carried out elsewhere.[22] The parity-violating amplitudes are dominated by the octet and decuplet poles. The small scalar-meson couplings, which result from the t-channel pole in the parity-violating amplitudes, are compensated in the parity-conserving amplitudes by an enhancement of the $\kappa \rightarrow$ vacuum coupling. Therefore, we maintain the previous predictions and success[2,20] of the tadpole model for the nonleptonic decays.

In conclusion, we would like to point out again that the tadpole model does allow one to explain the ratios of the parity-violating amplitudes and the vanishing of $V(\Sigma_+{}^+)$ in a natural way. These conclusions are based on the strong-interaction coupling constants and there are no fitted parameters in these results. To the extent that there is a nonzero $K_1 \rightarrow$ vacuum coupling, the success of the tadpole model has to be considered in any analysis of the nonleptonic decays.[23]

ACKNOWLEDGMENTS

I would like to express my thanks to Professor Jogesh C. Pati, who suggested this problem, for his constant encouragement and guidance. I would also like to acknowledge many useful discussions with Professor Sadao Oneda and Professor Samir K. Bose.

[22] D. Loebbaka, thesis, University of Maryland Technical Report No. 624 (unpublished).

[23] See A. Kumar and J. C. Pati, Ref. 3.

PHYSICAL RLVIEW VOLUME 169, NUMBER 5 25 MAY 1968

Nonanalyticity of the Scattering Amplitude at $u=0$ in a Relativistic Harmonic-Oscillator Model

G. Cocho*

International Centre for Theoretical Physics, Trieste, Italy

(Received 6 July 1967; revised manuscript received 26 January 1968)

Within the framework of a model based on the noncompact group $U(3,1)$, scattering amplitudes are calculated in the Born approximation using a phenomenological propagator in which an infinite number of one-particle states are exchanged. The amplitude is found to be nonanalytic at the point where the squared mass of the internal line changes sign ($u=0$). The possibility of testing such anomalous behavior in pion-nucleon backward scattering and in other processes is discussed.

1. INTRODUCTION

Recent work on current algebra and superconvergent amplitudes[1] has suggested that in order to saturate such relations with a finite or infinite number of one-particle states, the crossing symmetry of the theory must be different from the crossing given by the ordinary local finite-component field theory. (In a dispersion language, one needs different assumptions about the behavior of the kinematical singularities.) It is well known that in the infinite-component field theory with "local" coupling, such "anomalous" crossing is present,[2,3] and that the only known examples of consistent saturation of the current-algebra commutation relations have been obtained by the use of unitary representations of a noncompact group.[4] The lack of conventional crossing in the infinite-component field theories leads us to think that in such theories the scattering amplitude might be nonanalytic wherever the squared mass of an internal line changes sign. In this work we compute scattering amplitudes in the Born approximation in a model based on the noncompact group $U(3,1)$, and we find such a lack of analyticity.

Born diagrams including the exchange of an infinite number of particles have been discussed by Van Hove,[5] and in the framework of the noncompact group $O(3,1)$ by Cocho and Harum Ar-Rashid[6] and by Fronsdal.[7] In this work, we will compute Born approximations including the exchange of an infinite number of particles in the framework of the relativistic harmonic-oscillator model discussed in a previous work.[8]

Although $U(3,1)$ might not be the right group for elementary particles [and in particular the extensive work of Barut *et al.*[9] seems to suggest that $O(4,2)$ might be a better candidate], the calculations are simpler in $U(3,1)$, it is easier to obtain answers in a closed form, and we believe that such a possibility is worth explor-

* On leave of abscence from Instituto de Física, and Comisión Nacional de Energía Nuclear de México.

[1] I. T. Grodsky, Phys. Letters **25B**, 149 (1967).

[2] C. Fronsdal, Phys. Rev. **156**, 1653 (1967).

[3] C. Fronsdal and R. White Phys. Rev. **163**, 1835 (1967).

[4] S. Fubini, invited talk, in *Proceedings of the Fourth Coral Gables Conference on Symmetry Principles at High Energy, 1967* (W. H. Freeman and Co., San Francisco, 1967).

[5] L. Van Hove, Phys. Letters **24B**, 183 (1967).

[6] G. Cocho and Harum Ar-Rashid, Nuovo Cimento **47**, 874 (1967).

[7] C. Fronsdal, Phys. Rev. **168**, 1845 (1968).

