and cutoff-independent results even if we included in our scheme higher resonances corresponding to a higher cutoff of  $N$  and more channels in the higher-order approximation.

*Note added in proof.* After completing the manuscript I found that M. Ademollo, H. R. Rubinstein, G.

Veneziano, and M. A. Virasoro [Phys. Rev. Letters **19**, 1402 (1967)] considered a similar problem and derived bootstrap conditions based on GSCR. Their approach, however, is not one of a systematic bootstrap.

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### $K_{13}$ Decay Form Factors and the Current Algebra\*

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The  $K_{13}$  decay form factors are studied by using a current algebra. Compared with the previous approach, the following points are new. We use the pion partially conserved axial-vector current (PCAC) hypothesis instead of kaon PCAC. We use a smaller off-mass-shell extrapolation  $-q_\mu^2 = m_\pi^2 \rightarrow 0$  instead of the usual soft-pion extrapolation  $q_\mu \rightarrow 0$ . We apply a dispersion technique to a direct calculation of the form factors  $F_+(s)$  and  $F_-(s)$ , and utilize the current algebra to fix the absolute scale as well as the energy dependence with an approximation for vector currents proposed before. For the  $F_+(s)$ , the  $K^*$ -meson contribution gives a result consistent with the Ademollo-Gatto theorem at  $s=0$ . For the  $F_-(s)$ , both the  $K^*$  and the  $I=\frac{1}{2}$ ,  $0^+$  meson,  $\kappa$ , contribute. Our expression for the  $F_-(s)$  depends only on the mass of the  $\kappa$  meson (not on its width) and satisfies explicitly the requirement that  $F_-(s) \rightarrow 0$  in the  $SU(3)$  limit. The parameter  $\xi$  is given by  $\xi = F_-(0)/F_+(0) = -0.026 - (0.061 \text{ GeV}^2)/m_\kappa^2$ . Therefore, we predict a small negative value for  $\xi$ . Our method gives information on  $F_+(s)$  and  $F_-(s)$  for all the physical values of  $s$ , including the most important region around  $s=0$ , whereas the soft-pion approach gives information only at the rather unphysical point  $s=m_\kappa^2$ . However, at this very point, our results on  $F_+(m_\kappa^2) + F_-(m_\kappa^2)$  agrees with that of the soft-pion approach independently of the parameter  $m_\kappa$ . Therefore, our results include the soft-pion result at  $s=m_\kappa^2$  and seem to give a consistent description of the  $K_{13}$  decay form factors.

THE form factors  $F_+(s)$  and  $F_-(s)$  of the  $K_{13}$  decays contain many interesting clues to the problems of  $SU(3)$  symmetry. Precise determination of the value of  $F_+(0)$ , taking into account the form-factor effect, will give us a first measurement of the second-order  $SU(3)$  symmetry-breaking effect. In the  $SU(3)$  symmetry limit, the  $F_-(s)$  vanishes, and its actual value, measured by the parameter  $\xi$  defined as  $\xi \equiv F_-(0)/F_+(0)$ , will therefore provide important information on first-order  $SU(3)$  breaking. Recent calculations of these form factors on the basis of the algebra of currents together with the hypothesis of partially conserved axial-vector currents (PCAC) may be categorized as follows:

(a) Relate the  $[F_+(m_\kappa^2) + F_-(m_\kappa^2)]$  to the  $K_{\mu 2}$  decay by using a soft-pion-emission technique,  $q_\mu \rightarrow 0$ , where  $q_\mu$  is the pion four-momentum.<sup>1</sup>

(b) Determine the parameters of the  $I=\frac{1}{2}$ ,  $Y=\pm 1$  possible scalar resonance  $\kappa$  by studying resonance satu-

ration of axial charge commutators<sup>2,3</sup> with the less radical extrapolation,  $q^2 = (-m_\pi^2) \rightarrow 0$ .

(c) Apply dispersion techniques to a direct calculation of  $F_+(s)$  and  $F_-(s)$ , and utilize the charge-current algebra to fix the absolute scale as well as the energy dependence. Although this approach is very interesting, the results obtained<sup>4</sup> do not seem very satisfactory. First, the calculation involves soft-kaon emission as well as soft-pion emission, by letting the mass of the kaon as well as that of the pion go to zero. Secondly, the  $F_-(s)$  thus obtained, which includes the contribution of the  $\kappa$  meson, does not explicitly vanish if we take an  $SU(3)$  limit in its expression.

