

Superconvergent Bootstrap of the Vector and Tensor Regge Trajectories

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A dynamical scheme proposed by Mandelstam, based on rising Regge trajectories, the narrow-resonance approximation, and generalized superconvergence relations, is applied within exact $SU(3)$ to the amplitude for $V+P \rightarrow P+P$, where P and V stand for pseudoscalar and vector mesons. It is shown that the coupled vector- and tensor-octet trajectories can indeed bootstrap themselves satisfactorily. The vector-octet trajectory is also shown to bootstrap itself. The interesting mass formula $3\mu_V^2 = \mu_T^2 + 3\mu_P^2$ is also obtained, where μ_M^2 represents the average mass squared of the M -meson octet, and T stands for tensor mesons.

I. INTRODUCTION

RECENTLY, Mandelstam¹ has given a dynamical scheme based on rising Regge trajectories. The fundamental approximation is that the scattering amplitude can be approximated by the contribution of a finite number of Regge poles. In the first approximation, the Regge trajectories are simply assumed to be straight lines, or, equivalently, the scattering amplitude is assumed to be dominated by narrow resonances and unitarity determines the Regge residues up to an entire function, which can then be approximated by a polynomial. Crossing is imposed by the generalized superconvergence relations (GSCR) which Igi and the present author²⁻⁴ have proposed to use as a guide to high-energy scattering of hadrons. GSCR is a consequence of Regge asymptotic behavior and the usual analyticity properties of the scattering amplitude. The above approximation, which can be systematically improved, allows one to derive algebraic equations for a finite number of the Regge parameters in the direct and crossed channels. The equations may be sufficient to determine these parameters self-consistently.

We apply the above scheme to the scattering amplitudes of $V_{\alpha(\lambda)} + P_{\beta} \rightarrow P_{\tau} + P_{\sigma}$ and $\omega_{(\lambda)} + P_{\beta} \rightarrow P_{\tau} + P_{\sigma}$, where P_{α} is the pseudoscalar octet with $SU(3)$ quantum number α and even C -conjugation parity; $V_{\beta(\lambda)}$ is the vector octet with $SU(3)$ quantum number β , C odd, and helicity λ ; and $\omega_{(\lambda)}$ is the vector singlet with C odd and helicity λ . We assume exact $SU(3)$. It can easily be seen that in the first reaction only the familiar vector- and tensor-octet trajectories can contribute as high-ranking trajectories in all channels, while in the latter only the vector-octet trajectory can be exchanged as a dominant trajectory in each channel. The requirement of C -conserving couplings of Regge trajectories forbids any

trajectory of a given C -conjugation parity to be exchanged in the $SU(3)$ eigenchannels of **1**, **8_{ff}**, **8_{dd}**, and **27** in the above reactions. Although the Regge-pole exchange in the **10** (or **10***) eigenchannel may be possible in the reaction $V_{\alpha(\lambda)} + P_{\beta} \rightarrow P_{\tau} + P_{\sigma}$, the lack of experimental evidence for any high-ranking meson trajectory (or, equivalently, any low-lying mesons) assigned to a **10**- (or **10***)plet makes it very reasonable to neglect the **10** (or **10***) eigenchannel in our lowest approximation. Therefore we have only to take into account the well-known vector- and tensor-octet trajectories. It is shown that these trajectories can indeed bootstrap themselves, and that we obtain very reasonable results for the Regge parameters.

II. BOOTSTRAP OF THE COUPLED VECTOR- AND TENSOR-OCTET TRAJECTORIES

In the reaction $V_{\alpha(\lambda)} + P_{\beta} \rightarrow P_{\tau} + P_{\sigma}$ there is only one kinematically independent amplitude^{5,6} which we can take to be the t -channel helicity amplitude $\tilde{f}_{\tau\bar{\alpha}(\lambda); \beta\beta^t}(s, t, u)$, $\lambda = \pm 1$ (Fig. 1).⁶ This amplitude is free of kinematic singularities in s and u for fixed t , and defined from the usual helicity amplitude $f_{\tau\bar{\alpha}(\lambda); \beta\beta^t}(s, t)$ with $\lambda = \pm 1$ by^{6,7}