[8] G. Cocho, C. Fronsdal, I. T. Grodsky, and R. White, Phys. Rev. **162**, 1662 (1967).

[9] A. O. Barut and H. Kleinert, Phys. Rev. **156**, 1541 (1967); **157**, 1180 (1967); **160**, 1149 (1967); **161**, 1464 (1967); A. O. Barut, D. Corrigan, and H. Kleinert, Phys. Rev. Letters **20**, 167 (1968); Phys. Rev. **167**, 1527 (1968).

ing. Also, as has been suggested by Nambu,[10] if one considers the baryons as built out of three quarks interacting through harmonic-oscillator forces, then after extracting the motion of the center of mass, the system is described by two independent harmonic oscillators, and the content of such a pair of $U(3,1)$ representations is the same as that of the $O(4,1)$ H-atom-like representation used for elementary particles. Hence, the question is still not settled.

In this work we shall compute Born diagrams with "local vertices" and a phenomenological propagator which includes the linear mass spectrum suggested by the nonrelativistic harmonic oscillator. We find a discontinuity in the derivatives of the scattering amplitude when the four-momentum of the internal line changes from spacelike to timelike. However, even if our model is the correct one, such a Born approximation is not going to give the whole scattering amplitude. As the kinematics of our examples is the same as in meson-nucleon elastic backward scattering; we suggest looking at pion-nucleon and kaon-nucleon backward scattering to see if "cusps" of such a kind are present.

2. RELATIVISTIC HARMONIC OSCILLATOR

Let us consider the $U(3,1)$ algebra S with generators $C_{\mu\nu}$ satisfying the commutation relations

$$[C_{\mu\nu},C_{\lambda\rho}]=-g_{\nu\lambda}C_{\mu\rho}+g_{\mu\rho}C_{\lambda\nu} \tag{1}$$

and the Hermiticity condition

$$C_{\mu\nu}{}^{\dagger}=C_{\nu\mu}. \tag{2}$$

The symmetry algebra is the compact subalgebra $U(3)$, and $\zeta_{\mu\nu}=i(C_{\mu\nu}-C_{\nu\mu})$ is isomorphic to the homogeneous Lorentz group. The states are described by an infinite set of fields $\psi_\sigma(x)$, where x_μ is the center-of-mass coordinate whose conjugate momentum is the total momentum p_μ, and σ is an index that takes on an infinity of values. The operators of S act on the index only, not on the argument.

A local nonderivative interaction between two, three, or more fields is an S-invariant coupling of the form

$$\sum_{\sigma\lambda\tau}\psi_\sigma(x)\varphi_\lambda(x)\chi_\tau(x)C_{\sigma\lambda\tau}.$$

In a previous work, the "degenerate" $U(3,1)$ "representations," which are realized by the three-dimensional harmonic oscillator, were discussed, and the scalar form factor which appears in the coupling of the ground state of two of these towers to a scalar field [trival representation of $U(3,1)$] was computed.

We obtained for such a kinematical form factor the expression

$$F(t)=\left(\frac{p\cdot p'}{m^2}\right)^N=\left(1-\frac{t}{2m^2}\right)^N, \tag{3}$$

where p (p') is the four-momentum of the incoming (outgoing) particle, m is its mass, and N is a negative real number which labels the $U(3,1)$ unitary representation.

If

$$N=-\frac{m^2C^2}{2\mu\omega}, \tag{4}$$

then, as $C\to\infty$,

$$t=\frac{(p'-p)^2}{C^2}\to -k^2$$

and

$$F(t)=\left(1-\frac{t}{2m^2C^2}\right)^N\to\exp\left(-\frac{k^2}{4\mu\omega}\right),$$

which is the form factor of the nonrelativistic harmonic oscillator.

3. BORN DIAGRAMS

In this section we shall compute scattering and annihilation Born diagrams in which a scalar particle S ($N=0$) of mass μ and the ground state of a $U(3,1)$ tower ($N\neq 0$) of mass m interact. We shall use the local $G\bar{\psi}(p')\psi(p)S(p'-p)$ and a phenomenological propagator. In order to build such a propagator we shall assume: (i) The propagator includes the linear mass spectrum suggested by the nonrelativistic harmonic oscillator. (ii) It will be built with the operators p_μ and $C_{\mu\nu}$, with p_μ the four-momentum operator and $C_{\mu\nu}$ as given in the previous section. (iii) Such a phenomenological propagator will be valid both for spacelike and timelike momenta.

