Decuplet Poles in the Tadpole Model for Nonleptonic Hyperon Decays*

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Using a previously discussed dynamical model, we show that the baryon octet and decuplet poles alone lead to a simple explanation of the observed parity-violating nonleptonic hyperon decay amplitudes, essentially without involving any unknown parameters, provided that the strong interactions are $SU(3)$-invariant.

It is the purpose of this paper to extend a previous discussion[1,2] of a dynamical model for the nonleptonic hyperon decays. We wish to point out that the decuplet poles can play a significant role in the analysis. In fact, with the baryon octet and decuplet poles alone, it is possible to understand, in a simple way, the observed ratios of the parity-violating amplitudes, as well as the vanishing of the parity-violating amplitude for $\Sigma^+ \to n+\pi^+$ without involving any unknown parameters.

This approach can be compared to the recent calculations[3] of the nonleptonic hyperon decays using current-algebra techniques. With the assumption of octet dominance, one obtains for the parity-violating amplitudes the Lee-Sugawara[4] sum rule[5]

$$A(\Lambda_-^0)+2A(\Xi_-^-)=\sqrt{3}A(\Sigma_0^+) \quad \text{and} \quad V(\Sigma_+^+)=0.$$

There are two parameters in the theory to describe the remaining two parity-violating amplitudes; an over-all scale factor and a weak D/F $SU(3)$ mixing parameter. Therefore, the ratios of the parity-violating amplitudes are to be fitted within this theory.[6]

We assume that the weak $Y \to N+\pi$ amplitude can be related to the amplitude for inverse associated production $Y+N \to K+\pi$ through the dominance of the $K_1^0 \to$ vacuum transition.[7] The production amplitudes are estimated from the relevant poles in the s, t, and u channels (see Fig. 1). We assume $SU(3)$-symmetric couplings for all strong vertices, but use physical masses for initial, final, and intermediate states. With baryon octet and decuplet poles the ratios of the parity-violating nonleptonic hyperon decay amplitudes are determined by the *strong* coupling constants for the decuplet–baryon–pseudoscalar-meson vertex and the baryon-baryon-pseudoscalar vertex and the *strong* D/F $SU(3)$ mixing parameter for the $\bar{B}BP$ vertex. Using the recent analysis[8] of the d/f parameters from *strong* interaction data, we can predict all the ratios of the parity-violating hyperon decay amplitudes.

Let us define the matrix element for the decay by[9]

$$M(Y \to N+\pi)=i(2\pi)^4\delta(P_Y-P_N-P_\pi) \times \bar{U}_N(V-i\gamma_5 C)U_Y, \quad (1)$$

where V is the parity-violating (S-wave) amplitude. The necessary couplings are defined by the Lagrangian

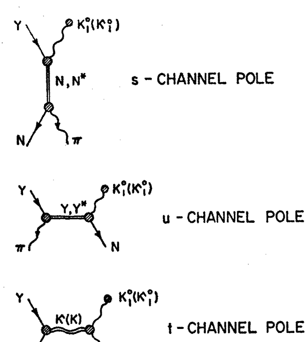

FIG. 1. Pole contributions to nonleptonic decays.

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5 The notation on particle symbols denotes the charge of decaying particle (superscript) and charge of emitted pion (subscript).

6 The appeal of current algebra at this point is that we can predict the parity-conserving amplitudes from the Born terms using the $\bar{B}B$ weak vertex determined from the parity-violating amplitudes.

7 A. Salam and J. C. Ward, Phys. Rev. Letters **5**, 390 (1960); A. Salam, Phys. Letters **8**, 217 (1964).

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9 For γ matrices, etc., we use the convention of S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson and Co., New York, 1961).

[assuming $SU(3)$ invariance for the strong interactions]

$$L=\sqrt{2}g((1-f)\,\mathrm{Tr}\{\bar{B}\gamma_5,B\}P-f\,\mathrm{Tr}[\bar{B}\gamma_5,B]P)+\begin{pmatrix}8&8&10\\i&j&k\end{pmatrix}\frac{G_D}{m_\pi}\phi_\pi{}^i\overleftrightarrow{\partial_\mu}\bar{\psi}_B{}^j\theta_D{}^{\mu,k}+\mathrm{H.c.}+if_Km_K{}^3\phi_{K_1^0}. \quad (2)$$

The normalization of the $\bar{B}BP$ coupling[10] is chosen to give the usual pion-nucleon coupling constant $g^2/4\pi=15$. The field for the decuplet, $\theta_D{}^\mu$, is defined by the Fierz-Pauli[11] formalism, with a positive-energy projection operator[12] given by

$$\sum_{\mathrm{spin}}\theta_\mu\bar{\theta}_\nu=-\tfrac{1}{3}[3g_{\mu\nu}-\gamma_\mu\gamma_\nu+(1/M^2)\times(\gamma_\mu PP_\nu+P_\mu P\gamma_\nu-4P_\mu P_\nu)](P+M). \quad (3)$$