$$\begin{aligned} \tilde{f}_{\tau\bar{\alpha}(\lambda); \beta\beta^t}(s, t, u) &= (\sin\theta_t)^{-1} f_{\tau\bar{\alpha}(\lambda); \beta\beta^t}(s, t) + (\sin\theta_t)^{-1} f_{\tau\bar{\alpha}(-\lambda); \beta\beta^t}(s, t) \\ &= 2(\sin\theta_t)^{-1} f_{\tau\bar{\alpha}(\lambda); \beta\beta^t}(s, t) \\ &= \sum_J (2J+1) F_{\tau\bar{\alpha}(\lambda); \beta\beta^t}^J(t) e_{0-\lambda}^J(t) (\cos\theta_t), \end{aligned} \quad (1)$$

where⁷

$$e_{01}^{J,+}(\cos\theta) = -e_{0-1}^{J,+}(\cos\theta) = P_J'(\cos\theta)/[J(J+1)]^{1/2}.$$

Here we have^{6,8}

$$\begin{aligned} f_{\tau\bar{\alpha}(\lambda); \beta\beta^t}(s, t) &= f_{\tau\bar{\alpha}(-\lambda); \beta\beta^t}(s, t) \\ &= (-1)^{Q_{\alpha}+Q_{\sigma}} f_{\tau\sigma; \alpha(\lambda)\beta^t}(s, t). \end{aligned} \quad (2)$$

⁵ M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

⁶ L. L. Wang, Phys. Rev. **142**, 1187 (1966).

⁷ M. Gell-Mann, M. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

⁸ P. Carruthers, *Introduction to Unitary Symmetry* (John Wiley & Sons, Inc., New York, 1966), p. 165.

¹ S. Mandelstam, Phys. Rev. **166**, 1539 (1968).

² K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967); University of Tokyo Report, 1967 (unpublished); Phys. Rev. **163**, 1622 (1967); A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); D. Horn and C. Schmid, California Institute of Technology Report No. CALT-68-127 (unpublished).

³ S. Matsuda and K. Igi, Phys. Rev. Letters **19**, 928 (1967).

⁴ K. Igi and S. Matsuda, in Proceedings of Topical Conference on High-Energy Collisions of Hadrons, CERN, Geneva, 1968 (unpublished).

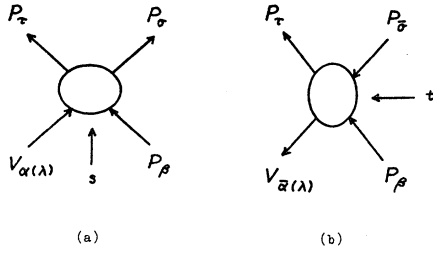


FIG. 1. (a) Reaction $V_{\alpha(\lambda)} + P_{\beta} \rightarrow P_{\tau} + P_{\sigma}$ in the s channel; (b) reaction $P_{\tau} + P_{\beta} \rightarrow P_{\tau} + V_{\alpha(\lambda)}$ in the t channel.

As stated above, we have only to consider the 8_{df} and 8_{fd} eigenamplitudes in the direct and crossed channels, which we denote as

$$\tilde{f}_{0\lambda;00}^{t,+}(s,t,u;8_{df}) = +\tilde{f}_{0\lambda;00}^{t,+}(u,t,s;8_{df}) \quad (3)$$

for the vector exchange in the t channel, and

$$\tilde{f}_{0\lambda;00}^{t,+}(s,t,u;8_{fd}) = -\tilde{f}_{0\lambda;00}^{t,+}(u,t,s;8_{fd}) \quad (4)$$

for the tensor exchange in the t channel. We have shown explicitly for later convenience the crossing of the s and u channels. Corresponding to these eigenamplitudes, we have the partial-wave helicity eigenamplitudes, $F_{0\lambda;00}^{J,+}(t;8_{df})$ and $F_{0\lambda;00}^{J,+}(t;8_{fd})$, which are defined similarly as in Eq. (1).