(1966); M. Suzuki, *ibid.* **16**, 212 (1966); V. S. Mathur, S. Okubo, and L. K. Pandit, *ibid.* **16**, 371 (1966).

<sup>2</sup> V. S. Mathur, S. Okubo, and L. K. Pandit, Phys. Rev. Letters **16**, 371 (1966); **16**, 601(E) (1966); V. S. Mathur and L. K. Pandit, Phys. Rev. **143**, 1216 (1966); K. Kawarabayashi, W. D. McGlinn, and W. W. Wada, Phys. Rev. Letters **15**, 897 (1965).

<sup>3</sup> S. Matsuda, S. Oneda, and J. Scuser, Phys. Rev. **159**, 1247 (1967).

<sup>4</sup> V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters **16**, 947 (1966); **16**, 1135(E) (1966).

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<sup>1</sup> C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153

The main purpose of this paper is to report a fresh attempt to evaluate the  $K_{13}$  decay form factors from approach (c) which is free from the two difficulties mentioned above.<sup>4</sup> We also discuss the relation of our results to approach (a).

Our approach consists in the use of appropriate charge-charge-density and charge-charge commutators recently studied by the present authors.<sup>5,6</sup> By this procedure, we obtain enough information to evaluate coupling constants, which can be fed into the usual dispersion integrals of the  $K_{13}$  decay form factors. Therefore, we first discuss consequences of the use of charge-charge-density and charge-charge commutators.

(A) The combined use of the charge commutators<sup>7</sup> typified by  $[V_K, A_\pi] \propto A_K$  and the PCAC hypothesis has been shown to be useful in discussing the coupling-constant relations in the broken  $SU(3)$  symmetry. We have proposed<sup>5,6</sup> an approximation that at zero momentum transfer the renormalization due to the symmetry breaking of the matrix elements of the *vector current*  $V_\mu^{K^*}(x)$  is small and practically negligible. Except for this approximation (at zero-momentum transfer) we use observed values for other quantities, which naturally include the main effects of  $SU(3)$  symmetry breaking. We have, for example, obtained<sup>5</sup> a relation between the  $K^*K\pi$  and  $\rho\pi\pi$  couplings which are defined with one of the pions off the mass shell ( $m_\pi \rightarrow 0$ ):

$$G_{K^*0\pi^+K^-} = (m_\rho^2 + m_{K^*}^2/2m_\rho^2)G_{\rho^0\pi^+\pi^-}. \quad (1)$$

If  $\Gamma(\rho \rightarrow \pi + \pi) \simeq 120$  MeV, this relation agrees very well with experiment. Extending the same approximation to the charge-current commutators typified by  $[V_K, V_0^\pi(x)] = V_0^K(x)$ , we also obtained<sup>6</sup>

$$G_{K^*} = (m_\rho^2 + m_{K^*}^2/2m_\rho^2)G_\rho, \quad (2)$$

where  $G_{\rho(K^*)}$  are defined as  $(2q_0)^{1/2} \langle 0 | V_{\mu(0)}^{\pi(K)} | \rho(K^*) \rangle \equiv G_{\rho(K^*)} \epsilon_{\mu\rho(K^*)}$ . Incidentally, this sum rule is essentially identical to the first sum rule given by the spectral-function approach.<sup>8</sup> However, we emphasize that since we always deal with charge-current commutators in our approach, the situation with respect to the Schwinger

<sup>5</sup> S. Matsuda and S. Oneda, Phys. Rev. **158**, 1594 (1967). More accurately, the approximation assumes that the matrix elements  $\langle B | V_K | A \rangle$  of  $V_K$  take on the  $SU(3)$  values at the infinite momentum limit of the particles appearing in the states  $A$  and  $B$ . Namely, we assume that the  $V_K$  behaves like an  $SU(3)$  generator in this limit. This prescription might correspond to the so-called asymptotic  $SU(3)$  symmetry. However, this does not give a very strict restriction on the form factors of  $\langle B | V_\mu^K(x) | A \rangle$ . For example, for the  $K_{13}$  decay form factors defined in Eq. (10), our approximation implies only  $F_+(0) \simeq 1$ , but no restrictions are imposed on  $F_-(0)$ , nor on  $F_+(s)$  or  $F_-(s)$ , at  $s \neq 0$ . Therefore, we are still entitled to compute  $F_+(s)$  and  $F_-(s)$  under our approximation.

<sup>6</sup> S. Matsuda and S. Oneda, Phys. Rev. **165**, 1749 (1968).