For the nonrelativistic harmonic oscillator, we have the mass spectrum

$$m=m_0+B\tau,\quad m^2=(m_0+B\tau)^2,\quad \tau=0,1,\cdots.$$

If we remember[8] that the eigenvalues of C_{00} are $\tau-N$, we may write the preceding equation as

$$m^2=(m_0+BN+BC_{00})^2,$$

which can be written in an invariant way

$$p^2=\left(m_0+BN+B\frac{p_\mu C^\mu{}_\nu p^\nu}{p^2}\right)^2.$$

Therefore, we will assume the phenomenological propagator

$$\frac{1}{p^2-[A+Bp_\mu C^\mu{}_\nu p^\nu/p^2]^2}$$

with $A=m_0+BN$. If p_μ is spacelike, we may take $p_0=p_1=p_2=0$. In order that the denominator of the

[10] Y. Nambu (to be published).

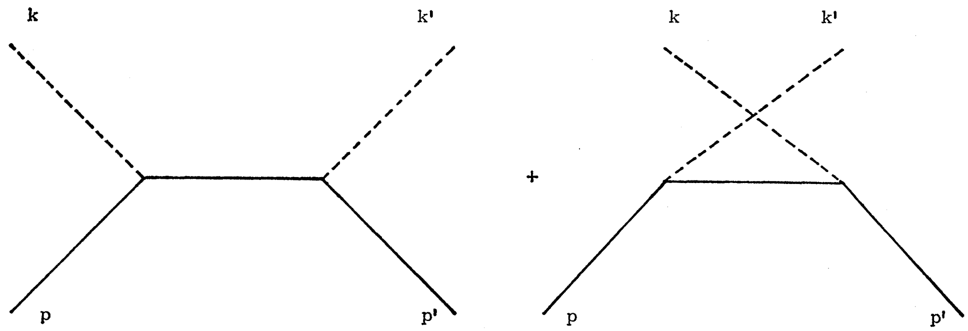

FIG. 1. Born diagrams for the scattering of a scalar particle ($N=0$) by a tower particle ($N\neq 0$).

propagator vanish, one needs

$$-p_3^2-(A-BC_{33})^2=-p_3^2-(A-B\tau)^2=0$$

which is not possible. There are no poles for spacelike momentum transfer.

Note the difference between the brackets in the timelike and the spacelike cases. That is the origin of the discontinuities which we shall compute next.

A. Compton-Like Scattering

Let us compute the Compton-like Born diagram of Fig. 1. The expression to compute is

$$M=\Bigg\{G^2\psi(p')\times\frac{(p+k)^4}{(p+k)^6-[A(p+k)^2+B(p+k)^\mu C_\mu{}^\nu(p+k)_\nu]^2}\psi(p)+(p+k)\rightarrow(p-k')\Bigg\}\equiv M_{\rm I}+M_{\rm II}. \quad (5)$$

When the incoming and outgoing particles are in the ground state, Eq. (5) may be formally written as

$$M=G^2\psi_0(p')\psi_0(p)\left(p'^\mu\frac{\partial}{\partial z^\mu}\right)^N\times\frac{(p+k)^4}{(p+k)^6-[A(p+k)^2+B(p+k)^\mu(z_\mu\partial/\partial z_\nu)(p+k)_\nu]^2}\times(p_\mu z^\mu)^N+(p+k)\rightarrow(p-k')\equiv M_{\rm I}+M_{\rm II}. \quad (6)$$

For the $M_{\rm I}$ amplitude $p+k$ is timelike. For $M_{\rm II}$, $p-k'$ may be timelike or spacelike. For the timelike case, the expression (6) may be evaluated in the center-of-mass system ($\mathbf{p}+\mathbf{k}=0$ for $M_{\rm I}$ and $\mathbf{p}-\mathbf{k}'=0$ for $M_{\rm II}$).