The basic assumption is that $F_{0\lambda;00}^{J,+}(t;8_{df})$ and $F_{0\lambda;00}^{J,+}(t;8_{fd})$ can be approximated by one vector- and one tensor-octet trajectory, respectively,

$$F_{0\lambda;00}^{J,+}(t;8_{df}) = \frac{\beta_{0-\lambda}^V(t)}{J - \alpha_V(t)} \quad (5)$$

and

$$F_{0\lambda;00}^{J,+}(t;8_{fd}) = \frac{\beta_{0-\lambda}^T(t)}{J - \alpha_T(t)}, \quad (6)$$

and that $\alpha_V(t) = a_V t + b_V$ and $\alpha_T(t) = a_T t + b_T$. Unitarity and analyticity in the narrow-resonance approximation then determine, in addition to the t -kinematic factors,⁶ the form of $\beta_{0-\lambda}^i(t)$ up to an entire function $E_i(t)$ ($i = V$ or T),

$$\begin{aligned} \beta_{0-\lambda}^i(t) &= (4q_t q_{t'}) \sqrt{t} \left(\frac{4a_i q_t q_{t'}}{e} \right)^{\alpha_i(t)-1} \\ &\times \frac{[\alpha_i(t)(\alpha_i(t)+1)]^{1/2}}{\Gamma[\alpha_i(t)+\frac{3}{2}]} E_i(t), \quad (7) \end{aligned}$$

where q_t ($q_{t'}$) is the c.m. momentum in the initial (final) state. This form exhibits the threshold behavior of $\beta_{0-\lambda}^i(t)$, the nonsense-eliminating factors⁹ at $\alpha_i(t) = 0, -1$, and at negative half-integers, and a scale factor $e/4a_i$ which is inserted in order that the left side of

⁹ Roughly speaking, the factor $\{\alpha_i(t)[\alpha_i(t)+i]\}^{1/2}$ of Eq. (7) comes from the fact that $\alpha_i=0$ is a sense-nonsense value for $F_{0\lambda;00}^{\alpha_i(t),+}(t)$. Note that the sense-nonsense distinction is in addition to the signature distinction between even and odd J .

Eq. (7), without the factor $(\sqrt{t})E_i(t)$, should not increase exponentially when t approaches $\pm\infty$.

After considering the crossing of the s and u channels in Eqs. (3) and (4) and Reggeizing according to the method of Ref. 7, we then obtain for high s

$$\begin{aligned} \tilde{f}_{01;00}^{t,+}(s,t,u;8_{df}) &\approx -(4q_t q_{t'} \sqrt{t}) \\ &\times \frac{e^{-i\pi\alpha_V(t)} - 1}{2 \sin \pi \alpha_V(t)} \frac{4\sqrt{\pi}}{\Gamma(\alpha_V(t))} \left(\frac{4a_V s}{e} \right)^{\alpha_V(t)-1} E_V(t) \quad (8) \end{aligned}$$

and

$$\begin{aligned} \tilde{f}_{01;00}^{t,+}(s,t,u;8_{fd}) &\approx -(4q_t q_{t'} \sqrt{t}) \\ &\times \frac{e^{-i\pi\alpha_T(t)} + 1}{2 \sin \pi \alpha_T(t)} \frac{4\sqrt{\pi}}{\Gamma(\alpha_T(t))} \left(\frac{4a_T s}{e} \right)^{\alpha_T(t)-1} E_T(t). \quad (9) \end{aligned}$$

We now impose crossing in the s and t channels by means of the simplest nontrivial GSCR which are in the finite-energy form

$$\begin{aligned} \int^N ds \frac{1}{2}(s-u) \operatorname{Im} \tilde{f}_{01;00}^{t,+}(s,t,u;8_{df}) &= (4q_t q_{t'} \sqrt{t}) \\ &\times \frac{(\sqrt{\pi})eN}{2a_V[\alpha_V(t)+1]\Gamma(\alpha_V(t))} \left(\frac{4a_V N}{e} \right)^{\alpha_V(t)} E_V(t), \quad (10) \end{aligned}$$

$$\begin{aligned} \int^N ds \operatorname{Im} \tilde{f}_{01;00}^{t,+}(s,t,u;8_{fd}) &= (4q_t q_{t'} \sqrt{t}) \\ &\times \frac{(\sqrt{\pi})e}{2a_T\alpha_T(t)\Gamma(\alpha_T(t))} \left(\frac{4a_T N}{e} \right)^{\alpha_T(t)} E_T(t), \quad (11) \end{aligned}$$