<sup>7</sup> Vector and axial-vector currents are denoted by  $V_\mu^{\pi^+}(x)$ ,  $V_\mu^{K^+}(x)$ ,  $\dots$ , and  $A_\mu^{\pi^+}(x)$ ,  $A_\mu^{K^+}(x)$ ,  $\dots$ , respectively, normalized so that in a quark model we would have, e.g.,  $V_\mu^{\pi^+}(x) = i\bar{q}\gamma_\mu \times (\lambda_1 + i\lambda) \frac{1}{2}q$ ,  $A_\mu^{K^+}(x) = i\bar{q}\gamma_\mu (\lambda_1 + i\lambda) \frac{1}{2}q$ , etc. The space integral of, say,  $V_0^\pi(x, 0)$  is denoted as  $V_\pi^+$ .

<sup>8</sup> T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, *ibid.* **19**, 137 (1967).

terms is much better than in the spectral-function approach, which deals directly with products of currents.

We note that, in the approximation mentioned above, the contribution of the  $\kappa$  meson is assumed to be unimportant. We have further extended<sup>6</sup> the approach to the charge-current commutator involving the axial charge, for example,  $[A_\pi, V_0^\pi(x)] = A_0^\pi(x)$ . If we consider matrix elements of these types of charge-charge-density commutators between the vacuum state and a state with momentum  $|\mathbf{q}| = \infty$ , then the only single-particle (or resonance) intermediate states which can contribute are either the spin-0 or the spin-1 states, so that one can expect a quick saturation (as in the case of spectral-function sum rules). Indeed, if we assume that the pion electromagnetic form factor satisfies an unsubtracted dispersion relation, we obtain<sup>6</sup> the Gell-Mann-Zachariasen relation<sup>9</sup> using pion PCAC,

$$G_\rho G_{\rho^0\pi^+\pi^-} = m_\rho^2, \quad (3)$$

with a small extrapolation:  $G_{\rho\pi\pi}$  is defined with one of the pions off the mass shell ( $m_\pi \rightarrow 0$ ).

If we now take the commutator  $[A_{\pi^+}, V_0^{K^0}(x)] = -A_0^{K^+}(x)$ , consider the matrix element between the vacuum and the  $K^-(q)$  state with  $|\mathbf{q}| = \infty$ , and assume that the  $K_{13}$  decay form factors  $F_+$  and  $F_-$  satisfy unsubtracted dispersion relations, we then obtain an analogous relation to Eq. (3) using pion PCAC, namely,

$$G_{K^*} G_{K^*0K^-\pi^+} = m_{K^*}^2 \left[ \left( \frac{f_K}{f_\pi} \right) - \frac{F_\kappa G_{\kappa^0 K^+ \pi^-}}{m_\kappa^2 - m_{K^*}^2} \right]. \quad (4)$$

$f_\pi$  and  $f_K$  are the form factors of the  $\pi \rightarrow \mu + \nu$  and  $K \rightarrow \mu + \nu$  decays, respectively.  $F_\kappa$  is defined by  $(2q_0)^{1/2} \times \langle 0 | V_\mu^{K^*} | \kappa(q) \rangle = F_\kappa q_\mu$ . The coupling constants  $G_{K^*K\pi}$  and  $G_{\kappa K\pi}$  are defined with a pion off the mass shell ( $m_\pi \rightarrow 0$ ). We have to retain the  $\kappa$  contribution in Eq. (4), since if the  $\kappa$  meson exists, there is no *a priori* reason that the  $\kappa$  contribution should be much smaller than the  $K^*$  one. This is contrary to the cases of Eqs. (1) and (2), where we deal with the operator  $V_K$ .

If, for the time being, we neglect the  $\kappa$ -meson contribution in (4), then we obtain from (4)

$$G_{K^*} G_{K^*0K^-\pi^+} = (f_K/f_\pi) m_{K^*}^2 = 1.28 m_{K^*}^2. \quad (5)$$

On the other hand, by combining (1)–(3), we obtain

$$G_{K^*} G_{K^*0K^-\pi^+} = (m_\rho^2 + m_{K^*}^2/2m_\rho^2) m_\rho^2 \simeq m_{K^*}^2. \quad (6)$$

The last equation of Eq. (6) is a numerical one that holds within 1%. Therefore, Eqs. (5) and (6) are not compatible. We can avoid this difficulty if we keep the contribution of the  $\kappa$  meson in Eq. (4), which is probably the simplest and most natural way to explain the difference. If we take this point of view, we replace Eq.