The decuplet coupling 3-j symbol was evaluated from the tables of McNamee and Chilton.[13]

We can now evaluate the contributions to the parity-violating amplitudes resulting from the octet and decuplet pole terms given in Fig. 1. The octet pole terms give

$$V(\Lambda_-{}^0)_8=\tfrac{1}{3}f_Km_Kg^2\left[-\frac{(1+2f)}{\Lambda+N}+\frac{2(1-2f)(1-f)}{\Sigma+N}\right],$$

$$V(\Sigma_+{}^+)_8=\sqrt{2}f_Km_Kg^2\left[\frac{(1-2f)(1+f)}{\Sigma+N}-\frac{1}{3}\frac{(1+2f)(1-f)}{\Lambda+N}\right], \quad (4)$$

$$V(\Sigma_-{}^-)_8=\sqrt{2}f_Km_Kg^2\left[-\frac{(1-2f)f}{\Sigma+N}-\frac{(1+2f)(1-f)}{\Lambda+N}\right],$$

$$V(\Xi_-{}^-)_8=\tfrac{1}{3}f_Km_Kg^2\left[\frac{2(1-f)}{\Xi+\Sigma}-\frac{(1-4f)(1-2f)}{\Xi+\Lambda}\right].$$

The decuplet poles give

$$V(Y\to N+\pi)_{10}=\tfrac{1}{3}\sqrt{2}f_Km_K(G_D/m_\pi m_D)^2\left\{\begin{pmatrix}8&8&\overline{10}\\Y&K&D\end{pmatrix}\begin{pmatrix}8&8&10\\N&\pi&D\end{pmatrix}m_Y(m_D+m_Y)(3E_N-m_D-m_Y)+\begin{pmatrix}8&8&\overline{10}\\Y&\pi&D\end{pmatrix}\begin{pmatrix}8&8&10\\N&K&D\end{pmatrix}[(3m_YE_N-m_N{}^2)(m_N+m_D)-m_Nm_Y(2E_N+m_D-m_N)]\right\}. \quad (5)$$

Therefore the parity-violating amplitudes can be expressed, in this model, in the form

$$V(Y\to N+\pi)_{8+10}=f_Km_Kg^2B(m_Y,m_N,m_I,f)+G_D{}^2D(m_Y,m_N,m_\pi,m_I{}^*). \quad (6)$$

The functions B and D are known functions of the physical masses and f [which gives the d/f ratio in the $SU(3)$ $\bar{B}BP$ coupling]. The function D can be evaluated immediately (in units of MeV^{-1}):

$$\begin{aligned}D(\Lambda_-{}^0)&=-0.043,\\D(\Sigma_+{}^+)&=-0.0134,\\D(\Sigma_-{}^-)&=-0.124,\\D(\Xi_-{}^-)&=+0.110.\end{aligned} \quad (7)$$

To proceed, it is useful to have some information on the decuplet coupling constant G_D. We can estimate this from the known decuplet decay widths. Using the Lagrangian in Eq. (2), the decay width is given by

$$\Gamma(D\to N+\pi)=\left[\begin{pmatrix}8&8&10\\\pi&N&D\end{pmatrix}\frac{G_D}{m_\pi}\right]^2\frac{1}{3\pi}\left(\frac{E_N+m_N}{m_D}\right)|\mathbf{p}|^3. \quad (8)$$

For a $N^*(1236)$ width of 120 MeV,[14] we obtain a value for $(\sqrt{3}G_D)^2$ of 0.825 or $G_D{}^2=0.375$. Experimental values for other decuplet decays result in a value of $G_D{}^2$ ranging from 0.2 to 0.3. Hence the value of the $SU(3)$-symmetric decuplet–baryon–pseudoscalar-meson coupling is not too well determined in this approximation; however, it is limited to a small enough range to be useful for our purposes.

The function B in Eq. (6) depends not only on the physical masses but also on the $SU(3)$ mixing parameter f. Knowing the pion-nucleon coupling constant g, and having information on the decuplet coupling constant G_D, we calculate the ratios of the parity-violating amplitudes as a function of f. Experimentally, there seems to be good evidence[15] that $V(\Sigma_+{}^+)=0$. This solution occurs in our model for $f=0.35$.[16] (Because of

[10] The $SU(3)$ mixing parameter is normalized so that $f+d=1$. The curly- and square-bracket notation stands for the anticommutator and commutator, respectively, of the usual 3×3 matrices for the baryon octets.

[11] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939).