where N is a suitable cutoff, to be discussed below. For the vector exchange we use the first-moment (but not the zero-moment) GSCR, since we are not willing to worry about any contributions from the fixed poles in the J plane at nonsense wrong-signature integers.¹ In our approximation there is another finite-energy GSCR with the **10** (or **10***) eigenamplitude in the t channel:

$$\int^N ds \frac{1}{2}(s-u) \operatorname{Im} \tilde{f}_{01;00}^{t,+}(s,t,u;10(10^*)) = 0. \quad (12)$$

Applying the $SU(3)$ crossing matrix and saturating the left sides¹⁰ of Eqs. (10)–(12) by the first particles on the vector- and tensor-octet trajectories, at $J=1$, and $J=2$, respectively, we derive the three equations after dropping the t -kinematic factors:

$$\begin{aligned} &(\mu_V^2 + \frac{1}{2}t - \frac{1}{2}\Sigma) \frac{E_V(\mu_V^2)}{a_V} + \frac{2}{e} \\ &\times (\mu_T^2 + \frac{1}{2}t - \frac{1}{2}\Sigma)(\mu_T^2 + 2t - \Sigma) E_T(\mu_T^2) \\ &= \frac{eN\alpha_V(t)}{4a_V\Gamma(\alpha_V(t)+2)} \left(\frac{4a_V N}{e} \right)^{\alpha_V(t)} E_V(t), \quad (13) \end{aligned}$$

¹⁰ We should be careful of the crossing of the s and u channels in integrating the left sides of Eqs. (10)–(12).

$$\begin{aligned} & \frac{E_V(\mu_V^2)}{a_V} + \frac{2}{e}(\mu_T^2 + 2t - \Sigma)E_T(\mu_T^2) \\ &= \frac{e(\alpha_T(t) + 1)}{4a_T\Gamma(\alpha_T(t) + 2)} \left(\frac{4a_T N}{e} \right)^{\alpha_T(t)} E_T(t), \quad (14) \\ & (\mu_V^2 + \frac{1}{2}t - \frac{1}{2}\Sigma) \frac{E_V(\mu_V^2)}{a_V} - \frac{2}{e} \\ & \times (\mu_T^2 + \frac{1}{2}t - \frac{1}{2}\Sigma)(\mu_T^2 + 2t - \Sigma)E_T(\mu_T^2) = 0, \quad (15) \end{aligned}$$

where $\Sigma = \mu_V^2 + 3\mu_P^2$, and μ_P^2 , $\mu_V^2 = (1 - b_V)/a_V$, and $\mu_T^2 = (2 - b_T)/a_T$ are the masses squared of the pseudo-scalar, vector, and tensor octets. In general, in this form of self-consistent bootstrap, N is not arbitrary; it must surely lie in the interval

$$\frac{2 - b_T}{a_T} < N < \frac{3 - b_V}{a_V} \quad (16)$$

since we have included in the integrations the first particles on the vector- and tensor-octet trajectories at $J=1$ and $J=2$, respectively, but not the second and higher particles.

First, let us consider the algebraic equations (13)–(15) at $t=0$,¹¹ taking the functions $E_i(t)$ in Eq. (7) to be constants in the lowest approximation. The only possible solution in this case contains as a consequence the “exchange degeneracy”,¹² and the results are

$$3\mu_P^2 = \mu_V^2 = \frac{1}{2}\mu_T^2 = 1/a, \quad E_V/E_T = \frac{1}{4}e, \quad (17)$$

with $a_V = a_T = a$ and $b_V = b_T = b = 0$. Note that these results are completely independent of the value of N . This suggests that the higher order of approximation will also lead to a solution completely or almost independent of N , although it may include a slight change in numerical values.