If $p-k'$ is spacelike it is better to evaluate $M_{\rm II}$ in the Breit system $p_0=k_0'$. One obtains $M=\sum M_i$, with

$$M_i=\left(\frac{p_0p_0'}{m^2}\right)^N\sum_{\tau=0}^{\infty}\binom{N}{\tau}f_i(\tau)\left(-\frac{p_3p_3'}{p_0p_0'}\right)^\tau, \quad (7)$$

where

$$\begin{aligned}f_1&=\{s-[A+B(\tau-N)]^2\}^{-1},\\ f_2&=\{u-[A+B(\tau-N)]^2\}^{-1}\Theta(u),\\ f_3&=\{u-[A-B\tau]^2\}^{-1}\Theta(-u),\end{aligned} \quad (8)$$

where $\mathbf{p}=p_3\hat{p}_3$, $\mathbf{p}'=p_3'\hat{p}_3+\mathbf{p}_\perp'$, and Θ is the unit step function. (Note that $|p_3p_3'/p_0p_0'|$ is always less than 1.) As a function of the Mandelstam variables

$$s=(p+k)^2,\quad t=(p'-p)^2,\quad u=(p-k')^2,$$

M may be written as

$$\begin{aligned}G^{-2}M=&\left[\frac{(s+m^2-\mu^2)^2}{4m^2s}\right]^N\frac{1}{2s^{1/2}}\Bigg\{\frac{1}{m+s^{1/2}}\,{}_2F_1\left(-N,\frac{m+s^{1/2}}{B};\frac{m+s^{1/2}}{B}+1;1-x\right)\\&-\frac{1}{m-s^{1/2}}\,{}_2F_1\left(-N,\frac{m-s^{1/2}}{B};\frac{m-s^{1/2}}{B}+1;1-x\right)\Bigg\}+\Theta(u)\left[\frac{(u+m^2-\mu^2)^2}{4m^2u}\right]^N\frac{1}{2u^{1/2}}\\&\times\Bigg\{\frac{1}{m+u^{1/2}}\,{}_2F_1\left(-N,\frac{m+u^{1/2}}{B};\frac{m+u^{1/2}}{B}+1;1-y\right)-\frac{1}{m-u^{1/2}}\,{}_2F_1\left(-N,\frac{m-u^{1/2}}{B};\frac{m-u^{1/2}}{B}+1;1-y\right)\Bigg\}\\&+\Theta(-u)\left[\frac{s-t-(m^2-\mu^2)^2/u}{4m^2}\right]^N\frac{1}{2u^{1/2}}\Bigg\{\frac{1}{m+BN+u^{1/2}}\,{}_2F_1\left(-N,-\frac{m+BN+u^{1/2}}{B};-\frac{m+BN+u^{1/2}}{B}+1;\frac{1}{1-y}\right)\\&-\frac{1}{m+BN-u^{1/2}}\,{}_2F_1\left(-N,-\frac{m+BN-u^{1/2}}{B};-\frac{m+BN-u^{1/2}}{B}+1;\frac{1}{1-y}\right)\Bigg\},\end{aligned} \quad (9)$$

with

$$x=\frac{s(4m^2-2t)}{[s+m^2-\mu^2]^2},\quad y=\frac{u(4m^2-2t)}{[u+m^2-\mu^2]^2}. \tag{10}$$

Note that the conventional crossing relation $M_{\rm I}(s,u)=M_{\rm II}(u,s)$ is valid only if in $M_{\rm II}$, $u>0$. By using transformation formulas of the hypergeometric functions, M becomes

$$\begin{aligned}G^{-2}M=&\left(\frac{2m^2-t}{2m^2}\right)^N\frac{1}{2s^{1/2}}\left[\frac{1}{m+s^{1/2}}\,{}_2F_1\left(-N,1;\frac{m+s^{1/2}}{B}+1;\frac{x-1}{x}\right)-\frac{1}{m-s^{1/2}}\,{}_2F_1\left(-N,1;\frac{m-s^{1/2}}{B}+1;\frac{x-1}{x}\right)\right]\\&+\left(\frac{2m^2-t}{2m^2}\right)^N\frac{1}{2u^{1/2}}\left[\frac{1}{m+BN+u^{1/2}}\,{}_2F_1\left(-N,1;-\frac{m+BN+u^{1/2}}{B}+1;\frac{1}{y}\right)\right.\\&\left.-\frac{1}{m+BN-u^{1/2}}\,{}_2F_1\left(-N,1;-\frac{m+BN-u^{1/2}}{B}+1;\frac{1}{y}\right)\right]+\Theta(u)\frac{[4m^2u/(u+m^2-\mu^2)^2]^{-N}}{\Gamma(-N)2u^{1/2}}(1-y)^{-m/B}\\&\times\left[\Gamma\left(\frac{m+u^{1/2}}{B}\right)\Gamma\left(-N-\frac{m+u^{1/2}}{B}\right)(1-y)^{-u^{1/2}/B}-\Gamma\left(\frac{m-u^{1/2}}{B}\right)\Gamma\left(-N-\frac{m-u^{1/2}}{B}\right)(1-y)^{u^{1/2}/B}\right].\end{aligned} \tag{11}$$