[12] For one derivation of the projection operator see R. E. Behrends and C. Fronsdal, Phys. Rev. **106**, 345 (1957). Our sign convention comes from the positive-definiteness requirement. Note also, we do not include the effects of the finite width of the $\frac{3}{2}$ intermediate state.

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the small decuplet pole contribution to the $\Sigma^+ \to n+\pi^+$ parity-violating amplitude, this solution is almost independent[17] of G_D.) Instead of solving for f from the condition $V(\Sigma_+^+)=0$, we can use the solutions for f obtained from strong-interaction data ($f=0.41\pm0.07$).[18] While the cancellation is somewhat sensitive to the exact value of f, throughout this entire range of f, $V(\Sigma_+^+)$ is suppressed relative to the remaining amplitudes. To the extent that $f=0.35$ is consistent with the determinations of f from *strong* interaction data, we can say that we predict $V(\Sigma_+^+)=0$.

Using $f=0.35$, we can evaluate the B functions in Eq. (6) (in units of MeV^{-1}):

$$\begin{aligned} g^2B(\Lambda_-^0)&=-0.070\,,\\ g^2B(\Sigma_+^+)&=+0.003\,,\\ g^2B(\Sigma_-^-)&=-0.061\,,\\ g^2B(\Xi_-^-)&=+0.062\,. \end{aligned} \tag{9}$$

Therefore, Eq. (6) can be written in the form

$$V(I)=f_Km_Kb_I+G_D{}^2d_I\,. \tag{10}$$

The b_I and d_I are *constants*[19] given by Eqs. (9) and (7), respectively. This result gives us the ratios of the parity-violating amplitudes as unique linear functions of $G_D{}^2$. This dependence is shown in Fig. 2. Experimental values for these ratios are[14]

$$V(\Lambda_-^0):V(\Sigma_-^-):V(\Xi_-^-)=-1.0:-1.2:+1.3 \tag{11}$$

(where these values have approximately a 5% error). As can be seen from Fig. 2, the values with no decuplet poles ($G_D=0$) has relatively too large a $V(\Lambda^0)$ amplitude. However, in the range of G_D indicated by experiment, there is satisfactory agreement. Given this range of G_D from *strong* interaction data, we would *predict* the ratios of the parity-violating amplitudes consistent with the data. Assuming the physical amplitudes are dominated entirely by these poles, the value of the tadpole coupling is

$$f_Km_K\sim0.4\times10^{-5}\ \text{MeV}\,. \tag{12}$$

This value is consistent with the range obtained from the analysis of $K_1\to2\pi$ and K_1-K_2 mass difference.[20]

Comparing these results with our previous discussion,[2] we must discuss the possible contribution of scalar mesons in the t channel (see Fig. 1). This contribution depends on the scalar-meson coupling to the baryon-baryon system [with the associated d'/f' $SU(3)$ mixing parameter] and the two-pseudoscalar-meson system. Because we have been able to fit the physical amplitude ratios with baryon octet and decuplet poles, the contribution of the scalar-meson poles should not disturb this ratio. This can happen in two ways: (1) The scalar-meson couplings can be so small that these poles are not significant; or (2) the scalar-meson poles contribute in the same ratio as the observed amplitudes.

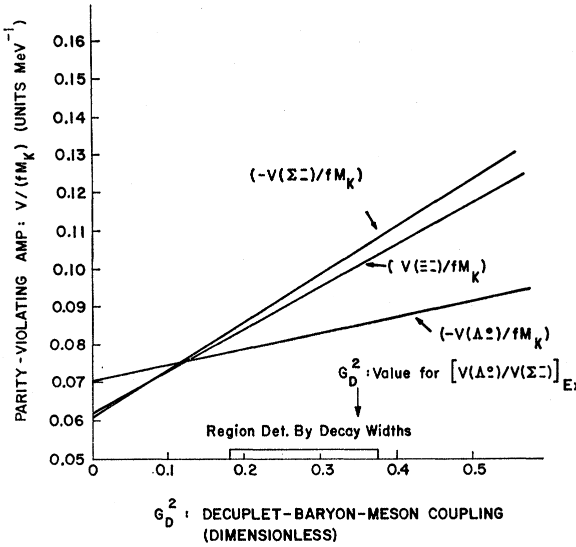

FIG. 2. Parity-violating decay amplitudes as a function of dimensionless decuplet coupling constant.