<sup>9</sup> M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

(5) by Eq. (4) and from Eqs. (4) and (6) we obtain

$$F_{\kappa} G_{\kappa^0 K^+ \pi^-} = \left[ \left( \frac{f_K}{f_{\pi}} \right) - \left( \frac{m_{\rho}^2 + m_{K^{*2}}}{2m_{\rho}^2} \right) \left( \frac{m_{\rho}^2}{m_{K^{*2}}} \right) \right] (m_{\kappa^2} - m_{K^2}). \quad (7)$$

Equations (6) and (7) constitute the essential part of the following discussion of the  $K_{l3}$  decay form factors. Before going further, we emphasize two points. First, in deriving (6) and (7) we did not use any specific models except for the use of current algebra. Secondly, since  $\partial_{\mu} V_{\mu}^K = 0$  in the  $SU(3)$  limit,  $F_{\kappa}$  must approach zero in this limit. This is explicitly borne out in Eq. (7). ( $f_K = f_{\pi}$  and  $m_{\rho} = m_{K^*}$  in the symmetry limit.) This limit consistency should be contrasted with previous results.<sup>4,10</sup>

(B) We are now in a position to discuss the  $K_{l3}$  decays with the usual model that the form factors are dominated by the  $K^*$  meson and the  $\kappa$  meson. We have already assumed in deriving Eq. (4) that these form factors satisfy unsubtracted dispersion relations. Following the standard method, we then obtain<sup>4</sup>

$$F_{+}(s) = G_{K^*} G_{K^0 K^+ \pi^-} / (m_{K^{*2}} - s) \quad (8)$$

and

$$F_{-}(s) = - \left( \frac{m_{K^2} - m_{\pi^2}}{m_{K^{*2}}} \right) F_{+}(s) + \frac{F_{\kappa} G_{\kappa^0 K^+ \pi^-}}{m_{\kappa^2} - s}, \quad (9)$$

where  $F_{+}(s)$  and  $F_{-}(s)$  are defined as

$$\langle \pi^0(q') | V_{\mu}^{K^-} | K^+(q) \rangle = (4q_0 q'_0)^{-1/2} \frac{1}{2} \sqrt{2} [F_{+}(s)(q+q')_{\mu} + F_{-}(s)(q-q')_{\mu}], \quad (10)$$

with

$$s = -(q-q')^2.$$

By substituting Eqs. (6) and (7) into Eqs. (8) and (9), respectively, we obtain

$$F_{+}(s) = \left( \frac{m_{\rho}}{m_{K^*}} \right)^2 \left( \frac{m_{\rho}^2 + m_{K^{*2}}}{2m_{\rho}^2} \right)^2 \left( \frac{m_{K^{*2}}}{m_{K^{*2}} - s} \right) \simeq 1.03 \frac{m_{K^{*2}}}{m_{K^{*2}} - s}. \quad (11)$$

We note that

$$(m_{\rho}/m_{K^*})^2 (m_{\rho}^2 + m_{K^{*2}}/2m_{\rho}^2)^2 = 1 + O(\epsilon^2),$$

where  $O(\epsilon^2)$  is a quantity of second order in the symmetry-breaking interaction. [Indeed,  $O(\epsilon^2)$  is small:  $O(\epsilon^2) \simeq 0.03$ .] Thus Eq. (11) explicitly satisfies the Ademollo-Gatto theorem,  $F_{+}(0) = 1 + O(\epsilon^2)$ . This was

<sup>10</sup> Our results also disagree with the result of L. K. Pandit, Phys. Rev. Letters **19**, 263 (1967). They obtained  $G_{\rho} G_{\rho^+ \pi^0 \pi^+} = 2G_{K^*} \times G_{K^* \pi^0 K^+}$  from the superconvergence assumption.

not realized in Ref. 10. For the  $F_{-}(s)$ , we obtain

$$F_{-}(s) = - \left( \frac{m_{K^2} - m_{\pi^2}}{m_{K^{*2}}} \right) F_{+}(s) + \left[ \left( \frac{f_K}{f_{\pi}} \right) - \left( \frac{m_{\rho}^2 + m_{K^{*2}}}{2m_{\rho}^2} \right) \left( \frac{m_{\rho}^2}{m_{K^{*2}}} \right) \right] \frac{m_{\kappa^2} - m_{K^2}}{m_{\kappa^2} - s}, \quad (12)$$

$$\simeq - \frac{m_{K^2} - m_{\pi^2}}{m_{K^{*2}}} F_{+}(s) + \left( \frac{f_K}{f_{\pi}} - 1 \right) \frac{m_{\kappa^2} - m_{K^2}}{m_{\kappa^2} - s}. \quad (12')$$