From Eq. (11) it is possible to see the following:

(i) If $B\to 0$ (equal-mass limit), then

$$M\to G^2\left(\frac{2m^2-t}{2m^2}\right)^N\left(\frac{1}{s-m^2}+\frac{1}{u-m^2}\right). \tag{12}$$

Note that in such a limit the scalar form factor discussed in Sec. 2, Eq. (3) appears as a multiplicative factor.

(ii) For small u, the discontinuity term $D(u)$ in (11) may be written

$$D(u)\simeq\frac{\Theta(u)}{\Gamma(-N)}\left(\frac{u}{m^2-\mu^2}\right)^{-N}H(0), \tag{13}$$

with

$$H(0)=\frac{\Gamma(m/B)\psi(m/B)\Gamma(-N-m/B)\psi(-N-m/B)}{B[4m^2/(m^2-\mu^2)]^N}.$$

If N is a positive integer (in such a case the representation is not unitary), the discontinuity term vanishes and we have ordinary crossing. If $N<0$ (unitary representation), there is no discontinuity in the amplitude in $u=0$, but there is a change in the derivatives. (Such an effect will be easier to see if $-1<N<0$ than if $N<-1$.) If we allow N to be positive but not an integer (in such a case the representation is not unitary) there will be a discontinuity in the amplitude itself.

(iii) If u is small, but y large (such is the case if $m^2\simeq\mu^2$ or if s is large), the second term in expression (11) becomes

$$\left(\frac{2m^2-t}{2m^2}\right)^N\left[\frac{1}{u-(m+BN)^2}+O(y)\right].$$

Note that the u pole is not in the same place as the s pole. If we remember that N is negative we see that the nearest pole at $u=(m+BN)^2$ is shifted to a lower value than that of the lowest mass m.

B. Annihilation Diagram

In a similar way one may compute the annihilation Born diagram of Fig. 2. One obtains

$$\begin{aligned}M=&G^2\frac{1}{2s^{1/2}}\left(\frac{t-2m^2}{2m^2}\right)^N\left[\frac{1}{m+BN+s^{1/2}}\right.\\&\times{}_2F_1\left(-N,1;-\frac{m+BN+s^{1/2}}{B}+1;\frac{1}{z}\right)-\frac{1}{m+BN-s^{1/2}}\\&\left.\times{}_2F_1\left(-N,1;-\frac{m+BN-s^{1/2}}{B}+1;\frac{1}{z}\right)\right],\end{aligned} \tag{14}$$

where

$$z=\frac{s(2t-4m^2)}{[s+m^2-\mu^2]^2},\quad t=(p+p')^2,\quad s=(p-k)^2.$$

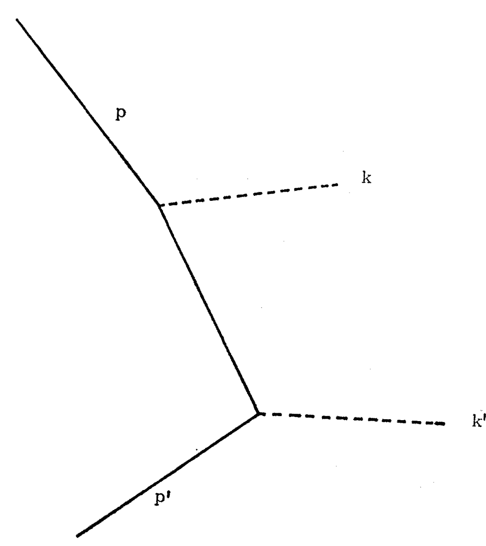

FIG. 2. Born diagram for the annihilation of two tower particles into two scalar particles.

If $B \to 0$,

$$M \to G^2\left(\frac{t-2m^2}{2m^2}\right)^N \frac{1}{m^2-s}. \tag{15}$$

4. DISCUSSION

It follows from the previous example that in infinite-component theories with nondegenerate mass spectra one might find discontinuities in the amplitude or in some of its derivatives whenever the four-momentum configuration of the external lines allows the four-momentum of an internal line to change from spacelike to timelike.