Next we go one step further taking into account the t dependence of the function $E_i(t)$ and evaluating the algebraic equations at $t = -b_V/a_V$, $\alpha_V(t) = 0$ and at $t = (-1 - b_T)/a_T$, $\alpha_T(t) = -1$. We can obtain the remarkable solution, again with the exchange degeneracy¹² as a consequence:

$$\alpha_V(t) = \alpha_T(t) = at + \frac{1}{2}(1 - 3a\mu_P^2) \quad (18)$$

and

$$\frac{E_V(\mu_V^2 - 2/a)}{E_V(\mu_V^2)} = \frac{E_T(\mu_T^2 - 2/a)}{E_T(\mu_T^2)} = \frac{16}{e^2}, \quad \frac{E_V(\mu_V^2)}{E_T(\mu_T^2)} = \frac{4}{e}, \quad (19)$$

with $b = \frac{1}{2}(1 - 3a\mu_P^2)$. From Eq. (18) we have the mass formula

$$3\mu_V^2 = \mu_T^2 + 3\mu_P^2, \quad (20)$$

¹¹ For a reasonable argument that the choice $t=0$ probably provides the best mean, see Ref. 1.

¹² R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).

which is rather well satisfied experimentally¹³ (the left side = $3 \times 0.73 = 2.19$ GeV² and the right side = $1.91 + 3 \times 0.17 = 2.42$ GeV²). Assuming the linear t dependence of $E_i(t)$, we can obtain from Eq. (19)

$$\begin{aligned} E_V(t) &= E_V(\mu_V^2) [1 - \frac{1}{2}a(16/e^2 - 1)(t - \mu_V^2)], \\ E_T(t) &= \frac{1}{4}e E_V(\mu_V^2) [1 - \frac{1}{2}a(16/e^2 - 1)(t - \mu_T^2)]. \quad (21) \end{aligned}$$

Note that these results are again independent of the value of N .

III. BOOTSTRAP OF THE VECTOR-OCTET TRAJECTORY

In the reaction $\omega_{(A)} + P_\beta \rightarrow P_\tau + P_\sigma$ only the vector-octet trajectory can be exchanged dominantly in each channel. Then, in the same approximation as before, we derive the bootstrap equation including only the vector trajectory

$$2(\mu_V^2 + \frac{1}{2}t - \frac{1}{2}\Sigma_\omega) \frac{E_V'(\mu_V^2)}{a_V}$$

$$= \frac{eN'\alpha_V(t)}{4a_V\Gamma(\alpha_V(t) + 2)} \left(\frac{4a_V N'}{e} \right)^{\alpha_V(t)} E_V'(t), \quad (22)$$

where $\Sigma_\omega = \mu_\omega^2 + 3\mu_P^2$ and N' must lie in this case between

$$\frac{1 - b_V}{a_V} < N' < \frac{3 - b_V}{a_V}. \quad (23)$$

Evaluating the bootstrap equation at $t = -b_V/a_V$, $\alpha_V(t) = 0$, we have

$$\alpha_V(t) = at + \frac{1}{2}(2 - a_V\Sigma_\omega), \quad (24)$$

which is independent of the value of N' . Note that when the vector-octet and singlet are degenerate in mass (i.e., $\mu_V^2 = \mu_\omega^2$), or, equivalently, when they form a nonet, this bootstrap equation becomes equivalent to the appropriate combination of Eqs. (13) and (15) if we take $N = N'$ and $E_V'(t) = \text{const} \times E_V(t)$. In other words, the compatibility of all our bootstrap equations [(13)–(15) and (22)] requires the degeneracy in mass and the same t dependence of $E(t)$ of the vector-octet and singlet.

IV. CONCLUSIONS

We have applied Mandelstam's dynamical scheme based on rising Regge trajectories and GSCR to the cases of complete bootstrap of one vector-octet trajectory and the coupled vector- and tensor-octet trajectories, and have shown that in these cases the trajectories can indeed bootstrap themselves and that

¹³ A. H. Rosenfeld *et al.*, University of California Radiation Laboratory Report No. UCRL 8030 (revised September, 1967) (unpublished). We have evaluated the average mass μ^2 of a meson octet by $\mu^2 = \frac{1}{8}(4\mu_K^2 + 3\mu_\pi^2 + \mu_8^2)$, where μ_8^2 is calculated from $4\mu_K^2 = 3\mu_8^2 + \mu_\pi^2$.

very satisfactory results including the interesting mass formula Eq. (20) are obtained independent of the choice of the cutoff. However, it is not clear, and still remains to be investigated, whether we could obtain self-consistent and cutoff-independent results even if we included in our scheme higher resonances corresponding to a higher cutoff of N and more channels in the higher-order approximation.