We can write Eq. (12) in the form (12') without affecting much the numerical results if we neglect the above  $O(\epsilon^2)$  term. In terms of the mass of  $\kappa$  meson, we obtain an expression for  $F_{-}(0)$  from Eq. (12),

$$F_{-}(0) = -0.027 - 0.062 \text{ GeV}^2/m_{\kappa^2}. \quad (13)$$

Therefore, we predict for the parameter  $\xi$

$$\xi = F_{-}(0)/F_{+}(0) = -0.026 - 0.061 \text{ GeV}^2/m_{\kappa^2}. \quad (14)$$

Note that our results for  $F_{-}$  or  $\xi$  depend only on the mass of  $\kappa$  (not on its decay width), which represents the effect of the  $s$ -wave  $K\pi$  interaction. We also note that Eq. (12) for  $F_{-}(s)$  becomes independent of the parameter  $m_{\kappa}$  at the point  $s = m_{K^2}$ .

If we use the value of the vector Cabibbo angle,  $\theta_V = 0.21$ , which resolves the discrepancy between the coupling constants of the  $\mu$  decay and the vector  $\beta$  decay, and if we use  $\Gamma(K^+ \rightarrow e^+ + \nu + \pi^0) = 3.88 \times 10^6 \text{ sec}^{-1}$  given by Rosenfeld *et al.*,<sup>11</sup> then we obtain, assuming that  $F_{+}(s)$  has a dependence on  $s$  given by (11),  $F_{+}(0) \simeq 1.05$ .<sup>12</sup> We have started with the assumption of small renormalization of the vector current at zero momentum transfer, and we indeed obtain  $F_{+}(0) \simeq 1.03$ . As mentioned before, Eq. (11) explicitly satisfies the Ademollo-Gatto theorem. Therefore, our approximation has an internal consistency, and is also consistent with the present  $K_{e3}$  decay rate within, say, 5%. The mass of the  $\kappa$  meson must be greater than that of the  $K$  meson. In the range  $m_K < m_{\kappa} < \infty$ , Eq. (14) limits  $\xi$  to the range  $-0.28 < \xi < -0.026$ . Note that we can get only negative values for the value of  $\xi$  from our approach. If we take, for example,  $m_{\kappa} \simeq 725 \text{ MeV}$ , then we obtain  $\xi = -0.16$ . We remark that if we calculate the form factor of the  $\pi_{e3}$  decay  $f_{+}(0)$ , by assuming that the  $\rho$  meson dominates the dispersion integral, we obtain using Eq. (3),  $f_{+}(0) = 1.0$ , which is indeed the desired result.

<sup>11</sup> A. H. Rosenfeld, A. Barbaro Galtieri, W. J. Podolsky, L. R. Price, M. Roos, P. Soding, W. J. Wills, and C. G. Wohl, Rev. Mod. Phys. **39**, 1 (1967).

<sup>12</sup> If we use  $\Gamma(K^+ \rightarrow e^+ + \nu + \pi^0) = (3.61 \pm 0.20) \times 10^6 \text{ sec}^{-1}$ , as compiled by G. H. Trilling, in Proceedings of the Argonne International Conference on Weak Interactions [Argonne National Laboratory, Report No. ANL-7130, p. 115, (unpublished)], we obtain  $F_{+}(0) \simeq 1.02$  with the neglect of electromagnetic corrections. S. Oneda and J. Sucher, Phys. Rev. Letters **15**, 927 (1965); **15**, 1049(E) (1965).

(C) Finally, we would like to make a short comment on the soft-pion-emission approach. The extrapolation  $q_\mu \rightarrow 0$  is certainly more hazardous than the one<sup>13</sup> we are using,  $-q^2 = m_\pi^2 \rightarrow 0$ . By using the soft-pion technique, Callan and Treiman obtained,<sup>11</sup> in our notation,