Although our result depends on the model we have used, and although the Born approximation (which is real in our case) is not the whole scattering amplitude, we believe that it is worth while to look at processes where the kinematics is the same as in our example, to see if cusps near $u=0$ are present. In particular, in meson-nucleon elastic backward scattering the kinematics is similar. Although preliminary evidence seems to show peaks near $u=0$ in pion-proton[11] and kaon-proton[12] elastic backward scattering, better data are needed.

Finally, it is worth remarking that the shifting of the effective position of the pole in the u channel with respect to the position of the pole in the s channel [see Sec. 3, (iii)] might be considered also in the exchange of bosons—in particular, in the vector-meson-dominance model for the electromagnetic form factors and in the one-boson-exchange baryon-baryon potentials.

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam and Professor P. Budini as well as the IAEA for the hospitality kindly extended to him at the International Centre for Theoretical Physics, Trieste. He is also grateful to Professor C. Fronsdal and Professor I. T. Grodsky for valuable discussions.

[11] A. Ashmore *et al.*, Phys. Rev. Letters **19**, 460 (1967).

[12] D. Cline, C. Moore, and D. Reeder, Phys. Rev. Letters **19**, 675 (1967).

PHYSICAL REVIEW VOLUME 169, NUMBER 5 25 MAY 1968

Determination of the Nucleon-Nucleon Scattering Matrix. VII. (p,p) Analysis from 0 to 400 MeV*

MALCOLM H. MACGREGOR, RICHARD A. ARNDT,† AND ROBERT M. WRIGHT

Lawrence Radiation Laboratory, University of California, Livermore, California

(Received 23 October 1967)

All of the available (p,p) scattering data from 1 to 400 MeV have been analyzed, and a self-consistent set of 839 data has been chosen. Using this data selection, we investigated a number of different forms for the phase-shift energy dependence. The correct number of free parameters to use with each form was studied. The most suitable form, form A, gave the least-squares values $\chi^2=810$ and $\chi^2=858$ for 30- and 23-parameter solutions, respectively. A subset of 588 data in six narrow energy bands was used to obtain single-energy solutions. It is shown that this subset contains most of the physical content of the full set of 839 data. The value $g^2=14.72\pm0.83$ was obtained for the pion-nucleon coupling constant.

I. INTRODUCTION

In previous papers in this series,[1–6] we have discussed phase-shift analyses of (p,p) and (n,p) data from 25 to 350 MeV. Subsequent to the publishing of these papers, a considerable amount of new data has become available,[7,8] both in the energy range we had previously considered and also at the higher energies. Thus it seemed to us worthwhile to update the previous analyses and to extend them to higher energies.

The (p,p) data in the elastic energy range up to about 400 MeV are now reasonably complete and accurate. Thus the isotopic spin $I=1$ scattering matrix can be reliably determined in this energy range. The aim of the present paper (paper VII) is to give the best possible values for the $I=1$ phase shifts from 0 to 400

* Work performed under the auspices of the U. S. Atomic Energy Commission.

† Present address: Virginia Polytechnic Institute, Blacksburg, Va.

[1] M. H. MacGregor, R. A. Arndt, and A. A. Dubow, Phys. Rev. **135**, B628 (1964).

[2] M. H. MacGregor and R. A. Arndt, Phys. Rev. **139**, B362 (1965).

[3] H. P. Noyes, D. S. Bailey, R. A. Arndt, and M. H. MacGregor, Phys. Rev. **139**, B380 (1965).

[4] R. A. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966).

[5] R. A. Arndt and M. H. MacGregor, Phys. Rev. **154**, 1549 (1967).

[6] R. M. Wright, M. H. MacGregor, and R. A. Arndt, Phys. Rev. **159**, 1422 (1967).

[7] The current status of the nucleon-nucleon experimental situation was reviewed by a number of speakers, in *Proceedings of the International Conference on Nucleon-Nucleon Interactions, University of Florida, Gainesville, 1967* [Rev. Mod. Phys. **39**, 495–717 (1967)]. A summary of the conference is given by M. H. MacGregor, Phys. Today **20**, 111 (1967).

[8] The existing (p,p) and (n,p) experimental data from 0 to 400 MeV are illustrated in graphical form in Figs. 1 and 2 of M. H. MacGregor, Rev. Mod. Phys. **39**, 556 (1967).