If we take the second point of view, so that the scalar mesons are significant, the observed amplitude ratios determine f' [$SU(3)$ mixing parameter for $\bar{B}BS$ coupling] as 1.4. Interestingly enough, this is just the value required by the tadpole model of Coleman and Glashow[21] for the medium-strong *mass splittings*. As they show, in this case the scalar-meson tadpole contribution to the *parity-conserving amplitudes* (P-wave) can be removed. In other words, a significant contribution for scalar-meson pole in the t channel of the parity-violating amplitudes results in a f' value which removes the mechanism giving rise to our successful explanation[2] of the parity-conserving amplitudes (κ tadpole). However, any nontadpole contribution (which could still allow f' *near* 1.4) causes the argument to break down. If we assume such a nontadpole contribution to the mass splittings, we can maintain the tadpole mechanism for the parity-conserving nonleptonic decay amplitudes even near $f'=1.4$ (where there is a singularity structure in determining the coupling constants, see Ref. 2). A complete analysis of the tadpole model for both parity-violating and parity-conserving amplitudes has been

[16] Because of the quadratic dependence on f in Eq. (4), there are really two solutions: $f=0.35$ and $f=-1.4$. The second can be discarded as it is inconsistent with the determinations in Ref. 8 or because it leads to unphysical $V(\Sigma_-^-)$; see Ref. 2.

[17] Over the range of G_D indicated by decay widths, f varies from 0.35 to 0.355. In other words, the value of f giving $V(\Sigma_+^+)=0$ is determined by masses in octet pole terms.

[18] This is the result of Kim in Ref. 8. The other values are (Martin and Wali) $f\sim0.25$ and (Ebel and James) $f\gtrsim0.35$.

[19] The b_I could be a function of G_D indirectly as its f dependence comes from $V(\Sigma_+^+)=0$. However, as pointed out before (Ref. 17), the value of f for $V(\Sigma_+^+)=0$ is *essentially* independent of G_D.

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[21] S. Coleman and S. Glashow, Phys. Rev. **134**, B671 (1964).

carried out elsewhere.[22] The parity-violating amplitudes are dominated by the octet and decuplet poles. The small scalar-meson couplings, which result from the t-channel pole in the parity-violating amplitudes, are compensated in the parity-conserving amplitudes by an enhancement of the $\kappa \rightarrow$ vacuum coupling. Therefore, we maintain the previous predictions and success[2,20] of the tadpole model for the nonleptonic decays.

In conclusion, we would like to point out again that the tadpole model does allow one to explain the ratios of the parity-violating amplitudes and the vanishing of $V(\Sigma_+{}^+)$ in a natural way. These conclusions are based on the strong-interaction coupling constants and there are no fitted parameters in these results. To the extent that there is a nonzero $K_1 \rightarrow$ vacuum coupling, the success of the tadpole model has to be considered in any analysis of the nonleptonic decays.[23]

[22] D. Loebbaka, thesis, University of Maryland Technical Report No. 624 (unpublished).

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[23] See A. Kumar and J. C. Pati, Ref. 3.

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Nonanalyticity of the Scattering Amplitude at $u=0$ in a Relativistic Harmonic-Oscillator Model

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Within the framework of a model based on the noncompact group $U(3,1)$, scattering amplitudes are calculated in the Born approximation using a phenomenological propagator in which an infinite number of one-particle states are exchanged. The amplitude is found to be nonanalytic at the point where the squared mass of the internal line changes sign ($u=0$). The possibility of testing such anomalous behavior in pion-nucleon backward scattering and in other processes is discussed.

1. INTRODUCTION

Recent work on current algebra and superconvergent amplitudes[1] has suggested that in order to saturate such relations with a finite or infinite number of one-particle states, the crossing symmetry of the theory must be different from the crossing given by the ordinary local finite-component field theory. (In a dispersion language, one needs different assumptions about the behavior of the kinematical singularities.) It is well known that in the infinite-component field theory with "local" coupling, such "anomalous" crossing is present,[2,3] and that the only known examples of consistent saturation of the current-algebra commutation relations have been obtained by the use of unitary representations of a noncompact group.[4] The lack of conventional crossing in the infinite-component field theories leads us to think that in such theories the scattering amplitude might be nonanalytic wherever the squared mass of an internal line changes sign. In this work we compute scattering amplitudes in the Born approximation in a model based on the noncompact group $U(3,1)$, and we find such a lack of analyticity.

Born diagrams including the exchange of an infinite number of particles have been discussed by Van Hove,[5] and in the framework of the noncompact group $O(3,1)$ by Cocho and Harum Ar-Rashid[6] and by Fronsdal.[7] In this work, we will compute Born approximations including the exchange of an infinite number of particles in the framework of the relativistic harmonic-oscillator model discussed in a previous work.[8]

Although $U(3,1)$ might not be the right group for elementary particles [and in particular the extensive work of Barut *et al.*[9] seems to suggest that $O(4,2)$ might be a better candidate], the calculations are simpler in $U(3,1)$, it is easier to obtain answers in a closed form, and we believe that such a possibility is worth explor-

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