$$F_+(m_K^2) + F_-(m_K^2) = (f_K/f_\pi)(\sin\theta_A)/\sin\theta_V. \quad (15)$$

The values of  $F$ 's at  $s = m_K^2$  are not physical for the  $K_{13}$  decay. The  $K_{e3}$  decay energy spectrum has a maximum around  $q^2 = 0$ . Therefore, physically important form factors are  $F_+(0)$  and  $F_-(0)$ . Since there is no convincing way of making an extrapolation in the  $q_\mu \rightarrow 0$  approach, let us assume, as in Ref. 1,  $F_+(m_K^2) \simeq F_+(0)$  and  $F_-(m_K^2) \simeq F_-(0)$ . If we take, for example,  $F_+(0) \simeq 1.05$  and assume  $\theta_A = \theta_V$ , we obtain, from (15),  $F_-(0) = 0.23$  and therefore  $\xi = +0.22$ . This must be compared with our prediction  $-0.28 < \xi < -0.026$ . Of course, the value of  $\xi$ ,  $+0.22$ , is not very trustworthy, since there is no guarantee that the extrapolation  $F_+(m_K^2) \simeq F_+(0)$  and  $F_-(m_K^2) \simeq F_-(0)$  is very good. However, since the  $s$  dependence of the  $F_+$  and  $F_-$  form factors does not seem to be very large, both experimentally and theoretically the above extrapolation should not be very bad. Therefore, we think that the Callan-Treiman rela-

tion already indicates that the value of  $\xi$  is small, i.e.,  $|\xi| < 1$ . We now wish to show that our prediction of a small negative value of  $\xi$  is consistent with the Callan-Treiman relation obtained at  $s = m_K^2$ . Let us compute  $F_+(m_K^2) + F_-(m_K^2)$  by using Eqs. (11) and (12'). Neglecting the  $O(\epsilon^2)$  term, we obtain

$$F_+(m_K^2) + F_-(m_K^2) = \frac{f_K}{f_\pi} + \frac{m_\pi^2}{m_K^2 - m_\pi^2} \simeq 1.30, \quad (16)$$

which is indeed very close to the Callan-Treiman relation (15). We particularly note that Eq. (16) is independent of the parameter  $m_K$ . We therefore claim that our results include the soft-pion result given at  $s = m_K^2$ . It will be interesting to check our prediction (especially the sign of the parameter  $\xi$ ) by more precise experiments. It is certainly encouraging to observe that all the polarization experiments of  $K_{\mu 3}$  decay (which seem cleaner than other types of  $K_{13}$  decay experiments) give negative values for the parameter  $\xi$ .<sup>14</sup>

We wish to thank Professor J. Sucher for his stimulating discussions in the early stage of this work. We also thank Professor G. Snow for reading the manuscript.

<sup>13</sup> This approach has also been attempted recently by Okubo *et al.* from a different standpoint. S. Okubo, in *Proceedings of the 1967 International Conference on Particles and Fields* (Interscience Publishers Inc., New York, 1967), p. 469.

<sup>14</sup> At present, experiments do not seem to exclude the possibility  $\xi < 0$ . J. W. Cronin, in *Proceedings of the 1967 International Conference on Particles and Fields* (Interscience Publishers Inc., New York, 1967), p. 1.

## Quark Models, Current Algebras, and Radiative Corrections to Beta Decay\*

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We study radiative corrections to semileptonic decays. Special commutation relations between the weak and electromagnetic currents reduce the question of finiteness of  $i \rightarrow f + e^- + \bar{\nu}$  to that of the finiteness of  $\langle f | J_\mu^W | i \rangle / \sqrt{Z_2^e}$ . In a quark model with interactions properly chosen, standard renormalization theory is then used to prove that these quantities, hence the order- $\alpha$  radiative corrections to any  $\beta$  decay, are finite. We conclude that commutators are not enough and that some small amount of dynamics is needed to remove divergent radiative corrections.

IT has recently been shown that, with the assumption of certain equal-time commutation relations between the electromagnetic and weak hadronic currents, the order- $\alpha$  radiative corrections to pion  $\beta$  decay are finite.<sup>1,2</sup> The algebra required differs from the standard current

algebra only in the space-space part, where one needs

$$\delta(x_0)[J_i^{e.m.}(x), J_j(0)] = \delta(x)[\delta_{ij}J_0^W(0) - i\epsilon_{ijk}J_k^W(0)].$$

Such relations can actually be realized in several field-theory models involving fundamental integrally charged triplets rather than quarks, but which seem capable of reproducing all the known features of strong interactions. What remains to be shown is that this same scheme is sufficient to treat the order- $\alpha$  corrections to any  $\beta$  decay—for example, that the corrections to  $g_A$  are finite. In this article we address the question and

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<sup>1</sup> N. Cabibbo, L. Maiani, and G. Preparata, *Phys. Letters* **25B**, 132 (1967).

<sup>2</sup> K. Johnson, F. Low, and H. Suura, *Phys. Rev. Letters* **18**, 1224 (1967).