Note added in proof. After completing the manuscript I found that M. Ademollo, H. R. Rubinstein, G.

Veneziano, and M. A. Virasoro [Phys. Rev. Letters **19**, 1402 (1967)] considered a similar problem and derived bootstrap conditions based on GSCR. Their approach, however, is not one of a systematic bootstrap.

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K_{13} Decay Form Factors and the Current Algebra*

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The K_{13} decay form factors are studied by using a current algebra. Compared with the previous approach, the following points are new. We use the pion partially conserved axial-vector current (PCAC) hypothesis instead of kaon PCAC. We use a smaller off-mass-shell extrapolation $-q_\mu^2 = m_\pi^2 \rightarrow 0$ instead of the usual soft-pion extrapolation $q_\mu \rightarrow 0$. We apply a dispersion technique to a direct calculation of the form factors $F_+(s)$ and $F_-(s)$, and utilize the current algebra to fix the absolute scale as well as the energy dependence with an approximation for vector currents proposed before. For the $F_+(s)$, the K^* -meson contribution gives a result consistent with the Ademollo-Gatto theorem at $s=0$. For the $F_-(s)$, both the K^* and the $I=\frac{1}{2}$, 0^+ meson, κ , contribute. Our expression for the $F_-(s)$ depends only on the mass of the κ meson (not on its width) and satisfies explicitly the requirement that $F_-(s) \rightarrow 0$ in the $SU(3)$ limit. The parameter ξ is given by $\xi = F_-(0)/F_+(0) = -0.026 - (0.061 \text{ GeV}^2)/m_\kappa^2$. Therefore, we predict a small negative value for ξ . Our method gives information on $F_+(s)$ and $F_-(s)$ for all the physical values of s , including the most important region around $s=0$, whereas the soft-pion approach gives information only at the rather unphysical point $s=m_\kappa^2$. However, at this very point, our results on $F_+(m_\kappa^2) + F_-(m_\kappa^2)$ agrees with that of the soft-pion approach independently of the parameter m_κ . Therefore, our results include the soft-pion result at $s=m_\kappa^2$ and seem to give a consistent description of the K_{13} decay form factors.

THE form factors $F_+(s)$ and $F_-(s)$ of the K_{13} decays contain many interesting clues to the problems of $SU(3)$ symmetry. Precise determination of the value of $F_+(0)$, taking into account the form-factor effect, will give us a first measurement of the second-order $SU(3)$ symmetry-breaking effect. In the $SU(3)$ symmetry limit, the $F_-(s)$ vanishes, and its actual value, measured by the parameter ξ defined as $\xi \equiv F_-(0)/F_+(0)$, will therefore provide important information on first-order $SU(3)$ breaking. Recent calculations of these form factors on the basis of the algebra of currents together with the hypothesis of partially conserved axial-vector currents (PCAC) may be categorized as follows:

(a) Relate the $[F_+(m_\kappa^2) + F_-(m_\kappa^2)]$ to the $K_{\mu 2}$ decay by using a soft-pion-emission technique, $q_\mu \rightarrow 0$, where q_μ is the pion four-momentum.¹

(b) Determine the parameters of the $I=\frac{1}{2}$, $Y=\pm 1$ possible scalar resonance κ by studying resonance satu-

ration of axial charge commutators^{2,3} with the less radical extrapolation, $q^2 = (-m_\pi^2) \rightarrow 0$.

(c) Apply dispersion techniques to a direct calculation of $F_+(s)$ and $F_-(s)$, and utilize the charge-current algebra to fix the absolute scale as well as the energy dependence. Although this approach is very interesting, the results obtained⁴ do not seem very satisfactory. First, the calculation involves soft-kaon emission as well as soft-pion emission, by letting the mass of the kaon as well as that of the pion go to zero. Secondly, the $F_-(s)$ thus obtained, which includes the contribution of the κ meson, does not explicitly vanish if we take an $SU(3)$ limit in its expression.

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